HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 11 Online

Assessment 3 Term 3 2021

STUDENT NUMBER: ____

General Instructions:

- Working time – 60 minutes
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total Marks: 38

Section I – 8 marks (pages 2–5)

- Attempt Questions 1–8
- Allow about 15 minutes for this section

Section II – 30 marks (pages 6–12)

- Attempt Questions 9–19
- Allow about 45 minutes for this section

Section I

8 marks
Attempt Questions 1 – 8
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 8

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1 What is the solution to the equation $\cos 2x = -\frac{1}{2}$ in the domain $0 \le x \le 2\pi$?

(A) $x = \pi, 2\pi$

(B) $x = \frac{\pi}{3}, \frac{2\pi}{3}$

(C) $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(D) $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}$

2 Find $\lim\limits_{x \to -4} \frac{2x+8}{3x^2-48}$

(A) $\frac{1}{12}$

(B) $-\frac{1}{12}$

(C) 12

(D) Undefined

3 A particle is moving on a straight line with displacement $x = \frac{15}{t}$, where x is measured in metres and t is the time in seconds. What is the acceleration of the particle after 6 seconds?

(A) $-5m/s^2$

(B) $\frac{5}{72}m/s^2$

(C) $\frac{-5}{36}m/s^2$

(D) $\frac{5}{36}m/s^2$

4 If the volume of water, V litres, in a tank after t minutes, is given by $V = 600t - t^3$, then after five minutes:

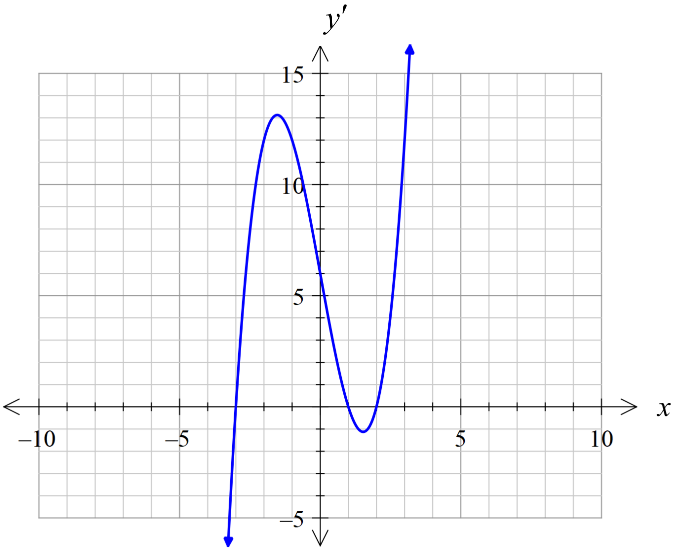
(A) The volume of water is decreasing at a rate of 525 litres per minute.

(B) The volume of water is increasing at a rate of 525 litres per minute.

(C) The volume of water is decreasing at a rate 2 875 litres per minute.

(D) The volume of water is increasing at a rate of 2 875 litres per minute.

5 The graph of the <u>gradient</u> function, $y' = f(x)$, is shown below.



For what values of x is the function, $y = f(x)$, stationary?

(A) $x = -1.5$ and $x = 1.5$

(B) $x = -3$, $x = 1$ and $x = 2$

(C) $x = -3$, $x = -1.5$, $x = 1$, $x = 1.5$ and $x = 2$

(D) $x = 1.5$ only

6 For what value(s) of x is the tangent to the function $y = \frac{1}{3}x^3 - 4x^2 - 9x + 10$ parallel to the line $y = 11x + 3$?

(A) $x = 9, x = 1$

(B) $x = -10, x = 2$

(C) $x = -9, x = -1$

(D) $x = 10, x = -2$

7 A function has equation:

$$f(x)=\begin{cases} x^2+6x-10, & \text{for } x<1 \\ kx+4, & \text{for } x\geq 1 \end{cases}$$

For what value of k is $f(x)$ continuous?

