HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 11 Yearly Examination
Assessment 3 Term 3 2020

STUDENT NUMBER: ____________________

General Instructions:	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen • Calculators approved by NESA may be used • For questions in Section II, show relevant mathematical reasoning and/ or calculations • Students are permitted to refer to one double-sided A4 page of handwritten topic summaries which they prepared prior to this assessment
Total Marks: 75	**Section I – 10 marks** (pages 2–6) • Attempt Questions 1–10 • Allow about 15 minutes for this section **Section II – 65 marks** (pages 7–17) • Attempt Questions 11–30 • Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 5

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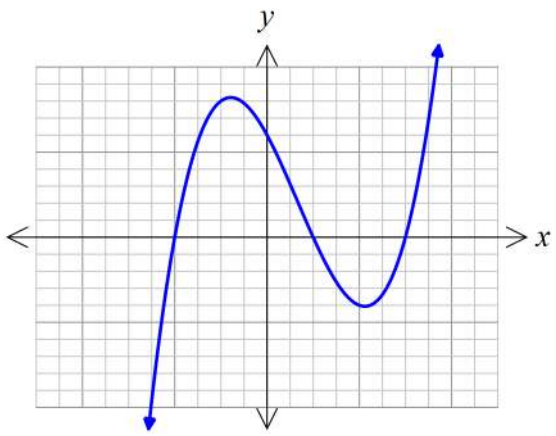
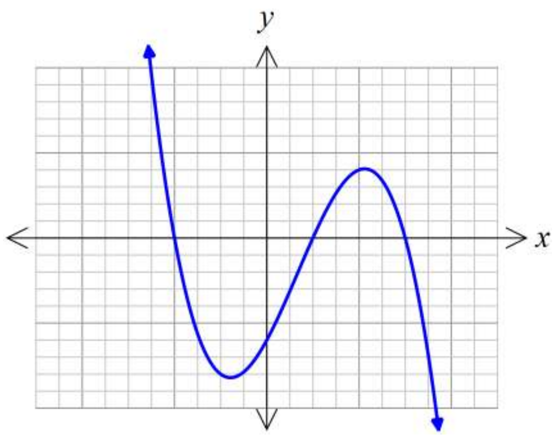
1 The graph of $y = f(x)$ is shown below:

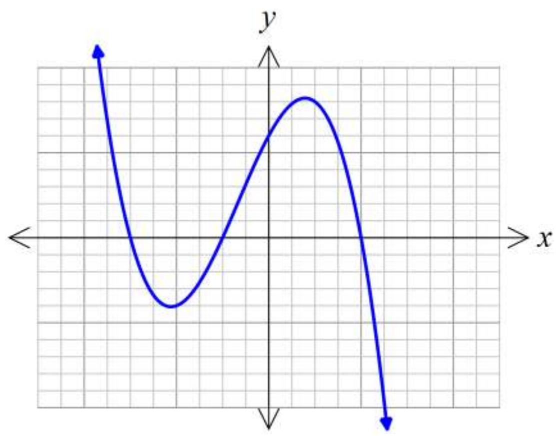


Which graph shows $y = -f(x)$?

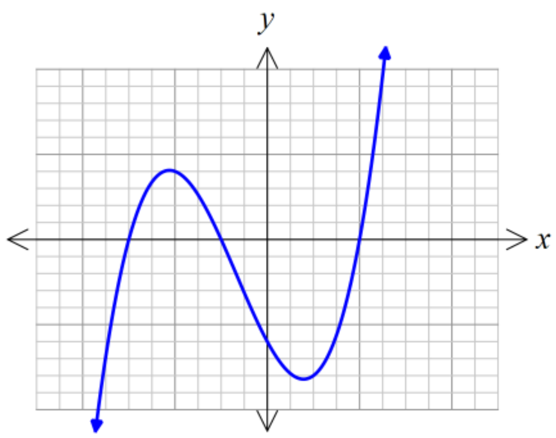
(A)



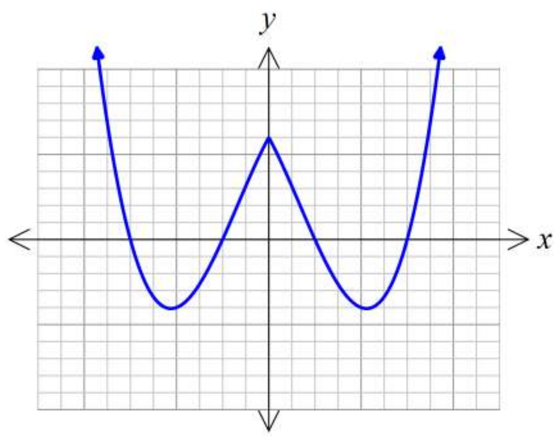
(B)



(C)



(D)



2 If $f(x) = 2x + 1$ and $g(x) = x^2$, the value of $f(g(2))$ is?

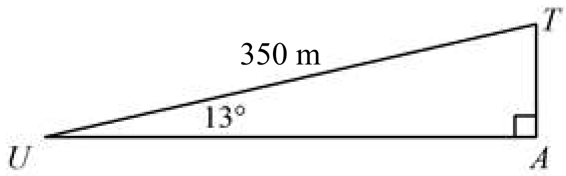
(A) 4

(B) 5

(C) 9

(D) 25

3 Uma is lying down on flat ground. The distance from Uma to the top of the television antenna, *AT*, is 350 m.



NOT TO SCALE

If the angle of elevation from Uma to the top of the mast is $13°$, then the height of the antenna is closest to:

(A) 80 m

(B) 81 m

(C) 78 m

(D) 79 m

4 $80°$ expressed as radians is closest to:

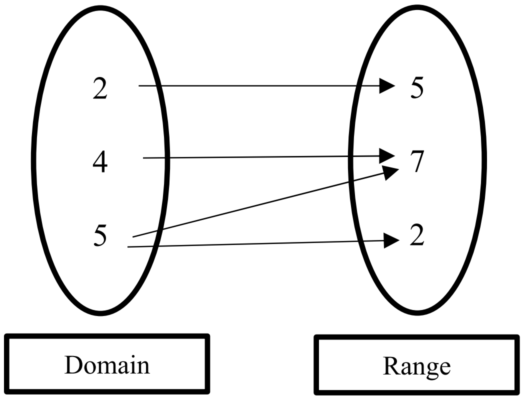
(A) $\dfrac{4\pi}{5}$

(B) 1.4

(C) 80π

(D) 25

5



The mapping diagram shows a relation that is:

(A) One-to-one

(B) One-to-many

(C) Many-to-one

(D) Many-to-many

6 The volume of water, V litres, in a tank after t minutes, is given by $V = 4 - t^2$, then after five minutes:

(A) The volume of water is decreasing at a rate of 21 litres per minute.

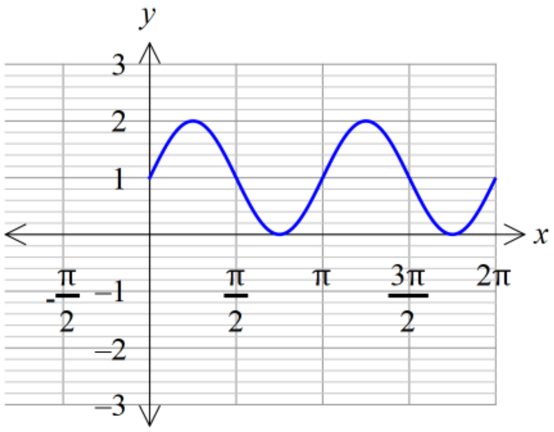
(B) The volume of water is increasing at a rate of 21 litres per minute.

