STUDENT NAME/NUMBER: .....

GOSFORD HIGH SCHOOL

September 2019

Preliminary Mathematics

Assessment Task 3

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II show relevant mathematical reasoning and/or calculations.

Total Marks – 70

Section I - 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II - 60 marks

- Attempt Questions 11-32
- Allow about 1 hour and 45 minutes for this section.

2019 Preliminary Mathematics

Student Name/Number:_____

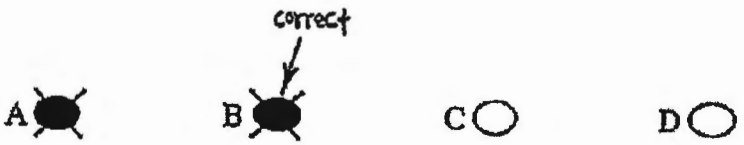
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2+4=$ (A) 2 (B) 6 (C) 8 (D) 9

A○ B● C○ D○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



1. A○ B○ C○ D○
2. A○ B○ C○ D○
3. A○ B○ C○ D○
4. A○ B○ C○ D○
5. A○ B○ C○ D○
6. A○ B○ C○ D○
7. A○ B○ C○ D○
8. A○ B○ C○ D○
9. A○ B○ C○ D○
10. A○ B○ C○ D○

Student name / number _____

Marks

Section I

10 Marks
Attempt Questions 1-10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1-10.

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1. What is the value of $\frac{x^6}{y^6} \div x^2 y^{-3}$? **1**

   (A) $\frac{x^3}{y^2}$

   (B) $\frac{x^4}{y^2}$

   (C) $\frac{x^4}{y^3}$

   (D) $\frac{x^4}{y^9}$

2. What is the minimum value of the expression $x^2 + 4x - 1$? **1**

   (A) -1

   (B) -5

   (C) -9

   (D) -13

3. What is the solution of the equation $\left|\frac{x}{2} - 1\right| - 3 = 0$? **1**

   (A) $x = -4$ *or* $x = -8$

   (B) $x = 4$ *or* $x = -8$

   (C) $x = -4$ *or* $x = 8$

   (D) $x = 4$ *or* $x = 8$

4. The discrete random variable X denotes the score when a fair die is rolled once. **1**
   What is the probability that $4X + 1$ is a perfect square?

   (A) $\frac{1}{6}$

   (B) $\frac{1}{3}$

   (C) $\frac{1}{2}$

   (D) $\frac{2}{3}$

Student name / number _____

Marks

5. What is the solution of the equation $\tan^2 x - 3 = 0$ for $0 \le x \le \pi$? **1**

(A) $x = \frac{\pi}{6}$ only

(B) $x = \frac{\pi}{3}$ only

(C) $x = \frac{\pi}{6}$ *or* $x = \frac{5\pi}{6}$

(D) $x = \frac{\pi}{3}$ *or* $x = \frac{2\pi}{3}$

6. A school has 2 student councils, council A has 4 boys and 3 girls, while council B has 2 boys and 5 girls. To lead the school, 1 student from council A and 1 student from council B is to be selected to be co-presidents. What is the probability that the co-presidents are both girls? **1**

(A) $\frac{15}{49}$

(B) $\frac{20}{49}$

(C) $\frac{3}{7}$

(D) $\frac{5}{7}$

7. Which one of the following functions is *not* one-to-one? **1**

(A) $f(x) = 4 - x^2, x \ge 0$

(B) $f(x) = \sqrt{4 - x^2}$

(C) $f(x) = 1 - 4x$

(D) $f(x) = \frac{4}{x}$

Student name / number _____

Marks

8. Which of the following equations has solutions $x = 2$ and $x = -3$ 1

(A) $x^2 - 5x + 6 = 0$

(B) $x^2 + 5x - 6 = 0$

(C) $x^2 + x - 6 = 0$

(D) $x^2 - x - 6 = 0$

9. If $f(x) = \dfrac{1}{x-3}$ and $g(x) = \sqrt{9-4x^2}$, what is the domain of $f(g(x))$? 1

(A) $\left(-\infty, -\dfrac{3}{2}\right] \cup \left[\dfrac{3}{2}, \infty\right)$

(B) $\left(-\infty, -\dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right)$