(A) $k=-7$

(B) $k=-1$

(C) $k=2$

(D) $k=-10$

8 What is the area, in cm^2, of the sector with radius 8 cm and arc length 20π cm?

(A) 80π

(B) 80

(C) 160π

(D) 20π

End of Section I

Section II

30 marks
Attempt Questions 9-19
Allow about 45 minutes for this section

Question 9 (3 marks)

Given $\cot\theta = \frac{\sqrt{21}}{5}$ and θ lies in the third quadrant, find the exact value of $\sin^2\theta$.

Question 10 (2 marks)

Simplify: $\tan(2\pi - x)\ \mathrm{cosec}(\pi - x)$

Question 11 (3 marks)

Solve for x: $0° \le x \le 360°$, $2\sin x\cos x = \sin x$

Question 12 (2 marks)

Prove: $\tan x + \cot x = \sec x\ \mathrm{cosec}\,x$

Question 13 (3 marks)

Differentiate the following:

(a) $y = 3x^3 + 4x^5$

(b) $y = 5x^{-2} + 3x^{-4}$. Express your answer without negative indices.

Question 14 (2 marks)

Find the derivative of $f\left(x\right) = \dfrac{6}{x}$ using the first principles method.

Question 15 (2 marks)

Differentiate $y = (2x+1)\sqrt{x}$

Question 16 (3 marks)

In an assessment task, Jack was asked to find the derivative of the function $y = \dfrac{4x-1}{3-2x^2}$.
He proceeded to answer the question. His first line of working out was as follows:

$$\frac{dy}{dx} = \frac{(3-2x^2)(4)-(4x-1)(-4)}{(3-2x^2)^2}$$

(a) Explain the mistake that Jack made in his first line of working out.

(b) Write the correct solution for Jack's first line of working out and then complete the solution for the derivative of the function.

Question 17 (5 marks)

The population of a town is given by $P = 45\ 000\ (3t+2)^2$, where t is the time in years.

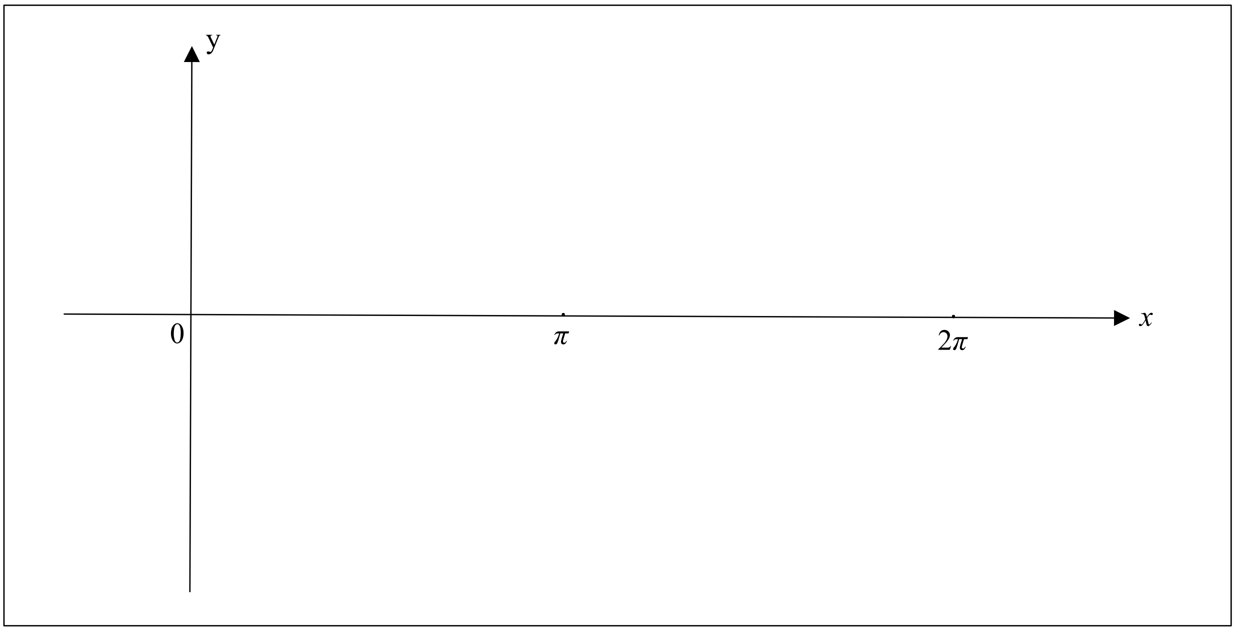
(a) What is the initial population of the town?

