STUDENT'S NAME: ______________________________

TEACHER'S NAME: ______________________________

2022

HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 11 ASSESSMENT TASK 3

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a black or blue pen
- NESA approved calculators may be used
- A reference sheet is provided in the Section I booklet
- For questions in Section II, show all relevant mathematical reasoning and/or calculations
- This examination paper is not to be removed from the examination centre

Total marks: 70

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10. The multiple-choice answer sheet has been provided
- Allow about 15 minutes for this section

Section II – 60 marks (pages 9– 30)

- Attempt Questions 11 – 15, write your solutions in the spaces provided.
- There are 5 separate question/answer booklets. Extra working pages are available if required.
- Allow about 1 hour and 45 minutes for this section.

SECTION I (10 marks)

Attempt Questions 1 - 10

Allow about 15 minutes on this section.

Use the multiple-choice answer sheet provided for Questions 1 – 10.

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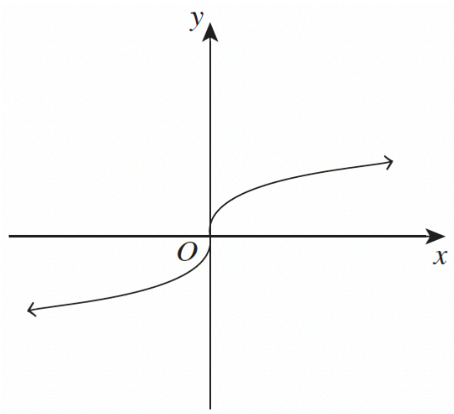
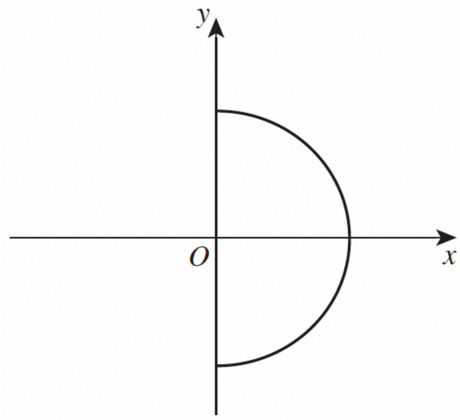
1. Which one of the following graphs represents a one-to-many relationship?

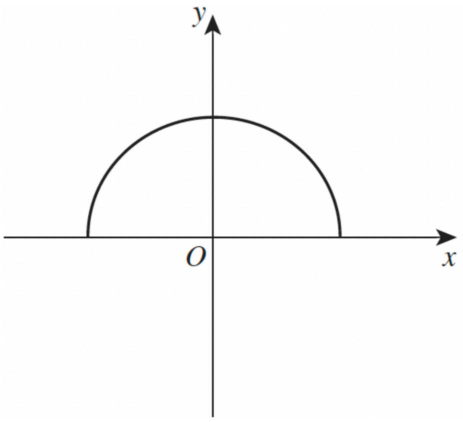
(A)



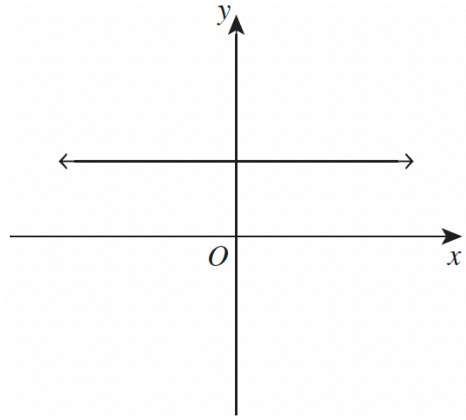
(B)



(C)



(D)



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2. Which one of the following is ***not*** a discrete random variable?

(A) The number of kilometres travelled per litre of petrol.

(B) The number of motorists buying petrol at a service station.

(C) The number of passengers in a car filling up with petrol at a service station.

(D) The number of times per week a motorist needs to refill their car with petrol.

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3. What is the derivative with respect to x of the expression $\dfrac{2x^3 - 5x^2 + x}{x}$?

(A) $8x - 30 + \dfrac{2}{x}$

(B) $6x^2 - 10x + 1$

(C) $4x^3 - 5x^2$

(D) $4x - 5$

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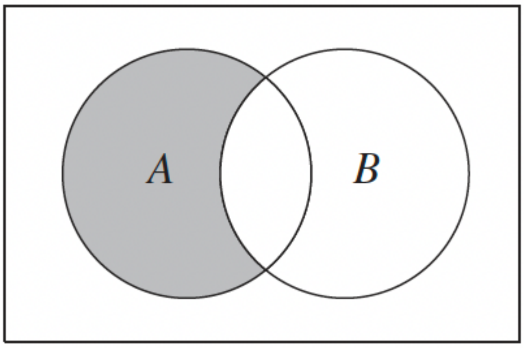
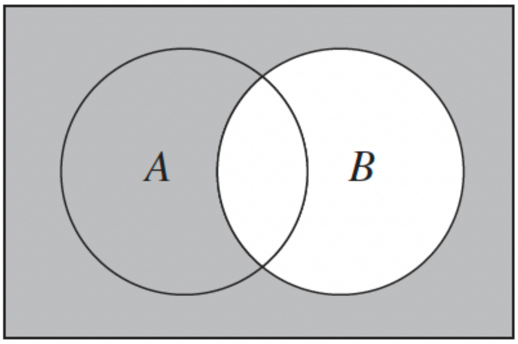
4. Which one of the following Venn diagrams represents $A \cup \overline{B}$?