(C) $\left[-\dfrac{3}{2}, 0\right) \cup \left(0, \dfrac{3}{2}\right]$

(D) $\left[-\dfrac{3}{2}, 0\right] \cup \left[0, \dfrac{3}{2}\right]$

10. If $y = \sqrt{x}\, e^{2x}$ what is an expression for $\dfrac{dy}{dx}$? 1

(A) $\dfrac{dy}{dx} = \dfrac{e^{2x}}{\sqrt{x}}$

(B) $\dfrac{dy}{dx} = \dfrac{x+1}{2\sqrt{x}}\, e^{2x}$

(C) $\dfrac{dy}{dx} = \dfrac{2x+1}{2\sqrt{x}}\, e^{2x}$

(D) $\dfrac{dy}{dx} = \dfrac{4x+1}{2\sqrt{x}}\, e^{2x}$

Student name / number _____

Marks

Section II

60 Marks
Attempt Questions 11-3[illegible]
Allow about 1 hour and 45 minutes for this section.

Answer the questions in the spaces provided. Sufficient space is provided for typical responses.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of this booklet.
If you use this space, clearly indicate which question you are answering

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Question 11 (2 marks)

$A(3,-1)$ is a vertex of the rectangle $ABCD$ whose side AD has equation $2x-y-7=0$. **2**

Find in general form the equation of the side AB.

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Question 12 (2 marks)

ABC is a triangle in which $\angle ABC = 30°$, $\angle ACB = 45°$ and $AC = 8$ cm. **2**

Find the exact length of AB.

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Student name / number _____

Marks

Question 13 (2 marks)

A fair die is rolled once. A is the event *the score is even* and B is the event *the score is 5 or 6.* **2**
Determine, giving reasons, whether or not the events A and B are independent.

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Question 14 (2 marks)

The number of bats, N, in a park at time t years, is given by the formula:

$N = N_0\, e^{0.347t}$ where N_0 is a constant.

Initially there were 40 bats in the park.

(a) Find the value of N_0. **1**

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(b) How many bats were in the park after 12 years? **1**

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Question 15 (2 marks)

Solve $9^x = \sqrt{3}$. **2**

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Student name / number _____

Marks

Question 16 (3 marks)

A class has 25 students. 19 of them study at least one of Art and Music, and 16 of them study at most one of Art and Music. Two students are chosen at random from the class.

(a) Draw a Venn diagram to represent the above information showing the number studying both Art and Music and the number showing neither Art nor Music. 1
It is not necessary to show the number studying Art only and Music only.

(b)Find the probability that both students chosen study exactly one of Art and Music. 2

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Question 17 (2 marks)

Find the values of x where the graph of the function $f(x)=\frac{1}{x^2-4x}$ is discontinuous. 2

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Student name / number _____

Marks

Question 18 (2 marks) 2

By using $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ show that the gradient function of $f(x) = 4x^2 + 3x$ is $8x + 3$.

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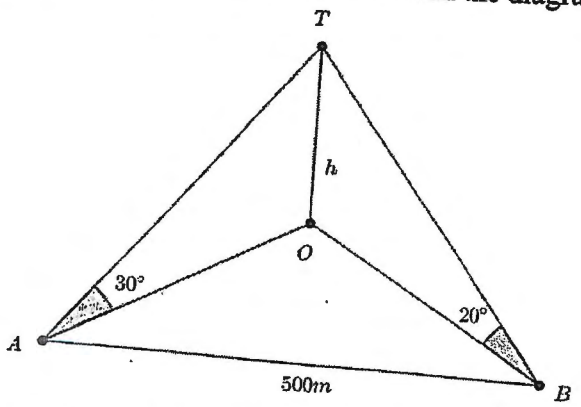
Question 19 (3 marks) 3

Sketch the graph of $y = x(x-1)(x+2)$, showing all intercepts.

Question 20 (4 marks)

Aaron (A) and Ben (B) are 500 metres apart. They can both be seen from the top of a tower (T) with height h metres as shown in the diagram below.



The angle of elevation of T from A is $30°$ while the angle of elevation of T from B is $20°$. The tower is on a bearing of 300°T from Ben and 30°T from Aaron.

(i) Show that the triangle AOB is right-angled. **1**

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(ii) Show that $OA = h\cot(30°)$. **1**

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(iii) Hence, or otherwise, find the height of the tower OT correct to 2 decimal places. **2**

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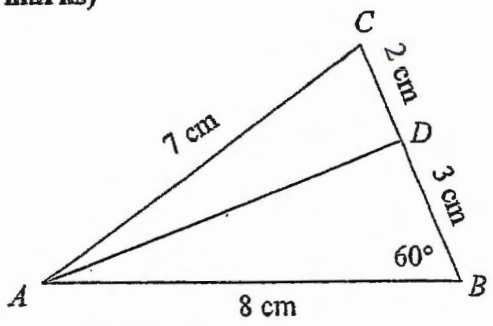
Student name / number _____

Marks

Question 21 (3 marks)



NOT TO SCALE

(a) Find the exact area of ΔABC. **1**

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(b) Find the exact area of ΔADC. **2**

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Question 22 (3 marks)

Differentiate the following with respect to x.

