STUDENT NAME: _____

TEACHER: _____

HURLSTONE
AGRICULTURAL
HIGH
SCHOOL

2020

Year 11

Task 3

Mathematics Advanced

Examiners

- Mr. D. Potaczala, Mr. R. Rawson, Mr. S. Faulds, Ms. T. Tarannum

General Instructions

- Reading time – 3 minutes
- Working time – 1 hour and 30 minutes
- Write using black pen
- NESA-approved calculators may be used
- A Reference sheet is provided for your use. This is included in the question booklet on pages 29 – 32
- In Questions 9 – 12, show relevant mathematical reasoning and/or calculations

Total marks: 72

Section I – 8 marks (pages 2 – 5)

- Attempt Questions 1 – 8
- Allow about 10 minutes for this section

Section II – 64 marks (pages 7 – 28)

- Attempt Questions 9 – 12
- Allow about 1 hour and 20 minutes for this section

SECTION I

8 marks

Attempt Questions 1 – 8

Allow about 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 8

Question 1

What is the domain of $y = \dfrac{g(x)}{f(x)}$, given $g(x) = \sqrt{x+5}$ and $f(x) = x - 2$?

A. $(-5, 2) \cup (2, \infty)$

B. $[-5, 2] \cup [2, \infty)$

C. $[-5, 2) \cup (2, \infty)$

D. $(-5, \infty)$

Question 2

Which equation represents the line perpendicular to $2x - 3y = 8$, passing through the point (2, 0)?

A. $3x + 2y = 4$

B. $3x + 2y = 6$

C. $3x - 2y = -4$

D. $3x - 2y = 6$

Question 3

In radians, what is the exact value of 210º?

A. 1.047

B. $\frac{\pi}{3}$

C. 3.665

D. $\frac{7\pi}{6}$

Question 4

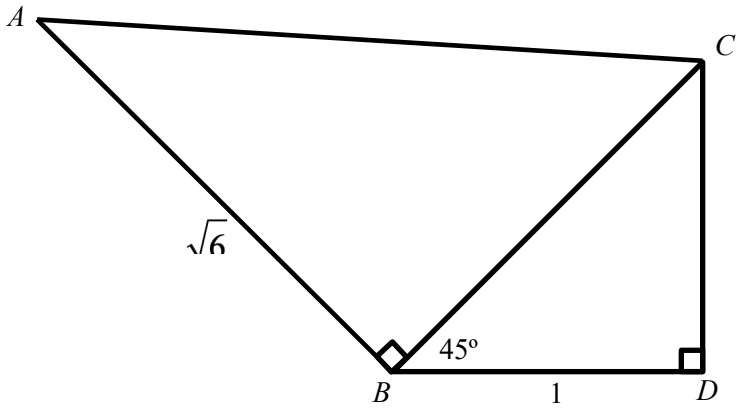

Diagram is not to scale.

Dimensions are in units.

What is the size of $\angle ACB$ in the above diagram?

A. 30º

B. 37.5 º

C. 60 º

D. 75 º

Question 5

When some concentrated chemical solutions are allowed to evaporate slowly, crystals are formed. The surface area of a particular crystal is given by $A = 0.7\sqrt{t^3}$, where A is in mm^2 and t is in days. What is the rate at which the surface is increasing after 4 days?

A. 2.01 mm^2 / day

B. 2.1 mm^2 / day

C. 1.05 mm^2 / day

D. 1.5 mm^2 / day

Question 6

For the function $f(x) = x + \frac{1}{x}$, which of the following is true at $x = 3$?

A. The graph is increasing

B. The graph is decreasing

C. The graph is stationary

D. The graph is discontinuous.

Question 7

Lou and Ali are on a fitness program for one month. The probability that Lou and Ali will finish the program successfully is 0.7 and 0.6 respectively. What is the probability that only one of them will be successful?

A. 0.18

B. 0.28

C. 0.42

D. 0.46

Question 8

A deck of 52 playing cards contains 12 picture cards. A card is chosen from the deck at random. What is the probability, correct to four decimal places, that a picture card or black card is chosen?

A. 0.5340

B. 0.3787

C. 0.7308

D. 0.6154

End of Section I

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Q9

Student Name: _____

Teacher: _____

SECTION II

64 marks

Attempt Questions 9 – 12

Allow about 1 hour and 20 minutes for this section

In Questions 9 – 12, your responses should include relevant mathematical reasoning and/or calculations.