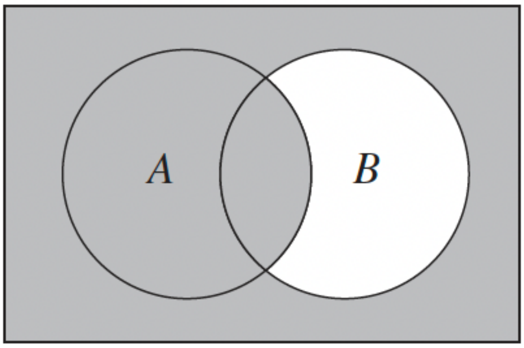
(A)



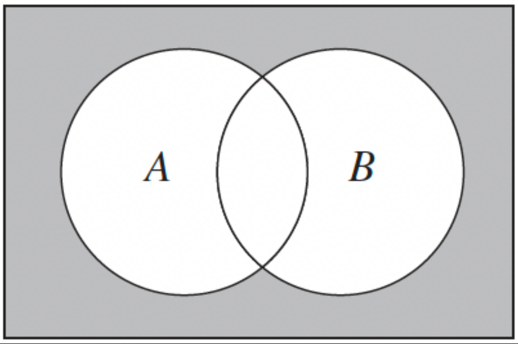
(B)



(C)



(D)



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5. Which one of the following sets of ordered pairs represents an odd function?

(A) {(2, 3), (3, 2), (5, 7), (7, 5)}

(B) {(–2, 3), (2, 3), (–5, 7), (7, 5)}

(C) {(2, 3), (–2, –3), (5, 7), (–5, –7)}

(D) {(2, 3), (3, 5), (5, 7), (7, 2)}

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6. Given $\cos\theta = -\dfrac{8}{17}$ and $\sin\theta$ is also negative, what is the value of $\tan\theta$?

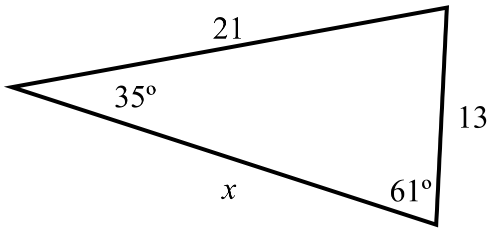
(A) $\dfrac{8}{15}$

(B) $-\dfrac{8}{15}$

(C) $-\dfrac{17}{15}$

(D) $\dfrac{15}{8}$

7.



NOT TO SCALE
Dimensions are in units.

For the above triangle, which expression will give the correct value for x?

(A) $\dfrac{13\sin 61°}{\sin 35°}$ (B) $13\cos 61°$

(C) $21\sin 84°$ (D) $\sqrt{13^2+21^2-2\times 13\times 21\times \cos 84°}$

8. What is the angle of inclination to the positive x axis, to the nearest degree, of the tangent to the curve $y=\dfrac{x^3}{6}+2x$ at the point where $x = 1$?

(A) 65° (B) 68°

(C) 79° (D) 112°

9. Over the domain $[0,2\pi]$, how many solutions are there for the equation $\sin^2 x = 0$?

(A) 4 (B) 3

(C) 2 (D) 1

10.

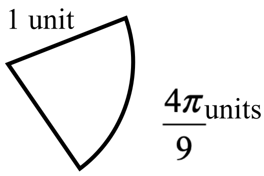


Diagram not to scale.

An arc on the unit circle, of length $\dfrac{4\pi}{9}$ units, is shown on the diagram above. What is the size of the angle subtended at the centre of the unit circle by this arc in degrees?

(A) 80° (B) 40°

(C) 20° (D) 10°

End of Section A

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \ r \neq 1$$

$$S = \frac{a}{1-r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

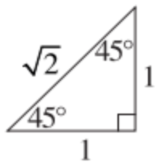
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

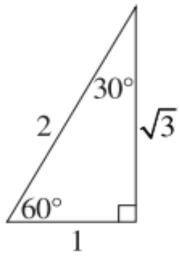


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

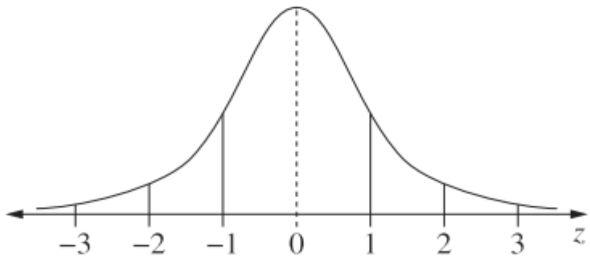
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\Big\{f(a) + f(b) + 2\big[f(x_1) + \cdots + f(x_{n-1})\big]\Big\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

SECTION II **Name:** ______________________________

70 marks

Attempt Questions 11 - 15

Allow about 1 hours and 45 minutes on this section.