(b) What is the rate of change of the population after 10 years?

(c) Calculate the average rate of change of the population in the first five years.

Question 18 (2 marks)

(a) In the box below, neatly draw the graph for $y = \cos x$ in the domain $0 \le x \le 2\pi$.



(b) On the same graph above, draw $y = \sec x$ in the domain $0 \le x \le 2\pi$, clearly showing all asymptotes.

Question 19 (3 marks)

A particle moves in a straight line so that its displacement $x(t)$ from a fixed point on the line at time $t \geq 0$ is given by $x(t) = 6 + 8t - 4\sqrt{t^2 + 16}$. Show that the particle never comes to rest.

End of Assessment Task

Extra Working Space

Mathematics Advanced

Year 11 Online Assessment Task 3

Term 3 2021

STUDENT NUMBER: ____________________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1.	A ○	B ○	C ○	D ○	6.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○	7.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○	8.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○					
5.	A ○	B ○	C ○	D ○					

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Mathematics Advanced

Year 11 Online
Assessment 3 Term 3 2021

STUDENT NUMBER: Solutions

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Total Marks: 38

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Section I

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1 What is the solution to the equation $\cos 2x = -\frac{1}{2}$ in the domain $0 \le x \le 2\pi$?

(A) $x = \pi, 2\pi$

(B) $x = \frac{\pi}{3}, \frac{2\pi}{3}$

(C) $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(D) $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}$

$\cos 2x = -\frac{1}{2}$

acute $x = \frac{\pi}{3}$

$2x = (\pi - \frac{\pi}{3}), (\pi + \frac{\pi}{3})$

$2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

$+2\pi$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

2 Find $\lim\limits_{x \to -4} \frac{2x+8}{3x^2-48}$

(A) $\frac{1}{12}$

(B) $-\frac{1}{12}$

(C) 12

(D) Undefined

$\Rightarrow \lim\limits_{x \to -4} \frac{2(x+4)}{3(x^2-16)}$

$= \lim\limits_{x \to -4} \frac{2\cancel{(x+4)}}{3\cancel{(x+4)}(x-4)}$

$= \frac{2}{3(-8)}$

3 A particle is moving on a straight line with displacement $x = \frac{15}{t}$, where x is measured in metres and t is the time in seconds. What is the acceleration of the particle after 6 seconds?

(A) $-5m/s^2$

(B) $\frac{5}{72}m/s^2$

(C) $\frac{-5}{36}m/s^2$

(D) $\frac{5}{36}m/s^2$

$x = 15t^{-1}$

$\dot{x} = -15t^{-2}$

$\ddot{x} = 30t^{-3}$

$\ddot{x} = \frac{30}{t^3}$

sub. in $t=6$, $\ddot{x} = \frac{30}{6^3}$

$\ddot{x} = \frac{5}{36}$

4 If the volume of water, V litres, in a tank after t minutes, is given by $V = 600t - t^3$, then after five minutes:

(A) The volume of water is decreasing at a rate of 525 litres per minute.

(B) The volume of water is increasing at a rate of 525 litres per minute.

(C) The volume of water is decreasing at a rate 2 875 litres per minute.

(D) The volume of water is increasing at a rate of 2 875 litres per minute.

