HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 11 Assessment 3 Yearly Examination Term 3 2019

STUDENT NUMBER: ____________________

General Instructions

- Reading Time – 10 minutes
- Working Time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators and drawing templates may be used
- In Questions 11 – 15, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks – 77

Section I Pages 2 – 5

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

Section II Pages 6 – 11

67 marks

Attempt Questions 11 – 16

Question	***1-10***	***11***	***12***	***13***	***14***	***15***	***16***	***Total***
Total	*/10*	*/11*	*/11*	*/12*	*/11*	*/11*	*/11*	*/77*

This assessment task constitutes 40% of the Preliminary Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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1 Which one, of the following sets of points does NOT represent a function?

(A) $\{(-6,\ 4),\ (-5,\ 1),\ (-1,\ 6),\ (0,\ 5),\ (6,\ 3)\}$

(B) $\{(-3,\ 0),\ (4,\ 2),\ (-1,\ 1),\ (0,-3),\ (4,\ 3)\}$

(C) $\{(-3,\ 0),\ (-5,\ 2),\ (-1,\ 0),\ (1,-3),\ (4,\ 3)\}$

(D) $\{(0,\ 0),(\pm 1,\ 0),(\pm 2,\ \sqrt{3}),(\pm 3,\ \sqrt{8})\}$

2 Which expression is NOT equivalent to $a^{-1\frac{1}{2}}$?

(A) $\dfrac{1}{a^2\sqrt{a}}$

(B) $a^{-\frac{3}{2}}$

(C) $\dfrac{1}{a\sqrt{a}}$

(D) $\dfrac{\sqrt{a}}{a^2}$

3 Which of the following is an even function?

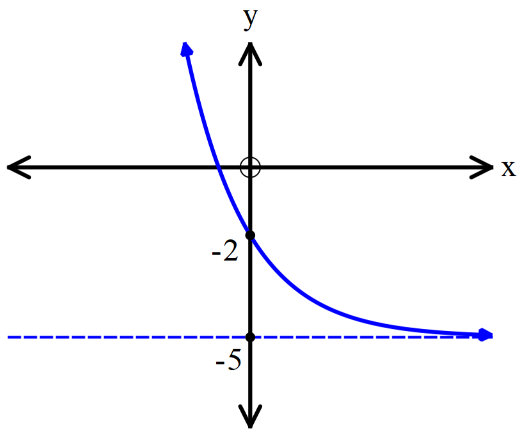
(A) $y = x^3 + x$

(B) $y = |x+1|$

(C) $y^2 = 9 - x^2$

(D) $y = |x| - 1$

4 Which of the following functions represent the one shown in the graph?



(A) $y = 2^{-x} - 2$

(B) $y = 2^{-x} - 5$

(C) $y = 3 \times 2^{-x} - 5$

(D) $y = 3 \times 2^{-x} - 2$

5 What are the solutions of $|x^2 - 3| = 6$

(A) 3 and -3

(B) 3 and $\sqrt{3}$

(C) ± 3 and $\pm\sqrt{3}$

(D) $\sqrt{3}$ and $-\sqrt{3}$

6 A horizontal line is tangent to $y = 2x^2 + 2$ at point P. The coordinates of P are:

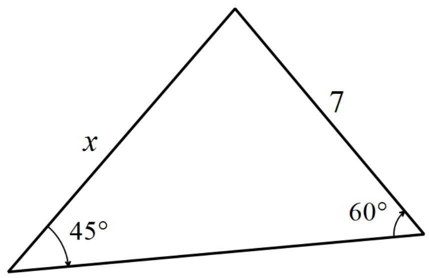
(A) $(0,\ 0)$

(B) $(0,\ 2)$

(C) $(0, -2)$

(D) $(2,\ 0)$

7 Find x, expressing your answer as an exact value:



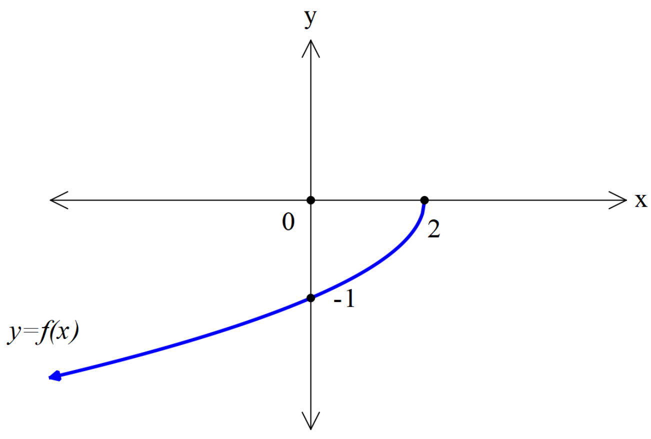
(A) $\frac{7\sqrt{6}}{2}$

(B) $\frac{\sqrt{6}}{14}$

(C) $\frac{7\sqrt{2}}{2}$

(D) $\frac{7\sqrt{6}}{3}$

8 What is the domain $D(f)$ and range $R(f)$ of the following function:



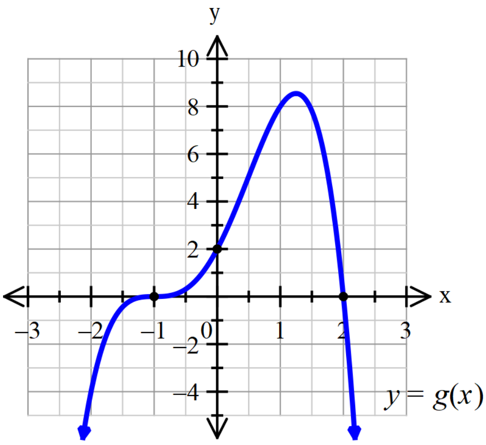
(A) $D(f)=(2,\infty), R(f)=(-\infty,0)$

(B) $D(f)=(-\infty,0], R(f)=(-\infty,2)$

(C) $D(f)=(-\infty,2], R(f)=(-\infty,0]$

(D) $D(f)=[2,-\infty), R(f)=[0,-\infty)$

9 Which of the following functions represents the graph shown below?



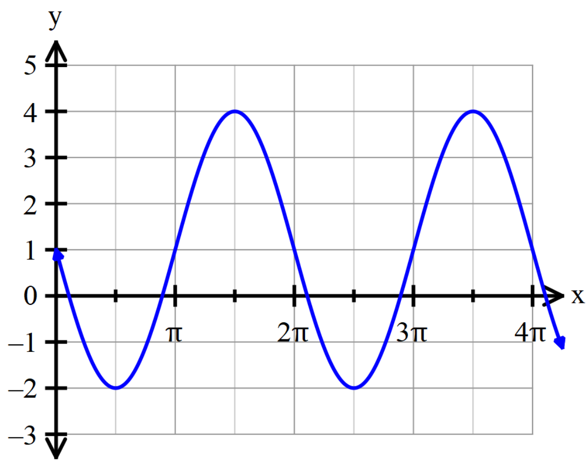
(A) $g(x)=(x-1)^2(x-2)$

(B) $g(x)=(x+1)^3(x-2)$

(C) $g(x)=-(x+1)^2(x-2)^2$

(D) $g(x)=-(x+2)^3(x-1)$

10 Which of the following functions represents the graph shown below?



(A) $y=3\sin x+1$

(B) $y=3\cos x-1$

(C) $y=1-3\sin x$

(D) $y=1-3\cos x$

End of Section I

Section II
67 marks
Attempt Questions 11-16

Question 11 (11 marks)

(a) Simplify $\dfrac{x^{-1}-x^{-2}}{x-x^{2}}$, giving your answer in fraction form with positive indices. **2**

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(b) Solve $\log_2\left(x^3\right)+5=11$. **2**

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(c) Given $\cos\theta=-\dfrac{2}{3}$, and $\tan\theta>0$, find the EXACT value of $\sin\theta$. **2**

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(d) Solve $6^x + \frac{36}{6^x} = 37$. Leave your answer in exact form. **2**

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(e) A circle has diameter AB, where $A = (2,5)$ and $B = (-4,1)$.

(i) Find the radius of the circle in simplest surd form. **1**

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(ii) Find the centre of the circle. **1**