- **Write your name at the top of each booklet.** Answer questions in the space provided. These spaces provide guidance for the expected length of response.
- Extra writing space is provided at the back of each question booklet. If you use this space, clearly indicate which part of the question you are answering.

Question 9 (16 marks)

(a) Sketch the curve $y = (2 - x)^3$, clearly indicating any x or y intercepts. **2 marks**

(b) Let $h(x) = 3x + 7$ and $g(x) = x^2 - 2$ **2 marks**

Show that $g(h(x)) = 9x^2 + 42x + 47$.

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(c) Determine whether the function $f(x) = \dfrac{3^x + 3^{-x}}{x^3}$ is odd, even or neither. **2 marks**

Justify your answer.

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(d) (i) Sketch the curve $x^2 - 6x + y^2 + 8y = 0$ **3 marks**

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(ii) With reference to your sketch, state why the above relation is/is not a function. **1 mark**

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(e) Let $f(x) = |2x - 5| - 2$.

(i) Sketch the graph of $y = f(x)$. **2 marks**

(ii) Hence, sketch the graph of $y = f(-x)$. **1 mark**

(f) Find the point(s) of intersection of the curves $y = 5x + 4$ and $y = 2x^2 + 4x - 2$. **3 marks**

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End of Question 9

Extra writing space

If you use this space, indicate which part of the question you are answering.

Student Name: _____

Teacher: _____

- **Write your name at the top of each booklet.** Answer questions in the space provided. These spaces provide guidance for the expected length of response.
- Extra writing space is provided at the back of each question booklet. If you use this space, clearly indicate which part of the question you are answering.

Question 10 (16 marks)

(a) ΔPQR has side lengths PR = 21.4 cm and QR = 18.6 cm and $\sin\angle QPR = 0.5592$. Find the two possible sizes of $\angle PQR$ to the nearest degree. **2 marks**

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(b) If $\sin\theta = -\frac{\sqrt{7}}{5}$ and $\cos\theta$ is negative, find exact values for $\tan\theta$ and $\sec\theta$. **2 marks**

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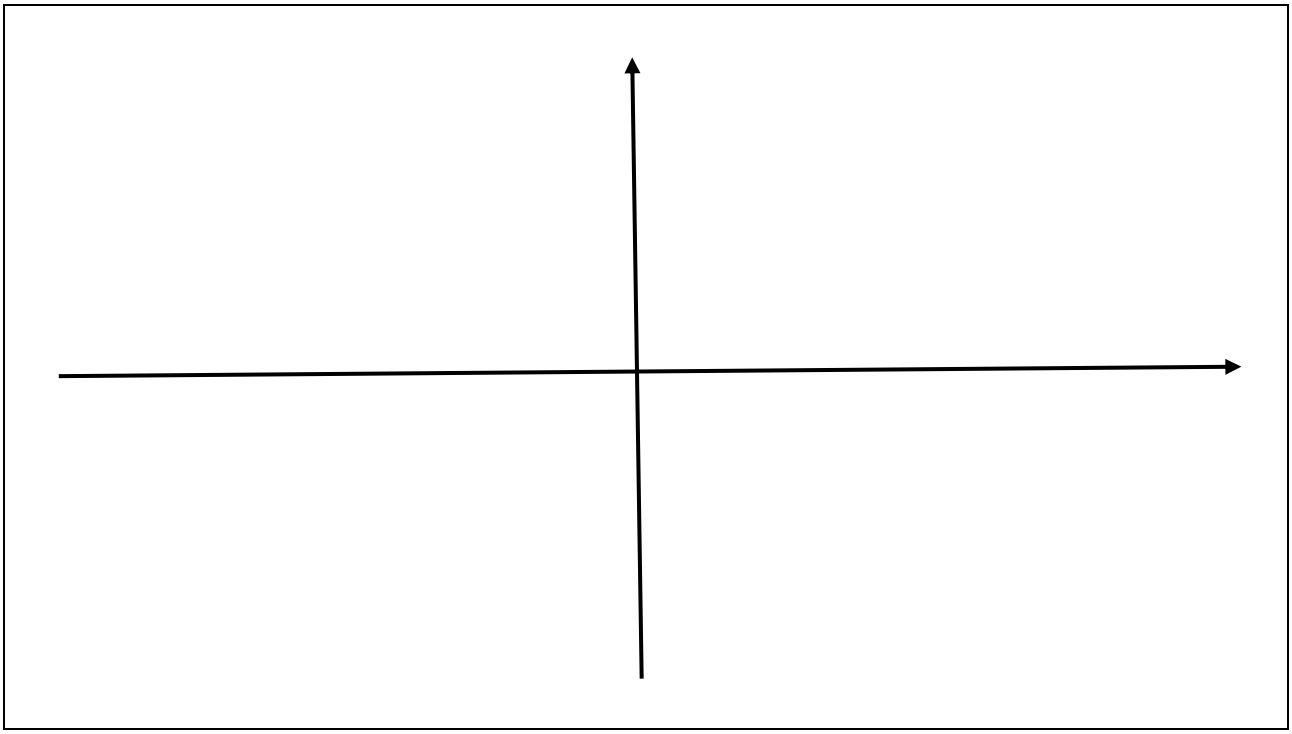
(c) On the set of axes below, draw neat sketches of the graphs of **4 marks**
$y = \cos x,\ 0 \le x \le 2\pi$ and $y = \sec x,\ 0 \le x \le 2\pi$.