(a) $y = 5x^4 + 7x^2 + 1$ **1**

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Student name / number _____

Marks

(b) $y = \dfrac{3}{(2x+5)^2}$ 2

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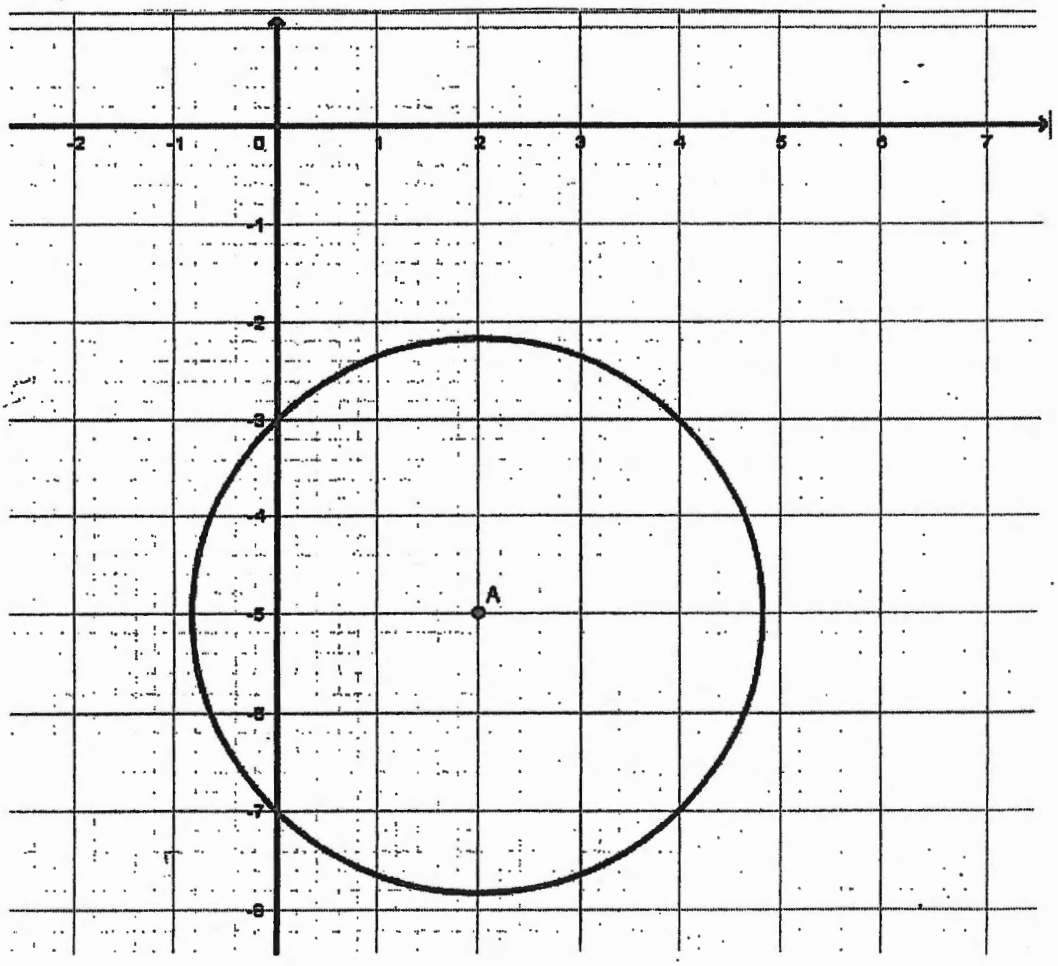
Question 23 (2 marks) 2



The circle above has equation $(x-a)^2+(y-b)^2=r^2$.

Find the values of a, b and r.

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Student name / number _____

Question 24 (3 marks) **Marks**

Lottie has 3 tickets in a raffle in which there are 25 tickets and 2 prizes.

(a) Draw a probability tree diagram to represent the above information. 1

(b) Find the probability that Lottie wins the second prize. 2

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Question 25 (3 marks)

Show that $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$. 3

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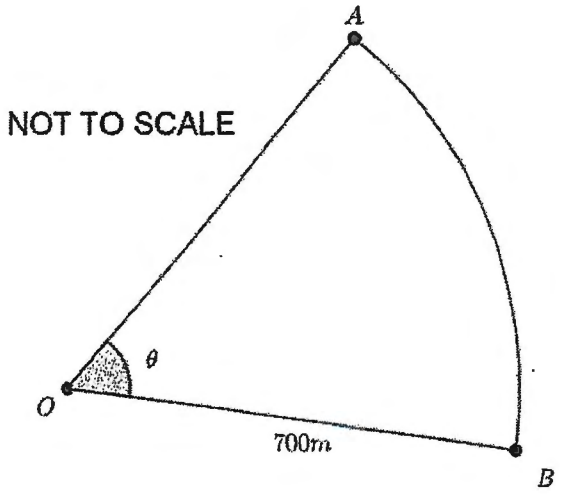
Question 26 (4 marks)

The diagram below illustrates sector AOB.