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(iii) Hence, write the equation of the circle. **1**

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Question 12 (11 marks)

(a) Use first principles , $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, to find the gradient function of $f(x)=\frac{1}{x}$. **2**

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(b) Prove that $\sin^2 x\tan^2 x+2\sin^2 x+\cos^2 x=\sec^2 x$. **2**

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(c) Solve $\operatorname{cosec}(\theta-30°)=3$ for $0°\le\theta\le 180°$. Answer correct to the nearest degree. **2**

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(d) Given $f(x)=\begin{cases} 1-x^2, & \text{for } x\le 1 \\ kx+1, & \text{for } x>1 \end{cases}$

(i) Find the value of k for which $f(x)$ is continuous at $x=1$. **1**

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(ii) If $k=-1$, find the value of $f(-3)+f(2)$. **1**

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(iii) If $k=-1$, find the range of $f(x)$. **1**

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(e) If the minimum value of parabola $y=x^2-3x+k$ is 5, find the value of k. **2**

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Question 13 (12 marks)

(a) Differentiate the following functions, leaving your answers in simplest factorised form where possible:

(i) $y = \dfrac{2}{\sqrt{x}}$ **2**

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(ii) $y = (e^{2x} - 1)^3$ **2**

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(iii) $f(x) = x^3\sqrt{x^2 - 1}$ **2**

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(b) Consider the function $f(x)=\frac{1}{3}x^3-x^2-15x+3$.

(i) Find $f'(x)$. **1**

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(ii) Find the values of x such that $f'(x)=0$. **2**

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(iii) Find the set of values of x for which $f(x)$ is decreasing. **1**

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(iv) Find the gradient of the tangent and normal of $f(x)$ at $x=0$. **2**

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Question 14 (11 marks)

(a) Consider the equation $x+\frac{2}{x}=p$, find the values of p for which the equation has real roots. **2**

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(b) For $f(x)=\ |x+2|-3$,

(i) Sketch the function, showing any intercepts. **3**

(ii) Hence, or otherwise, find the values for $f(x)\geq 1$. **1**

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(c) A bacteria replicated itself according to the equation $N(t) = 140 \times 2^{3.5t}$, where $N(t)$ is the number of bacteria present in a test tube t hours after the experiment began.

(i) What is the initial number of bacteria added to the test tube? **1**

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(ii) How many bacteria are there after 10 hours? **2**

Answer in scientific notation correct to 3 significant figures.

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(iii) When the number of bacteria reaches 3×10^{25} in the test tube, it will stop replicating. How long before this number is reached?

Give your answer correct to the nearest minute. **2**

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Question 15 (11 marks)

(a) Given that $f(x) = \dfrac{2x+1}{3x-4}$, $\left(x \neq \dfrac{4}{3}\right)$, find $f'(-2)$. **2**

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(b) Solve $\log_2(x+4) + \log_2(x-1) - \log_2(x+2) = 2$. **3**

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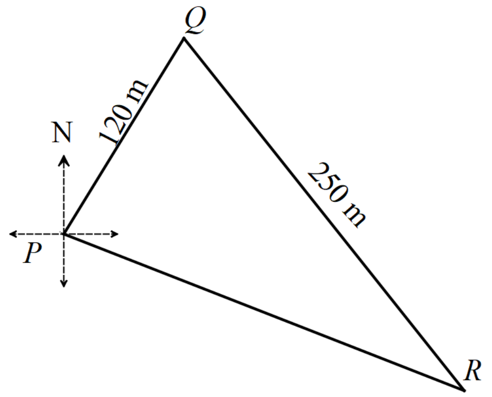
(c) A farmer walked 120 metres from P to Q on a bearing of $040°\text{T}$.

He then walked 250 meters from Q to R on a bearing of $140°\text{T}$.