Answer each question in the spaces provided. Extra working space is available after each question. If you need to use this extra space, you must clearly indicate this in the main solution space, and then clearly indicate the question number and part that you are answering in the extra space.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

2022 Year 11 Mathematics Advanced Task 3 Examination Section II

Question 11 (12 marks) **Marks**

(a) Differentiate the expression $\left(x^2-5\right)\left(9x-2x^3\right)$ with respect to x. **2**

(b) Differentiate $f\left(x\right)=x^2+5x$ using $f'\left(x\right)=\lim\limits_{h\to 0}\dfrac{f\left(x+h\right)-f\left(x\right)}{h}$. **2**

Marks

(c) Find $\frac{dy}{dx}$ if $y=\left(2x^2-3\right)^5$. **2**

(d) Find the exact gradient of the tangent to the curve $y=\frac{5x+2}{3x+1}$ when $x=1$. **3**

Marks

(e) Find the equation of the normal to the curve $y = x^2 - 4x + 4$ at the point $(3, 1)$. **3**

End of Question 11

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: ______________________________

Question 12 (12 marks) **Marks**

(a) Find the domain of the function $f(x)=\dfrac{x}{x-3}+\sqrt{x}$. **2**

....

....

(b) Determine whether the function $f(x)=\dfrac{1}{x^3-4x}$ is odd, even or neither. **2**

Justify your answer.

....

....

....

....

....

....

(c) **(i)** Show that $\dfrac{2x^2+x-1}{x^2-1}=2+\dfrac{1}{x-1}$. **2**

....

....

....

....

....

Marks

(ii) Use the result shown in part **(i)** to help sketch a neat graph of $y = \dfrac{2x^2 + x - 1}{x^2 - 1}$ labelling any intercepts, discontinuities and other key features. **3**

(d) Consider the functions $f(x) = x^2 - 9$ and $g(x) = 3x + 1$.

(i) Find a simplified expression for $h(x) = f(g(x))$. **2**

(ii) Find the value of $h(2)$. **1**

End of Question 12

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

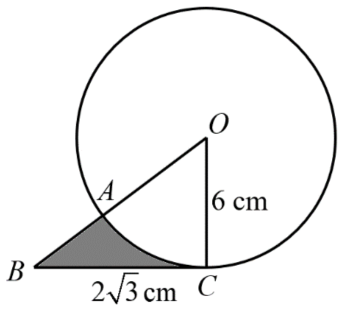
Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: ____________________________

Question 13 (12 marks) **Marks**

(a) The diagram shows the circle with centre *O*. *OAB* is a straight line, *BC* is the tangent at *C* with length $2\sqrt{3}$ cm and radius 6 cm.



(i) Show $\angle BOC$ is $\frac{\pi}{6}$. **1**

....

....

....

(ii) Find the area of sector OAC in terms of π. **1**

....

....

....

(iii) Hence, find the area of the shaded region. **2**
(Give your answer correct to 2 decimal places.)

....

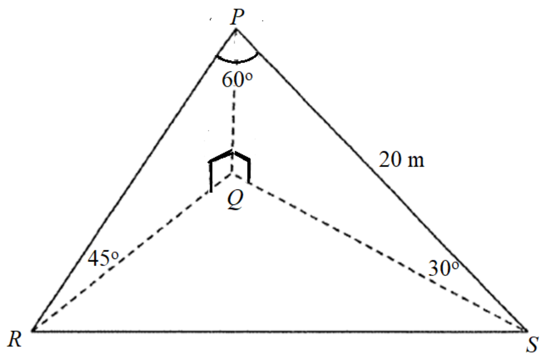
....

....

....

Marks

(b) The diagram shows *R, S, Q* are three points on the same horizontal ground. *PQ* is a vertical pole on the plane *RSQ*. *PR* and *PS* are ropes with $PB = 20$ m and $\angle RPS = 60°$.



(i) Show that *PQ* is 10 m. **1**

(ii) Find *PR*, giving your answer as an exact value. **1**

(iii) Find the distance between the points *R* and *S*. **2**
(Give your answer correct to 1 decimal place.)

Marks

(iv) Find the size of $\angle RQS$ correct to the nearest degree. **3**

(v) Given that R is due south of Q, find the bearing of Q from S to the nearest degree. **1**

End of Question 13

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: ______________________________

Question 14 (12 marks) **Marks**

(a) Solve: **2**

$$2\cos\theta = -\sqrt{3} \text{ for } -\pi \le \theta \le \pi.$$

(b) For what values of x is $y = \tan x$ undefined for $0° \le x \le 720°$? **2**

(c) Prove the identity: **3**

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$$

Marks

(d) Solve: $2\cos^2 2x = 6\sin^2 2x, 0° \le x \le 90°$ **3**

(e) Draw a neat sketch of the graph of $y = -4\cos(2x), 0 \le x \le 2\pi$. **2**

End of Question 14

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: ______________________________