(a) Let A = area of the sector and ℓ = length of arc AB. 2

Given that $A:\ell^2 = 9:4\pi$, show that $\theta = \dfrac{2\pi}{9}$.

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(b) Find the length of the chord AB, correct to 3 significant figures. 2

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Student name / number _____

Marks

Question 27 (4 marks)

(a) Find the equation of the tangent to the curve $y=(2x-3)^3$ at the point where $x=1$. 3

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(b) Find the angle of inclination of this tangent line correct to the nearest degree. 1

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Question 28 (4 marks)

Two straight roads intersect at right angles at a point O. At time t minutes after observation begins, car A on one road has displacement $x=t-2$ kilometres from O, and car B on the other road has displacement $y=9-2t$ kilometres from O.

(a) Show that at time t minutes after observation begins, the distance d kilometres between the two cars is given by $d^2=5t^2-40t+85$. 2

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(b) Hence find when the distance between the two cars is 5 km. 2

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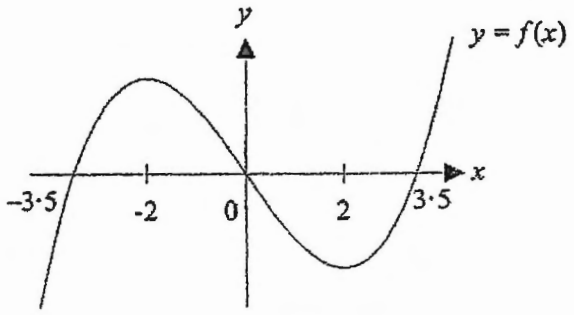
Student name / number _____

Marks

Question 29 (2 marks)



Sketch the curve $y = f'(x)$ showing the shape of the curve and the x intercepts. 2

Question 30 (3 marks)

A discrete random variable X with mean 2·1 has probability distribution 3

x	1	2	3
$P(X=x)$	0·2	a	b

for some constants a and b. Find the values of a and b.

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Question 31 (5 marks)

(a) By letting $v = sin(x)$ or otherwise, solve $2sin^2(x) - 5sin(x) + 2 = 0$ in the domain $0 \leq x \leq 2\pi$. **3**

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(b) Let $f(x)$ be an odd function and $g(x)$ be an even function. **2**

Prove that $g(f(x))$ is even.

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Student name / number _____

Marks

Section II Extra writing space.

If you use this space, clearly indicate which question you are answering.

STUDENT NAME/NUMBER: ..... SOLUTIONS .....

GOSFORD HIGH SCHOOL

September 2019

Preliminary Mathematics

Assessment Task 3

General Instructions

- Reading time – 5 minutes
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Total Marks – 70

Section I - 10 marks

- Attempt Questions 1-10
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Section II - 60 marks

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2019 Preliminary Mathematics

Student Name/Number:_____

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A○ B● C○ D○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A● B⊗ C○ D○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A● correct B● C○ D○

1. A○ B○ C● D○
2. A○ B● C○ D○
3. A○ B○ C● D○
4. A○ B● C○ D○
5. A○ B○ C○ D●
6. A● B○ C○ D○
7. A○ B● C○ D○
8. A○ B○ C● D○
9. A○ B○ C● D○
10. A○ B○ C○ D●

Student name / number _____

Marks

Section I

10 Marks
Attempt Questions 1-10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1-10.

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1. What is the value of $\frac{x^6}{y^6}+x^2y^{-3}$? 1

(A) $\frac{x^3}{y^2}$

(B) $\frac{x^4}{y^2}$

(C) $\frac{x^4}{y^3}$

(D) $\frac{x^4}{y^9}$

2. What is the minimum value of the expression x^2+4x-1 ? 1

(A) -1

(B) -5

(C) -9

(D) -13

3. What is the solution of the equation $\left|\frac{x}{2}-1\right|-3=0$? 1

(A) $x=-4$ *or* $x=-8$

(B) $x=4$ *or* $x=-8$

(C) $x=-4$ *or* $x=8$

(D) $x=4$ *or* $x=8$

4. The discrete random variable X denotes the score when a fair die is rolled once. 1
What is the probability that $4X+1$ is a perfect square?