(i) Find the value of $\angle PQR$. **1**

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(ii) Find the length of PR, correct to the nearest metre. **2**

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(iii) Find the bearing of R from P, correct to the nearest degree. **2**

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(iv) Find the area of ΔPQR. Answer correct to the nearest m^2. **1**

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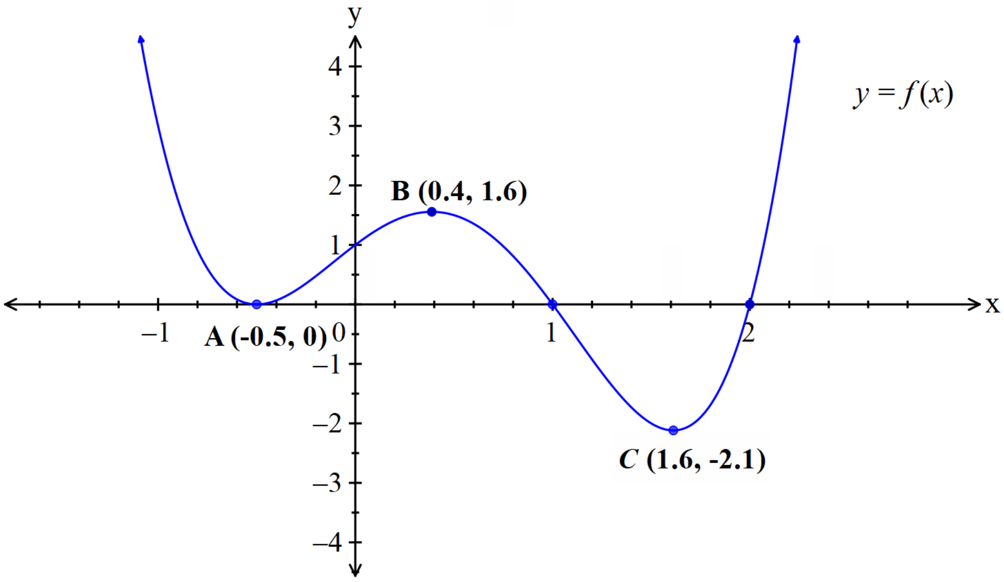
Question 16 (11 marks)

(a) The diagram below shows the graph of a function $y = f(x)$. Point A, B and C are stationary points at which the derivatives are zero.



(i) Sketch the graph of $f'(x)$. Clearly label all essential information. **3**

(ii) State the values of x for which $f(x)$ is increasing. **1**

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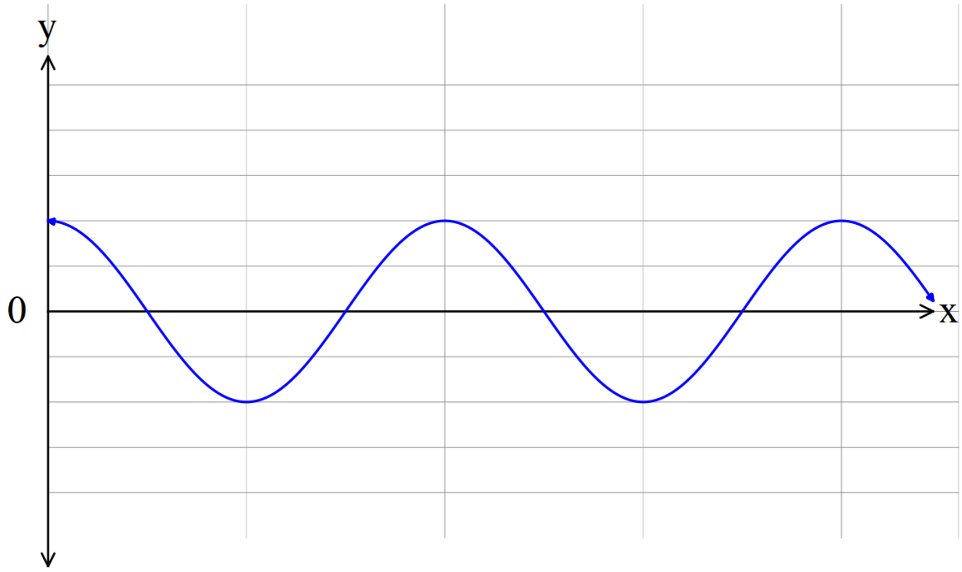
(b) The graph of $y = 2\cos(2x)$ is shown below, where x is in radians.



(i) Find the period and amplitude. **2**

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(ii) On the above graph, label all the x- and y- intercepts on both axes. **2**

(iii) Draw $y = 2\sec(2x)$ on the above graph, clearly showing any asymptotes. **1**

(iv) Find the exact solutions for $2\sec(2x) = 4$ for $0 \le x \le 2\pi$. **2**

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End of Examination

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HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 11 Assessment 3 Yearly Examination Term 3 2019

STUDENT NUMBER: ____________SOLUTION______________________

General Instructions

- Reading Time – 10 minutes
- Working Time – 2 hours
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Total marks – 77

Section I Pages 2 – 5

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

Section II Pages 6 – 11

67 marks

Attempt Questions 11 – 16

Question	***1-10***	***11***	***12***	***13***	***14***	***15***	***16***	***Total***
Total	*/10*	*/11*	*/11*	*/12*	*/11*	*/11*	*/10*	*/76*

This assessment task constitutes 40% of the Preliminary Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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1 Which one, of the following sets of points does NOT represent a function?

