Hurlstone Agricultural High School

Student Name: _____

Class Teacher: _____

2021

Year 11
Evidence of Learning Task
Mathematics Advanced

General Instructions

- Working time – 1 hour and 15 minutes
  (Multiple Choice – 45 minutes, Extended Response – 30 minutes)
- Scanning and Uploading time – 15 minutes
- Write using black or blue pen
- Calculators approved by NESA may be used
- A reference sheet is attached for your use at the end of the paper
- This examination may be downloaded and printed, using spaces provided for your answers. Alternatively, it may be read on screen with answers completed on your own paper.
- Scan and upload a PDF copy of your Multiple Choice Answer sheet, Extended Response solutions and All My Own Work Declaration to the assignment box in your Class Teams page
- Make sure your document is accessible and your writing is legible.

Part A
Multiple Choice
Marks: 30

- Answer Questions 1 – 30 on the Multiple Choice Answer sheet provided at the end of this paper **OR** write answers on your own single sheet of paper clearly labelled with the question number
- No working out is required to be shown for this section

Part B
Extended Response
Marks: 18

- Answer Questions 31 – 34 in the spaces provided in the question paper OR write answers on your own paper clearly labelled with the question and part number. You should start a new sheet of paper for each new question.
  The questions are not of equal value.
- Show all relevant mathematical reasoning and/or calculations for each question

Total Marks: 48

Part A – Multiple Choice (30 questions worth 1 mark each)

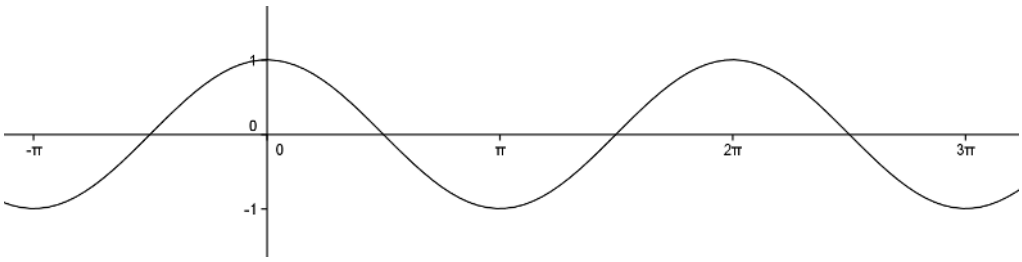
Answer Questions 1 – 30 on the Multiple Choice Answer sheet provided OR write answers on your own single sheet of paper with each answer clearly labelled with the question number.

No working out is required to be shown for this section. Allow about 45 minutes for this section.

1. In radians, which of the following is equal to 150°?

A $\frac{2\pi}{3}$ **B** $\frac{5\pi}{12}$ **C** $\frac{5\pi}{6}$ **D** $\frac{3\pi}{4}$

2.



The above graph is defined by which one of these equations?

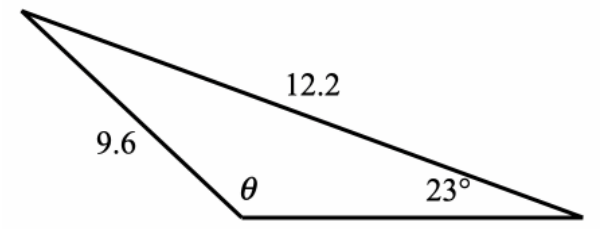
A $y = \sin x°$ **B** $y = \cos x°$ **C** $y = \sin x$ **D** $y = \cos x$

3. When simplified, $\sin(180+\theta)+\cos(360-\theta)$ is equal to which of the following?

A $\sin\theta+\cos\theta$ **B** $\sin\theta-\cos\theta$

C $-\sin\theta-\cos\theta$ **D** $-\sin\theta+\cos\theta$

4. In the triangle shown, θ is obtuse. To the nearest degree, what is the size of θ?



A 147° **B** 150° **C** 153° **D** 156°

5. The number of solutions, for $0° \le \theta \le 360°$, to the equation $\sin\theta\left(2\cos\theta + 1\right) = 0,$ is which of the following?

A 2 **B** 3 **C** 4 **D** 5

6. $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta}$ simplifies to which of the following?