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

Student name / number _____

Marks

5. What is the solution of the equation $\tan^2 x-3=0$ for $0\le x\le\pi$? 1

(A) $x=\frac{\pi}{6}$ only

(B) $x=\frac{\pi}{3}$ only

(C) $x=\frac{\pi}{6}$ *or* $x=\frac{5\pi}{6}$

(D) $x=\frac{\pi}{3}$ *or* $x=\frac{2\pi}{3}$

6. A school has 2 student councils, council A has 4 boys and 3 girls, while council B has 2 boys and 5 girls. To lead the school, 1 student from council A and 1 student from council B is to be selected to be co-presidents. What is the probability that the co-presidents are both girls? 1

(A) $\frac{15}{49}$

(B) $\frac{20}{49}$

(C) $\frac{3}{7}$

(D) $\frac{5}{7}$

7. Which one of the following functions is *not* one-to-one? 1

(A) $f(x)=4-x^2, x\ge 0$

(B) $f(x)=\sqrt{4-x^2}$

(C) $f(x)=1-4x$

(D) $f(x)=\frac{4}{x}$

Student name / number _____

Marks

8. Which of the following equations has solutions $x=2$ and $x=-3$ 1

(A) $x^2-5x+6=0$

(B) $x^2+5x-6=0$

(C) $x^2+x-6=0$

(D) $x^2-x-6=0$

9. If $f(x)=\frac{1}{x-3}$ and $g(x)=\sqrt{9-4x^2}$, what is the domain of $f(g(x))$? 1

(A) $\left(-\infty,-\frac{3}{2}\right]\cup\left[\frac{3}{2},\infty\right)$

(B) $\left(-\infty,-\frac{3}{2}\right)\cup\left(\frac{3}{2},\infty\right)$

(C) $\left[-\frac{3}{2},0\right)\cup\left(0,\frac{3}{2}\right]$

(D) $\left[-\frac{3}{2},0\right]\cup\left[0,\frac{3}{2}\right]$

10. If $y=\sqrt{x}\,e^{2x}$ what is an expression for $\frac{dy}{dx}$? 1

(A) $\frac{dy}{dx}=\frac{e^{2x}}{\sqrt{x}}$

(B) $\frac{dy}{dx}=\frac{x+1}{2\sqrt{x}}e^{2x}$

(C) $\frac{dy}{dx}=\frac{2x+1}{2\sqrt{x}}e^{2x}$

(D) $\frac{dy}{dx}=\frac{4x+1}{2\sqrt{x}}e^{2x}$

Student name / number _____

Marks

Section II

60 Marks
Attempt Questions 11-3[illegible]
Allow about 1 hour and 45 minutes for this section.

Answer the questions in the spaces provided. Sufficient space is provided for typical responses.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of this booklet.
If you use this space, clearly indicate which question you are answering

Question 11 (2 marks)

$A(3,-1)$ is a vertex of the rectangle $ABCD$ whose side AD has equation $2x-y-7=0$. 2
Find in general form the equation of the side AB.

AD: $y=2x-7$ $\therefore m_{AB}=-\frac{1}{2}$

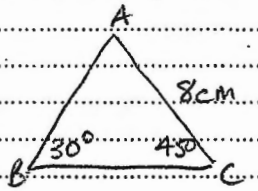
eqn of AB: $y+1=-\frac{1}{2}(x-3)$

$2y+2=-x+3$

$x+2y-1=0$

Question 12 (2 marks)

ABC is a triangle in which $\angle ABC=30°$, $\angle ACB=45°$ and $AC=8\,\text{cm}$. 2
Find the exact length of AB.



$\frac{AB}{\sin 45°}=\frac{8}{\sin 30°}$

$AB=\frac{8\times\frac{1}{\sqrt{2}}}{\frac{1}{2}}$

$=8\sqrt{2}$

Student name / number _____

Marks

Question 13 (2 marks)

A fair die is rolled once. A is the event *the score is even* and B is the event *the score is 5 or 6*. Determine, giving reasons, whether or not the events A and B are independent. 2

$P(A)=\frac{1}{2}$ $P(B)=\frac{1}{3}$ $\therefore P(A)P(B)=\frac{1}{6}$

and $P(A\cap B)=\frac{1}{6}$

$\therefore$ independent

Question 14 (2 marks)

The number of bats, N, in a park at time t years, is given by the formula:

$N = N_0 e^{0.347t}$ where N_0 is a constant.

Initially there were 40 bats in the park.

