STUDENT'S NAME: _____

TEACHER'S NAME: _____

2023

HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 11 ASSESSMENT TASK 3

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a black or blue pen
- NESA approved calculators may be used
- A reference sheet is provided in the Section I booklet
- For questions in Section II, show all relevant mathematical reasoning and/or calculations
- This examination paper is not to be removed from the examination centre

Total marks: 70

Section I – 10 marks

- Attempt Questions 1 – 10. The multiple-choice answer sheet has been provided
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt Questions 11 – 15, write your solutions in the spaces provided.
- There are 5 separate question/answer booklets. Extra working pages are available if required.
- Allow about 1 hour and 45 minutes for this section.

SECTION I - 10 marks

Attempt Questions 1 - 10

Allow about 15 minutes on this section.

Use the multiple-choice answer sheet provided for Questions 1 – 10.

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1. Which of the following is the range of the function $f(x) = x^2 + 1$?

   (A) $[1, \infty)$

   (B) $(-\infty, 1]$

   (C) $[-1, 1]$

   (D) $(-\infty, \infty)$

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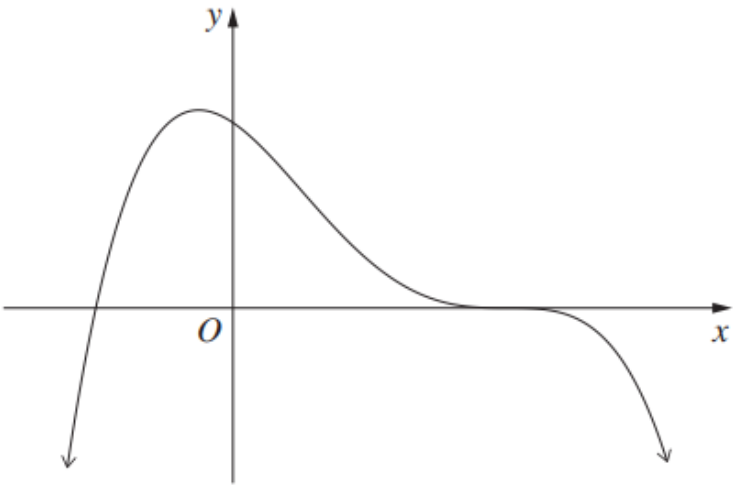
2. The graph of $y = f(x)$ is shown below.



   Which of the following could be the equation of this graph?

   (A) $y = (1 - x)(2 + x)^3$

   (B) $y = (1 + x)(2 - x)^3$

   (C) $y = (x + 1)(x - 2)^3$

   (D) $y = (x - 1)(2 + x)^3$

Section I continues on the next page ...

3. What is the period of $y = \tan x$?

(A) 3π

(B) 2π

(C) π

(D) $\frac{\pi}{2}$

4. Which of the following is the correct solution to the equation $2\cos x = \sqrt{3}$, $0 \leq x \leq 2\pi$?

(A) $x = \frac{\pi}{3}, \frac{2\pi}{3}$

(B) $x = \frac{\pi}{3}$

(C) $x = \frac{\pi}{6}, \frac{11\pi}{6}$

(D) $x = \frac{\pi}{6}, \frac{5\pi}{6}$

5. For the function $f(x) = 2x^3 - x^2 + 3x - 5$, what is the value of $f'(-1)$?

(A) 11

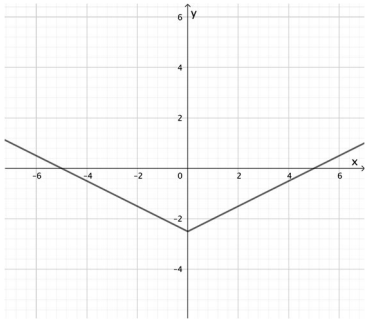
(B) 7

(C) -9

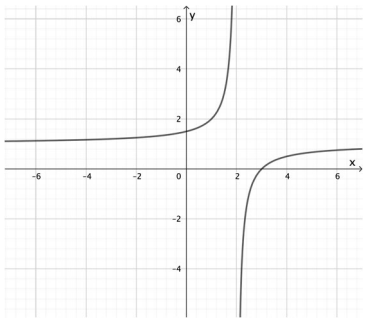
(D) -11

Section I continues on the next page ...

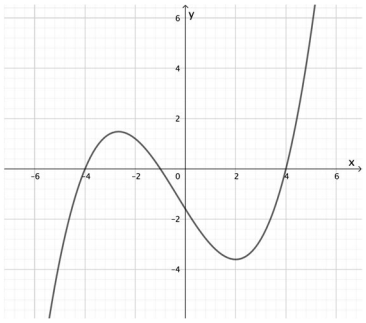
6. Three different functions are shown on the graphs below.



Graph I



Graph II



Graph III

Which of the graphs is continuous for the domain $-4 \leq x \leq 4$?

(A) Graph I only

(B) Graph II only

(C) Graph III only

(D) Graphs I and III

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7. A class of 27 students contains 14 students who swim, 11 who play football and 3 who both swim and play football.

If one student is chosen at random, what is the probability that this student either swims or plays football, but not both?