Question 15 (12 marks) **Marks**

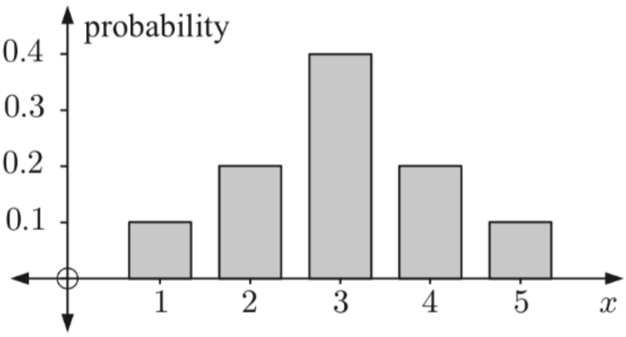
(a) A game is played where a person rolls a fair, six-sided die. If the die lands on the number 2, the person playing wins and the game ends.

(i) By using a probability tree diagram or otherwise, calculate the probability that the game ends after the third roll. **2**

(ii) Calculate the probability that the person rolls the die no more than three times in a single game. **1**

Marks

(b) A discrete random variable X has the probability distribution shown below.



(i) Find $E(X)$. **1**

(ii) Find the standard deviation. **2**

(iii) Find $P(x > \mu + \sigma)$. **1**

Marks

(c) The probability that Michael will score ***more than*** 100 points in a game of bowling is $\dfrac{31}{40}$. **1**

A commentator states that the probability that Michael will score ***less than*** 100 points in a game of bowling is $\dfrac{9}{40}$.

Is the commentator correct? Give a reason for your answer.

(d) The events X and Y are independent. Given that $P(X) = 0.3$ and $P(Y) = 0.42$, find $P(X \cup Y)$. **2**

Marks

(e) A discrete random variable X has the probability distribution shown below. **2**

Y	-1	0	1	2
$\mathrm{P}(Y = y)$	0.1	a	0.3	b

Find a and b if $E(X) = 0.9$.

End of Question 15

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Year 11 Advanced Mathematics Yearly Examination (Task 3) 2022

Question No. 1 – 10 (Multiple Choice) Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems

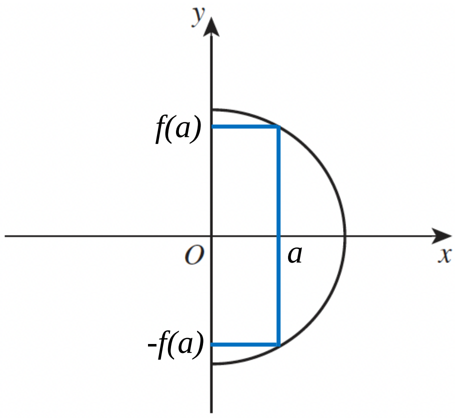
MA11-2 uses the concepts of functions and relations to model, analyse and solve practical problems

MA11-3 uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes

MA11-4 uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities

MA11-5 interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems

MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions

Outcome	Solutions	Marking Guidelines
MA11-2	**1. B**  From the diagram for **B** it can be seen that **one** value of *x*, in this case a, will map to **many** function values, in this case *f(a)* and -*f(a)*.	**1 mark** Correct Answer
MA11-7	**2. A** The number of kilometres travelled is not measured in discrete units. The number can be measured to the desired level of accuracy, hence it is a continuous variable. The number of motorists (B), the number of passengers (C) and number of times per week (D) can only be measured as discrete variables as in each case the variables can only be measured as whole numbers.	**1 mark** Correct Answer
MA11-5	**3. D** As the expression is a fraction with a monomial denominator, simplify the fraction before differentiation. $\frac{2x^3 - 5x^2 + x}{x} = 2x^2 - 5x + 1$ $\frac{d}{dx}(2x^2 - 5x + 1) = 4x - 5$	**1 mark** Correct Answer

MA11-7

4. A

$A \cap \bar{B}$ means we need to shade everything in set A that overlaps with everything that is *not* in B, hence the answer A.
Answer B represents $\bar{B}$.
Answer C represents $A \cup \bar{B}$.
Answer D represents $(\overline{A \cup B})$

1 mark
Correct Answer

MA11-2

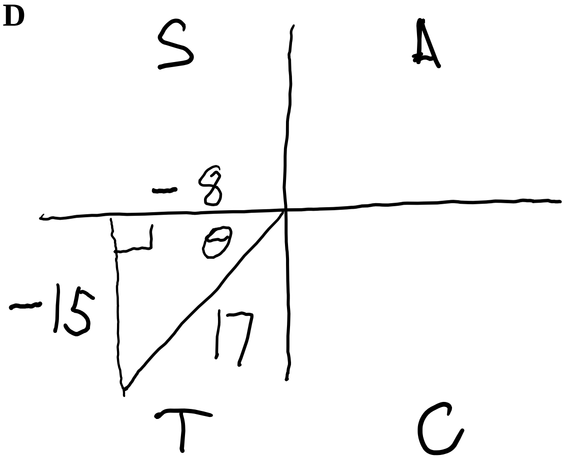
5. C

For an odd function $f(x) = -f(-x)$
Since the sets consist of individual points, if we have the point (2, 3) the set must also contain (-2, -3). If the set contains (5, 7), it must also contain (-5, -7).
Answer C shows the only set with this property.