(C) The volume of water is decreasing at a rate 10 litres per minute.

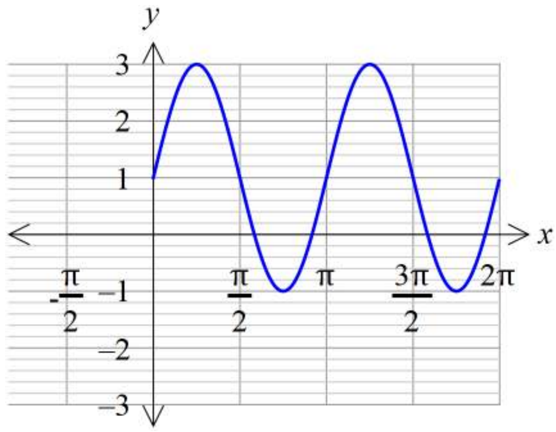
(D) The volume of water is increasing at a rate of 10 litres per minute.

7 The graph of $y = 2\sin 2x + 1$ for $0 \le x \le 2\pi$ is:

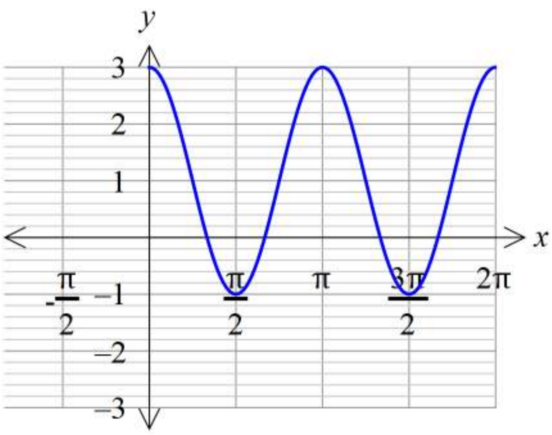
(A)



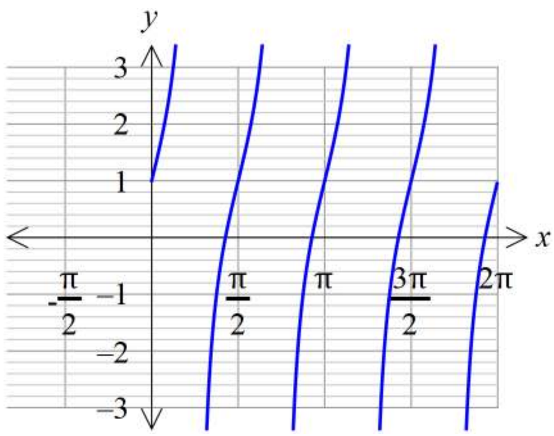
(B)



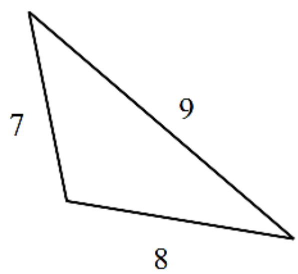
(C)



(D)



8 The size of the largest angle in triangle ABC is closest to:



NOT TO SCALE

(A) 48°

(B) 58°

(C) 73°

(D) 110°

9 Given that $\cos 40° = 0.766$, which of the following statements are true?

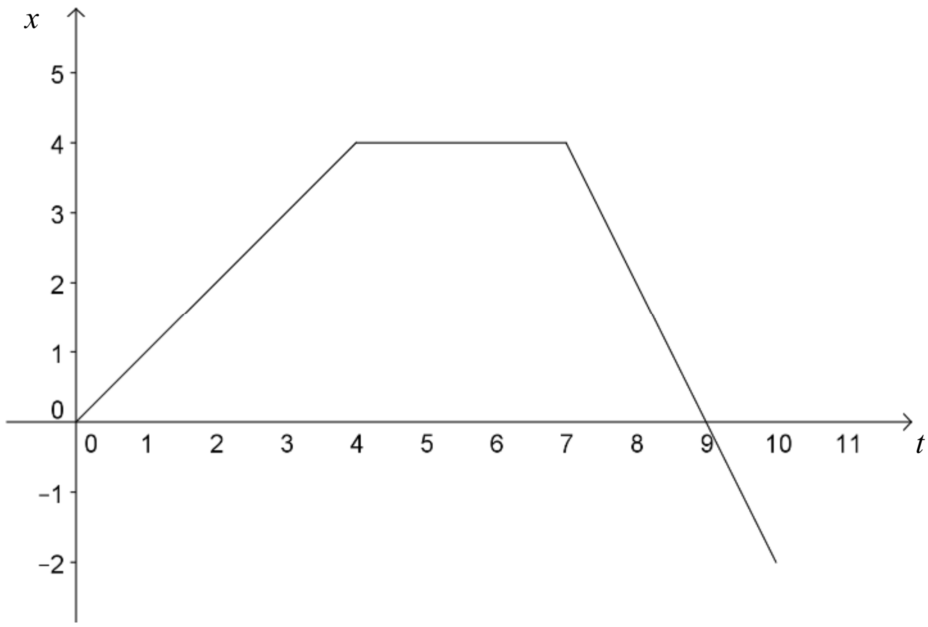
(A) $\cos 140° = -0.766$ and $\sin 130° = 0.766$

(B) $\cos 140° = -0.766$ and $\sin 50° = -0.766$

(C) $\cos 320° = 0.766$ and $\sin 130° = -0.766$

(D) $\cos 220° = 0.766$ and $\sin 50° = 0.766$

10 The displacement-time graph of an object's motion is shown below, where displacement x is measured in metres, and time t in seconds.



The distance travelled by the object in the first 10 seconds is:

(A) -2 metres

(B) 4 metres

(C) 8 metres

(D) 10 metres

End of Section I

Section II

65 marks
Attempt Questions 11-30
Allow about 1 hour and 45 minutes for this section

Question 11 (2 marks)

Show that $2\sin 210° + \tan 225° = 0$

Question 12 (2 marks)

Simplify $\dfrac{y}{y+3} \div \dfrac{y^2}{y^2+4y+3}$.`

Question 13 (2 marks)

Solve $3x^2 + 2x - 3 = 0$, giving your answers in simplest exact form.

Question 14 (2 marks)

Solve $|2x+1|=5$.

Question 15 (2 marks)

Solve $\sqrt{3}\tan x+1=0$ for $0\le x\le 2\pi$.

Question 16 (2 marks)

Find the derivative of $f(x)=x^2+3x+2$ by first principles.

Question 17 (7 marks)

Differentiate the following functions

(a) $y = \dfrac{x-2}{x^2}$ **2**

(b) $y = x^2\left(3x-2\right)^2$ **2**

(c) $f\left(x\right) = \dfrac{4}{\sqrt{x}}$ **1**

(d) $y = \dfrac{x}{2x+5}$ **2**

Question 18 (2 marks)

Find the exact value of $\cos x$, given that $\sin x = \frac{1}{5}$ and $\tan x < 0$.

Question 19 (5 marks)

Consider the polynomial $f(x) = (x-2)(x+1)(x-1)(x+2)$.