A $2\sec^2\theta$ **B** $2\cos^2\theta$ **C** $2\sin^2\theta$ **D** $2\operatorname{cosec}^2\theta$

7. Given the function $y = 4x^5$, which one of the following is correct?

A $\dfrac{dy}{dx} = 4x^4$ **B** $\dfrac{dy}{dx} = 5x^4$ **C** $\dfrac{dy}{dx} = 1024x^4$ **D** $\dfrac{dy}{dx} = 20x^4$

8. The derivative of $y = 3x^2\left(2x^3 + 1\right)^3$ is given by which of the following?

A $6x\left(2x^3 + 1\right)^2\left(9x^3 + 1\right)$ **B** $6x\left(2x^3 + 1\right)^2\left(9x^3 + 2x + 1\right)$

C $6x\left(2x^3 + 1\right)^2\left(11x^3 + 1\right)$ **D** $6x\left(2x^3 + 1\right)^2\left(11x^3 + 2x + 1\right)$

9. If $y = \sqrt{2x+3}$, what is the value of $\dfrac{dy}{dx}$ when $x = 3$?

A $-\dfrac{1}{3}$ **B** $\dfrac{1}{3}$ **C** $-\dfrac{1}{6}$ **D** $\dfrac{1}{6}$

10. What is the first derivative of $P\left(x\right) = \dfrac{4x^{10} + 3x^6 - 3x^3}{x^3}$?

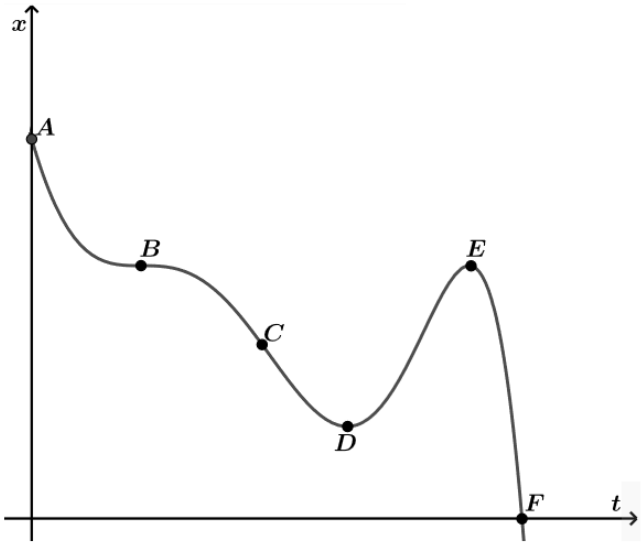
A $28x^6 + 9x^2$ **B** $28x^6 + 9x^2 - 3$

C $40x^6 + 18x^2$ **D** $40x^6 + 18x^2 - 3$

11. A particle's speed decreases from 34 ms^{-1} to 8 ms^{-1} in 4 seconds. What was the average rate of change of its speed?

A 6.5 ms^{-2} **B** – 6.5 ms^{-2} **C** –13 ms^{-2} **D** 13 ms^{-2}

12. For the displacement graph below, at which of the points will the particle be at rest?



A *B, C, D* and *E* only **B** *A* and *F* only

C *D* and *E* only **D** *B, D* and *E* only

13. For the equation $y = 3x^2 - 4$, what is the angle of inclination of the tangent to the curve at the point where $x = 1$?

A 8^o **B** 6^o **C** 61^o **D** 81^o

14. For the function $f(x) = \frac{4}{3x^3}$, what is the derivative function?

A $f'(x) = -\frac{2}{3x^2}$ **B** $f'(x) = -\frac{4}{x^4}$

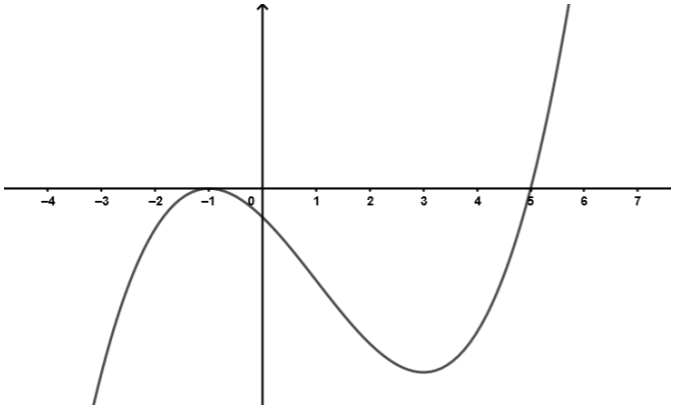
C $f'(x) = -\frac{4}{x^2}$ **D** $f'(x) = -\frac{36}{x^4}$

15. For what values of x is the function $f(x) = \frac{1}{x^2 - 4x}$ not continuous?

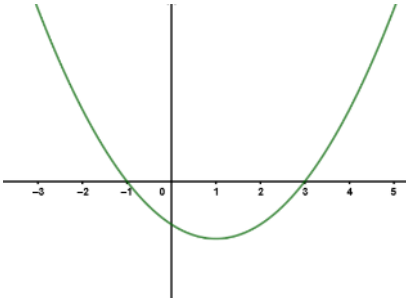
A $x = 0, x = 4$ **B** $0 < x < 4$

C $x < 0, x > 4$ **D** Continuous for all values of x

16. Which of the following is the graph of $\frac{dy}{dx}$ for the graph shown below?



A



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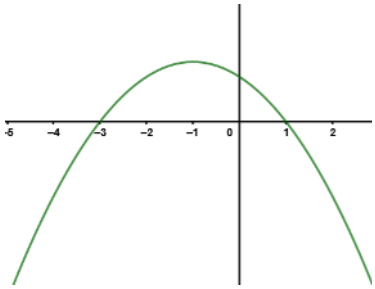