Clearly indicate on your graphs the following features:

- any x intercepts.
- the position of any asymptotes.
- any points the graphs have in common.



(d) Show that $\text{cosec}^2\alpha - \dfrac{1}{\tan^2\alpha} = 1.$ **2 marks**

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(e) Two surveillance drones leave their base (B) with the first (D_1) flying for 15 km on a bearing of 291° before landing and the second (D_2) flying for 36 km on a bearing of 035º before landing.

(i) In the space below, draw a neat sketch of the information given. **1 mark**

(ii) Calculate the distance between the two landing sites. **2 marks**

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(f) Solve $2\sin^2\theta - \sin\theta - 1 = 0, 0° \le \theta \le 360°$. **3 marks**

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End of Question 10

Extra writing space

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Student Name: _____

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- **Write your name at the top of each booklet.** Answer questions in the space provided. These spaces provide guidance for the expected length of response.
- Extra writing space is provided at the back of each question booklet. If you use this space, clearly indicate which part of the question you are answering.

Question 11 (16 marks)

(a) Differentiate $f(x) = 2x^{-3}$. **1 mark**

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(b) Differentiate and fully factorise $y = 3x(4-x)^3$. **3 marks**

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(c) Use first principles differentiation to find the derivative, $f'(x)$, **2 marks**

of $f(x) = 5x^2 + 3x + 4$.

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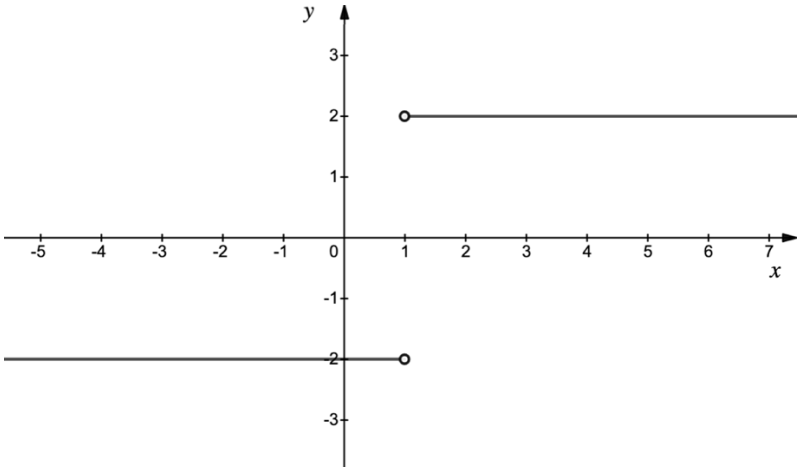
(d) Sketch a possible function which could have the gradient function as graphed below. **2 marks**



(e) Find the equation of the tangent to the curve $y = x^3 + 2x - 3$ at the point where $x = -2$ **2 marks**

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(f) A particle is moving along the x-axis and is initially at the origin. Its velocity v metres per second at time t seconds is given by $v = \dfrac{2t}{9+t^2}$ for $t \geq 0$.