(a) State the degree of the polynomial. **1**

(b) What is the constant term of the polynomial? **1**

(c) Sketch the graph of $y = f(x)$ showing the x- and y- intercepts. **3**

Question 20 (4 marks)

Consider the function, $f(x) = x^3 - 3x^2$

(a) Show that the equation of the normal to the curve $y = f(x)$ at the point $(1, -2)$ is $x - 3y - 7 = 0$. **3**

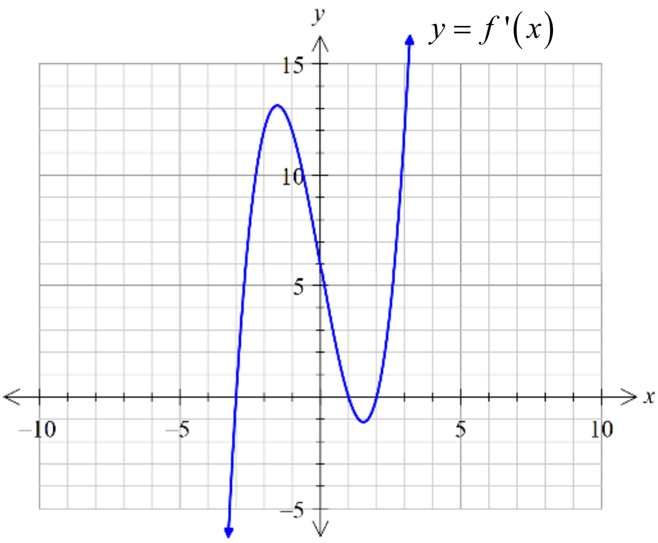
(b) Find the x-values of the point(s) on the curve $y = f(x)$ where the tangent to the curve is horizontal. **1**

Question 21 (2 marks)

Prove the identity $\sin x \cos x = \dfrac{\tan x}{1 + \tan^2 x}$.

Question 22 (3 mark)

The graph of $y = f'(x)$ is given below.



(a) State the x-values of the stationary points of $y = f(x)$. **1**

(b) State the values of x, using interval notation, for which the graph of $y = f(x)$ is increasing. **2**

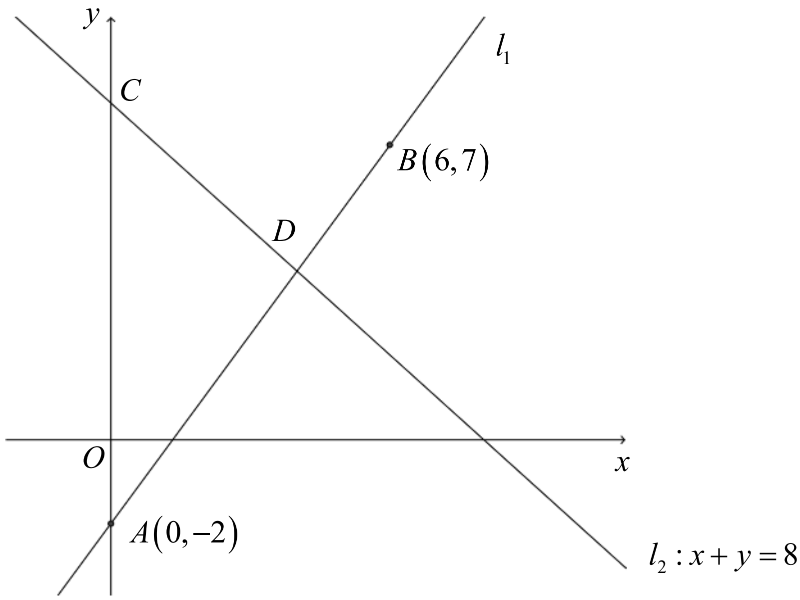
Question 23 (2 marks)

Solve $\sin 2\theta = \frac{1}{2}$ for $0° \le \theta \le 360°$

Question 24 **(6 marks)**

The straight-line l_1 passes through the points A and B with coordinates $(0,-2)$ and $(6,7)$ respectively. The straight line l_2 with equation $x+y=8$ cuts the y-axis at the point C.

The lines l_1 and l_2 intersect at the point D.



NOT TO SCALE

(a) Find the equation of the line l_1 in the form $y = mx + c$ **2**

(b) Find the coordinates of D. **2**

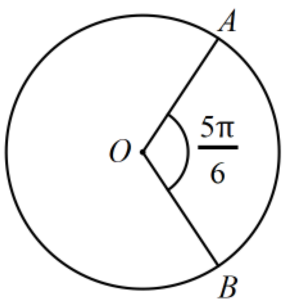
(c) Calculate the area of ΔACD. **2**

Question 25 (2 marks)

Prove that $f\left(x\right)=\dfrac{6x^3}{x^4+x^2}$ is an odd function.

Question 26 (3 marks)

Two points A and B lie on the circumference of a circle of radius 5 cm, with centre $\dfrac{5\pi}{6}$.



NOT TO SCALE

(a) Find the exact area of the minor sector AOB. **1**

(b) Find the area of the minor segment bounded by the chord AB and the arc AB, correct to nearest square centimetre. **2**

Question 27 (6 marks)

A football is kicked vertically upwards. Its height h metres above the ground after t seconds is described by $h = 14t - 5t^2$.

(a) Differentiate h with respect to t to find $\frac{dh}{dt}$. **1**

(b) Given that $\frac{dh}{dt}$ represents the velocity of the ball at a given instant, find the velocity of the ball after 0.5 seconds. **1**

(c) Find the greatest height the ball reaches. **2**

(d) At what time is the ball travelling upwards with a velocity of 3 metres per second? **1**

(e) What is the acceleration of the ball? **1**

Question 28 (2 marks)

A sector of a circle has perimeter 12 m and an angle of 0.4 radians at the centre.

Find the radius of the circle.

Question 29 (3 marks)

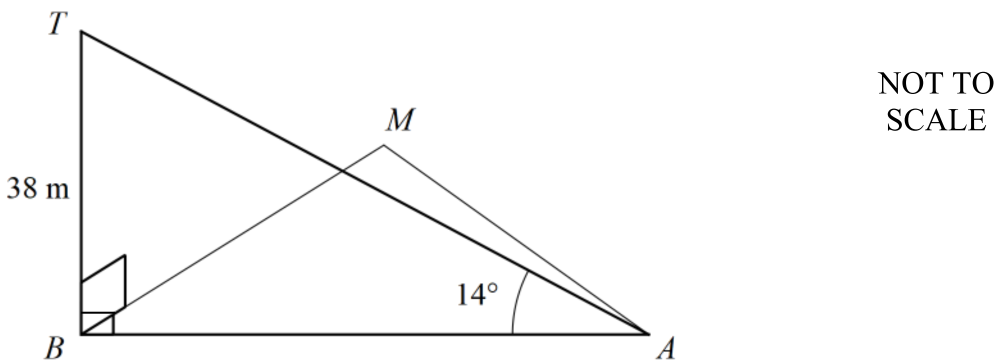
Solve $2\cos^2\theta + 3\cos\theta - 2 = 0$ for $0° \le \theta \le 180°$.

Question 30 (6 marks)

A vertical tree of height 38 metres stands on horizontal ground. The bottom of the tree is at the point *B*. Observer *M*, standing on the ground, is due North of the bottom of the tree.

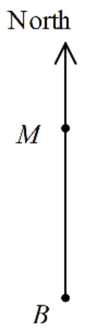
Observer *A*, standing on the ground, sees the top of the tree, *T*, at an angle of elevation of $14°$. The true bearing of Observer *M* from Observer *A* is $290°$.

Observers *A* and *M* are standing the same distance away from the tree.