(a) Find the value of N_0. 1

when $t=0$, $N=40$ $\therefore 40 = N_0 e^0$

ie $N_0 = 40$

(b) How many bats were in the park after 12 years? 1

$N = 40e^{4.164}$

$= 2573$

Question 15 (2 marks)

Solve $9^x = \sqrt{3}$. 2

$3^{2x} = 3^{1/2}$

$\therefore 2x = \frac{1}{2}$

$x = \frac{1}{4}$

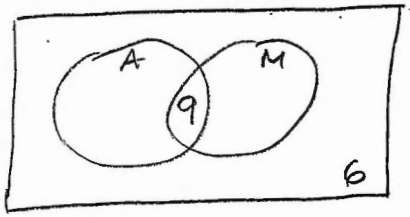
Student name / number _____

Marks

Question 16 (3 marks)

A class has 25 students. 19 of them study at least one of Art and Music, and 16 of them study at most one of Art and Music. Two students are chosen at random from the class.

(a) Draw a Venn diagram to represent the above information showing the number studying both Art and Music and the number showing neither Art nor Music. 1
It is not necessary to show the number studying Art only and Music only.



(b) Find the probability that both students chosen study exactly one of Art and Music. 2

$\frac{10}{25} \cdot \frac{9}{24} = \frac{3}{20}$

Question 17 (2 marks)

Find the values of x where the graph of the function $f(x)=\frac{1}{x^2-4x}$ is discontinuous. 2

$f(x) = \frac{1}{x(x-4)}$ $\therefore$ discontinuous at $x = 0, 4$

Student name / number _____

Marks

Question 18 (2 marks) 2

By using $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ show that the gradient function of $f(x)=4x^2+3x$ is $8x+3$.

$f(x+h) = 4(x+h)^2+3(x+h)$

$= 4x^2+8xh+4h^2+3x+3h$

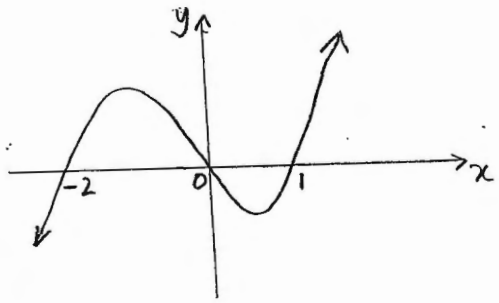
$f'(x) = \lim_{h\to 0}\frac{4x^2+8xh+4h^2+3x+3h-4x^2-3x}{h}$

$= \lim_{h\to 0}\frac{h(8x+4h+3)}{h}$

$= 8x+3$

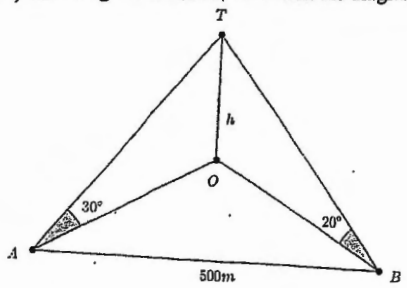
Question 19 (3 marks) 3

Sketch the graph of $y = x(x-1)(x+2)$, showing all intercepts.



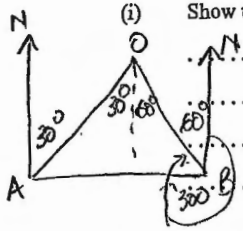
Question 20 (4 marks)

Aaron (A) and Ben (B) are 500 metres apart. They can both be seen from the top of a tower (T) with height h metres as shown in the diagram below.



The angle of elevation of T from A is 30° while the angle of elevation of T from B is 20°. The tower is on a bearing of 300°T from Ben and 30°T from Aaron.

(i) Show that the triangle AOB is right-angled. 1



$\angle AOB = 30^\circ + 60^\circ$

$\therefore \triangle AOB$ is right angled

(ii) Show that $OA = h\cot(30^\circ)$. 1

In $\triangle AOT$: $\cot 30^\circ = \frac{OA}{h}$

$\therefore OA = h\cot 30^\circ$

(iii) Hence, or otherwise, find the height of the tower OT correct to 2 decimal places. 2

$OB = h\cot 20^\circ$ $\quad \triangle OAB$ right angled

$\therefore h^2\cot^2 30^\circ + h^2\cot^2 20^\circ = 500^2$

$h^2(\cot^2 30^\circ + \cot^2 20^\circ) = 500^2$

$h^2 = \frac{500^2}{\cot^2 30^\circ + \cot^2 20^\circ}$

$h = \sqrt{23699.75519\ldots}$

$= 153.95\text{m}$

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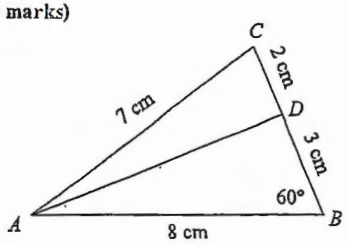
Question 21 (3 marks)



NOT TO SCALE

(a) Find the exact area of ΔABC. 1

$A = \frac{1}{2} \times 8 \times 5 \times \sin 60^\circ$

$= 10\sqrt{3}$ cm^2

(b) Find the exact area of ΔADC. 2

$\frac{\sin C}{8} = \frac{\sin 60^\circ}{7}$

$\sin C = \frac{8 \times \frac{\sqrt{3}}{2}}{7}$

$= \frac{4\sqrt{3}}{7}$

$\text{Area} = \frac{1}{2} \times 7 \times 2 \times \frac{4\sqrt{3}}{7}$

$= 4\sqrt{3}$ cm^2

Question 22 (3 marks)

Differentiate the following with respect to x.