1 mark
Correct Answer

MA11-3

6. D



From the given information the angle θ must be in the third quadrant as both sine and cosine are negative.
Constructing the diagram above using the given cosine ratio and Pythagoras' Theorem we can then see that $\tan\theta = \frac{15}{8}$.

1 mark
Correct Answer

MA11-3

7. D

Deducing that the missing angle in the triangle is 84° (using angle sum of a triangle), we can then apply the cosine rule to find an unknown side:

$$x = \sqrt{13^2 + 21^2 - 2 \times 13 \times 21 \times \cos 84^{\circ}}$$

1 mark
Correct Answer

MA11-5	**8. B** $y = \frac{x^3}{6} + 2x$ $\frac{dy}{dx} = \frac{3x^2}{6} + 2$ $= \frac{x^2}{2} + 2$ The tangent ratio of the angle of inclination is the same as the gradient of the curve. At $x = 1, \frac{dy}{dx} = \frac{1}{2} + 2 = 2.5$ ie. $\tan\theta = 2.5$ $\therefore \theta \approx 68°$	**1 mark** Correct Answer
MA11-4	**9. B** $\sin^2 x = 0$ $\therefore \sin x = 0$ $x = 0, \pi, 2\pi \quad [0, 2\pi]$ ie. 3 solutions	**1 mark** Correct Answer
MA11-4	**10. A** On the unit circle, an arc of length 1 unit subtends an arc of 1 radian. It then follows that an arc of length $\frac{4\pi}{9}$ units would subtend an angle of $\frac{4\pi}{9}$ radians. Converting this angle to degrees: $\frac{4\pi}{9} \times \frac{180}{\pi} = 80°$	**1 mark** Correct Answer
MA11-1	This outcome is generally assessed in all questions in this section.	

Year 11 Advanced Mathematics Yearly Examination (Task 3) 2022		
Question No. Solutions and Marking Guidelines		
Outcomes Addressed in this Question		
For example: MA11-5 interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems		
Outcome	**Solutions**	**Marking Guidelines**
	Question 11	
MA11-5	a) $\frac{dy}{dx}=\left(x^2-5\right)\left(9-6x^2\right)+\left(9x-2x^3\right)\left(2x\right)$ Or $\frac{dy}{dx}=-10x^4+57x^2-45$	**2 marks** Correct solution, giving correct derivative of function. **1 mark** Some progress towards a correct solution.
MA11-5	b) $f'(x)=\lim_{h\to 0}\frac{x^2+2hx+h^2+5x+5h-x^2-5x}{h}$ $=\lim_{h\to 0}\frac{2hx+h^2+5h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h+5)}{h}$ $=\lim_{h\to 0}2x+h+5$ $=2x+5$	**2 marks** Correct solution, giving correct substitution into $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ and correct solution $f'(x)=2x+5$. **1 mark** Some progress towards a correct solution, giving correct substitution into $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. .
MA11-5	c) $\frac{dy}{dx}=5\left(2x^2-3\right)\left(4x\right)$ Or $\frac{dy}{dx}=\left(20x\right)\left(2x^2-3\right)$	**2 marks** Correct solution, giving correct derivative of function. **1 mark** Some progress towards a correct solution.
MA11-5	d) $\frac{dy}{dx}=\frac{(3x+1)(5)+(5x+2)(3)}{(3x+1)^2}$ Or $\frac{dy}{dx}=\frac{-1}{(3x+1)^2}$ At $x=1$ $\frac{dy}{dx}=\frac{-1}{(3(1)+1)^2}$ $=-\frac{1}{16}$ $\therefore$ gradient of tangent $=-\frac{1}{16}$ when $x=1$	**3 marks** Correct solution, giving correct derivative of function and finding $m=-\frac{1}{16}$. **2 marks** Some progress towards correct solution, giving correct derivative of function. **1 mark** Some progress towards a correct solution.

MA11-5	e) $\frac{dy}{dx} = 2x - 4$ at $x = 3, \frac{dy}{dx} = 2(3) - 4 = 2$ $\therefore$ gradient of tangent to curve at $x = 3$ is $m_T = 2$ $\therefore$ gradient of normal to curve at $x = 3$ is $m_N = -\frac{1}{2}$ $\therefore$ Equation of normal is: $y - 1 = -\frac{1}{2}(x - 3)$ $x + 2y - 5 = 0$	**3 marks** Correct solution, giving correct derivative of function, finding gradient of normal at $(3,1)$ and equation of normal at $(3,1)$. **2 marks** Some progress towards correct solution, giving correct derivative of function and finding gradient of normal at $(3,1)$. **1 mark** Some progress towards a correct solution, giving correct derivative of function and finding gradient of tangent.