(a) Show that $AB = 38\cot 14°$ **1**

(b) Complete the diagram of the top view of the scenario below, showing the point A and the bearing of M from A. **1**



(c) Show that $\angle MBA = 40°$. **2**

(d) Find the distance between A and M. **2**

End of Assessment Task

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Mathematics Advanced

Year 11 Yearly Examination
Assessment 3 Term 3 2020

STUDENT NUMBER: ____

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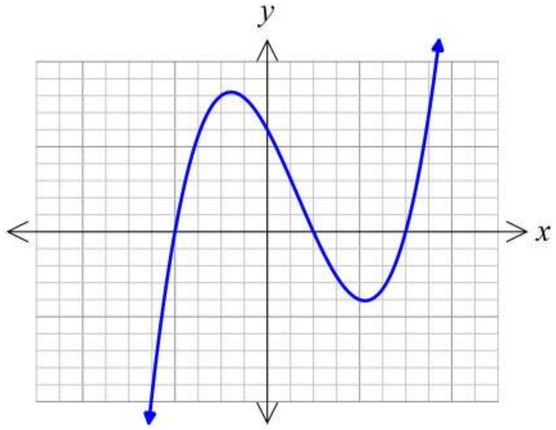
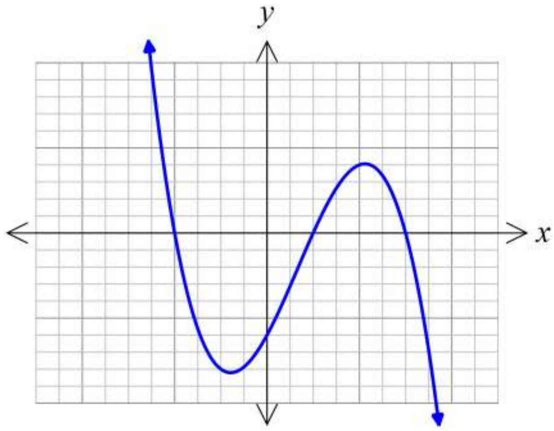


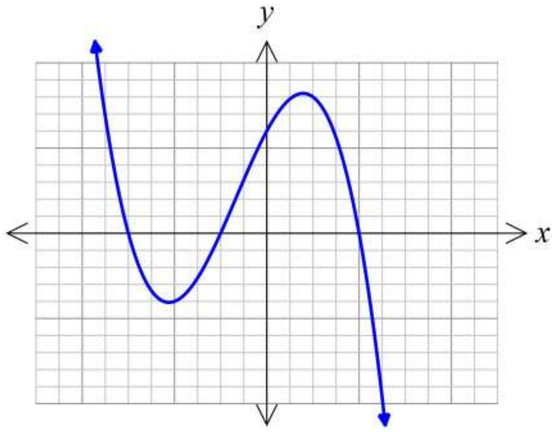
Question 1
$y = -f(x)$ is a reflection of $y = f(x)$ across $x-$ axis
Answer: **A**

Which graph shows $y = -f(x)$?

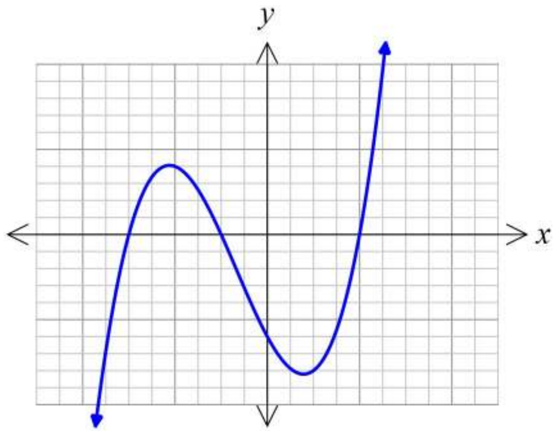
(A)



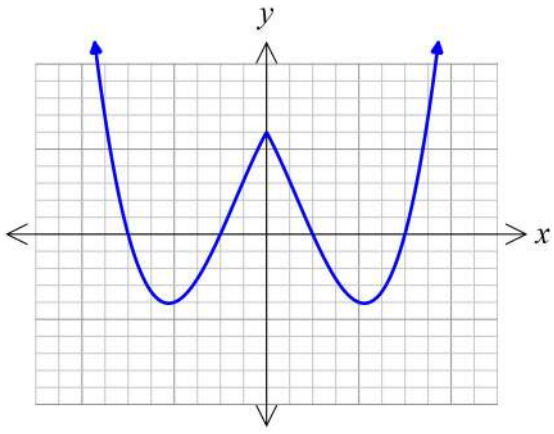
(B)



(C)



(D)



2 If $f(x)=2x+1$ and $g(x)=x^2$, the value of $f(g(2))$ is?

(A) 4

(B) 5

(C) 9

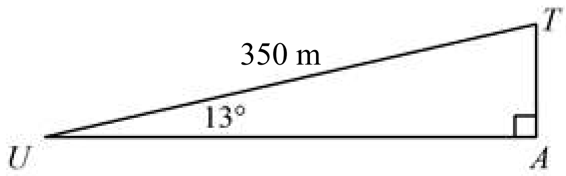
(D) 25

Question 2

$$f(g(2))=f(4)$$
$$=2\times4+1$$
$$=9$$

Answer**: C**

3 Uma is lying down on flat ground. The distance from Uma to the top of the television antenna, AT, is 350 m.



NOT TO SCALE

If the angle of elevation from Uma to the top of the mast is $13°$, then the height of the antenna is closest to:

(A) 80 m

(B) 81 m

(C) 78 m

(D) 79 m

Question3

$$\sin 13° = \frac{AT}{350}$$
$$AT = 350\sin 13°$$
$$= 78.732...$$

Answer: **D**

4 $80°$ expressed as radians is closest to:

(A) $\frac{4\pi}{5}$

(B) 1.4

(C) 80π

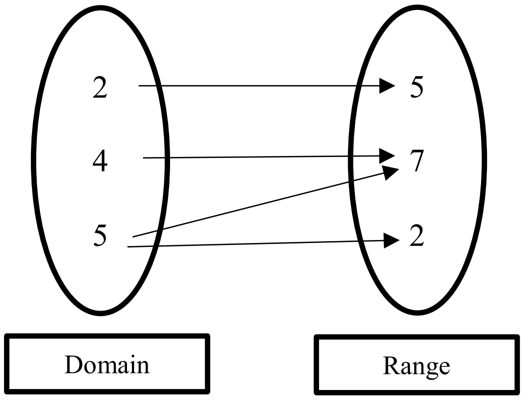
(D) 25

Question 4

$$180° = \pi$$
$$1° = \frac{\pi}{180}$$
$$80° = \frac{80\pi}{180}$$
$$= \frac{4}{9}\pi$$
$$= 1.39626...$$

Answer**: B**

5



The mapping diagram shows a relation that is:

(A) One-to-one

(B) One-to-many

(C) Many-to-one

(D) Many-to-many

Question 5

Answer: **B**

Many students were unable to determine the type of relation correctly.

6 The volume of water, V litres, in a tank after t minutes, is given by $V = 4 - t^2$, then after five minutes:

(A) The volume of water is decreasing at a rate of 21 litres per minute.

(B) The volume of water is increasing at a rate of 21 litres per minute.

(C) The volume of water is decreasing at a rate 10 litres per minute.

(D) The volume of water is increasing at a rate of 10 litres per minute.

Question 6

$$\frac{dV}{dt} = -2t$$

When $t = 5$, $\frac{dV}{dt} = -10$

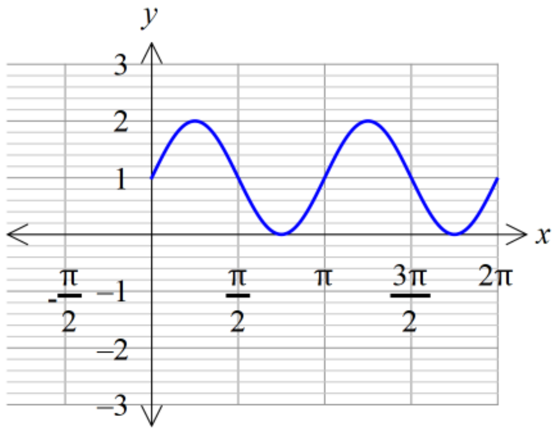
Answer: **C**

This question was not done well.