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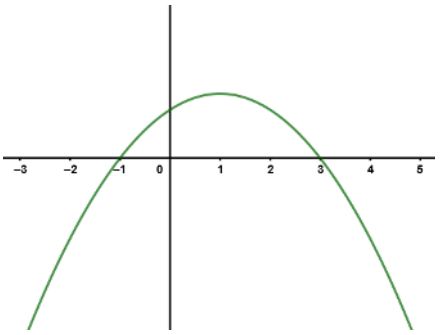
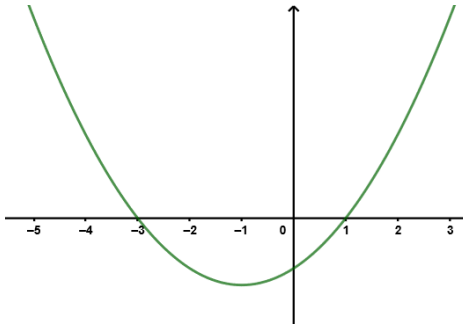


D



17. A survey of 60 consumers about how they pay for their purchases found that 42 used a credit/debit card and 31 used cash. What is the probability that a consumer chosen at random from this group will use both cash and card for their purchases?

A $\frac{13}{60}$ **B** $\frac{11}{60}$ **C** $\frac{11}{71}$ **D** $\frac{73}{71}$

18. A uniform discrete variable B has values $b = 0, 1, 2, 3, 4, 5$.
What is the value of $E(b)$?

A $\frac{1}{6}$ **B** $\frac{1}{2}$ **C** $\frac{5}{6}$ **D** $\frac{5}{2}$

19. The table below shows the results of a survey conducted on a Year 11 class that asks how many siblings each student has.

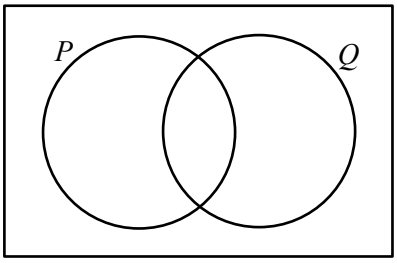
Number of Siblings	Frequency
0	2
1	8
2	5
3	3
4	1

What is the relative frequency of students in this class that have 2 siblings?

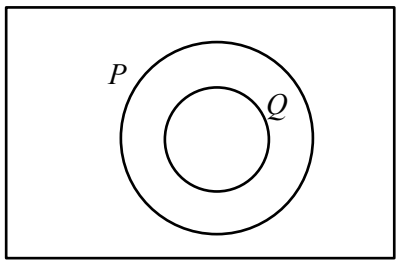
A $\frac{1}{2}$ **B** $\frac{5}{19}$ **C** 5 **D** 15

20. Two events, P and Q, are mutually exclusive. Which of the following Venn Diagrams illustrates this scenario?

A



B



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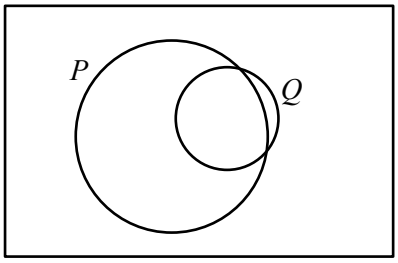
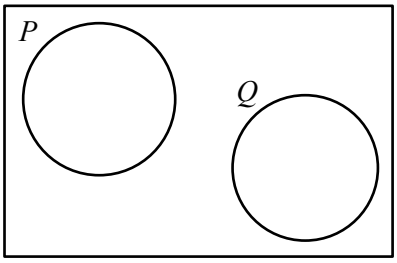
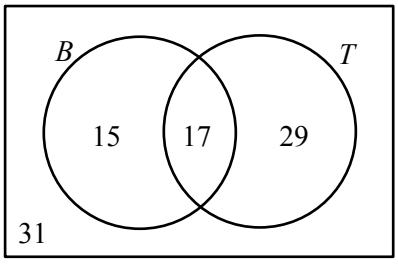


D



21. The Venn Diagram below shows the number of employees at a city based business and their mode of transport to work, where B = catches a bus and T = catches a train.



What is the probability that a worker chosen at random will catch a bus ***or*** train to work?