Outcomes Addressed in this Question

MA11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems

MA11-2 uses the concepts of functions and relations to model, analyse and solve practical problems

Outcome	Solutions	Marking Guidelines
	For example: (a) D: $x \geq 0, x \neq 3$ *alternative notation*: $[0, 3) \cup (3, \infty)$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution.
	(b) $f(x) = \dfrac{1}{x^3 - 4x}$ $f(-x) = \dfrac{1}{(-x)^3 - 4(-x)}$ $= \dfrac{1}{-x^3 + 4x}$ $= -\dfrac{1}{x^3 - 4x}$ $f(-x) = -f(x)$ $\therefore$ odd function	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution.
	(c)(i) $\text{LHS} = \dfrac{2x^2 + x - 1}{x^2 - 1}$ $= \dfrac{(2x-1)(x+1)}{(x-1)(x+1)}$ $= \dfrac{2x-1}{x-1}$ $= \dfrac{2x-2+1}{x-1}$ (this is a key step to be *shown*) $= \dfrac{2(x-1)+1}{x-1}$ $= 2 + \dfrac{1}{x-1}$ $= \text{RHS}$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution.

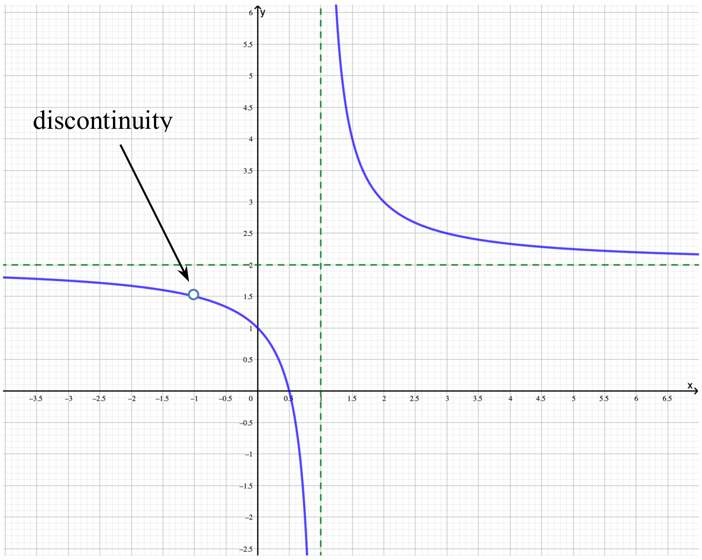
Question 12 continued…

(c)(ii) Note:

$$D: x^2 - 1 \neq 0$$

$$\therefore x \neq \pm 1$$

$$\Rightarrow \text{asymptote at } x = 1, \text{ discontinuity at } x = -1$$



3 marks
Correct solution, giving correct asymptotes and intercepts.
*Note: open circle at (-1, 1.5) was not required for full marks in this question… **this will not always be the case***
2 marks
Substantially correct solution.
1 mark
Some progress towards a correct solution.

(d)(i) $f(x) = x^2 - 9$

$$f(g(x)) = [g(x)]^2 - 9$$
$$= [3x+1]^2 - 9$$
$$= 9x^2 + 6x + 1 - 9$$
$$= 9x^2 + 6x - 8$$

2 marks
Correct solution
1 mark
Substantial progress towards correct solution.

(d)(ii) $h(2) = 9(2)^2 + 6(2) - 8$

$$= 36 + 12 - 8$$
$$= 40$$

1 mark
Correct solution

Year 11 Advanced Mathematics Yearly Examination (Task 3) 2022

Question No. 13 Solutions and Marking Guidelines

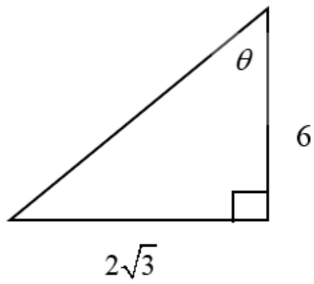
Outcomes Addressed in this Question

MA11-3 uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes

Outcome	Solutions	Marking Guidelines
MA11-3	(a)(i)	

$$\tan \angle BOC = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$$

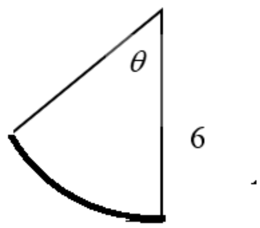
$$\angle BOC = 30^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{6}$$



1 mark
Correct solution

(ii) Area of sector

$$= \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 6^2 \times \frac{\pi}{6}$$
$$= 3\pi \text{ cm}^2$$



1 mark
Correct solution

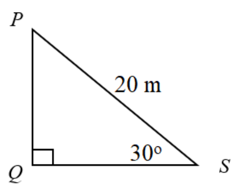
(iii) Area of shaded region $= \frac{1}{2} \times \text{base} \times \text{height} - \text{area of sector}$

$$= \frac{1}{2} \times 2\sqrt{3} \times 6 - 3\pi$$
$$= 0.97 \text{ cm}^2$$

2 marks
Correct solution
1 mark
Substantial progress towards correct solution.
or
Some progress towards a correct solution.