(A) $\{(-6,\ 4),\ (-5,\ 1),\ (-1,\ 6),\ (0,\ 5),\ (6,\ 3)\}$

(B) $\{(-3,\ 0),\ (4,\ 2),\ (-1,\ 1),\ (0,-3),\ (4,\ 3)\}$

(C) $\{(-3,\ 0),\ (-5,\ 2),\ (-1,\ 0),\ (1,-3),\ (4,\ 3)\}$

(D) $\left\{(0,\ 0),(\pm 1,\ 0),(\pm 2,\ \sqrt{3}),(\pm 3,\ \sqrt{8})\right\}$

2 Which expression is NOT equivalent to $a^{-1\frac{1}{2}}$?

(A) $\dfrac{1}{a^2\sqrt{a}}$

(B) $a^{-\frac{3}{2}}$

(C) $\dfrac{1}{a\sqrt{a}}$

(D) $\dfrac{\sqrt{a}}{a^2}$

3 Which of the following is an even function?

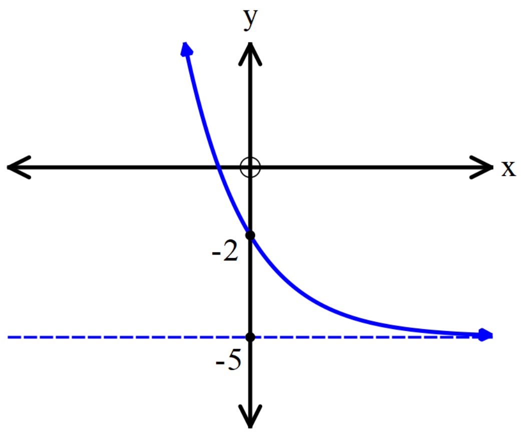
(A) $y = x^3 + x$

(B) $y = |x+1|$

(C) $y^2 = 9 - x^2$

(D) $y = |x| - 1$

4 Which of the following functions represent the one shown in the graph?



(A) $y = 2^{-x} - 2$

(B) $y = 2^{-x} - 5$

(C) $y = 3 \times 2^{-x} - 5$

(D) $y = 3 \times 2^{-x} - 2$

5 What are the solutions of $|x^2 - 3| = 6$

(A) 3 and -3

(B) 3 and $\sqrt{3}$

(C) ± 3 and $\pm\sqrt{3}$

(D) $\sqrt{3}$ and $-\sqrt{3}$

6 A horizontal line is tangent to $y = 2x^2 + 2$ at point P. The coordinates of P are:

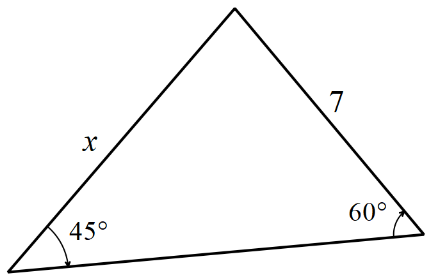
(A) $(0,\ 0)$

(B) $(0,\ 2)$

(C) $(0, -2)$

(D) $(2,\ 0)$

7 Find x, expressing your answer as an exact value:



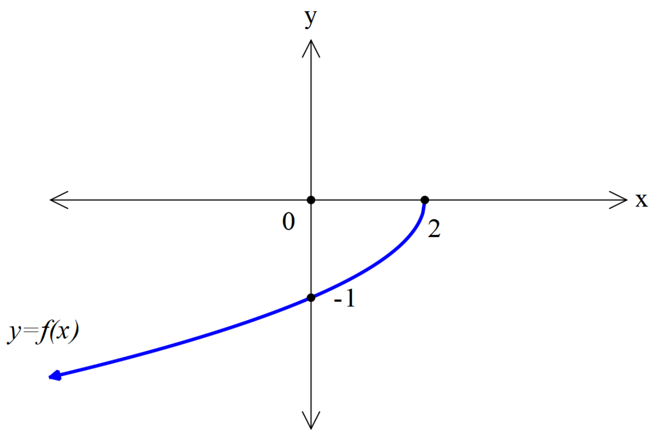
(A) $\frac{7\sqrt{6}}{2}$

(B) $\frac{\sqrt{6}}{14}$

(C) $\frac{7\sqrt{2}}{2}$

(D) $\frac{7\sqrt{6}}{3}$

8 What is the domain $D(f)$ and range $R(f)$ of the following function:



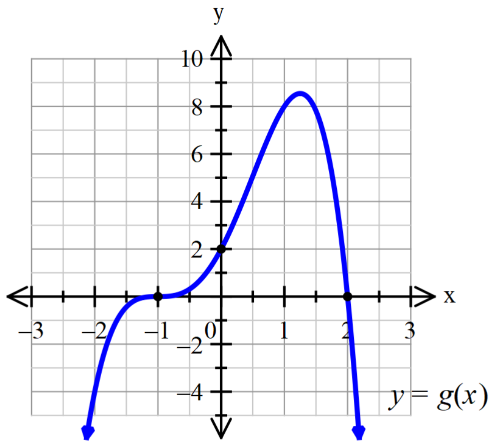
(A) $D(f) = (2, \infty), R(f) = (-\infty, 0)$

(B) $D(f) = (-\infty, 0], R(f) = (-\infty, 2)$

(C) $D(f) = (-\infty, 2], R(f) = (-\infty, 0]$

(D) $D(f) = [2, -\infty), R(f) = [0, -\infty)$

9 Which of the following functions represents the graph shown below?



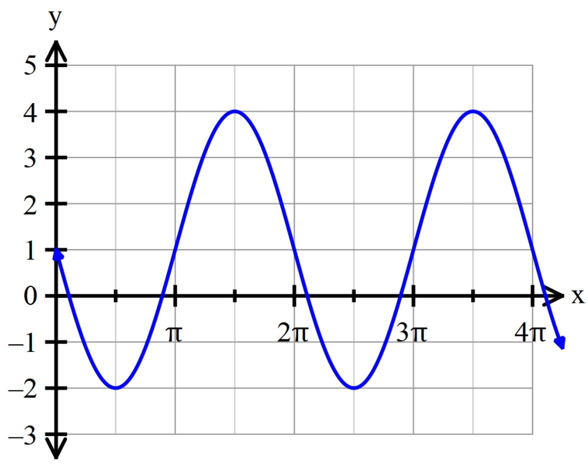
(A) $g(x) = -(x-1)^2(x-2)$

(B) $g(x) = -(x+1)^3(x-2)$

(C) $g(x) = -(x+1)^2(x-2)^2$

(D) $g(x) = -(x+2)^3(x-1)$

10 Which of the following functions represents the graph shown below?



(A) $y = 3\sin x + 1$

(B) $y = 3\cos x - 1$

(C) $y = 1 - 3\sin x$

(D) $y = 1 - 3\cos x$

End of Section I

Section II

67 marks
Attempt Questions 11-16

Question 11 (11 marks)