(A) $\frac{5}{27}$

(B) $\frac{19}{27}$

(C) $\frac{22}{27}$

(D) Cannot be calculated

Section I continues on the next page ...

8. The probability distribution for variable X is shown below.

x	0	1	2	3	4	5	6	7
$P(X=x)$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$

What is $P(1 < X \leq 4)$?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{5}{12}$

(D) $\frac{7}{12}$

---

9. What is the value of $\log_4 0.25$?

(A) 4

(B) 2

(C) 0

(D) -1

---

10. What is the solution to the equation $(3^x - 9)(5^x - 1) = 0$?

(A) $x = 0$ only

(B) $x = 0$, $x = 2$

(C) $x = 1$ only

(D) $x = 1$, $x = 2$

End of Section I

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NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

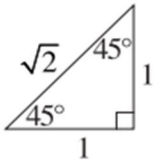
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

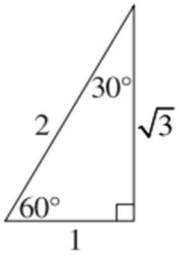
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

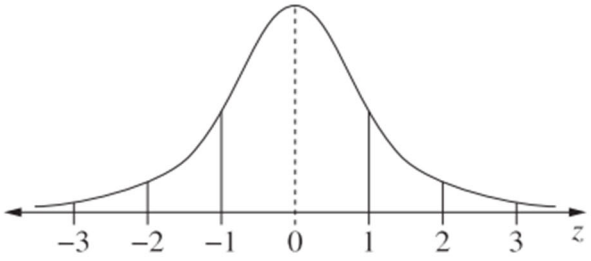
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2023 Year 11 Mathematics Advanced Task 3 Examination Section II

SECTION II **Student Name: _____**

60 marks **Teacher Name:_____**

Attempt Questions 11 - 15

Allow about 1 hours and 45 minutes on this section.

Answer each question in the spaces provided. Extra working space is available after each question. If you need to use this extra space, you must clearly indicate this in the main solution space, and then clearly indicate the question number and part that you are answering in the extra space.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (12 marks) **Marks**

(a) Consider $f(x) = x^2 - 1$ and $g(x) = x - 1$

(i) Find $f(g(x))$ in its simplest form. **1**

(ii) Determine whether $f(g(x))$ is odd, even or neither. **1**

Marks

(b) Solve $|x - 3| = 7$ **1**

(c) Simplify $\frac{-2}{x^2-4} \times \frac{x-2}{x+2}$ **2**

(d) Find the values of k for which the quadratic equation $3x^2 + 2x + k = 0$ has no real roots. **2**

Marks

(e) Find the equation of the circle with centre $A(1,0)$ that passes through the point $B(4,1)$. **2**

(f) Sketch $y = f(x)$ for the function $f(x) = (x+3)^2 + 1$ for the domain $(-4,-1]$. Show all important features. **3**

End of Question 11

Section II: Extra writing space

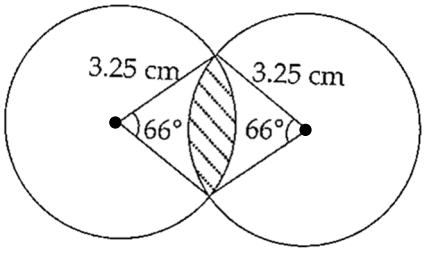
If you use this space, clearly indicate which question you are answering.

Student Name: _____

Teacher Name: _____

Question 12 (12 marks) **Marks**

(a) The image shown below has a vertical axis of symmetry. The centres of the circles are shown.



(i) Express 66° in radians. **1**

(ii) Find the area of the shaded section. **2**

(b) Given that $\sin\theta = \frac{5}{13}$, determine the value of $\cos\theta$, given $90° < \theta < 180°$. **1**

Marks

(c) Prove the following result. **3**

$$\frac{\operatorname{cosec} x + \cot x}{\tan x + \sin x} = \cot x \operatorname{cosec} x$$

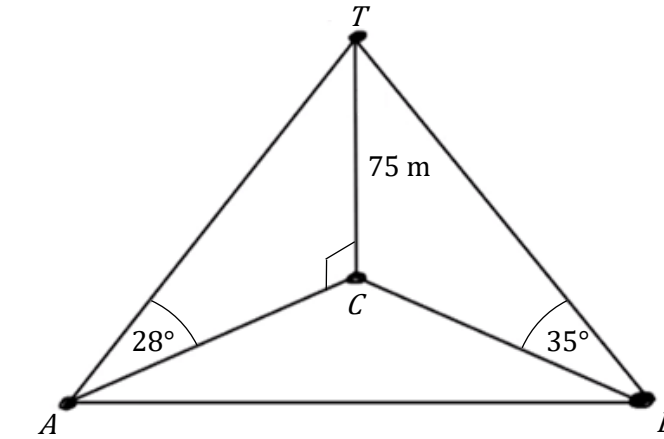
(d) A triangular garden bed has side lengths 5.6 m, 4.3 m and 4.7 m. **2**

Determine the size of the largest angle, correct to the nearest minute.

Marks

(e) The diagram shows a 75 m vertical air traffic control tower. Jaden the Air Traffic Controller is located at the top of the tower (T) and observes two planes on the ground. Plane 1 (located at A) is due west of plane 2 (located at B).