A $\frac{17}{61}$ **B** $\frac{44}{61}$ **C** $\frac{17}{92}$ **D** $\frac{61}{92}$

22. The results of a survey of P-plate drivers are shown in the two-way table below.

	Male (M)	Female (F)	Totals
Lost license for speeding $\left(S\right)$	13	5	18
Not lost license for speeding $\left(\overline{S}\right)$	51	50	101
Totals	64	55	119

What is $P\left(S\middle|M\right)$ as a fraction?

A $\frac{18}{101}$ **B** $\frac{13}{64}$ **C** $\frac{64}{119}$ **D** $\frac{13}{18}$

23. Which one of the probability distribution tables below shows a uniform distribution?

A

x	0	2	4
$p(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

B

x	0	1	2
$p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{4}{8}$

C

x	3	5	9
$p(x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

D

x	15	20	25
$p(x)$	0.1	0.8	0.15

24. For what value of ***a*** will the distribution below be a probability distribution?

x	1	2	3	4	5
$P(X=x)$	0.1	0.35	0.41	***a***	0.05

A 0.91 **B** 1.09

C 0.09 **D** cannot be determined

25. The table below shows the probability function for a random variable T.

t	3	5	7	9	11
$P\left(T=t\right)$	$\frac{1}{32}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{3}{32}$

What is the probability $P\left(5\le T<11\right)$?

A $\frac{7}{8}$ **B** $\frac{11}{32}$ **C** $\frac{7}{16}$ **D** $\frac{1}{32}$

26. Enter the data from the following probability distribution function onto your calculator.

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$

What is the value of $Var(x)$ for this distribution, correct to 3 decimal places.

A $Var(X)=3.667$ **B** $Var(X)=2.889$

C $Var(X)=1.700$ **D** $Var(X)=0.111$

27. What is the value of $10^{\log_{10}3}$ in its simplest form?

A 3 **B** 10 **C** 1 000 **D** $\frac{1}{3}$

28. Which of these single logarithms is equivalent to $2\log_a x-5\log_a y+3\log_a z$?

A $\log_a\left(\frac{x^2z^3}{y^5}\right)$ **B** $-30\log_a(xyz)$

C $-\log_a(x^2y^5z^3)$ **D** $\log_a\left(\frac{x^2y^5}{z^3}\right)$

29. If $\log x-\log 3=\log\left(\frac{x-3}{2}\right)$, then what is the value of x?

A 3 **B** 5 **C** 7 **D** 9

30. If $3^a=2$, then which of the expressions below is equivalent to $\log_3\frac{2}{3}$?

A $a-1$ **B** $3a-1$ **C** $2a-2$ **D** a^2+a-1

Part B – Extended Response (18 marks)

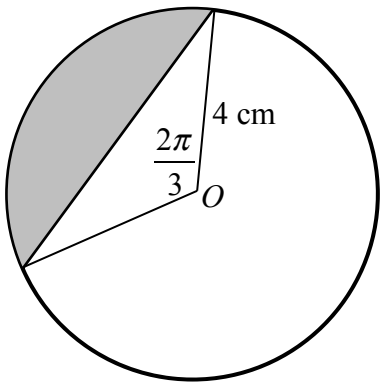
Answer Questions 31 – 34 in the spaces provided OR write your solutions on your own paper. Start each question on a new sheet of paper with each of your answers clearly labelled with the question number and your name.

All relevant working should be shown to justify your answer. Use black or blue pen.

Allow about 30 minutes for this section.

Question 31 (4 marks) **Marks**

(a) The diagram below shows a circle of radius 4 cm. Calculate the area of the **minor segment** shaded, if the endpoints of the chord subtend an angle at the centre of $\frac{2\pi}{3}$ radians. Give your answer to one decimal place. **2**



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Marks

(b) Solve the equation $\tan\theta = -3$, for $0° \le \theta \le 360°$, giving your answers to the nearest minute. **2**

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Question 32 (6 marks) **Marks**

(a) For the discrete random variable X, the probability distribution is given by:

$$p(x) = \begin{cases} kx, & x = 1,2,3,4 \\ k(9-x), & x = 5,6,7,8 \end{cases}$$

(i) Find the value of k and complete the following table to show the probability distribution of X. **2**

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x	1	2	3	4	5	6	7	8
$p(x)$								

(ii) Use your calculator, or otherwise, to find the value of $E(X)$ and the standard deviation of the distribution. **2**

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Marks

(b) Group *A* contains 10 gorillas and 5 chimpanzees. Group *B* contains 4 gorillas and 6 chimpanzees. Two apes are selected at random from the groups. Use a probability tree, or otherwise, to find the probability that one is a gorilla and the other is a chimpanzee if one ape is selected from each group. **2**

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Question 33 (5 marks)