(i) What is the initial velocity of the particle? **1 mark**

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(ii) Find an expression in its simplest form for the acceleration of the particle. **2 marks**

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(iii) When is the acceleration zero? **2 marks**

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(iv) What is the maximum velocity attained by the particle? **1 mark**

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End of Question 11

Extra writing space

If you use this space, indicate which part of the question you are answering.

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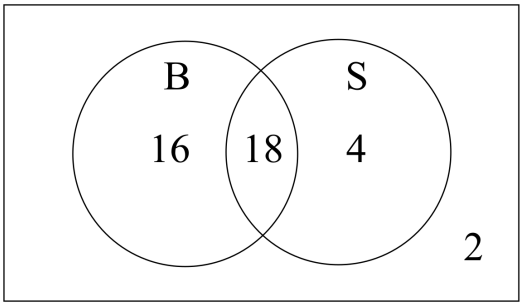
Student Name: _____

Teacher: _____

- **Write your name at the top of each booklet.** Answer questions in the space provided. These spaces provide guidance for the expected length of response.
- Extra writing space is provided at the back of each question booklet. If you use this space, clearly indicate which part of the question you are answering.

Question 12 (16 marks)

(a) The Venn diagram shows the results of a survey about whether students do Biology (B), Sport Science (S) or neither.



(i) Find the probability that a student selected at random does not do Sport Science. **1 mark**

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(ii) Find $P(S \cap B) + P(S' \cap B)$. **2 marks**

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(iii) Is doing Biology mutually exclusive to doing Sport Science? **1 mark**
Give a reason for your answer.

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(b) A bag contains two red balls, one black ball and one white ball. Andrew selects one ball from the bag and keeps it hidden. He then selects a second ball, also keeping it hidden.

(i) Draw a probability tree diagram to show all the possible outcomes. **2 marks**

(ii) Find the probability that both the selected balls are red. **1 mark**

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(iii) Find the probability that at least one of the selected balls is red **1 mark**

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(iv) Andrew drops one of the selected balls and we can see that it is red. **1 mark**
What is the probability that the ball that is still hidden is also red?

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(c) Suppose $P(A) = \frac{2}{5}$ and $P(A \mid B) = \frac{1}{3}$. Are A and B independent events? **1 mark**

Give a reason for your answer.

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(d) Pat and Chris each throw a fair die.

(i) Find the probability that they both throw the same number. **1 mark**

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(ii) Find the probability that the number thrown by Chris is greater than the number thrown by Pat. **2 marks**

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(e) An eight sided die is biased, so that the probability of a score x is given by $P(x) = k^2 x$, where $x = 1, 2, 3, 4, 5, 6, 7, 8$. Find *k*, giving a reason for your answer. **3 marks**

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End of Paper

NSW Education Standards Authority

2020 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$\left(x - h\right)^2 + \left(y - k\right)^2 = r^2$$

Financial Mathematics

$$A = P\left(1 + r\right)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r} = \frac{a\left(r^n - 1\right)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

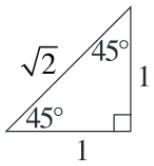
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

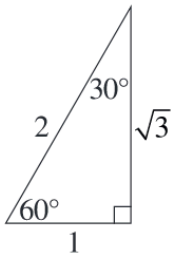
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

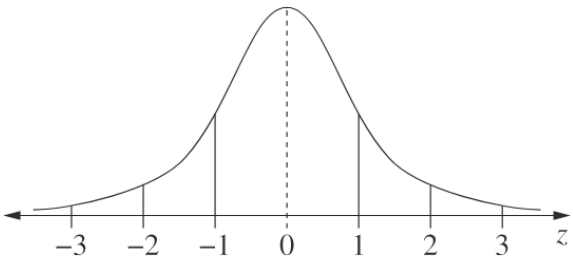
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

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Mathematics Advanced

MULTIPLE CHOICE ANSWER SHEET

2020 Year 11 Task 3

Student Name _____

Teacher _____

Select the letter A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 3 + 5 = (A) 7 (B) 8 (C) 9 (D) 10

A ⬭ **B** ⬬ **C** ⬭ **D** ⬭

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ⬭ **B** ⬬ **C** ⬭ **D** ⬭

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word '*correct'* and drawing an arrow as follows:

A ⬬ **B** ⬬ **C** ⬭ **D** ⬭

Correct

Question

1. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

2. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

3. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

4. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

5. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

6. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

7. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

8. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

Question	Marks
MC	/8
9	/16
10	/16
11	/16
12	/16
Total	/72

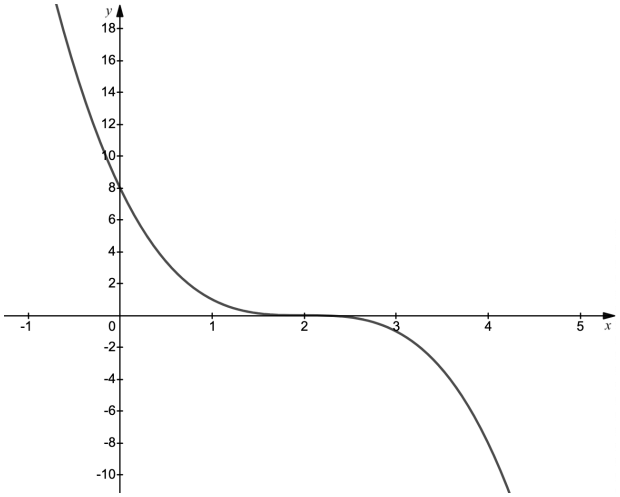
Year 11 Mathematics Advanced Task 3 Yearly Examination 2020

Multiple Choice

1) C **2)** B **3)** D **4)** C **5)** B **6)** A **7)** D **8)** D

Year 11 Question No.9	Mathematics Advanced Task 3 Solutions and Marking Guidelines	Examination 2020
MA 11-2	uses the concepts of functions and relations to model, analyse and solve practical problems	
Part	**Solutions**	**Marking Guidelines**

(a)



2 marks
Correct solution (including intercepts and an attempt to show the horizontal gradient at x-intercept)

1 mark
Substantially correct

(b)

$$g(x) = x^2 - 2$$

$$\begin{aligned} g(h(x)) &= (g(x))^2 - 2 \\ &= (3x+7)^2 - 2 \\ &= 9x^2 + 42x + 49 - 2 \\ &= 9x^2 + 42x + 47 \end{aligned}$$

2 marks
Correct solution

1 mark
Substantially correct

(c)

$$f(x) = \frac{3^x + 3^{-x}}{x^3}$$

$$f(-x) = \frac{3^{-x} + 3^{-(-x)}}{(-x)^3}$$

$$= \frac{3^{-x} + 3^x}{-x^3} = -\frac{3^x + 3^{-x}}{x^3}$$

$$f(-x) = -f(x)$$

$\therefore$ odd function

2 marks
Correct solution

1 mark
Substantially correct

Question 9 continued...

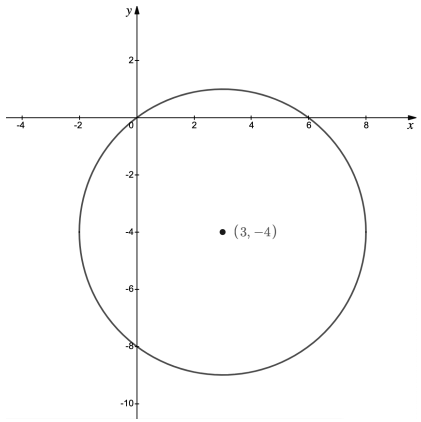
(d)(i)

$$x^2 - 6x + y^2 + 8y = 0$$

$$x^2 - 6x + 9 + y^2 + 8y + 16 = 9 + 16$$

$$(x-3)^2 + (y+4)^2 = 25$$

Circle, centre (3, – 4) and radius 5



3 marks
Correct solution (must show centre, and pass through (0,0))

2 marks
Substantially correct

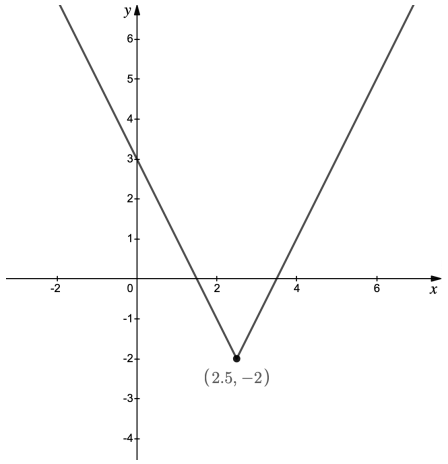
1 mark
Partial progress towards correct solution