The angle of elevation from plane 1 to the top of the tower is 28° and the angle of elevation from plane 2 to the top of the tower is 35°.



NOT TO SCALE

The bearing of the base of the tower (C) from plane 1 is 050°T, and the bearing of the base of the tower (C) from plane 2 is 300°T.

Show that $\angle CAB = 40°$ and hence find the distance between the two planes, correct to the nearest metre. **3**

End of Question 12

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: _____

Teacher Name:_____

Question 13 (12 marks) **Marks**

(a) Differentiate the function $f(x) = 4 + 7x - x^2$ from **first principles**. **2**

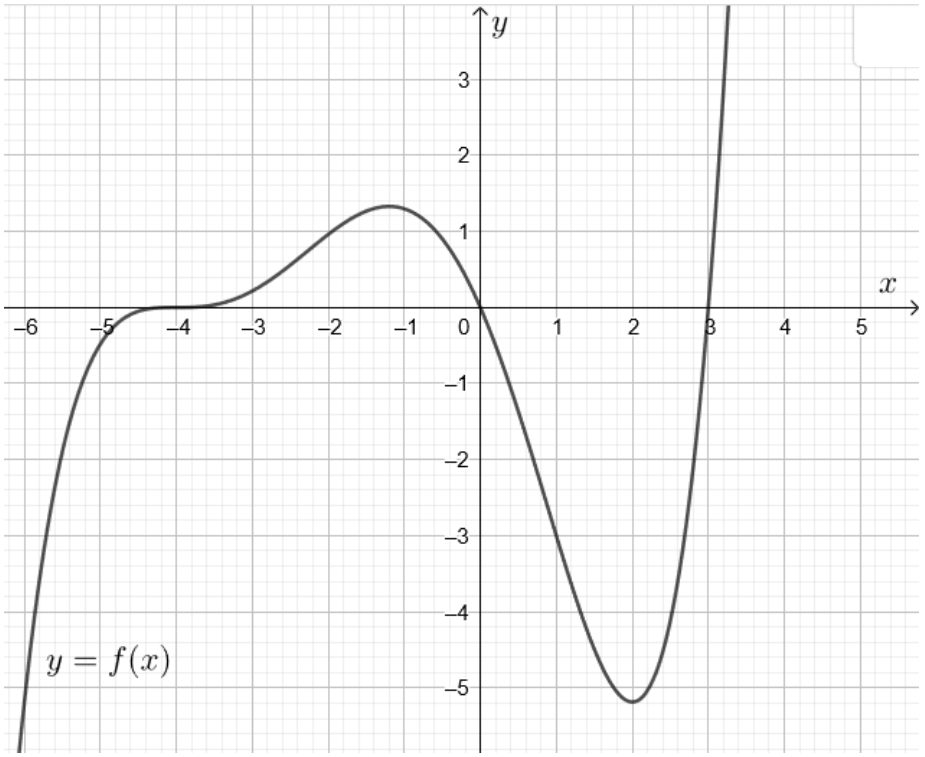
(b) Differentiate the following expressions with respect to t :

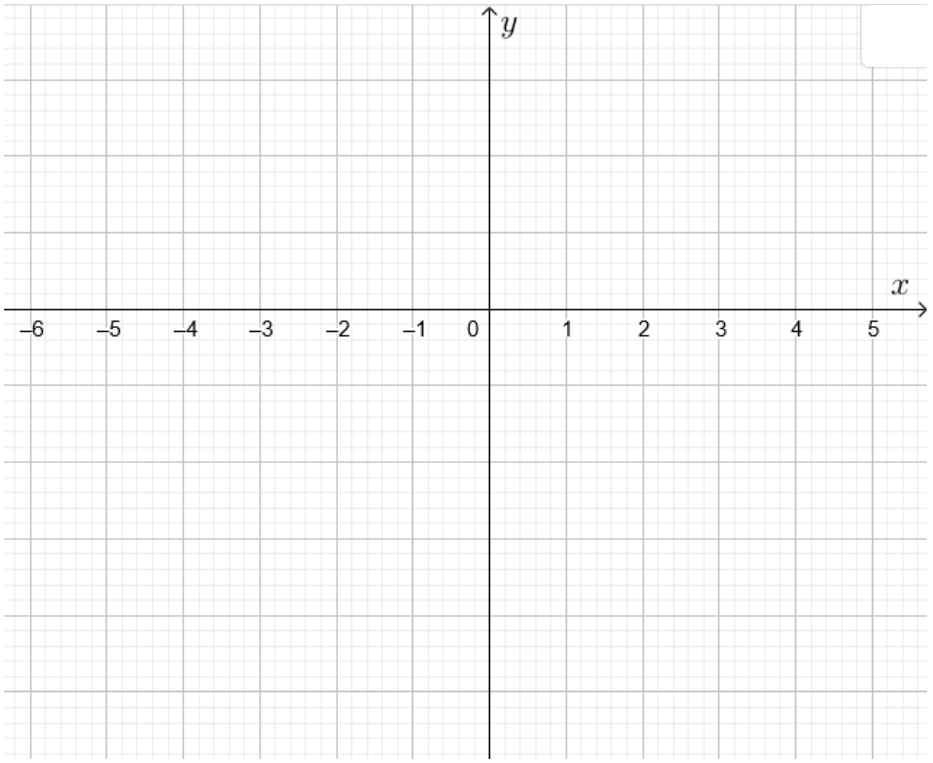
(i) $2(5t + 6)^8$ **1**

(ii) $\frac{t^4+1}{t}$ **2**

Marks

(c) The graph of $y = f(x)$ is shown below. **3**
On the second set of axes draw a neat sketch of the derivative function $y = f'(x)$.
Ensure that critical features are shown in an accurate position in relation to the x axis.