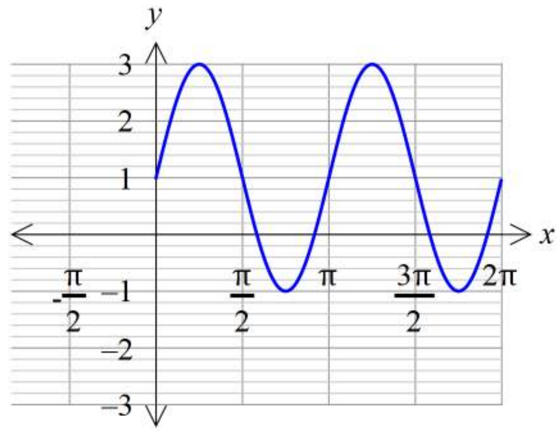
7 The graph of $y = 2\sin 2x + 1$ for $0 \le x \le 2\pi$ is:

Question 7
Answer: **B**

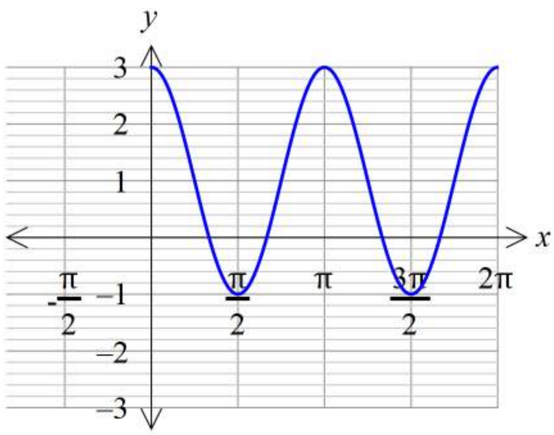
(A)



(B)



(C)



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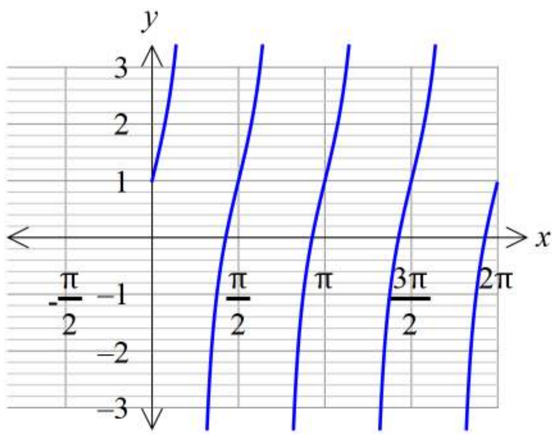
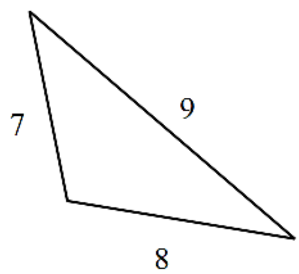


8 The size of the largest angle in triangle ABC is closest to:



NOT TO SCALE

(A) 48°

(B) 58°

(C) 73°

(D) 110°

Question 8

$$\angle = \cos^{-1}\left(\frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8}\right)$$
$$= 73°$$

Answer: **C**

9 Given that $\cos 40° = 0.766$, which of the following statements are true?

(A) $\cos 140° = -0.766$ and $\sin 130° = 0.766$

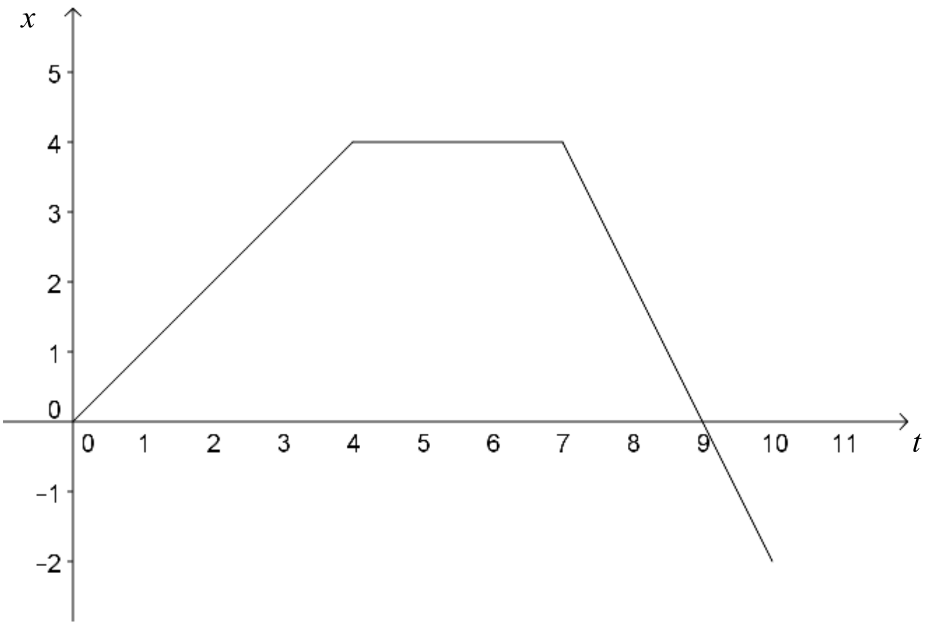
(B) $\cos 140° = -0.766$ and $\sin 50° = -0.766$

(C) $\cos 320° = 0.766$ and $\sin 130° = -0.766$

(D) $\cos 220° = 0.766$ and $\sin 50° = 0.766$

Question 9
By elimination
Answer: A

10 The displacement-time graph of an object's motion is shown below, where displacement x is measured in metres, and time t in seconds.



The distance travelled by the object in the first 10 seconds is:

(A) -2 metres

(B) 4 metres

(C) 8 metres

(D) 10 metres

Question 10
$d = 4 + 4 + 2$
$= 10$
Answer: D

End of Section I

Section II

65 marks
Attempt Questions 11-30
Allow about 1 hour and 45 minutes for this section

Question 11 (2 marks)

Show that $2\sin 210° + \tan 225° = 0$

$$\begin{aligned} 2\sin 210° + \tan 225° &= 2\times -\sin 30° + \tan 45° \\ &= 2\times -\frac{1}{2} + 1 \\ &= -1+1 \\ &= 0 \end{aligned}$$

Show could have been done better. Better solutions showed why $\sin 210° = -1$ and $\tan 225° = 1$ by relating back to acute angle. You need to show your steps.

Please set out by treating LHS independently from RHS.

Question 12 (2 marks)

Simplify $\frac{y}{y+3} \div \frac{y^2}{y^2+4y+3}$.`

$$\begin{aligned} \frac{y}{y+3} \div \frac{y^2}{y^2+4y+3} &= \frac{y}{y+3} \times \frac{y^2+4y+3}{y^2} \\ &= \frac{1}{y+3} \times \frac{(y+1)(y+3)}{y} \\ &= \frac{y+1}{y} \end{aligned}$$

Generally, done well

Question 13 (2 marks)

Solve $3x^2 + 2x - 3 = 0$, giving your answers in simplest exact form.

$$3x^2 + 2x - 3 = 0$$

$$a = 3, b = 2, c = -3$$

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{2^2 - 4\times 3\times -3}}{2\times 3} \\ &= \frac{-2\pm\sqrt{40}}{6} \\ &= \frac{-2\pm 2\sqrt{10}}{6} \\ &= \frac{-1\pm\sqrt{10}}{3} \end{aligned}$$

Students who approach by completing the square were less successful than those who used quadratic formula.