(d)(ii)

This is ***not*** a function as it fails the vertical line test (or is a many-to-many function)

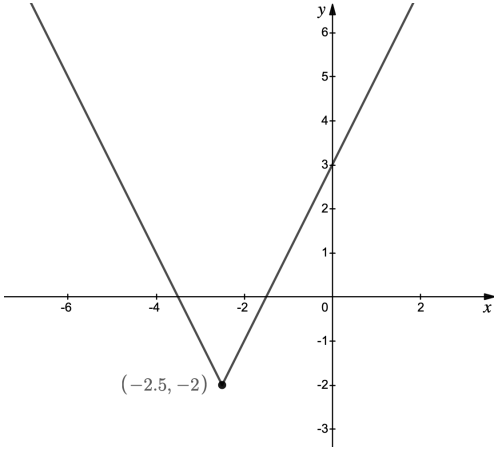
1 mark
Correct solution (which includes reason ***and*** conclusion

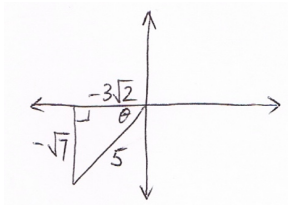
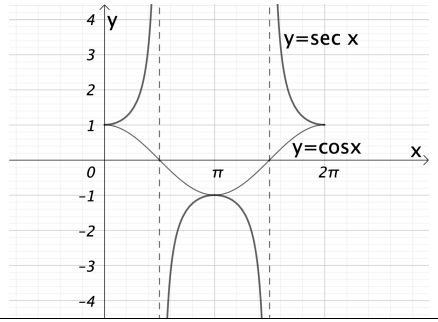
(e)(i)



2 marks
Correct solution (MUST include y-intercept)

1 mark
Partial progress towards correct solution

(e)(ii)	*Question 9 continued...*  	**1 mark** Correct solution
(f)	$2x^2+4x-2=5x+4$ $2x^2-x-6=0$ $(2x+3)(x-2)=0$ $x=-\frac{3}{2}, 2$ $y=-\frac{7}{2}, 14$ $\therefore$ the points of intersection are $\left(-\frac{3}{2},-\frac{7}{2}\right)$ and $(2,14)$	**3 marks** Correct solution **2 marks** Substantially correct **1 mark** Partial progress towards correct solution

Year 11 Mathematics Yearly Examination (Assessment Task 3) 2020		
Question No. 10	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		
MA11-3	uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes	
MA11-4	uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities	
Outcome	**Solutions**	**Marking Guidelines**
MA11-3	(a) $\frac{\sin Q}{PR}=\frac{\sin P}{QR}$ $\frac{\sin Q}{21.4}=\frac{0.5592}{18.6}$ $\sin Q=\frac{21.4\times 0.5592}{18.6}$ $=0.6434$ $\therefore Q=40°$ or $140°$	**2 marks** Correct solution showing full working, including both correct angles. **1mark** Substantial progress towards correct solution.
MA11-3	(b)  Since $\sin\theta$ and $\cos\theta$ are both negative, θ must be an angle in the third quadrant as shown. Using Pythagoras' Theorem, the adjacent side of the triangle is $-\sqrt{18}=-3\sqrt{2}$. Hence, $\tan\theta=\frac{\sqrt{7}}{3\sqrt{2}}$ and $\sec\theta=-\frac{5}{3\sqrt{2}}$.	**2 marks** Correct solution giving both ratios correctly in exact form. Denominator need not be rationalised. **1 mark** Substantial progress towards correct solution, including one ratio correctly stated in exact form.
MA11-4	(c)  	**4 marks** Correct solution, showing both graphs correctly with the correct shape, including where appropriate max./min. values, x and y intercepts, asymptotes **3 marks** One of the above features missing in either graph. **2 marks** One graph displayed correctly, OR Elements of the required features missing in both graphs. **1 mark** Some meaningful progress towards a correct sketch of one of the graphs.