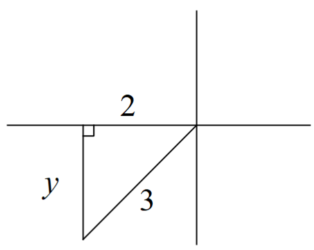
(a) Simplify $\dfrac{x^{-1}-x^{-2}}{x-x^2}$, giving your answer in fraction form with positive indices. **2**

$$\frac{x^{-1}-x^{-2}}{x-x^2}=\frac{\frac{1}{x}-\frac{1}{x^2}}{x-x^2}$$
$$=\frac{\frac{x-1}{x^2}\times x^2}{\left(x-x^2\right)\times x^2}$$
$$=\frac{x-1}{x^3(1-x)}$$
$$=\frac{-1}{x^3}$$

(b) Solve $\log_2\left(x^3\right)+5=11$. **2**

$$\log_2\left(x^3\right)=6$$
$$x^3=2^6$$
$$x^3=4^3$$
$$x=4$$

(c) Given $\cos\theta=-\dfrac{2}{3}$, and $\tan\theta>0$, find the EXACT value of $\sin\theta$. **2**



$y^2=3^2-2^2$

$|y|=\pm\sqrt{5}$

since θ is in Q3

$\therefore \sin\theta=\dfrac{-\sqrt{5}}{3}$

(d) Solve $6^x + \dfrac{36}{6^x} = 37$. Leave your answer in exact form. **2**

$$(6^x)^2 + 36 - 37 \times 6^x = 0$$
$$\left(6^x - 36\right)\left(6^x - 1\right) = 0$$
$$\left(6^x - 36\right) = 0,\ \left(6^x - 1\right) = 0$$
$$6^x = 36,\ 6^x = 1$$
$$x = 2, 0$$

(e) A circle has diameter AB, where $A = (2,5)$ and $B = (-4,1)$.

(i) Find the radius of the circle in simplest surd form. **1**

$$\text{diameter} = \sqrt{(-4-2)^2 + (1-5)^2}$$
$$= \sqrt{36+16}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$

$\therefore$ the radius is $\sqrt{13}$ units.

(ii) Find the centre of the circle. **1**

$$M_{AB} = \left(\frac{2-4}{2}, \frac{5+1}{2}\right)$$

$\therefore$ the midpoint is $(-1,3)$.

(iii) Hence, write the equation of the circle. **1**

$\therefore$ the equation of the circle is $(x+1)^2 + (y-3)^2 = 13$.

Question 12 (11 marks)

(a) Use first principles , $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, to find the gradient function of **2**

$f(x)=\dfrac{1}{x}$.

$$f'(x)=\lim_{h\to 0}\left(\frac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h}\right)$$
$$=\lim_{h\to 0}\left(\frac{x-(x+h)}{x(x+h)h}\right)$$
$$=\lim_{h\to 0}\left(\frac{-1}{x(x+h)}\right)$$
$$=\frac{-1}{x^2}$$

(b) Prove that $\sin^2 x\tan^2 x+2\sin^2 x+\cos^2 x=\sec^2 x$. **2**

$$LHS=\sin^2 x\frac{\sin^2 x}{\cos^2 x}+2\sin^2 x+\cos^2 x$$
$$=\frac{\sin^4 x+2\sin^2 x\cos^2 x+\cos^4 x}{\cos^2 x}$$
$$=\frac{\left(\sin^2 x+\cos^2 x\right)^2}{\cos^2 x}$$
$$=\frac{1}{\cos^2 x}$$
$$=\sec^2 x$$
$$=RHS$$

$$Or: LHS=\sin^2 x(\sec^2 x-1)+2\sin^2 x+\cos^2 x$$
$$=\sin^2 x\sec^2 x-\sin^2 x+2\sin^2 x+\cos^2 x$$
$$=\frac{\sin^2 x}{\cos^2 x}+\sin^2 x+\cos^2 x$$
$$=\tan^2 x+1$$
$$=\sec^2 x$$
$$=RHS$$

(c) Solve $\operatorname{cosec}\,(\theta-30°)=3$ for $0°\le\theta\le 180°$. Answer correct to the nearest degree. **2**

$\cos ec(\theta-30°)=3$

$\sin(\theta-30°)=\dfrac{1}{3}$

$-30°\le\theta-30°\le 150°$

$\theta-30°=19°,161°$

check the domain, only $19°$ is in the domain

$\therefore\theta=49°$

(d) Given $f(x)=\begin{cases}1-x^2, & \text{for } x\le 1\\ kx+1, & \text{for } x>1\end{cases}$

(i) Find the value of k for which $f(x)$ is continuous at $x=1$. **1**

$f(1)=1-1^2$

$\quad =0$

for the function to be continuous at $x=1$,

$k(1)+1=0$

$\therefore k=-1$

(ii) If $k=-1$, find the value of $f(-3)+f(2)$. **1**

if $k=-1$, $f(x)=\begin{cases}1-x^2, & \text{for } x\le 1\\ 1-x, & \text{for } x>1\end{cases}$

$f(-3)=1-(-3)^2$

$\quad =-8$

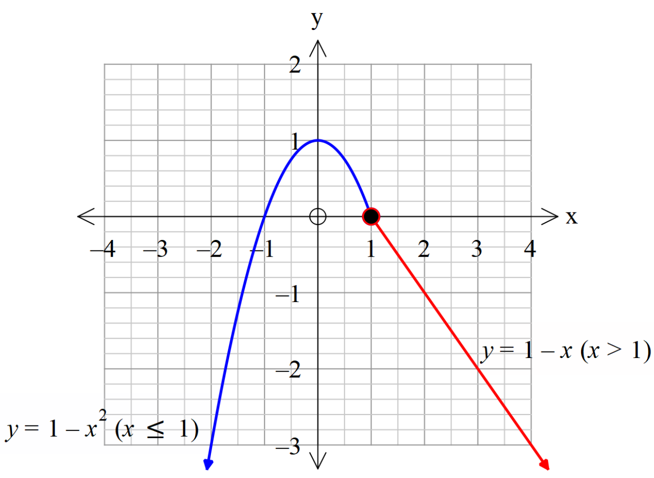
$f(2)=1-(2)$

$\quad =-1$

$\therefore f(-3)+f(2)=-8-1$

$\quad =-9$

(iii) If $k=-1$, find the range of $f(x)$. **1**



$\therefore$ the range of $f(x)$ is $y\le 1$.