Marks

(d)

(i) Find the equation of the tangent to the curve $y = x^3 - 4x + 3$ at the point where $x = 1$. **3**

(ii) Is the curve increasing or decreasing at the point where $x = 1$? Justify your answer. **1**

End of Question 13

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: _____

Teacher Name: _____

Question 14 (12 marks) **Marks**

(a) Three employees, Alice, Ben and Chloe, work for the same company.
They respectively handle 48%, 30% and 22% of the company's orders.
It has been found that they make mistakes in 10%, 5% and 2% of their orders respectively.

(i) Use a tree diagram to display this information. **2**

(ii) Calculate the probability that an order handled by the company has a mistake. **2**

Marks

(b) A website claims to be 98% accurate at detecting plagiarism.

That is, it will return the correct result for a paper,
"Plagiarised" or "Not Plagiarised," 98% of the time.

It is known that 5% of submitted papers at a school are plagiarised.

If a teacher checks a randomly selected paper using the website and gets the result "Plagiarised," what is the probability that it is a plagiarised paper?

2

(c) A discrete random variable X has the probability distribution table shown.

x	11	12	13	14
$P(X = x)$	0.2	0.3	0.1	0.4

(i) Calculate the expected value of X. **1**

(ii) Calculate the variance of X. **2**

Marks

(d) A discrete random variable X has the probability distribution table shown.

x	-3	-1	1	3	5
$P(X=x)$	a	$\frac{1}{5}$	$\frac{1}{15}$	b	$\frac{1}{5}$

Find the values of a and b given that $E(X) = 1\frac{4}{15}$. **3**

End of question 14

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Student Name: _____

Teacher Name: _____

Question 15 (12 marks) **Marks**

(a) Solve $2^{x+1} = 9$, give your answer correct to three decimal places. **2**

(b) Solve the equation $\log_{10} x = 2\log_{10} 3 + \log_{10} 5 - \log_{10} 2 - 1$ **2**

(c) The compound interest formula is $A = P(1 + \frac{r}{100})^n$ where P is the initial money invested and A is the amount that P grows to after n periods of time, with interest applied at r% per period.

If \$2000 is initially invested at 6% p.a, how long will it take for the investment to triple in its value? (Answer to the nearest year) **2**

Marks

(d) Sketch the graph of $y = \log_5 x$ showing all important features. **2**

Marks

(e) The magnitude of an earthquake is measured on the logarithmic scale known as the Richter scale. If an earthquake has intensity I, its magnitude $M(I)$ is defined as $M(I) = \log_{10}\left(\frac{I}{I_0}\right)$, where I_0 is a minimum level of intensity used for comparisons. In 1989 in Newcastle, Australia an earthquake measured 5.9 on the Richter scale and resulted in destruction and the loss of life. In February 2023, an earthquake of magnitude 7.8 devastated the country of Turkey.

(i) How many times more intense was the earthquake in Turkey than the one in Newcastle, Australia?

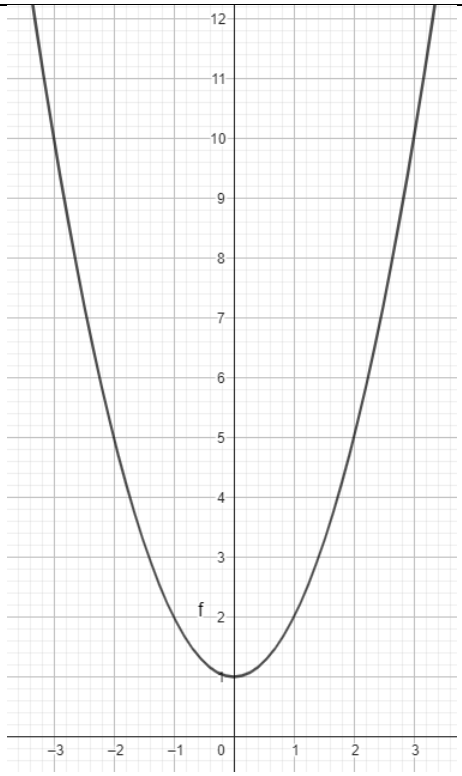
2

(ii) What is the Richter scale magnitude of an earthquake which is 20 times more intense than the earthquake in Newcastle, Australia? **2**