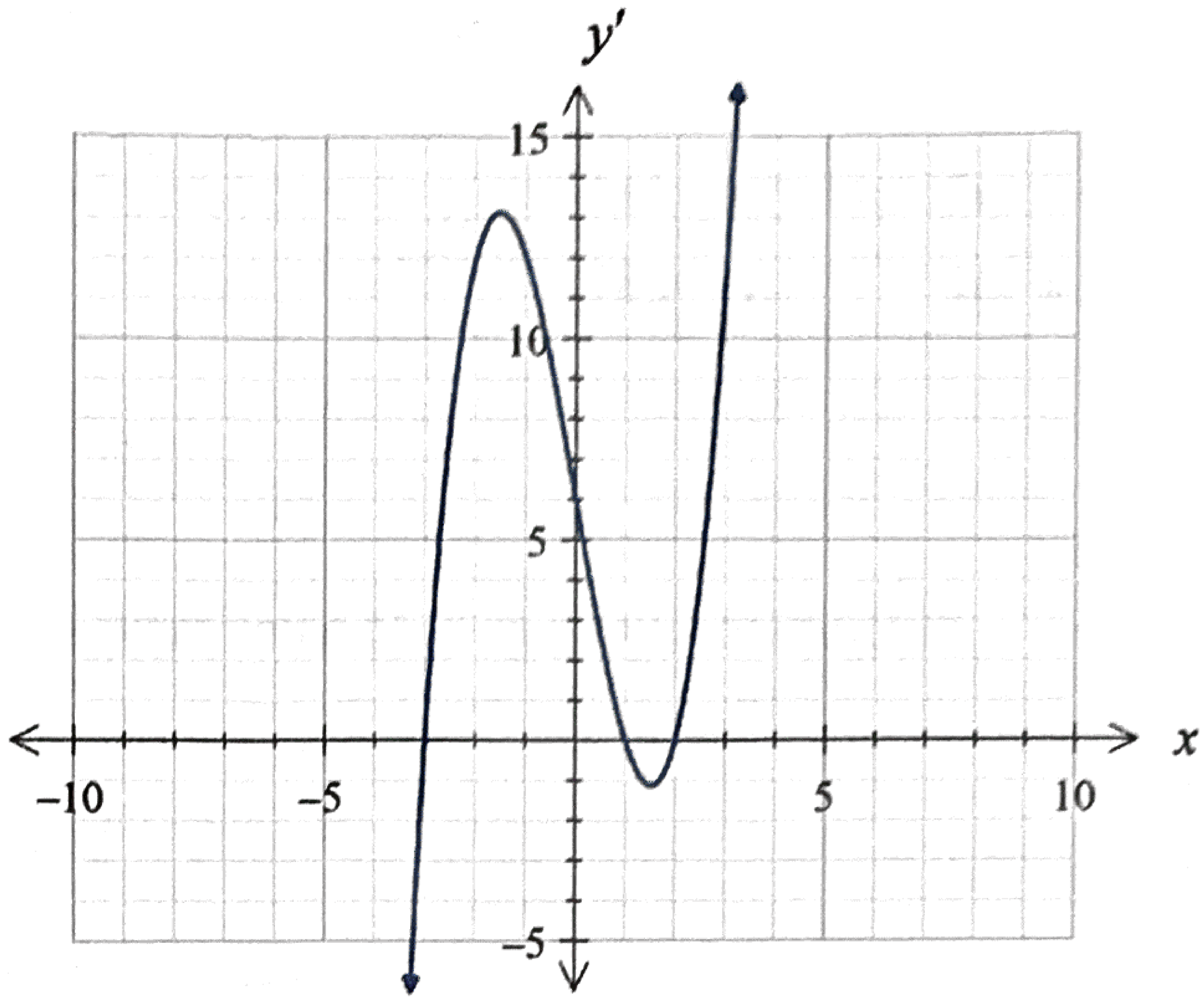
$V' = 600 - 3t^2$

sub in $t=5$

$V' = 600 - 3(25)$

$V' = 525$

5 The graph of the <u>gradient</u> function, $y' = f(x)$, is shown below.



For what values of x is the function, $y = f(x)$, stationary?