Marks

(a) Find the derivative, $f'(x)$, of the function $f(x) = x^2 - 5x$ from first principles. **2**

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Marks

(b) The point $P(1,-3)$ lies on the curve $y=-\frac{1}{9}(4-x)^3$. Find the equation of the normal at P. **3**

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Question 34 (3 marks) **Marks**

Solve the following equation: **3**

$$\log_{10}(5x-4)-\log_{10}(2x-1)=1+\log_{10}\frac{1}{5}$$

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~ End of Task ~

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

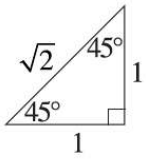
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

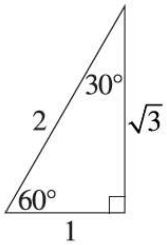
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

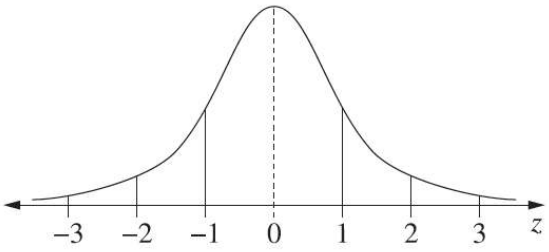
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$

HURLSTONE AGRICULTURAL HIGH SCHOOL

Year 11 Mathematics Advanced
2021 Evidence of Learning Task
(Task 3)
Multiple Choice Answer Sheet

NAME.....

EXAMPLE 2 + 4 =

A 2 **B** 4 **C** 6 **D** 8

Ⓐ Ⓑ ● Ⓓ

ATTEMPT ALL QUESTIONS

1	Ⓐ Ⓑ Ⓒ Ⓓ
2	Ⓐ Ⓑ Ⓒ Ⓓ
3	Ⓐ Ⓑ Ⓒ Ⓓ
4	Ⓐ Ⓑ Ⓒ Ⓓ
5	Ⓐ Ⓑ Ⓒ Ⓓ
6	Ⓐ Ⓑ Ⓒ Ⓓ
7	Ⓐ Ⓑ Ⓒ Ⓓ
8	Ⓐ Ⓑ Ⓒ Ⓓ
9	Ⓐ Ⓑ Ⓒ Ⓓ
10	Ⓐ Ⓑ Ⓒ Ⓓ
11	Ⓐ Ⓑ Ⓒ Ⓓ
12	Ⓐ Ⓑ Ⓒ Ⓓ
13	Ⓐ Ⓑ Ⓒ Ⓓ
14	Ⓐ Ⓑ Ⓒ Ⓓ
15	Ⓐ Ⓑ Ⓒ Ⓓ

16	Ⓐ Ⓑ Ⓒ Ⓓ
17	Ⓐ Ⓑ Ⓒ Ⓓ
18	Ⓐ Ⓑ Ⓒ Ⓓ
19	Ⓐ Ⓑ Ⓒ Ⓓ
20	Ⓐ Ⓑ Ⓒ Ⓓ
21	Ⓐ Ⓑ Ⓒ Ⓓ
22	Ⓐ Ⓑ Ⓒ Ⓓ
23	Ⓐ Ⓑ Ⓒ Ⓓ
24	Ⓐ Ⓑ Ⓒ Ⓓ
25	Ⓐ Ⓑ Ⓒ Ⓓ
26	Ⓐ Ⓑ Ⓒ Ⓓ
27	Ⓐ Ⓑ Ⓒ Ⓓ
28	Ⓐ Ⓑ Ⓒ Ⓓ
29	Ⓐ Ⓑ Ⓒ Ⓓ
30	Ⓐ Ⓑ Ⓒ Ⓓ

ALL MY OWN WORK DECLARATION

I declare that, to the best of my knowledge this assignment is my own work, all sources have been referenced, and the assignment contains no plagiarism. I further declare that I have not previously submitted this work or any version of it for assessment in any other subject.

Student's Name (please print in BLOCK letters):

Student's Signature:

Date of Signing:

Hurlstone Agricultural High School

Solutions
(and Marking Guidelines)

2021

Year 11
Evidence of Learning Task
Mathematics Advanced

Solutions
(and Marking Guidelines)

HURLSTONE AGRICULTURAL HIGH SCHOOL

Year 11 Mathematics Advanced
2021 Evidence of Learning Task
(Task 3)
Multiple Choice Answer Sheet

NAME.....