(b)(i) In ΔPQS

$$\sin 30^\circ = \frac{PQ}{20}$$
$$PQ = 20\sin 30^\circ$$
$$= 10 \text{ m}$$



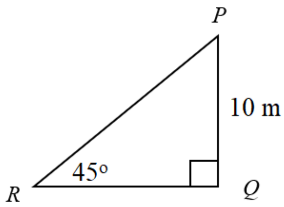
1 mark
Correct solution

(ii) In ΔPQR

$$\sin 45^\circ = \frac{PQ}{PR}$$

$$PR = \frac{10}{\sin 45^\circ}$$

$$= 10\sqrt{2}\ \text{m}$$



1 mark
Correct solution

(iii) In ΔPRS

$$RS^2 = \left(10\sqrt{2}\right)^2 + 20^2 - 2\left(10\sqrt{2}\right)(20)\cos 60^\circ$$

$$= 200 + 400 - 200\sqrt{2}$$

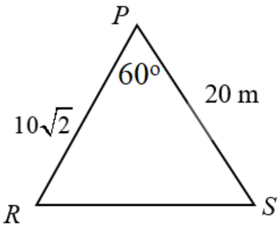
$$= 317.1573$$

$$RS = \sqrt{317.1573}$$

$$= 17.8\,\text{m}$$

$\therefore$ The distance RS is $17.8\,\text{m}$.

(Not a right angled triangle)



2 marks
Correct solution
1 mark
Substantial progress towards correct solution.
or
Some progress towards a correct solution.

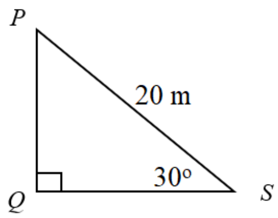
(iv) In ΔPQS

$$\cos 30^\circ = \frac{QS}{PS}$$

$$QS = 20\cos 30^\circ$$

$$= 10\sqrt{3}\ \text{m}$$

$$PQ = QR = 10\ \text{m}$$

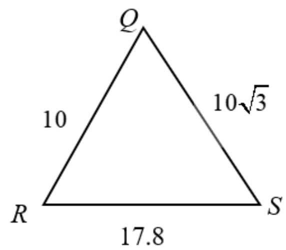


In ΔQRS

$$\cos\angle RQS = \frac{10^2 + \left(10\sqrt{3}\right)^2 - 17.8^2}{2(10)\left(10\sqrt{3}\right)}$$

$$= \frac{400 - 17.8^2}{200\sqrt{3}}$$

$$\angle RQS = 76.2^\circ$$

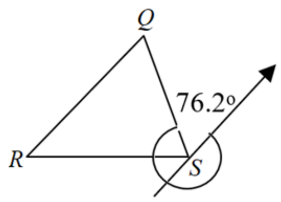


3 marks
Correct solution
2 marks
Substantial progress towards correct solution.
1 mark
Some progress towards a correct solution.