(a) $y = 5x^4 + 7x^2 + 1$ 1

$y' = 20x^3 + 14x$

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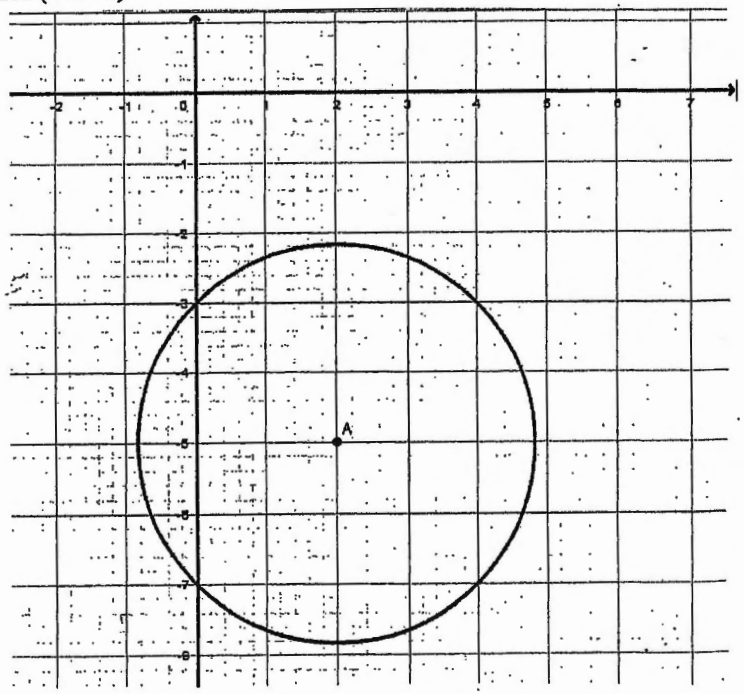
(b) $y = \frac{3}{(2x+5)^2}$ 2

$y = 3(2x+5)^{-2}$

$y' = -6(2x+5)^{-3} \cdot 2$

$= \frac{-12}{(2x+5)^3}$

Question 23 (2 marks) 2



The circle above has equation $(x-a)^2 + (y-b)^2 = r^2$.

Find the values of a, b and r.

$(x-2)^2 + (y+5)^2 = r^2$

$r = \sqrt{(2-0)^2 + (-5+3)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

$\therefore a = 2, b = -5, r = 2\sqrt{2}$

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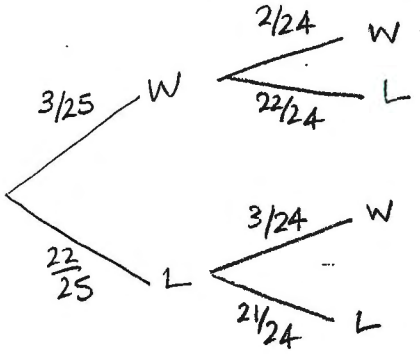
Question 24 (3 marks) Marks

Lottie has 3 tickets in a raffle in which there are 25 tickets and 2 prizes.

(a) Draw a probability tree diagram to represent the above information. 1



(b) Find the probability that Lottie wins the second prize. 2

$P(\text{2nd prize}) = \frac{3}{25} \cdot \frac{2}{24} + \frac{22}{25} \cdot \frac{3}{24}$

$= \frac{3}{25}$

Question 25 (3 marks)

Show that $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$. 3

$LHS = \frac{(1+\cos\theta)^2 + \sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{1+2\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}$

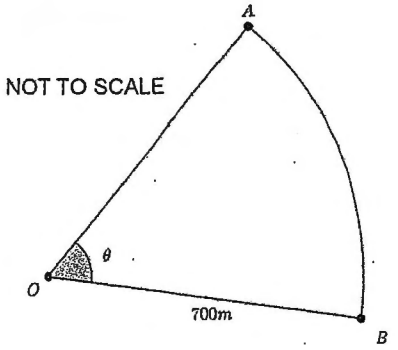
$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$

$= RHS$

Question 26 (4 marks)

The diagram below illustrates sector AOB.