End of question 15

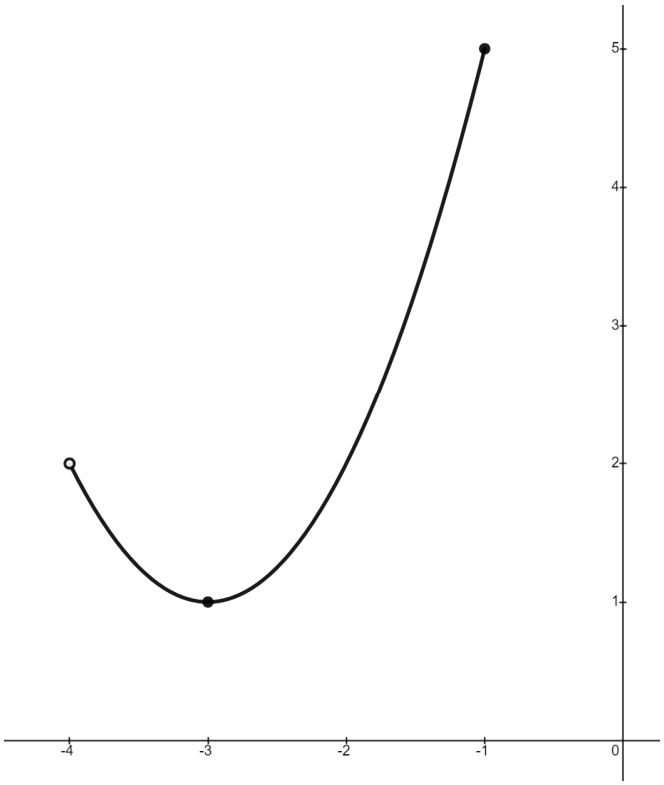
Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Year 11 Task 3 2023	
Multiple Choice - Solutions	
Question	**Solutions**
1	 The parabola is concave up and its vertex is at (0, 1) hence the smallest y-value is 1. **Answer: A**
2	The function is concave down, so options A or B are potential answers. Option B gives the roots in the correct relative positions according to the diagram. **Answer: B**
3	What is the period of $y = \tan x$ is π **Answer: C**
4	Solve $2\cos x = \sqrt{3}$ $\cos x = \frac{\sqrt{3}}{2}$ Cos is positive in the 1st and 4th quadrant. $\therefore x = \frac{\pi}{6}, \frac{11\pi}{6}$ **Answer: C**
5	$f(x) = 2x^3 - x^2 + 3x - 5$ $f'(x) = 6x^2 - 2x + 3$ $f'(-1) = 6(-1)^2 - 2(-1) + 3 = 11$ **Answer: A**

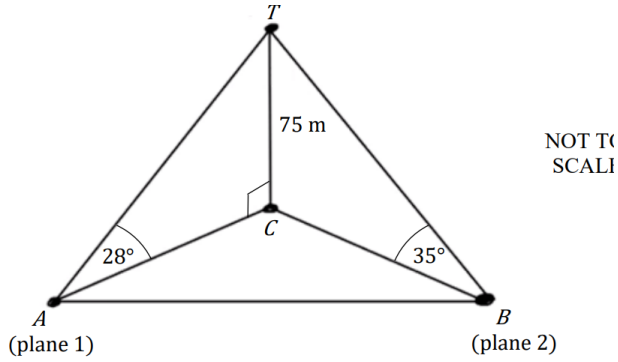
6	Continuous means with no gaps. **Answer: D**
7	$14 + 11 + 3 = 28$ Hence the 3 students who both swim and play football are also included in the totals given. Number of students who only swim is $(14 - 3 = 11)$ and the number of students who only play football is $(11 - 3 = 8)$. Therefore, the number of students that either swim of play football but not both is $11 + 8 = 19$ Therefore, the probability is $\frac{19}{27}$ **Answer: B**
8	$P(1 < X \leq 4) = \frac{1}{12} + \frac{1}{4} + \frac{1}{12} = \frac{5}{12}$ **Answer: C**
9	$\log_4 0.25 = \frac{\log_{10} 0.25}{\log_{10} 4} = -1$ OR $\log_4 0.25 = \log_4 \frac{1}{4}$ $= \log_4 4^{-1}$ $= -1 \log_4 4$ $= -1$ **Answer: D**
10	$(3^x - 9)(5^x - 1) = 0$ $3^x - 9 = 0 \quad , \quad 5^x - 1 = 0$ $3^x = 9 \quad , \quad 5^x = 1$ $\therefore x = 2 \quad , \quad x = 0$ **Answer: B**

Task 3 2023		
Question No. 11	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		
MA11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems		
Outcome	**Solutions**	**Marking Guidelines**
MA11-1	**(a)** i) $(x-1)^2-1=x^2-2x$ ii) $f(g(-a))=(-a)^2-2(-a)=a^2+2a$ $f(x)$ is neither odd or even as $f(-a)\neq f(a)$ and $f(-a)\neq -f(a)$	**1 mark** Correct solution **1 mark** Correct solution with complete reasoning
MA11-1	**(b)** $\lvert x-3\rvert=7$ Case 1: $x-3=7$ Case 2: $-x+3=7$ $x=10$ $x=-4$	**1 mark** Correct solution
MA11-1	**(c)** $\frac{-2}{x^2-4}\times\frac{x-2}{x+2}=\frac{-2}{(x-2)(x+2)}\times\frac{x-2}{x+2}$ $=\frac{-2}{(x+2)^2}$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution
MA11-1	**(d)** If no real roots, then $\Delta<0$ $2^2-4(3)k<0$ $k>\frac{1}{3}$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution
MA11-1	**(e)** 1) Centre at (1,0) gives $(x-1)^2+y^2=r$ At $(4,1)$ we have $(4-1)^2+1^2=r$ $r=10$ So the equation of the circle is $(x-1)^2+y^2=10$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution

MA11-1	**(f)**  	**3 marks** Correct graph drawn to scale with correct boundary points **2 marks** Correct graph with incorrect end points or other mistake **1 mark** One feature of the graph is correct

Year 11 Advanced Mathematics Task 3 2023		
Question 12	**Solutions and Marking Guidelines**	
Part	**Solutions**	**Marking Guidelines**
a.	i) $66° = 66 \times \frac{\pi}{180}$ $= \frac{66\,\pi}{180}$ $= \frac{11\,\pi}{30} = 1.15$ radians	1 mark for correct conversion
	ii) Area of shaded region $= 2 \times \frac{1}{2} r^2(\theta - \sin\theta)$ $A = 2 \times \frac{1}{2}(3.25)^2\left(\frac{11\,\pi}{30} - \sin\frac{11\,\pi}{30}\right)$ $A = 2.51$ cm^2	2 marks for correct solutions 1 mark for some progress towards correct solution
b.	Given that $\sin\theta = \frac{5}{12}$ $\cos\theta = -\frac{12}{13}$ as $90° \leq \theta \leq 180°$	1 mark for correct value of $\cos\theta$ in the interval given.
c.	$\text{LHS} = \frac{\operatorname{cosec} x + \cot x}{\tan x + \sin x} = \cot x \operatorname{cosec} x$ $= \frac{\frac{1}{\sin x} + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \sin x}$ $= \frac{\frac{1 + \cos x}{\sin x}}{\frac{\sin x + \cos x \sin x}{\cos x}}$ $= \frac{1 + \cos x}{\sin x} \times \frac{\cos x}{\sin x(1 + \cos x)}$ $= \frac{\cos x}{\sin x \ \sin x}$ $= \frac{\cos x}{\sin x} \times \frac{1}{\sin x}$ $= \cot x \operatorname{cosec} x = \text{RHS}$	3 marks for correct proof 2 marks significant progress towards correct solution 1 mark for limited progress towards correct solution

Question 12 continued **Solutions and Marking Guidelines**

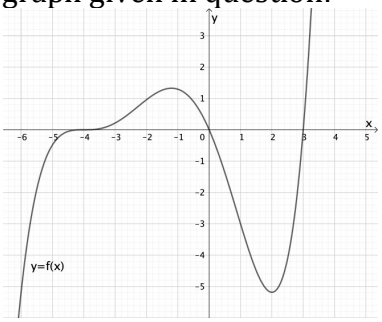
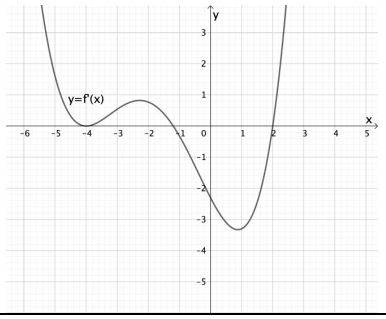
Part	Solutions	Marking Guidelines
d.	In a triangle the largest angle is opposite the largest time. $(5.6)^2 = (4.7)^2 + (4.3)^2 - 2(4.7)(4.3)\cos\vartheta$ $0.228 = \cos\vartheta$ $\vartheta = 76° 49'$	1 mark for correct substitution into cosine formula 1 mark for correct angle to nearest minute
e.	 NOT TO SCALE $\angle CAB = 90° - 50° = 40°$ $\angle CBA = 300° - 270° = 30°$ $\therefore \angle ACB = 180° - 40° - 30° = 110°$ $\tan 28 = \frac{75}{AC}$ $\therefore AC = \frac{75}{\tan 28} \approx 141$ m using the sine rule $\frac{AB}{\sin 110} = \frac{141}{\sin 30}$ $AB = 265$ $\therefore$ the distance between the two planes is 265 m	3 marks for correct solution 2 marks significant progress towards correct solution 1 mark for limited progress towards correct solution

Year 11 Mathematics Yearly Examination (Assessment Task 3) 2023

Question No. 13 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA11-5 interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems

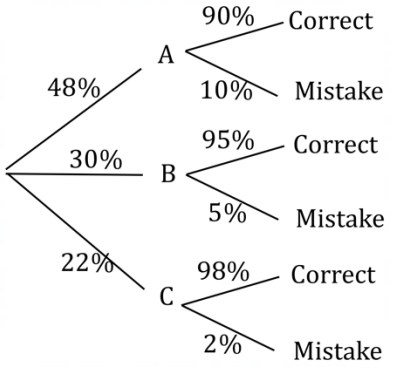
Outcome	Solutions	Marking Guidelines
MA11-5	**(a)** $f(x) = 4 + 7x - x^2$ $f(x+h) = 4 + 7(x+h) - (x+h)^2$ $= 4 + 7x + 7h - x^2 - 2xh - h^2$ $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0} \frac{4+7x+7h-x^2-2xh-h^2-(4+7x-x^2)}{h}$ $= \lim_{h\to 0} \frac{4+7x+7h-x^2-2xh-h^2-4-7x+x^2}{h}$ $= \lim_{h\to 0} \frac{7h-2xh-h^2}{h}$ $= \lim_{h\to 0} \frac{h(7-2x-h)}{h}$ $= \lim_{h\to 0} 7 - 2x - h$ $= 7 - 2x$	**2 marks** Correct solution showing full working. **1mark** Substantial progress towards correct solution.
MA11-5	**(b) (i)** $\frac{d}{dt} 2(5t+6)^8 = 16(5t+6)^7 \times 5$ $= 80(5t+6)^7$	**1 mark** Correct solution.
MA11-5	**(ii)** $\frac{d}{dt}\left(\frac{t^4+1}{t}\right) = \frac{d}{dt}\left(t^3 + \frac{1}{t}\right)$ $= \frac{d}{dt}(t^3 + t^{-1})$ $= 3t^2 - t^{-2}$ $= 3t^2 - \frac{1}{t^2}$ $= \frac{3t^4 - 1}{t^2}$	**2 marks** Correct solution showing full working. **1mark** Substantial progress towards correct solution.
MA11-5	**(c)** Function graph given in question:  Gradient function derived from above function graph.  	**3 marks** Graph shows these features: 1. Turning point at (-4, 0) derived from horizontal point of inflexion on function graph. 2. x-intercepts at (-1.2, 0) and (2, 0) derived from turning points on function graph. 3. Turning points at $x = -2.3$ and $x = 0.9$ derived from points of inflrexion on function graph **2 marks** Graph shows two of the three above features. **1 mark** Graph shows only 1 of the three features.

MA11-5	**(d) (i)** $y = x^3 - 4x + 3$ $\frac{dy}{dx} = 3x^2 - 4$ When $x = 1$, $\frac{dy}{dx} = 3 - 4 = -1$ and, $y = 1 - 4 + 3 = 0$ $\therefore$ Tangent has gradient, $m = -1$ and it passes through $(1, 0)$ Equation of tangent: $y - y_1 = m(x - x_1)$ $y - 0 = -1(x - 1)$ $= -x + 1$ $y = -x + 1$	**3 marks** Correct solution demonstrating all correct steps. **2 marks** Substantial progress towards full and correct solution. **1 mark** Some progress towards a correct solution.
MA11-5	**(ii)** Curve is decreasing, since the derivative/gradient of the tangent is negative there. ie. $m = -1$	**1 mark** Correct answer with justification.

Year 11	Mathematics Advanced	Assessment Task 3 2023
Question Number 14	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions

	Solutions	Marking Guidelines
(a) (i)	 Outcomes A Correct A Mistake B Correct B Mistake C Correct C Mistake	**2 marks:** Correct tree diagram and lists outcomes **1 mark:** Correctly sets up the branches of the tree diagram only **or** equivalent merit
(a) (ii)	$P(\text{Mistake}) = P(\text{A Mistake}) + P(\text{B Mistake}) + P(\text{C Mistake})$ $= 0.48 \times 0.10 + 0.30 \times 0.05 + 0.22 \times 0.02$ $= 0.0674$ (or 6.74%) $\therefore$ Probability of a mistake is 6.74%	**2 marks:** Correct answer. **1 mark:** Uses the probability rules appropriately, **or** equivalent merit.
(b)	$P(\text{plagiarised}\mid\text{positive result}) = \dfrac{P(\text{plagiarised} \cap \text{positive result})}{P(\text{positive result})}$ $= \dfrac{0.05 \times 0.98}{0.05 \times 0.98 + 0.95 \times 0.02}$ $= 0.72 = 72\%$ (Denominator must include both true positives and false positives)	**2 marks:** Correct solution **1 mark:** Attempts to establish conditional probability situation, **or** equivalent merit.
(c) (i)	$E(X) = \sum x\,p(x)$ $= 11 \times 0.2 + 12 \times 0.3 + 13 \times 0.1 + 14 \times 0.4$ $= \frac{127}{10} = 12.7$ or $E(X) = \frac{127}{10} = 12.7$, from a calculator	**1 mark:** Correct solution