(e) If the minimum value of parabola $y = x^2 - 3x + k$ is 5, find the value of k.
2

$$y = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + k$$

The minimum value $-\frac{9}{4} + k = 5$

$$\therefore k = \frac{29}{4}$$

Or: axis of symmetry at $x = \frac{-b}{2a} = \frac{3}{2}$

substitute $x = \frac{3}{2}$ into $5 = x^2 - 3x + k$

$$5 = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + k$$

$$k = 5 + \frac{9}{4} = \frac{29}{4}$$

Question 13 (12 marks)

(a) Differentiate the following functions, leaving your answers in simplest factorised form where possible:

(i) $y = \frac{2}{\sqrt{x}}$ **2**

$$y = 2x^{-\frac{1}{2}}$$
$$y' = 2\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$$
$$y' = \frac{-1}{x\sqrt{x}}$$

(ii) $y = (e^{2x} - 1)^3$ **2**

$$y' = 3(e^{2x} - 1)^2 \times 2e^{2x}$$
$$y' = 6e^{2x}(e^{2x} - 1)^2$$

(iii) $f(x) = x^3\sqrt{x^2 - 1}$ **2**

$$f'(x) = 3x^2\sqrt{x^2 - 1} + \frac{1}{2}\left(x^2 - 1\right)^{-\frac{1}{2}}x^3$$
$$= 3x^2\sqrt{x^2 - 1} + \frac{x^3}{2\sqrt{x^2 - 1}}$$
$$= \frac{3x^4 + x^3 - 3x^2}{2\sqrt{x^2 - 1}}$$

(b) Consider the function $f(x)=\frac{1}{3}x^3-x^2-15x+3$.

(i) Find $f'(x)$. **1**

$f'(x)=x^2-2x-15$

(ii) Find the values of x such that $f'(x)=0$. **2**

$x^2-2x-15=0$
$(x-5)(x+3)=0$
$\therefore x=5,-3$

(iii) Find the set of values of x for which $f(x)$ is decreasing. **1**

$(x-5)(x+3)<0$ for decreasing function
$\therefore$ $-3<x<5$

(iv) Find the gradient of the tangent and normal of $f(x)$ at $x=0$. **2**

$x=0, f'(0)=-15$
$\therefore m_{\text{tangent}}=-15$
$m_{normal}=\frac{1}{15}$

Question 14 (11 marks)

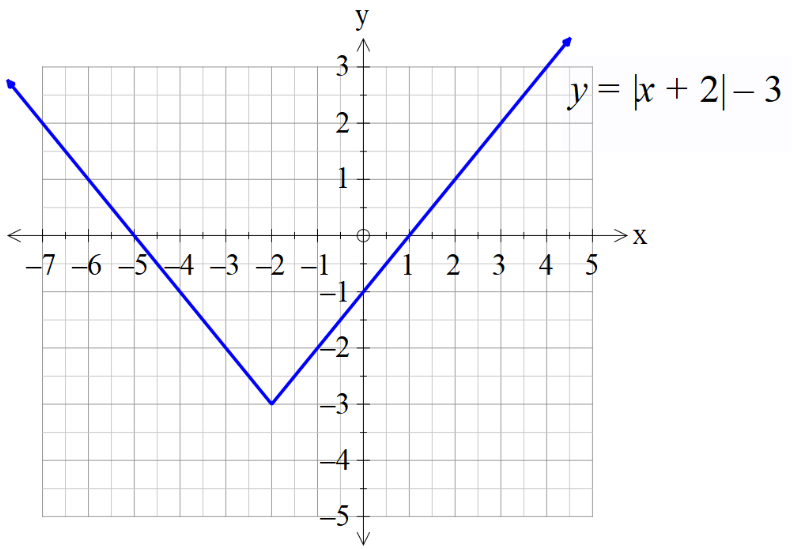
(a) Consider the equation $x+\frac{2}{x}=p$, find the values of p for which the equation has real roots. **2**

$x^2+2=px$
$x^2-px+2=0$
$\Delta\geq 0$ for real roots
$(-p)^2-4(1)(2)\geq 0$
$p^2-8\geq 0$
$p=\pm 2\sqrt{2}$
$\therefore p\leq -2\sqrt{2}, p\geq 2\sqrt{2}$

Comment: well done in the first section but poorly done at the end (last line incorrect).

(b) For $f(x) = |x+2|-3$,

(i) Sketch the function, showing any intercepts. **3**



Comment: well done!

(ii) Hence, or otherwise, find the values for $f(x) \geq 1$. **1**

Draw $y = 1$ on the above graph,
for $f(x) \geq 1$, $\quad x \leq -6, x \geq 2$.

or solve $|x+2|-3 \geq 1$ algebraically:
$|x+2| \geq 4$
$x+2 \leq -4, x+2 \geq 4$
$\therefore x \leq -6, x \geq 2$

Comment: mostly well done!

(c) A bacteria replicated itself according to the equation $N(t) = 140 \times 2^{3.5t}$, where $N(t)$ is the number of bacteria present in a test tube t hours after the experiment began.

(i) What is the initial number of bacteria added to the test tube? **1**

$$N(0) = 140 \times 2^0$$
$$= 140$$

Comment: well done!