Generally well done.

Common errors included incorrect simplification of $\sqrt{40}$, or incorrect cancelling, for example

$$\frac{-2\pm 2\sqrt{10}}{6} = \frac{-2\pm\sqrt{10}}{3}$$

Question 14 (2 marks)

Solve $|2x+1|=5$.

$|2x+1|=5$

$2x+1=5 \qquad 2x+1=-5$

$2x=4 \qquad 2x=-6$

$x=2 \qquad x=-3$

$\therefore x=2$ or $x=-3$

Mostly well done, however there were a few students who did not know how to set up the 2 cases correctly

Question 15 (2 marks)

Solve $\sqrt{3}\tan x+1=0$ for $0 \le x \le 2\pi$.

$\sqrt{3}\tan x+1=0$

$\tan x=-\dfrac{1}{\sqrt{3}}$

x in 2nd and 4th quadrant

Related acute angle is $\dfrac{\pi}{6}$

$$x=\pi-\frac{\pi}{6},\ 2\pi-\frac{\pi}{6}$$

$$=\frac{5\pi}{6},\ \frac{11\pi}{6}$$

There were a lot of students who did not pick the right quadrants for the solutions when TAN is NEG. Also, a few missed the neg. sign.

When a Domain is given in radians, it must be answered in radians.

Degrees should NOT be mentioned at all.

Question 16 (2 marks)

Find the derivative of $f(x)=x^2+3x+2$ by first principles.

$$f(x)=x^2+3x+2$$

$$f(x+h)=(x+h)^2+3(x+h)+2$$

$$=x^2+2xh+h^2+3x+3h+2$$

Mostly well done, although a few students need to Understand what f(x+h) means.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2+3x+3h+2-(x^2+3x+2)}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2+3h}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h+3)}{h}$$

$$=\lim_{h\to 0}(2x+h+3)$$

$$=2x+3$$

Question 17 (7 marks)

Differentiate the following functions

(a) $y = \dfrac{x-2}{x^2}$ **2**

$$y = \frac{x-2}{x^2} = \frac{1}{x} - \frac{2}{x^2} = x^{-1} - 2x^{-2}$$

$$\frac{dy}{dx} = -x^{-2} + 4x^{-3} = \frac{4}{x^3} - \frac{1}{x^2} = \frac{4-x}{x^3}$$

This could have also have been done using quotient rule. Some students had the wrong sign in the quotient rule. Some also thought the derivative of "1" was "1" unfortunately.

(b) $y = x^2(3x-2)^2$ **2**

$u = x^2$ $\quad v = (3x-2)^2$

$u' = 2x$ $\quad v' = 6(3x-2)$

$$\frac{dy}{dx} = 6x^2(3x-2) + 2x(3x-2)^2$$
$$= 2x(3x-2)(3x+3x-2)$$
$$= 2x(3x-2)(6x-2)$$
$$= 4x(3x-2)(3x-1)$$

Mostly well done. Some students need to realise that the best way to simplify these type of derivatives is by factorising rather than expanding

$\sqrt{x}$ **1**

$$f(x) = \frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}} = 4x^{\frac{-1}{2}}$$

$$f'(x) = 4 \times -\frac{1}{2} \times x^{\frac{-3}{2}} = \frac{-2}{x^{\frac{3}{2}}} = -\frac{2}{x\sqrt{x}}$$

Again quotient rule could have been used. Some students also derived "4" to be "1" instead of "0"

(d) $y = \dfrac{}{2x+5}$ **2**

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{(2x+5)\times 1 - 2\times x}{(2x+5)^2} = \frac{2x+5-2x}{(2x+5)^2} = \frac{5}{(2x+5)^2}$$

Quotient Rule:

$v = 2x+5$ $\quad u = x$

$v' = 2$ $\quad u' = 1$

$v^2 = (2x+5)^2$

Mostly well done question with not many errors other than occasional + instead of -

Question 18 (2 marks)

Find the exact value of $\cos x$, given that $\sin x = \frac{1}{5}$ and $\tan x < 0$.

$\sin x = \frac{1}{5}$ Sin positive, tan negative, $\therefore x$ in second quadrant.

$\cos^2 x = 1 - \sin^2 x$

$\cos^2 x = 1 - \frac{1}{25}$

Or draw right angled triangle and use Pythagoras' Theorem

$\cos x = \pm\sqrt{\frac{24}{25}}$

$\cos x = -\frac{2\sqrt{6}}{5}$ as x in 2nd quadrant

The most common error was to omit the negative

Question 19 (5 marks)

Consider the polynomial $f(x) = (x-2)(x+1)(x-1)(x+2)$.

(a) State the degree of the polynomial. **1**

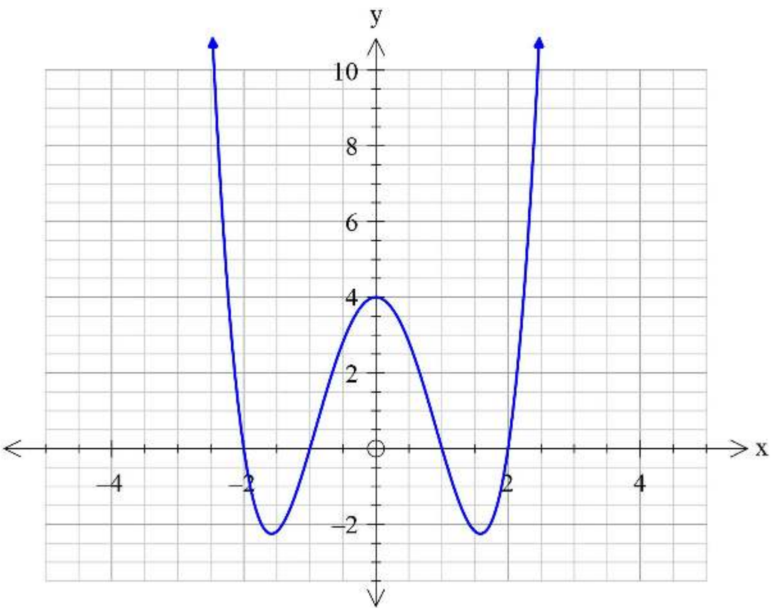
Degree of $f(x)$ is 4

Students who answered x^4 have not answered the question correctly

(b) What is the constant term of the polynomial? **1**

The constant term is $-2\times+1\times-1\times 2 = 4$

(c) Sketch the graph of $y = f(x)$ showing the x- and y- intercepts. **3**



Features should include
x intercepts
y intercepts
correct shape
symmetry
label axes
arrows

Question 20 (4 marks)

Consider the function, $f(x) = x^3 - 3x^2$

(a) Show that the equation of the normal to the curve $y = f(x)$ at the point $(1,-2)$ is $x - 3y - 7 = 0$. **3**

$f'(x) = 3x^2 - 6x$

$f'(1) = 3 - 6$

$= -3$

Gradient of tangent at $x = 1$ is -3.

Gradient of normal at $x = 1$ is $\frac{1}{3}$

Equation of normal:

$y - -2 = \frac{1}{3}(x - 1)$

$3y + 6 = x - 1$

$x - 3y - 7 = 0$

Or

$y = mx + b$

$-2 = \frac{1}{3} \times 1 + b$

$b = \frac{-7}{3}$

$y = \frac{1}{3}x - \frac{7}{3}$

$3y = x - 7$

A few students used the equation to state that $m_T = -3$ and $m_N = \frac{1}{3}$ without differentiating. This is assuming the equation is $x - 3y - 7 = 0$ when that is what we are aiming to **show** $\therefore$ not a valid method

(b) Find the x-values of the point(s) on the curve $y = f(x)$ where the tangent to the curve is horizontal. **1**

For horizontal tangent, $f'(x) = 0$

$0 = 3x^2 - 6x$

$0 = 3x(x - 2)$

$\therefore x = 0, x = 2$

Some errors from students not knowing how to attempt the questions.