(A) $x = -1.5$ and $x = 1.5$

(B) $x = -3, x = 1$ and $x = 2$

(C) $x = -3, x = -1.5, x = 1, x = 1.5$ and $x = 2$

(D) $x = 1.5$ only

6 For what value(s) of x is the tangent to the function $y = \frac{1}{3}x^3 - 4x^2 - 9x + 10$ parallel to the line $y = 11x + 3$?

(A) $x = 9, x = 1$

(B) $x = -10, x = 2$

(C) $x = -9, x = -1$

(D) $x = 10, x = -2$

$y' = x^2 - 8x - 9$

$m = 11$

$\therefore x^2 - 8x - 9 = 11$

$x^2 - 8x - 20 = 0$

$(x - 10)(x + 2) = 0$

$\therefore x = 10, x = -2$

7 A function has equation:

$$f(x)=\begin{cases} x^2+6x-10, & \text{for } x<1 \\ kx+4, & \text{for } x\geq 1 \end{cases}$$

For what value of k is $f(x)$ continuous?

(A) $k=-7$

(B) $k=-1$

(C) $k=2$

(D) $k=-10$

$f(1) = 1+6-10$
$= -3$

$f(1) = k+4$

$\therefore k+4 = -3$

$\therefore k = -7$

8 What is the area, in cm^2, of the sector with radius 8 cm and arc length 20π cm?

(A) 80π

(B) 80

(C) 160π

(D) 20π

$l = r\theta$

$20\pi = 8\theta$

$\therefore \theta = \frac{20\pi}{8}$

$\theta = \frac{5\pi}{2}$

$\therefore A = \frac{1}{2}(8)^2\left(\frac{5\pi}{2}\right)$

$A = 32\left(\frac{5\pi}{2}\right)$

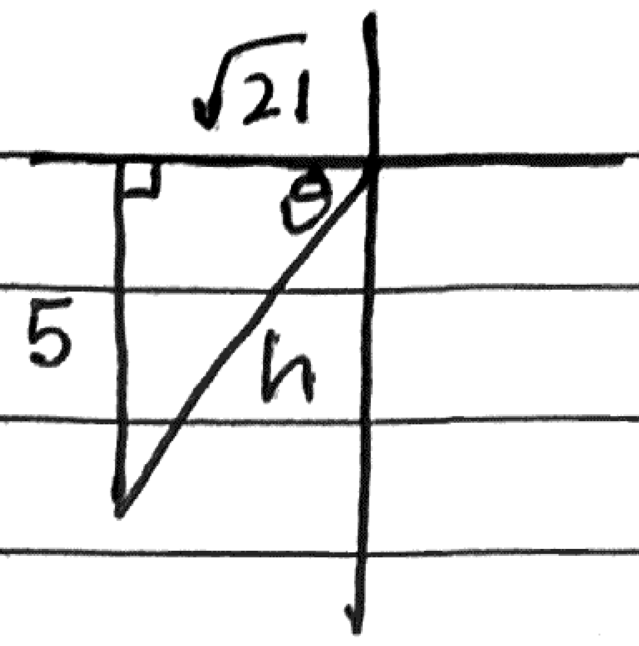
$A = 80\pi$

End of Section I

Section II
30 marks
Attempt Questions 9-19
Allow about 45 minutes for this section

Question 9 (3 marks)

Given $\cot\theta = \frac{\sqrt{21}}{5}$ and θ lies in the third quadrant, find the exact value of $\sin^2\theta$.



$h^2 = (\sqrt{21})^2 + (5)^2$

Marker's feedback:
Generally well done.
~~Some forgot to square.~~

$h^2 = 21 + 25$

$h^2 = 46$

$h = \sqrt{46}$, $h > 0$

$\therefore \sin^2\theta = \left(\frac{5}{\sqrt{46}}\right)^2$

$\sin^2\theta = \frac{25}{46}$

Question 10 (2 marks)

Simplify: $\tan(2\pi - x)\,\mathrm{cosec}(\pi - x)$

Marker's feedback:
Generally well done.
Some didn't simplify further.