MA11-4 (d)

$$\operatorname{cosec}^2\alpha - \frac{1}{\tan^2\alpha} = 1$$

$$LHS = \operatorname{cosec}^2\alpha - \frac{1}{\tan^2\alpha}$$

$$= \frac{1}{\sin^2\alpha} - \frac{1}{\sin^2\alpha \Big/ \cos^2\alpha}$$

$$= \frac{1}{\sin^2\alpha} - \frac{\cos^2\alpha}{\sin^2\alpha}$$

$$= \frac{1-\cos^2\alpha}{\sin^2\alpha}$$

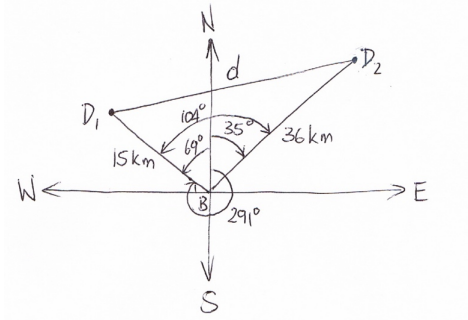
$$= \frac{\sin^2\alpha}{\sin^2\alpha}$$

$$= 1$$

$$= RHS$$

2 marks
Correct solution demonstrating all correct steps.
1 mark
Substantial progress towards full and correct solution.

MA11-3 (e)(i)



1 mark
Correct diagram showing the given information.

MA11-3

(ii) Using the Cosine Rule:

$$d^2 = 15^2 + 36^2 - 2\times 15\times 36\times \cos 104°$$

$$\approx 1782.275647$$

$$d = 42.2\,\text{km (1 dec. pl.)}$$

2 marks
Correct solution.
1 mark
Substantial progress towards full and correct solution.

MA11-4 (f)

$$2\sin^2\theta - \sin\theta - 1 = 0$$

$$2\sin^2\theta - 2\sin\theta + \sin\theta - 1 = 0$$

$$2\sin\theta(\sin\theta - 1) + (\sin\theta - 1) = 0$$

$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$

$$\therefore \sin\theta = -\frac{1}{2} \quad \text{OR} \quad \sin\theta = 1$$

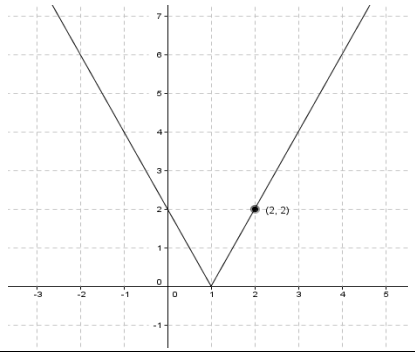
$$\theta = 210°, 330° \qquad \theta = 90°$$

3 marks
Full correct solution.
2 marks
Correct solution with one of the required answers missing or incorrect.
1 mark
Some progress towards correct solution, including correct factorisation.

Year 11 Mathematics Advanced Yearly Examination 2020
Question No. 11 Solutions and Marking Guidelines

Outcome Addressed in this Question

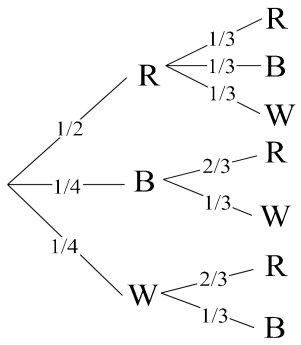
MA11-5 interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems

Part	Solutions	Marking Guidelines
(a)	$f'(x) = 2(-3)x^{-3-1} = -6x^{-4}$	**1 mark** Correct answer
(b)	$\frac{d}{dx}\left[3x(4-x)^3\right]$ $= 3(4-x)^3 + 3x\times 3(4-x)^2(-1)$ $= 3(4-x)^2\left((4-x) - 3x\right)$ $= 3(4-x)^2(4-4x)$ $= 12(4-x)^2(1-x)$ $= -12(4-x)^2(x-1)$	**3 marks** Correct answer **2 marks** Substantially correct **1 mark** Partial progress towards correct solution
(c)	$f'(x) = \lim_{h\to 0}\frac{5(x+h)^2 + 3(x+h) + 4 - (5x^2+3x+4)}{h}$ $= \lim_{h\to 0}\frac{5x^2 + 10xh + 5h^2 + 3x + 3h + 4 - 5x^2 - 3x - 4}{h}$ $= \lim_{h\to 0}\frac{10xh + 5h^2 + 3h}{h}$ $= \lim_{h\to 0}\frac{h(10x + 5h + 3)}{h}$ $= \lim_{h\to 0}(10x + 5h + 3)$ $= 10x + 3$	**2 marks** Correct answer **1 mark** substantially correct
(d)		**2 marks** Correct answer **1 mark** Substantially correct