EXAMPLE 2 + 4 =

A 2 **B** 4 **C** 6 **D** 8

Ⓐ Ⓑ ● Ⓓ

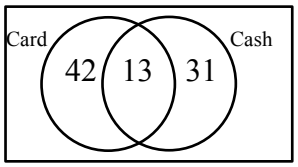
ATTEMPT ALL QUESTIONS

1	Ⓐ Ⓑ ● Ⓓ
2	Ⓐ Ⓑ Ⓒ ●
3	Ⓐ Ⓑ Ⓒ ●
4	Ⓐ ● Ⓒ Ⓓ
5	Ⓐ Ⓑ Ⓒ ●
6	● Ⓑ Ⓒ Ⓓ
7	Ⓐ Ⓑ Ⓒ ●
8	Ⓐ Ⓑ ● Ⓓ
9	Ⓐ ● Ⓒ Ⓓ
10	● Ⓑ Ⓒ Ⓓ
11	Ⓐ ● Ⓒ Ⓓ
12	Ⓐ Ⓑ Ⓒ ●
13	Ⓐ Ⓑ Ⓒ ●
14	Ⓐ ● Ⓒ Ⓓ
15	● Ⓑ Ⓒ Ⓓ

16	● Ⓑ Ⓒ Ⓓ
17	● Ⓑ Ⓒ Ⓓ
18	Ⓐ Ⓑ Ⓒ ●
19	Ⓐ ● Ⓒ Ⓓ
20	Ⓐ Ⓑ Ⓒ ●
21	Ⓐ Ⓑ Ⓒ ●
22	Ⓐ ● Ⓒ Ⓓ
23	Ⓐ Ⓑ ● Ⓓ
24	Ⓐ Ⓑ ● Ⓓ
25	● Ⓑ Ⓒ Ⓓ
26	Ⓐ ● Ⓒ Ⓓ
27	● Ⓑ Ⓒ Ⓓ
28	● Ⓑ Ⓒ Ⓓ
29	Ⓐ Ⓑ Ⓒ ●
30	● Ⓑ Ⓒ Ⓓ

2021 Y11 EOL Mathematics Advanced Multiple Choice Solutions (Questions 1 – 30)

Outcomes	Q No.	Solutions	Answer	Outcomes	Q No.	Solutions	Answer
	1	$\frac{150\times\pi}{180}$	C		2	Use calculator to test $y=\cos\theta$	D
	3	$=-\sin\theta+\cos\theta$	D		4	Sine Rule $\frac{\sin\theta}{12.2}=\frac{\sin 23^o}{9.6}$	B
	5	$\sin\theta(2\cos\theta+1)=$ $\sin\theta=0 \ or \ 2\cos\theta=-1$ $\cos\theta=\frac{-1}{2}$ $\theta=0,\pi,2\pi \quad \frac{2\pi}{3},\frac{4\pi}{3}$	D		6	$=\frac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}$ $=\frac{2}{\cos^2\theta}=2\sec^2\theta$	A
	7	$\frac{dy}{dx}=5\times4x^4$ $=20x^4$	D		8	$=3x^2\times3(2x^3+1)^2(3\times2x^2)$ $+(2x^3+1)^3\times6x$ $=6x(2x^3+1)^2(11x^3+1)$	C
	9	$\frac{dy}{dx}=\frac{1}{2}(2x+3)^{\frac{-1}{2}}\times2$ $=(2x+3)^{\frac{-1}{2}}$ $when\ x=3$ $\frac{dy}{dx}=(2(3)+3)^{\frac{-1}{2}}$ $=\frac{1}{3}$	B		10	$=\frac{4x^{10}}{x^3}-\frac{2x^9}{x^3}+\frac{3x^6}{x^3}+\frac{6x^4}{x^3}-\frac{3x^3}{x^3}$ $=4x^7-2x^6+3x^3+6x-3$ $\frac{dy}{dx}=28x^6-12x^5+9x^2+6$	A
	11	$=\frac{8-34}{4}$ $=-6.5ms^{-2}$	B		12	B= pt of inflection D= min stationary pt E= max stationary pt	D
	13	$\frac{dy}{dx}=3(2x-3)^2(2)$ $at\ x=1$ $\frac{dy}{dx}=6$ $\tan\theta=6$ $\theta=81^o$	D		14	$f'(x)=\frac{3x^3(0)-4(9x^2)}{(3x^3)^2}$ $=\frac{-36x^2}{9x^6}$ $=\frac{-4}{x^2}$	B