$= -\tan x \cdot \mathrm{cosec}\, x$

$= -\frac{\tan x}{\sin x}$

$= -\frac{\sin x}{\cos x} \times \frac{1}{\sin x}$

$= -\frac{1}{\cos x}$

$= -\sec x$

Question 11 (3 marks)

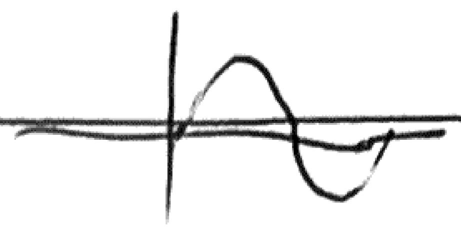
Marker's feedback:

Solve for x: $0° \leq x \leq 360°$, $2\sin x \cos x = \sin x$

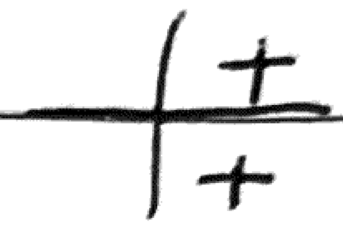
$$2\sin x\cos x - \sin x = 0$$

$$\sin x(2\cos x - 1) = 0$$

$$\sin x = 0 \quad , \quad 2\cos x = 1$$



$$\cos x = \frac{1}{2}$$



$$x = 0, 180°, 360°$$

$$x = 60°, 300°$$

$$\therefore x = 0°, 60°, 180°, 300°, 360°$$

Question 12 (2 marks)

Prove: $\tan x + \cot x = \sec x \operatorname{cosec} x$

Marker's feedback:

$$LHS = \tan x + \cot x$$

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$

$$= \frac{1}{\cos x \sin x}$$

$$= \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$

$$= \sec x \operatorname{cosec} x$$

$$= RHS$$

$$\therefore LHS = RHS$$

Question 13 (3 marks)

Differentiate the following:

(a) $y = 3x^3 + 4x^5$

Marker's feedback:
Well done question

$y' = 9x^2 + 20x^4$

(b) $y = 5x^{-2} + 3x^{-4}$. Express your answer without negative indices.

$y' = -10x^{-3} - 12x^{-5}$

$y' = \frac{-10}{x^3} - \frac{12}{x^5}$

Marker's feedback:
~~Mostly well done question.~~
A lot wasted time by combining the 2 fractions together which wasn't necessary in this case.

Question 14 (2 marks)

Find the derivative of $f(x) = \frac{6}{x}$ using the first principles method.

Marker's feedback:
First step was generally well done. Some had problems with the simplification and tried to expand (x+h)^-1 which can't be done.
Other's left of the word Lim at the start of each line until the substitution of h=0 could be made.

$f(x) = \frac{6}{x}$ $\quad f(x+h) = \frac{6}{x+h}$

$f'(x) = \lim_{h \to 0} \frac{\frac{6}{x+h} - \frac{6}{x}}{h}$

$= \lim_{h \to 0} \frac{\frac{6x - 6(x+h)}{x(x+h)}}{h}$

$= \lim_{h \to 0} \frac{6x - 6x - 6h}{x(x+h)} \times \frac{1}{h}$

$= \lim_{h \to 0} \frac{-6\cancel{h}}{x(x+h)} \times \frac{1}{\cancel{h}}$

$= \frac{-6}{x(x+0)}$

$= \frac{-6}{x^2}$

Marker's feedback:

Question 15 **(2 marks)**

Differentiate $y=(2x+1)\sqrt{x}$

$u = 2x+1$

$u' = 2$

$v = \sqrt{x}$

$v = x^{\frac{1}{2}}$

$v' = \frac{1}{2}x^{-\frac{1}{2}}$

$v' = \frac{1}{2\sqrt{x}}$

$y' = 2\sqrt{x} + (2x+1)\frac{1}{2\sqrt{x}}$

$y' = 2\sqrt{x} + \frac{2x+1}{2\sqrt{x}}$

$y' = \frac{2x+2x+1}{2\sqrt{x}} = \frac{4x+1}{2\sqrt{x}}$

Question 16 **(3 marks)**

In an assessment task, Jack was asked to find the derivative of the function $y=\frac{4x-1}{3-2x^2}$.
He proceeded to answer the question. His first line of working out was as follows:

$$\frac{dy}{dx}=\frac{(3-2x^2)(4)-(4x-1)(-4)}{(3-2x^2)^2}$$

Marker's feedback:

(a) Explain the mistake that Jack made in his first line of working out.