Some errors from using $f(x) = 0$ instead of $f'(x) = 0$.

Some errors from incorrect factorising.

Question 21 (2 marks)

Prove the identity $\sin x \cos x = \dfrac{\tan x}{1 + \tan^2 x}$.

$\sin x \cos x = \dfrac{\tan x}{1 + \tan^2 x}$

$RHS = \dfrac{\tan x}{1 + \tan^2 x}$

$= \dfrac{\tan x}{\sec^2 x}$ since $1 + \tan^2 x = \sec^2 x$

$= \tan x \times \cos^2 x$

$= \dfrac{\sin x}{\cos x} \times \cos^2 x$

$= \sin x \cos x$

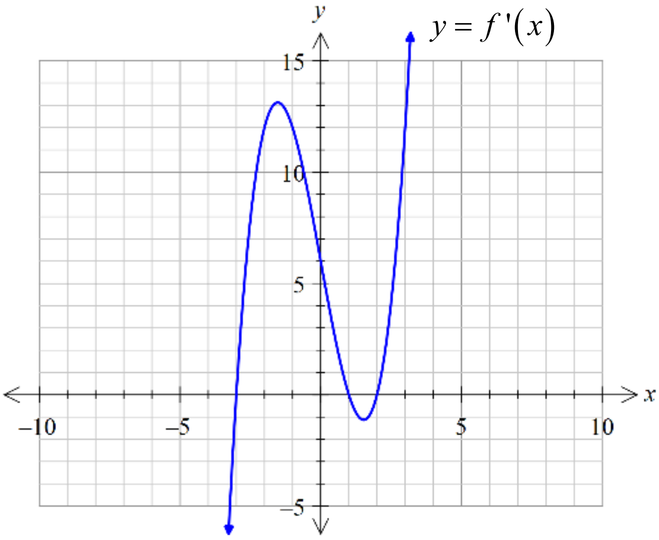
$= LHS$

Better responses had the justification $1 + \tan^2 x = \sec^2 x$

Most students, but not all, had good setting out with LHS and RHS.

Question 22 (3 mark)

The graph of $y = f'(x)$ is given below.



(a) State the x-values of the stationary points of $y = f(x)$. **1**

The stationary points are where $f'(x) = 0$ ie the y-intercepts of the gradient function

$x = -3,\ x = 1,\ x = 2$

Comment: Some students thought the graph is for $y = f(x)$, hence gave the wrong answers.

(b) State the values of x, using interval notation, for which the graph of $y = f(x)$ is increasing. **2**

$y = f(x)$ is increasing when $f'(x)$ is positive, ie the values of x where the graph of the gradient function is above the x-axis

$(-3,1)\ or\ (2,\infty)$

Comment: Some students didn't use interval notations. Some incorrectly using square brackets.

Question 23 (2 marks)

Solve $\sin 2\theta = \frac{1}{2}$ for $0° \le \theta \le 360°$

$\sin 2\theta = \frac{1}{2}$ $\qquad 0° \le \theta \le 360°$

$\qquad 0° \le 2\theta \le 720°$

2θ in first and second quadrant

Related acute angle is $30°$

$2\theta = 30°, 180° - 30°, 30° + 360°, 180° - 30° + 360°$

$2\theta = 30°, 150°, 390°, 510°$

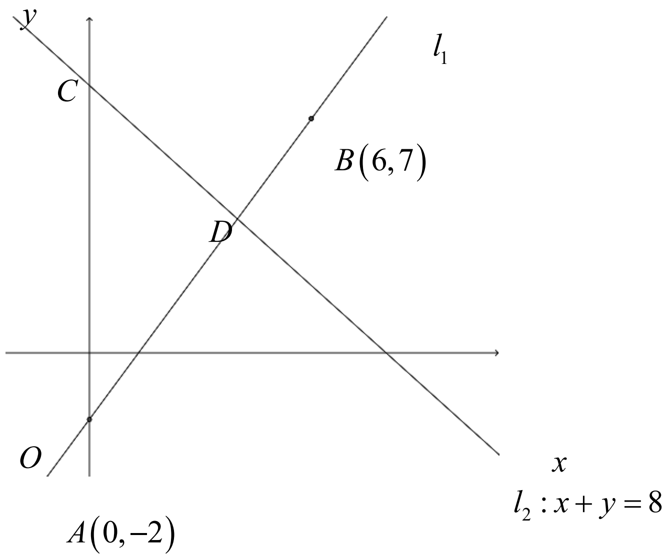
$\theta = 15°, 75°, 195°, 255°$

Comment: Some students didn't find all correct angles in the domain.

Question 24 (6 marks)

The straight-line l_1 passes through the points A and B with coordinates $(0,-2)$ and $(6,7)$ respectively. The straight line l_2 with equation $x+y=8$ cuts the y-axis at the point C.

The lines l_1 and l_2 intersect at the point D.



NOT TO SCALE

(a) Find the equation of the line l_1 in the form $y = mx + c$ **2**

(a)

$$m_{AB} = \frac{y_2 - y_2}{x_2 - x_1} = \frac{7--2}{6-0} = \frac{9}{6} = \frac{3}{2}$$

A is y-intercept

$$y = mx + b$$
$$y = \frac{3}{2}x - 2$$

Comment: Generally well done.

(b) Find the coordinates of D. **2**

$y = \frac{3}{2}x - 2$ ...(1) $x + y = 8$ ...(2)

From (2) $x = 8 - y$

$$y = \frac{3}{2}(8-y) - 2$$
$$2y = 24 - 3y - 4$$
$$5y = 20$$

Substitute into (1)

$$y = 4$$

From (2)

$$x = 8 - 4$$
$$x = 4$$
$$\therefore D(4,4)$$

Comment: Generally well done.

(c) Calculate the area of ΔACD. **2**

$$A = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times 4 \times (2+8)$$
$$= 20\, units^2$$

Comment: Generally well done. Using the area formula $A = \frac{1}{2}ab\,\mathbf{sin}\,c$ should give the same answer. Some student incorrectly assume the two lines are perpendicular, which is not true in this case.

Question 25 **(2 marks)**

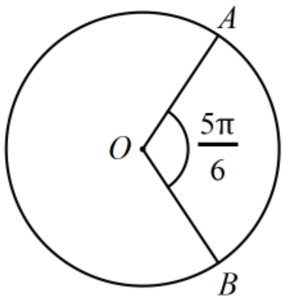
Prove that $f(x)=\dfrac{6x^3}{x^4+x^2}$ is an odd function.

$$f(x)=\frac{6x^3}{x^4+x^2}$$
$$f(-x)=\frac{6(-x)^3}{(-x)^4+(-x)^2}$$
$$=\frac{-6x^3}{x^4+x^2}$$
$$=-f(x)$$
$\therefore f(x)$ is an odd function

Comment: Students lost marks if they didn't show all of the substitution involving $-x$.

Question 26 **(3 marks)**

Two points A and B lie on the circumference of a circle of radius 5 cm, with centre $\dfrac{5\pi}{6}$.



NOT TO SCALE

(a) Find the exact area of the minor sector AOB. **1**

$$A=\frac{1}{2}r^2\theta$$
$$=\frac{1}{2}\times 5^2\times\frac{5\pi}{6}$$
$$=\frac{125}{12}\pi \text{ cm}^2$$

Comment: Well done.