(ii) How many bacteria are there after 10 hours? **2**

Answer in scientific notation correct to 3 significant figures.

$$\begin{aligned} N(0) &= 140\times 2^{3.5\times 10} \\ &= 140\times 2^{35} \\ &= 4.81\times 10^{12} \text{ (3 sig. fig.)} \end{aligned}$$

Comment: well done!

(iii) When the number of bacteria reaches 3×10^{25} in the test tube, it will stop replicating. How long before this number is reached?

Give your answer correct to the nearest minute. **2**

$$3\times 10^{25} = 140\times 2^{3.5t}$$

$$\frac{3\times 10^{25}}{140} = 2^{3.5t}$$

$$\ln\left(\frac{3\times 10^{25}}{140}\right) = \ln\left(2^{3.5t}\right)$$

$$3.5t = \frac{\ln\left(\frac{3\times 10^{25}}{140}\right)}{\ln 2}$$

$$\begin{aligned} t &= \frac{\ln\left(\frac{3\times 10^{25}}{140}\right)}{3.5\ln 2} \\ &= 22.1439...\,hours \\ &= 22 \text{ hours } 9 \text{ minutes (nearest minutes)} \end{aligned}$$

Comment: generally well done, however, some students used hours instead of minutes.

Question 15 (11 marks)

(a) Given that $f(x)=\dfrac{2x+1}{3x-4},\ \left(x\neq\dfrac{4}{3}\right)$, find $f'(-2)$. **2**

$$\begin{aligned} f'(x)&=\frac{(3x-4)(2)-3(2x+1)}{(3x-4)^2}\\ &=\frac{6x-8-6x-3}{(3x-4)^2}\\ &=\frac{-11}{(3x-4)^2}\\ f'(-2)&=\frac{-11}{\left(3(-2)-4\right)^2}\\ &=\frac{-11}{100}\end{aligned}$$

$or:\ f(x)=(2x+1)(3x-4)^{-1}$

$$\begin{aligned} f'(x)&=(3x-4)^{-1}\times 2+(2x+1)\times(-1)(3x-4)^{-2}\times 3\\ &=\frac{2}{3x-4}-\frac{3(2x+1)}{(3x-4)^2}\\ &=\frac{2(3x-4)-6x-3}{(3x-4)^2}\\ &=\frac{-11}{(3x-4)^2}\end{aligned}$$

Comments: lots of arithmetic errors. 1 mark for correct differentiation, 1 mark for correct answer.

(b) Solve $\log_2(x+4)+\log_2(x-1)-\log_2(x+2)=2$. **3**

$$\log_2\left(\frac{(x+4)(x-1)}{(x+2)}\right)=2$$

$$4=\frac{(x+4)(x-1)}{(x+2)}$$

$$4x+8=x^2+3x-4$$

$$x^2-x-12=0$$

$$(x-4)(x+3)=0$$

$$x=4,-3$$

Check the solutions: substitute $x=-3$ into the original equation,

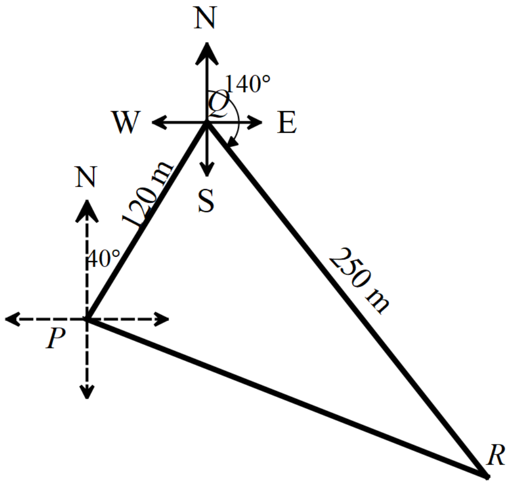
$\log_2(-3+4)+\log_2(-3-1)-\log_2(-3+2)$ has no solution as $\log_2(-3-1)$ is undefined.

$\therefore x=4$ is the only solution.

Comments: 1 mark for using log laws to find the quadratic, 1 mark for solving quadratic, 1 mark for correct answer. Many students missed the third mark.

(c) A farmer walked 120 metres from P to Q on a bearing of $040°\text{T}$.

He then walked 250 meters from Q to R on a bearing of $140°\text{T}$.



(i) Find the value of $\angle PQR$. **1**

$\angle PQR = 80°$ (use angles on parallel lines to deduce)

(ii) Find the length of PR, correct to the nearest metre. **2**

$PR^2 = 120^2 + 250^2 - 2\times120\times250\cos 80°$

$PR = \sqrt{120^2 + 250^2 - 2\times120\times250\cos 80°}$ $(PR > 0)$

$= 257.839...$

$= 258$ m (nearest metre)

Comments: some students were working in radians and got an answer of 289.0038661.

(iii) Find the bearing of R from P, correct to the nearest degree. **2**

$$\frac{\sin\angle QPR}{250} = \frac{\sin 80°}{258}$$

$$\sin\angle QPR = \frac{250\sin\angle QPR}{258}$$

$$\angle QPR = \sin^{-1}\left(\frac{250\sin\angle QPR}{258}\right)$$

$= 72°39'$

$\therefore$ the bearing of R from P is $72°39' + 40° = 112°39'$.

Comments: some students found the angle but failed to find the bearing.

(iv) Find the area of ΔPQR. Answer correct to the nearest m^2. **1**

$$Area = \frac{1}{2}(120)(250)\sin 80°$$
$$= 14772 \text{ m}^2 \quad (\text{nearest m}^2)$$

Question 16 (11 marks)

(a) The diagram below shows the graph of a function $y = f(x)$. Point A, B and C are stationary points at which the derivatives are zero.