(v) Bearing

$$= 360^\circ - 76.2^\circ$$

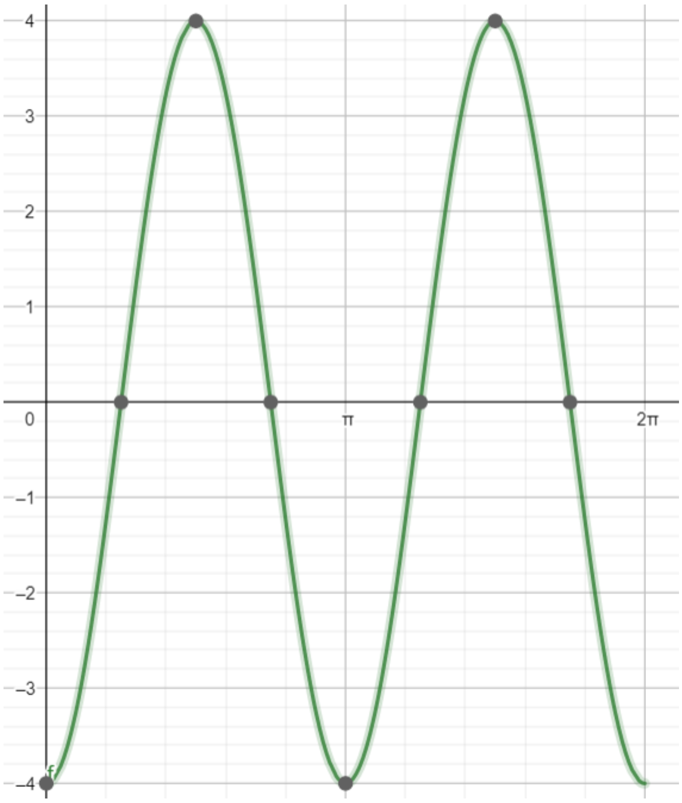
$$= 284^\circ$$



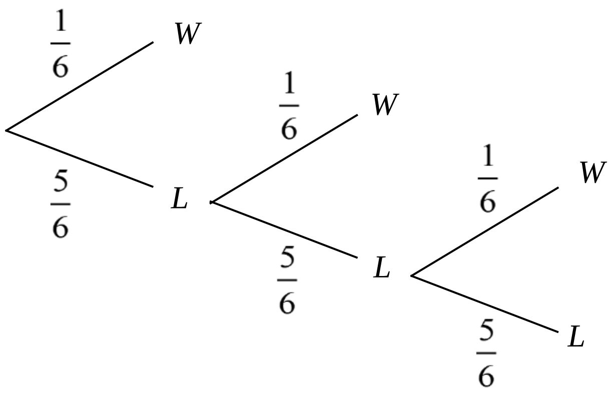
1 mark
Correct solution

Question No. 14 Solutions and Marking Guidelines

Outcomes Addressed in this Question		
MA11-4 uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities		
Question	**Solutions**	**Marking Guidelines**
(a)	$\cos\theta = -\frac{\sqrt{3}}{2}$ Related angle $= \frac{\pi}{6}$ $\theta = \pi - \frac{\pi}{6},\ \pi + \frac{\pi}{6}$ $\theta = \frac{5\pi}{6},\ -\frac{5\pi}{6}$	**2 marks** Correct solution (in radians) within the given range. **1 mark** Solution with incorrect range used OR in degrees OR substantial progress towards the correct solution.
(b)	$y = \tan x$ is undefined for: $x = 90°,\ 270°,\ 360° + 90°,\ 720° - 90°$ $x = 90°,\ 270°,\ 450°,\ 630°$	**2 marks** Provides the correct solution in degrees. **1 mark** Provides any TWO values of *x* in degrees
(c)	$\text{LHS} = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$ $= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$ $= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$ $= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$ $= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$ $= \frac{2}{\sin\theta}$ $= 2\mathrm{cosec}\,\theta$ $= \text{RHS}$	**3 Marks** Provides the correct solution. **2 Marks** Shows substantial progress by simplifying the numerator. **1 Mark** Writes the expression under a common denominator

(d)	$2\cos^2(2x) - 6\sin^2(2x) = 0$ $2\left(\cos^2(2x) - 3\sin^2(2x)\right) = 0$ $2\left(\cos(2x) + \sqrt{3}\sin(2x)\right)\left(\cos(2x) - \sqrt{3}\sin(2x)\right) = 0$ $\cos(2x) + \sqrt{3}\sin(2x) = 0$ *or* $\cos(2x) - \sqrt{3}\sin(2x) = 0$ $\tan(2x) = \frac{1}{\sqrt{3}}$ *or* $\tan(2x) = -\frac{1}{\sqrt{3}}$ $x = 75°$ *or* $x = 15°$	**3 Marks** Provides both solutions AND correct factorisation of trigonometric function. **2 Marks** Provides correct factorisation AND one other correct solution. **1 Mark** Provides one correct solution OR correct factorisation.
(e)		**2 Marks** Sketch is correct and neat with key intercept points clearly labelled. **1 Mark** Shape of the sketch is correct, however, messy or missing key intercept points.

Year 11 Question No.15	Mathematics Advanced Task 3 Solutions and Marking Guidelines	Examination 2022
MA11- 7	uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions	

Part	Solutions	Marking Guidelines
(a)	i)  $P(LLW)=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{25}{216}$	**2 marks** Correct solution **1 mark** Single error
	ii) $P(W)+P(LW)+P(LLW)=\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}+\frac{25}{216}$ $=\frac{91}{216}$	**1 mark** Correct solution
(b)	i) $E(X)=\sum f(x)x$ $=0.1\times 1+0.2\times 2+0.4\times 3+0.2\times 4+0.1\times 5$ $=3$	**1 mark** Correct Solution
	ii) $V(X)=E(X^2)-E(X)^2$ $=\sum f(x)^2x-3^2$ $=0.1^2\times 1+0.2^2\times 2+0.4^2\times 3+0.2^2\times 4+0.1^2\times 5-3^2$ $=1.2$ $\sigma=\sqrt{V(X)}$ $=1.095$	**2 marks** Correct solution **1 mark** Single error
	iii) $P(x>\mu+\sigma)=P(x>3+1.095)$ $=P(x>4.095)$ $=P(x=5)$ $=0.1$	**1 mark** Correct solution

(c)	No, since the commentator did not consider the probability of a score of ***exactly*** 100; it is not reasonable to assume it has a chance of 0%.	**1 mark** Correct conclusion with correct justification
(d)	If independent $P(X \cap Y) = P(X)P(Y)$ $= 0.3 \times 0.42$ $= 0.126$ $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$ $= 0.3 + 0.42 - 0.126$ $= 0.594$ NOTE: students should revise the deference between independent events and mutually exclusive events.	**2 marks** Correct solution **1 mark** Significant progress that would lead to a correct answer
(e)	$E(X) = \sum f(x)x$ $0.9 = 0.1 \times (-1) + a \times 0 + 0.3 \times 1 + b \times 2$ $b = 0.35$ $\sum f(x) = 1$ $0.1 + a + 0.3 + b = 1$ $0.1 + a + 0.3 + 0.35 = 1$ $a = 0.25$	**2 marks** Correct solution **1 mark** Significant progress that would lead to a correct answer