(a) Let A = area of the sector and ℓ = length of arc AB. 2

Given that $A : \ell^2 = 9 : 4\pi$, show that $\theta = \frac{2\pi}{9}$.

$\frac{\frac{1}{2}r^2\theta}{(r\theta)^2} = \frac{9}{4\pi}$

$\frac{1}{2\theta} = \frac{9}{4\pi}$

$2\theta = \frac{4\pi}{9} \quad \therefore \theta = \frac{2\pi}{9}$

(b) Find the length of the chord AB, correct to 3 significant figures. 2

$AB^2 = 700^2 + 700^2 - 2 \times 700 \times 700 \times \cos\left(\frac{2\pi}{9}\right)$

$= 478.8282\ldots$

$= 479$

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Question 27 (4 marks)

(a) Find the equation of the tangent to the curve $y=(2x-3)^3$ at the point where $x=1$. 3

$y' = 3(2x-3)^2 \cdot 2$

$= 6(2x-3)^2$

when $x=1$, $y'=6$ and $y=-1$

$\therefore$ eqn of tangent:

$y+1 = 6(x-1)$

$y+1 = 6x-6$

$y = 6x-7$

(b) Find the angle of inclination of this tangent line correct to the nearest degree. 1

$\tan\theta = 6$

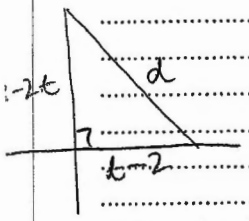
$\theta = \tan^{-1}6$

$= 81^\circ$

Question 28 (4 marks)

Two straight roads intersect at right angles at a point O. At time t minutes after observation begins, car A on one road has displacement $x=t-2$ kilometres from O, and car B on the other road has displacement $y=9-2t$ kilometres from O.

(a) Show that at time t minutes after observation begins, the distance d kilometres between the two cars is given by $d^2=5t^2-40t+85$. 2



$d^2 = (t-2)^2 + (9-2t)^2$

$= t^2 - 4t + 4 + 81 - 36t + 4t^2$

$= 5t^2 - 40t + 85$

(b) Hence find when the distance between the two cars is 5 km. 2

when $d=5$ $\quad 5t^2-40t+85 = 25$

$t^2 - 8t + 12 = 0$

$(t-6)(t-2) = 0$

$t = 6, 2$

$\therefore$ after 2 & 6 seconds

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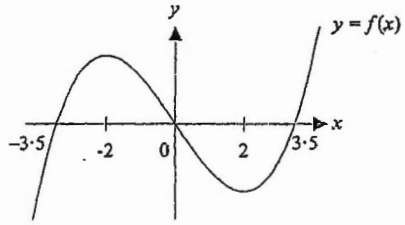
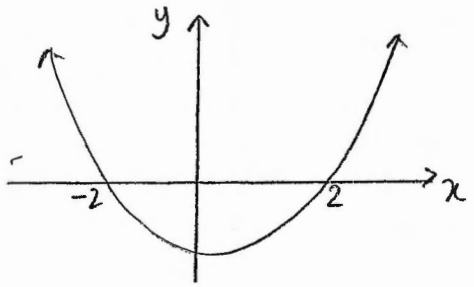
Question 29 (2 marks)



Sketch the curve $y=f'(x)$ showing the shape of the curve and the x intercepts. 2



Question 30 (3 marks)

A discrete random variable X with mean 2·1 has probability distribution 3

x	1	2	3
$P(X=x)$	0·2	a	b

for some constants a and b. Find the values of a and b.

$0.2 + 2a + 3b = 2.1$

$2a + 3b = 1.9$ — (1)

$a + b = 0.8$ — (2)

$\therefore 2a + 2b = 1.6$ — (3)

(1) − (3) $\quad b = 0.3$

$a = 0.5$

Question 31 (5 marks)

(a) By letting $v = sin(x)$ or otherwise, solve $2sin^2(x) - 5sin(x) + 2 = 0$ in the domain $0 \le x \le 2\pi$. 3

$2v^2 - 5v + 2 = 0$

$(2v-1)(v-2) = 0$

$v = \frac{1}{2}, 2$

$\therefore \sin x = \frac{1}{2}$ or $\sin x = 2$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ no solns

(b) Let $f(x)$ be an odd function and $g(x)$ be an even function. 2

Prove that $g(f(x))$ is even.

let $h(x) = g(f(x))$

$h(-x) = g(f(-x))$

$= g(-f(x))$ since $f(x)$ odd

$= g(f(x))$ since $g(x)$ even

$\therefore h(-x) = h(x)$

$\therefore g(f(x))$ is even