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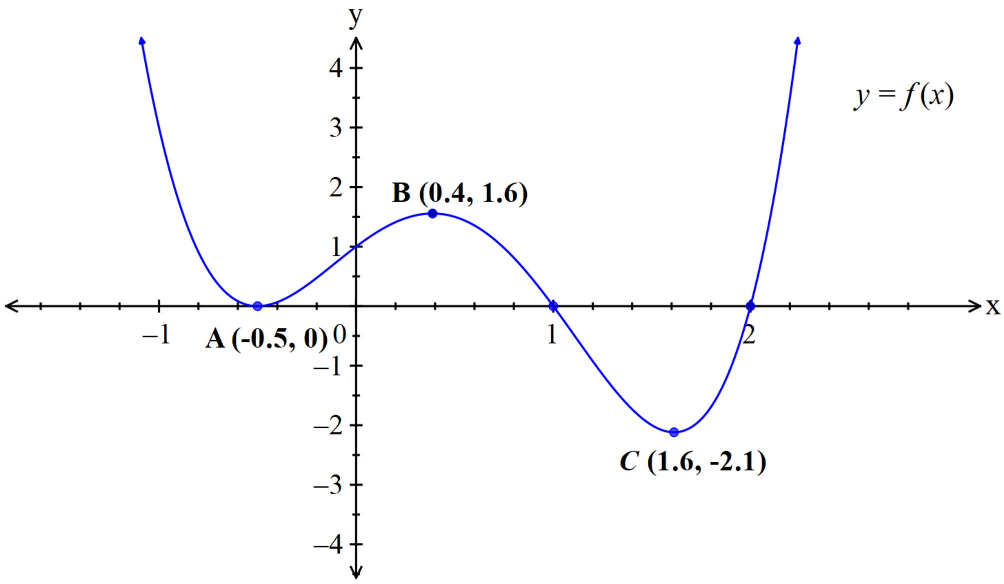
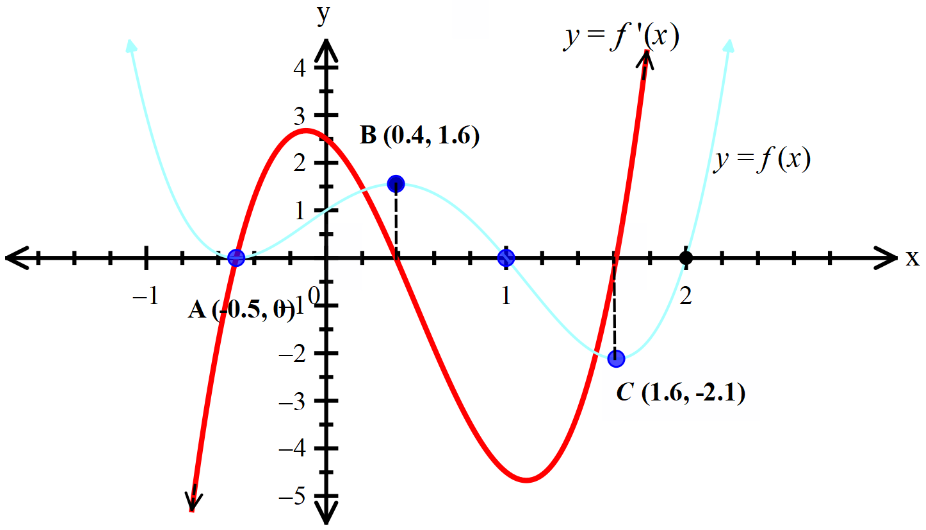


Comments:

- Many students need to learn to draw their derivative graphs correctly knowing what each key feature represent in the derivative function.
- Label the x and y axis correctly on the derivative graph.
- Scales need to be evenly distributed.

(i) Sketch the graph of $f'(x)$. Clearly label all essential information. **3**

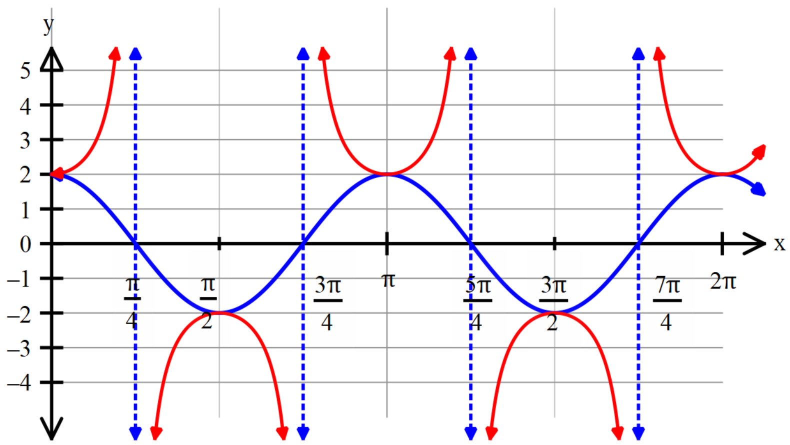


(ii) State the values of x for which the derivative of $f(x)$ is increasing. **1**

$-0.5 < x < 0.4, \quad x > 1.6$

This question has been excluded from this assessment as there is a mistake in the question. The intended question should be : State the values of x for which $f(x)$ is increasing. The total mark and student marks have been adjusted.

(b) The graph of $y = 2\cos(2x)$ is shown below, where x is in radians.



Comment:

- Many students do not understand the concept of amplitude and period.
- Many mistaken y=2sec2x being the reciprocal graph of y=2cos2x
- Many need to learn to solve their equations in radians and be comfortable with it.

(i) Find the period and amplitude. **2**

The amplitude is 2.
The period is π.

(ii) On the above graph, label all the x- and y- intercepts on both axes. **2**

(iii) Draw $y = 2\sec(2x)$ on the above graph, clearly showing any asymptotes. **1**

(iv) Find the exact solutions for $2\sec(2x) = 4$ for $0 \le x \le 2\pi$. **2**

$$\sec(2x) = 2$$
$$\cos(2x) = \frac{1}{2}$$
$$0 \le x \le 2\pi$$
$$0 \le 2x \le 4\pi$$
$$2x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$$
$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Comment: Many did not adjust the domain to accommodate the multiple angle, have missed the other 2 solutions.

End of Examination

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