The derivative of $3-2x^2$ is $-4x$, not -4.

(b) Write the correct solution for Jack's first line of working out and then complete the solution for the derivative of the function.

$$\frac{dy}{dx} = \frac{(3-2x^2)(4)-(4x-1)(-4x)}{(3-2x^2)^2}$$

$$= \frac{12-8x^2+16x^2-4x}{(3-2x^2)^2}$$

$$= \frac{8x^2-4x+12}{(3-2x^2)^2}$$

Marker's feedback:

Question 17 (5 marks)

The population of a town is given by $P = 45\ 000\ (3t+2)^2$, where t is the time in years.

(a) What is the initial population of the town?

Marker's feedback: Most students did well in this part

$P = 45000(3(0)+2)^2$

$P = 180\ 000$

(b) What is the rate of change of the population after 10 years?

$P' = 90\ 000(3t+2) \times 3$

$P' = 270\ 000(3t+2)$

Sub in $t = 10$

$P' = 270\ 000(3 \times 10 + 2)$

$P' = 8\ 640\ 000$

$\therefore$ rate of chge after 10 yrs is 8 640 000 people/year.

Marker's feedback: 1 mark for correct differention and 1 mark for correct answer. The "people/year" part was ignored.

(c) Calculate the average rate of change of the population in the first five years.