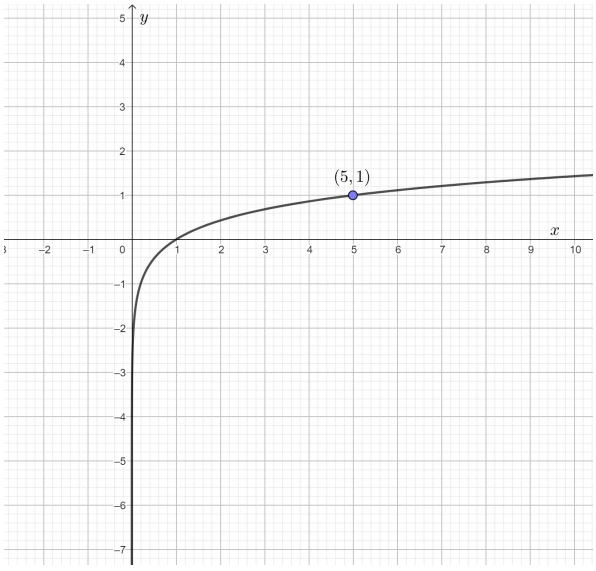
(c) (ii)	$Var(X) = \sum (x-\mu)^2 p(x)$ $= (11-12.7)^2 \times 0.2 + (12-12.7)^2 \times 0.3$ $+ (13-12.7)^2 \times 0.1 + (14-12.7)^2 \times 0.4$ $= \frac{141}{100} = 1.41$ or $Var(X) = \sum [x^2 p(x)] - \mu^2 = E(X^2) - \mu^2$ $= 11^2 \times 0.2 + 12^2 \times 0.3 + 13^2 \times 0.1 + 14^2 \times 0.4$ $- (12.7)^2$ $= \frac{141}{100} = 1.41$ or determines $Var(X) = 1.41$ using a calculator	**2 marks:** Correct solution **1 mark:** Attempts to use summation of at least two terms **or** uses a calculator and calculates the standard deviation only
(d)	$a + \frac{1}{5} + \frac{1}{15} + b + \frac{1}{5} = 1 \quad \ldots\ldots(1)$ $-3.a - 1.\frac{1}{5} + 1.\frac{1}{15} + 3.b + 5.\frac{1}{5} = 1\frac{4}{15} \quad \ldots\ldots(2)$ (1) becomes $a + b = \frac{8}{15} \quad \ldots\ldots(3)$ (2) becomes $-a + b = \frac{2}{15} \quad \ldots\ldots(4)$ (4) + (3) gives $2b = \frac{10}{15}$ $\therefore b = \frac{1}{3}$ Substitute $b = \frac{1}{3}$ into (3) $\therefore a = \frac{1}{5}$	**3 marks:** Correct solution **2 marks:** Correctly determines a pair of simultaneous equations involving a and b, **or** equivalent merit **1 mark:** Correctly determines only one equation involving a and b, **or** equivalent merit

Year 11 Mathematics Yearly Examination (Assessment Task 3) 2023

Question No. 15 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA11-6 manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems

Outcome	Solutions	Marking Guidelines
MA11-6	**(a)** $2^{x+1} = 9$ $\log_{10}2^{x+1} = \log_{10}9$ $(x+1)\log_{10}2 = \log_{10}9$ $x+1 = \frac{\log_{10}9}{\log_{10}2}$ $x = \frac{\log_{10}9}{\log_{10}2} - 1$ $\therefore x = 2.170\ (3\ dp)$	**2 marks** Correct solution showing full working. **1mark** Substantial progress towards correct solution.
MA11-6	**(b)** $\log_{10}x = 2\log_{10}3 + \log_{10}5 - \log_{10}2 - 1$ $= \log_{10}3^2 + \log_{10}5 - \log_{10}2 - \log_{10}10$ $= \log_{10}\left(\frac{9\times5}{2\times10}\right)$ $= \log_{10}\left(\frac{9}{4}\right)$ $\therefore x = \frac{9}{4}$	**2 marks** Correct solution showing full working. **1mark** Substantial progress towards correct solution.
MA11-6	**(c)** $A = P\left(1+\frac{r}{100}\right)^n$ $6000 = 2000(1+0.06)^n$ $3 = 1.06^n$ $\log_{1.06}3 = n$ $n = \frac{\log3}{\log1.06}$ $n = 18.85$ $\therefore n = 19$ years	**2 marks** Correct solution showing full working. **1mark** Substantial progress towards correct solution.
MA11-6	**(d)**  	**2 marks** Graph shows these features: 1. x intercept at (1,0) 2. Additional point e.g (5,1) 3. Correct shape **1 mark** Graph shows 2 of the three features.

MA11-6	**(e) (i)** $M(I_N) = 5.9$ $\log_{10}\frac{I_N}{I_0} = 5.9$ $10^{5.9} = \frac{I_N}{I_0}$ $I_N = 10^{5.9} \times I_0$ $M(I_T) = 7.8$ $\log_{10}\frac{I_T}{I_0} = 7.8$ $10^{7.8} = \frac{I_T}{I_0}$ $I_T = 10^{7.8} \times I_0$ $\frac{I_T}{I_N} = \frac{10^{7.8} \times I_0}{10^{5.9} \times I_0}$ $= 10^{1.9}$ $=79.4$ The earthquake in Turkey was approximately 79.4 times more intense than the earthquake in Newcastle. **(ii)** $I_N = 10^{5.9} \times I_0$ $I_x = 20 \times 10^{5.9} \times I_0$ $M(I_x) = \log_{10}\frac{I_x}{I_0}$ $M(I_x) = \log_{10}\frac{20 \times 10^{5.9} \times I_0}{I_0}$ $M(I_x) = \log_{10}(20 \times 10^{5.9})$ $M(I_x) = 7.20$ The Richter scale magnitude of an earthquake which is 20 times more intense than the earthquake in Newcastle is 7.2.	**2 marks** Correct solution showing full working. **1mark** Some progress towards correct solution. **2 marks** Correct solution showing full working. **1mark** Some progress towards correct solution.