(e)	**gradient of the tangent** $m=\frac{dy}{dx}$ $m=\frac{d}{dx}\left(x^3+2x-3\right)$ $m=3x^2+2$ **at** $x=-2$ $m=3(-2)^2+2=14$ **at** $x=-2$ $y=(-2)^3+2(-2)-3=-8-4-3=-15$ **Equation of the tangent :** $y-(-15)=14(x-(-2))$ $y+15=14x+28$ $y=14x+13$	**2 marks** Correct answer **1 mark** Substantially correct
(f) i)	$initial, t=0$ $v=\frac{2(0)}{9+0^2}=0m/\sec$	**1 mark** Correct answer
ii)	$a=\frac{dv}{dt}=\frac{2(9+t^2)-2t(2t)}{(9+t^2)^2}=\frac{18+2t^2-4t^2}{(9+t^2)^2}=\frac{18-2t^2}{(9+t^2)^2}$	**2 marks** Correct answer **1 mark** Substantially correct
iii)	$a=0$ $\frac{18-2t^2}{(9+t^2)^2}=0$ $18-2t^2=0$ $2t^2=18$ $t^2=9$ $t=\pm 3$ $\because t>0$ $\therefore t=3\sec$	**2 marks** Correct answer **1 mark** Substantially correct
iv)	Maximum velocity occurs when acceleration $a=0$. This happens at $t=3$. Maximum velocity $v=\frac{2(3)}{9+(3)^2}=\frac{6}{18}=\frac{1}{3}m/\sec$	**1 mark** Correct answer.

Year 11	Mathematics Advanced	Task 3	Examination 2020
Question No.12	Solutions and Marking Guidelines		

MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions

Part	Solutions	Marking Guidelines
a i)	$P(\tilde{S})=\frac{16+2}{40}=\frac{9}{20}$	**1 mark** Correct solution
ii)	$P(S\cap B)+P(\tilde{S}\cap B)=\frac{18}{40}+\frac{16}{40}=\frac{17}{20}$	**2 marks** Correct solution **1 mark** Substantially correct
iii)	Not mutually exclusive since $P(S\cap B)\neq 0$	**1 mark** Correct solution
b i)		**2 marks** Correct <u>probability</u> tree diagram **1 mark** Substantially correct
ii)	$P(RR)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$	**1 mark** Correct solution
iii)	$P(\text{at least 1 } R)=1-P(\text{no } R)$ $=1-\left(\frac{1}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{3}\right)$ $=1-\frac{1}{6}$ $=\frac{5}{6}$	**1 mark** Correct solution

iv)

$P(R \mid R) = \frac{1}{3}$

(i.e. the chance of the second R after the first R is already selected)

1 mark
Correct solution

c)

Not independent since $P(A) \neq P(A \mid B)$

1 mark
Correct solution

d i)

$P(\text{same number}) = 6 \times \frac{1}{6}$

1 mark
Correct solution

ii)

From the table, $P(\text{Chris} > \text{Pat}) = \frac{15}{36}$

		Chris					
		1	2	3	4	5	6
Pat	1		×	×	×	×	×
	2			×	×	×	×
	3				×	×	×
	4					×	×
	5						×
	6						

2 marks
Correct solution

1 mark
Substantially correct

e)

Probability of all outcomes is 1.

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) = 1$$
$$1 \times k^2 + 2 \times k^2 + 3 \times k^2 + 4 \times k^2 + 5 \times k^2 + 6 \times k^2 + 7 \times k^2 + 8 \times k^2 = 1$$
$$k^2(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = 1$$
$$k^2 = \frac{1}{36}$$
$$k = \pm\frac{1}{6}$$

k can take either $\pm\frac{1}{6}$ due to k^2 in $P(x) = k^2 x$

(i.e. square of any integer is postive)

2 marks
Correct solution (including the positive and negative values of k)

1 mark
Substantially correct