	15	$x(x-4)=0$ $x=0,\ x=4$	A		16	Gradient of curve is zero at $x=-1$ and $3\ \left(ie.\frac{dy}{dx}=0\right)$ $\Rightarrow$ A or C. Gradient of curve is positive $\left(ie.\frac{dy}{dx}>0\right)$ when $x<-1$ and $x>3$ $\Rightarrow$ A	A
	17	 From Venn diagram above: $\mathrm{P}(\mathrm{Card}\cap\mathrm{Cash})=\frac{13}{60}$	A		18	A uniform discrete variable will have equal probabilities for each outcome. $\therefore E(b)=\sum x.p(x)$ $=\frac{1}{6}\times0+\frac{1}{6}\times1+\frac{1}{6}\times2+\frac{1}{6}\times3+\frac{1}{6}\times4+\frac{1}{6}\times5$ $=\frac{15}{6}$ $=\frac{5}{2}$	D
	19	Frequency of 2 siblings = 5 Total students = 2+8+5+3+1 =19 $\therefore$ Relative Frequency $=\frac{5}{19}$	B		20	The occurrence of one event, automatically excludes the occurrence of the other, hence, circles cannot overlap.	D
	21	$P(B\cup T)=P(B)+P(T)-P(B\cap T)$ $=\frac{32}{92}+\frac{46}{92}-\frac{17}{92}$ $=\frac{61}{92}$	D		22	Probability that a driver has lost their license for speeding given they are male: There are 13 male speeders from the group of 64 males $\therefore P(S\|M)=\frac{13}{64}$	B
	23	A uniform distribution has equal probabilities for each outcome.	C		24	For a probability distribution: $\sum p(x)=1$ ie. $0.1+0.35+0.41+a+0.05=1$ $\therefore a=0.09$	C
	25	$P(T=5,7,9)=\frac{1}{4}+\frac{5}{15}+\frac{5}{15}$ $=\frac{7}{8}$	A		26	From calculator: $\sigma=1.699673$ $Var(X)=\sigma^2$ $=2.889$ (3 dec. pl.)	A

	27	Let $x = 10^{\log_{10} 3}$ In logarithmic form: $\log_{10} x = \log_{10} 3$ $\therefore x = 3$	A		28	$2\log_a x - 5\log_a y + 3\log_a z$ $= \log_a x^2 - \log_a y^5 + \log_a z^3$ $= \log_a \frac{x^2 z^3}{y^5}$	A
	29	$\log x - \log 3 = \log\left(\frac{x-3}{2}\right)$ $\log \frac{x}{3} = \log\left(\frac{x-3}{2}\right)$ $\frac{x}{3} = \frac{x-3}{2}$ $2x = 3(x-3)$ $2x = 3x - 9$ $x = 9$	D		30	$\because 3^a = 2$ $\therefore$ by definition, $a = \log_3 2$ $\log_3 \frac{2}{3} = \log_3 2 - \log_3 3 = a - 1$	A

Part B – Extended Response (18 marks)

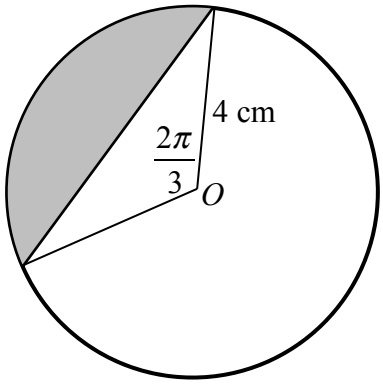
Answer Questions 31 – 34 in the spaces provided OR write your solutions on your own paper. Start each question on a new sheet of paper with each of your answers clearly labelled with the question number and your name.

All relevant working should be shown to justify your answer. Use black or blue pen.

Allow about 30 minutes for this section.

Question 31 (4 marks) **Marks**

(a) The diagram below shows a circle of radius 4 cm. Calculate the area of the **minor segment** shaded, if the endpoints of the chord subtend an angle at the centre of $\frac{2\pi}{3}$ radians. Give your answer to one decimal place. **2**



$$A = \frac{1}{2}r^2\left(\theta - \sin\theta\right)$$
$$= \frac{1}{2}\times 4^2 \times \left(\frac{2\pi}{3} - \sin\left(\frac{2\pi}{3}\right)\right)$$
$$= \frac{1}{2}\times 16 \times \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$
$$= 9.8\text{cm}^2$$

Marking Guideline

2 marks – correct solution
1 mark – substantial progress towards correct solution

Marks

(b) Solve the equation $\tan\theta = -3$, for $0° \le \theta \le 360°$, giving your answers to the nearest minute. **2**

$$\tan\theta = -3,\ 0° \le \theta \le 360°$$
$$\theta = \tan^{-1}(-3) \qquad (\theta \text{ in 2nd and fourth quadrants})$$
$$= 108°26', 288°26'$$

Marking Guideline
2 marks – correct solution including both answers to correct degree of accuracy
1 mark – substantial progress towards correct solution

Question 32 (6 marks) **Marks**

(a) For the discrete random variable X, the probability distribution is given by:

$$p(x) = \begin{cases} kx, & x = 1,2,3,4 \\ k(9-x), & x = 5,6,7,8 \end{cases}$$

(i) Find the value of k and complete the following table to show the probability distribution of X. **2**

$$k + 2k + 3k + 4k + 4k + 3k + 2k + k = 1$$
$$(\text{since the function is a probability distribution})$$
$$\therefore 20k = 1$$
$$k = \frac{1}{20}$$

x	1	2	3	4	5	6	7	8
$p(x)$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{3}{20}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{1}{10}$	$\frac{1}{20}$