$$\text{average rate of chge} = \frac{P_5 - P_0}{5 - 0}$$

$$= \frac{13\ 005\ 000 - 180\ 000}{5}$$

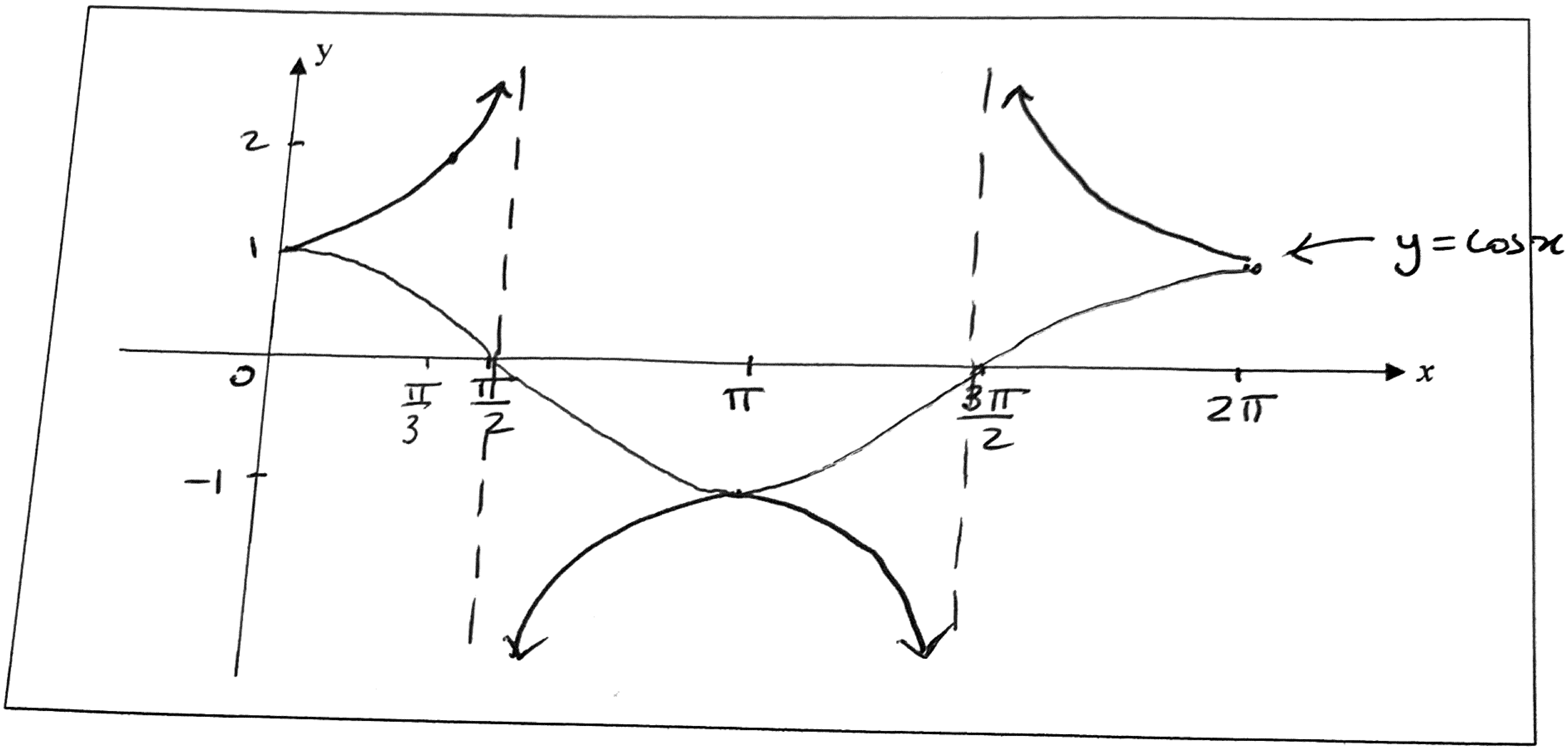
$$= \frac{12\ 825\ 000}{5}$$

$$= 2\ 565\ 000$$

Marker's feedback: Poorly done. One mark was given for something correct that made sense. Some students differentiated P and then used the derivative in the gradient formula.

Question 18 (2 marks)

(a) In the box below, neatly draw the graph for $y = \cos x$ in the domain $0 \le x \le 2\pi$.



(b) On the same graph above, draw $y = \sec x$ in the domain $0 \le x \le 2\pi$, clearly showing all asymptotes.

Marker's feedback:

Done well by most students

Students should label their graphs and ensure that the graph of y=secx approaches the asymptotes

A few students had little concept of y=secx and did not show points of intersection at (0,1), (pi,-1), (2pi,1)

Question 19 **(3 marks)**

A particle moves in a straight line so that its displacement $x(t)$ from a fixed point on the line at time $t \geq 0$ is given by $x(t) = 6 + 8t - 4\sqrt{t^2 + 16}$. Show that the particle never comes to rest.

$$x = 6 + 8t - 4(t^2+16)^{\frac{1}{2}}$$

$$\dot{x} = 8 - 2(t^2+16)^{-\frac{1}{2}} \times 2t$$

$$\dot{x} = 8 - 4t(t^2+16)^{-\frac{1}{2}}$$

$$\dot{x} = 8 - \frac{4t}{\sqrt{t^2+16}} = 0 \text{ for coming to rest}$$

$$8 = \frac{4t}{\sqrt{t^2+16}}$$

Marker's feedback:

sqrt(t^2+16) does NOT equal t+4

1 for differentiating. Many studdents forgot to multiply by the derivative of inside the bracket.

$$2 = \frac{t}{\sqrt{t^2+16}}$$

1 for demonstrating understanding that the particle comes to rest when x'(t)=0.

1 for SHOWING that there is no real solution for x'(t)=0, therefore, the particle never comes to rest.

$$4 = \frac{t^2}{t^2+16}$$

$$4t^2 + 64 = t^2$$

$$64 = -3t^2$$

$$t^2 = \frac{64}{-3}$$

$$t = \mp\sqrt{\frac{64}{-3}}$$

no soln

$\therefore$ particle never comes to rest.

Students who tried to take the approach of showing that the velocity was always positive, therefore, never eqaulling zero and being at rest needed to SHOW substantial working with t<sqrt(t^2+16) so 4<x'(t)<=8.

End of Assessment Task