(b) Find the area of the minor segment bounded by the chord AB and the arc AB, correct to nearest square centimetre. **2**

$A=$ sector-triangle
$$=\frac{125\pi}{12}-\frac{1}{2}\times 5^2\sin\frac{5\pi}{6}$$
$$=26.474...$$
$$=26\text{ cm}^2\ (\text{nearest square centimetre})$$

Comment: Generally well done but some students forgot to convert their calculator to radian mode.

Comment: All of this question was answered well.

Question 27 (6 marks)

A football is kicked vertically upwards. Its height h metres above the ground after t seconds is described by $h = 14t - 5t^2$.

(a) Differentiate h with respect to t to find $\frac{dh}{dt}$. **1**

$h = 14t - 5t^2$

$\frac{dh}{dt} = 14 - 10t$

(b) Given that $\frac{dh}{dt}$ represents the velocity of the ball at a given instant, find the velocity of the ball after 0.5 seconds. **1**

When $t = 0.5$,

$\frac{dh}{dt} = 14 - 10 \times 0.5$

$= 9$

Velocity is 9 metres per second.

(c) Find the greatest height the ball reaches. **2**

$h = t(14 - 5t)$ t-value of vertex $t = \frac{-b}{2a} = \frac{-14}{-10} = 1.4$

When $t = 1.4$,

$h = 14 \times 1.4 - 5 \times 1.4^2$

$= 9.8$

Max height is 9.8 metres

(d) At what time is the ball travelling upwards with a velocity of 3 metres per second? **1**

Let $\frac{dh}{dt} = 3$

$14 - 10t = 3$

$10t = 11$

$t = 1.1$

Ball is travelling at 3 metres per second upwards at 1.1 seconds

(e) What is the acceleration of the ball? **1**

$\frac{dv}{dt} = -10$

Acceleration is -10 metres per second per second

Question 28 **(2 marks)**

A sector of a circle has perimeter 12 m and an angle of 0.4 radians at the centre.

Find the radius of the circle.

$P = r + r + l$

$12 = 2r + 0.4r$

$12 = 2.4r$

$r = 5$

The radius is 5 metres.

Students could identify that the perimeter was the sum of two radii and arc length were then successful.
Common errors included changing 0.4 radians to $\frac{2}{5}\pi$ (?), or changing it to degrees and then using $l = r\theta$.

Students who simplified the problem to an arc length only problem could achieve any comparable marks to correct solution.

Question 29 **(3 marks)**

Solve $2\cos^2\theta + 3\cos\theta - 2 = 0$ for $0° \le \theta \le 180°$.

$2\cos^2\theta + 3\cos\theta - 2 = 0$

$2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$

$(2\cos\theta - 1)(\cos\theta + 2) = 0$

$\cos\theta = \frac{1}{2}$ $\qquad$ $\cos\theta = -2$

$\theta = 60°$ $\qquad$ no solutions, since $-1 \le \cos\theta \le 1$

$\therefore \theta = 60°$

Very well done.
Students who identified the equation as reducible to quadratic AND factorised were successful. Students who used quadratic formula rather than factoring the non-monic quadratic were more likely to make errors.

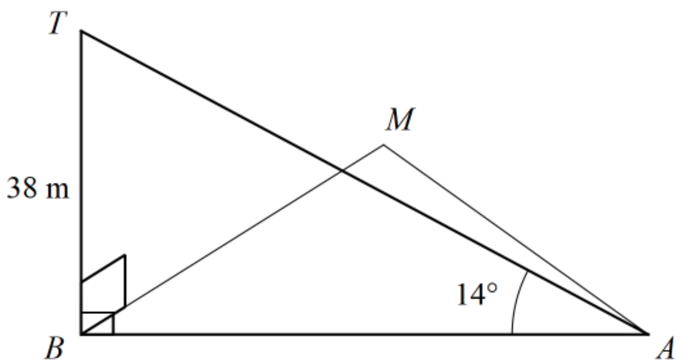
It was pleasing to see both equations $\cos\theta = \frac{1}{2}$, $\cos\theta = -2$ but it would be nice to have some mention of no solutions for $\cos\theta = -2$, and why no solutions

Question 30 **(6 marks)**

A vertical tree of height 38 metres stands on horizontal ground. The bottom of the tree is at the point B. Observer M, standing on the ground, is due North of the bottom of the tree.

Observer A, standing on the ground, sees the top of the tree, T, at an angle of elevation of $14°$. The true bearing of Observer M from Observer A is $290°$.

Observers A and M are standing the same distance away from the tree.



NOT TO SCALE

(a) Show that $AB = 38\cot 14°$ **1**

In ΔBAT

$$\tan 14° = \frac{BT}{AB}$$

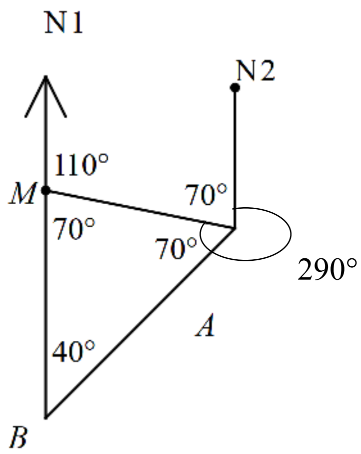
$$AB = \frac{38}{\tan 14°}$$

$$= 38\cot 14°$$

Very well done.
Some students tried to "work backwards" from what was given – they were unsuccessful in "showing".
Best approach to use tan, but some students used $\cot = \frac{adj}{opp}$ successfully.

(b) Complete the diagram of the top view of the scenario below, showing the point A and the bearing of M from A. **1**

Please make sure you read what you need to show.
Quite a number of students didn't mark in the bearing of M from A as asked.
The inclusion in the diagram of sizes of angles like $110°$ and $70°$ were useful for part (c) but finding the sizes of those angles should still be shown/explained as part of your solution in (c).
A number of students had diagrams with $\angle MBA$ as obtuse, or right-angled, when in the next part it is shown as $40°$.
Make sure you link the parts together and fix your diagram if you find in a later part it doesn't match.



(c) Show that $\angle MBA = 40°$. **2**

$$\angle N_2AM = 360 - 290° \text{ (angles at a point)}$$
$$= 70°$$
$$\angle N_1MA = 180° - 70°$$
$$(co-\text{interior angles on parallel lines})$$
$$= 110°$$
$$\angle BMA = 180 - 110° = 70° = \angle BAM \text{ (angles on straight line, base angles of isoceles triangle equ}$$
$$\angle MBA = 180° - 70° - 70° \text{ (angle sum of triangle)}$$
$$= 40°$$

This needs to be shown with clear logic – the main focus was the step by step to get the angles, not necessarily formal geometric reasoning (which was appreciated).
It was difficult to follow steps where the diagram was not labelled well above (either using points *A etc)* or introducing pronumerals (θ but not defining them)

(d) Find the distance between A and M. **2**

$$AM^2 = BM^2 + BA^2 - 2 \times BM \times BA \times \cos 40°$$
$$= \left(38\cot 14^\circ\right)^2 + \left(38\cot 14^\circ\right)^2 - 2\times\left(38\cot 14^\circ\right)\times\left(38\cot 14^\circ\right)\cos 40°$$
$$= 104.25...$$
$$= 104 \text{ m (nearest metre)}$$

Well done, both methods using sine rule or cosine rule.
Most difficulties occurred due to calculator errors, inputting in $\cot 14°$, or leaving off a square in the formula.

End of Assessment Task