Marking Guideline
2 marks – correct value for k and correctly filled table
1 mark – one of the above completed correctly (allow for CFPA)

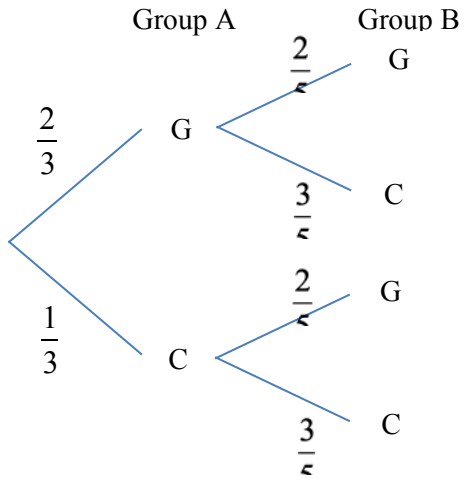
(ii) Use your calculator, or otherwise, to find the value of $E(X)$ and the standard deviation of the distribution. **2**

From the calculator:
$E(x) = 4.5$ $\qquad$ $\sigma = 1.80$

Marking Guideline
2 marks – values for both statistics correct
1 mark – only one of the values correct

Marks

(b) Group A contains 10 gorillas and 5 chimpanzees. Group B contains 4 gorillas and 6 chimpanzees. Two apes are selected at random from the groups. Use a probability tree, or otherwise, to find the probability that one is a gorilla and the other is a chimpanzee if one ape is selected from each group. **2**



$$P(GC \text{ or } CG) = \frac{2}{3}\times\frac{3}{5}+\frac{1}{3}\times\frac{2}{5}$$
$$= \frac{6}{15}+\frac{2}{15}$$
$$= \frac{8}{15}$$

A probability tree is helpful, but not essential.
Marking Guideline
2 marks – Correct solution
1 mark – substantial progress towards correct solution

Question 33 (5 marks)

Marks

(a) Find the derivative, $f'(x)$, of the function $f(x)=x^2-5x$ from first principles. **2**

$$f(x)=x^2-5x$$
$$f(x+h)=(x+h)^2-5(x+h)$$
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{(x+h)^2-5(x+h)-(x^2-5x)}{h}$$
$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-5x-5h-x^2+5x}{h}$$
$$=\lim_{h\to 0}\frac{2xh+h^2-5h}{h}$$
$$=\lim_{h\to 0}\frac{h(2x+h-5)}{h}$$
$$=\lim_{h\to 0}(2x+h-5)$$
$$=2x-5$$

Marking Guideline
2 marks – correct solution
1 mark – substantial progress towards correct solution

Marks

(b) The point $P(1,-3)$ lies on the curve $y=-\frac{1}{9}(4-x)^3$. Find the equation of the normal at P. **3**

$$\frac{dy}{dx}=-\frac{1}{3}(4-x)^2\times -1$$

$$=\frac{1}{3}(4-x)^2$$

At $P(1,-3)$ $\quad m_{\text{tangent}}=\frac{1}{3}(-3)^2$

$$=3$$

$$\therefore m_{\text{normal}}=-\frac{1}{3}$$

Equation of normal $\quad y-y_1=m(x-x_1)$

$$y+3=-\frac{1}{3}(x-1)$$

$$-3y-9=x-1$$

$$x+3y+8=0$$

Marking Guideline
3 marks – correct solution showing correct derivative, correct gradient of tangent and normal and equation of normal (allow for CFPA)
2 marks – 2 of the 3 components above present.
1 mark – only one of the three components above present.

Question 34 (3 marks)

Marks

Solve the following equation: **3**

$$\log_{10}(5x-4)-\log_{10}(2x-1)=1+\log_{10}\frac{1}{5}$$

$$\log_{10}(5x-4)-\log_{10}(2x-1)-\log_{10}\left(\frac{1}{5}\right)=1$$

$$\log_{10}(5x-4)-\log_{10}(2x-1)+\log_{10}(5)=\log_{10}10$$

$$\log_{10}\left(\frac{5(5x-4)}{2x-1}\right)=\log_{10}10$$

$$\frac{5(5x-4)}{2x-1}=10$$

$$25x-20=20x-10$$

$$5x=10$$

$$x=2$$

Marking Guideline
3 marks – correct solution
2 marks – substantial progress towards correct solution including knowledge of all required logarithm laws.
1 mark – some progress towards correct solution showing some knowledge of logarithm laws

~ End of Task ~