

HURLSTONE
AGRICULTURAL HIGH
SCHOOL

STUDENT'S NAME: _____

TEACHER'S NAME: _____

2024

ASSESSMENT TASK 3
YEARLY EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using a black or blue pen
- NESA approved calculators may be used
- A reference sheet has been provided in the Section I booklet
- For questions in Sections II – VI, **show all relevant mathematical reasoning and/or calculations**

Total marks:
70

Section I: 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10.
A multiple-choice answer sheet has been provided (page 9)
- Allow about 15 minutes for this section

Sections II – VI: 60 marks (pages 11 – 41)

- Attempt Questions 11 – 33.
Write your solutions in the spaces provided
- There are **5 separate question/answer booklets**
Extra working pages are available if required
- Allow about 1 hour and 45 minutes for these sections

This examination paper is not to be removed from the examination centre

Section I

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes on this section.

Use the multiple-choice answer sheet provided for Questions 1 – 10.

1. Which of the following is the solution to the equation $|x - 7| = 6$?

A. $x = 13$

B. $x = 1$ or $x = 13$

C. $x = -13$ or $x = -1$

D. $x = -13$ or $x = 13$

2. Given a set of coordinates $(-4, -2)$, $(-4, -5)$, $(-1, 0)$, $(3, -2)$ and $(5, -1)$, which of the following best describes this relation?

A. One-to-one

B. One-to-many

C. Many-to-one

D. Many-to-many

3. What is the exact value of $\sec 240^\circ$?

A. 2

B. -2

C. $\frac{2}{\sqrt{3}}$

D. $-\frac{2}{\sqrt{3}}$

4. What are the solutions to the equation $2 \sin x + \sqrt{3} = 0$ in the interval $-\pi \leq x \leq \pi$?

A. $x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

B. $x = -\frac{\pi}{6}$ or $x = -\frac{5\pi}{6}$

C. $x = \frac{\pi}{3}$ or $x = \frac{2\pi}{3}$

D. $x = -\frac{\pi}{3}$ or $x = -\frac{2\pi}{3}$

Section I continued on next page ...

5. Given $f(x) = -3x^4 + 6x - 1$, what is the value of $f'(1)$?

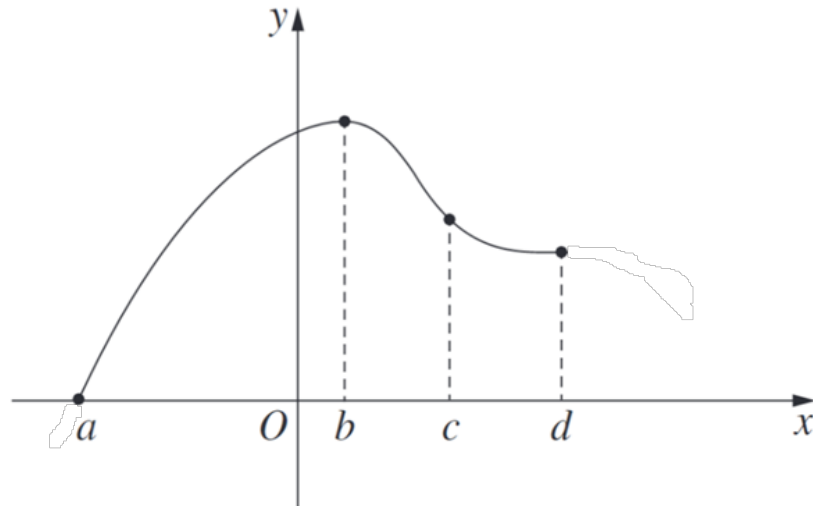
A. -18

B. -6

C. 2

D. 6

6. The diagram below shows the graph of a function $y = f(x)$.



For what values of x is the function increasing?

A. $a < x < b$

B. $b < x < d$

C. $x < b$

D. $x > a$

7. If x and y are positive numbers and $y \neq 1$, which of the following is/are true?

I. $\log_a(x+y) = \log_a x + \log_a y$

$$\text{II.} \quad \frac{\log_a x}{\log_a y} = \log_a \frac{x}{y}$$

III. $\log_a x^y = y \log_a x$

A. I only

B. II only

C. III only

D. I and III only

Section I continued on next page ...

8. If $4^{2x+1} = 8$, which of the following is the correct expression for 2^x ?

A. $\sqrt[4]{2}$

B. $\sqrt{2}$

C. $\sqrt{8}$

D. 2

9. In one class at HAHS, 24 out of 30 students surveyed said that they take a train to school. Based on this survey, how many out of the 960 students at HAHS take a train to school?

A. 576

B. 768

C. 800

D. 936

10. It is given that $P(A|B) = 0.6$ and $P(B) = 0.15$. Given that A and B are independent, what is the value of $P(A)$?

A. 0.09

B. 0.15

C. 0.25

D. 0.6

~ End of Section I ~

Mathematics Advanced

Mathematics Extension 1

Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

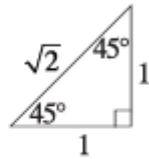
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1+t^2}$$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2} (1 - \cos 2nx)$$

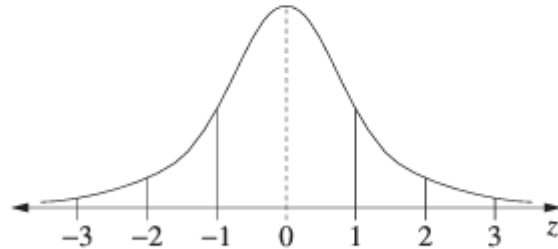
$$\cos^2 nx = \frac{1}{2} (1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z -scores between -1 and 1
- approximately 95% of scores have z -scores between -2 and 2
- approximately 99.7% of scores have z -scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function

Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \{f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})]\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underline{u}| = |x_1\underline{i} + y_1\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1x_2 + y_1y_2,$$

$$\text{where } \underline{u} = x_1\underline{i} + y_1\underline{j}$$

$$\text{and } \underline{v} = x_2\underline{i} + y_2\underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

Complex Numbers

$$\begin{aligned} z &= a + ib = r(\cos \theta + i \sin \theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n(\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$



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ASSESSMENT TASK 3
YEARLY EXAMINATION

Mathematics Advanced

Section I – MULTIPLE CHOICE ANSWER SHEET

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ☒ B ☒ ^{correct} C ☐ D ☐

1.	A	☐	B	☐	C	☐	D	☐
2.	A	☐	B	☐	C	☐	D	☐
3.	A	☐	B	☐	C	☐	D	☐
4.	A	☐	B	☐	C	☐	D	☐
5.	A	☐	B	☐	C	☐	D	☐
6.	A	☐	B	☐	C	☐	D	☐
7.	A	☐	B	☐	C	☐	D	☐
8.	A	☐	B	☐	C	☐	D	☐
9.	A	☐	B	☐	C	☐	D	☐
10.	A	☐	B	☐	C	☐	D	☐

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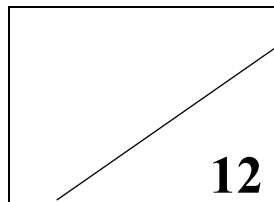
ASSESSMENT TASK 3
YEARLY EXAMINATION

Mathematics Advanced

Section II

Instructions

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- Write using black or blue pen
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- You may not remove any booklets, used or unused, from the examination room.



SECTION II

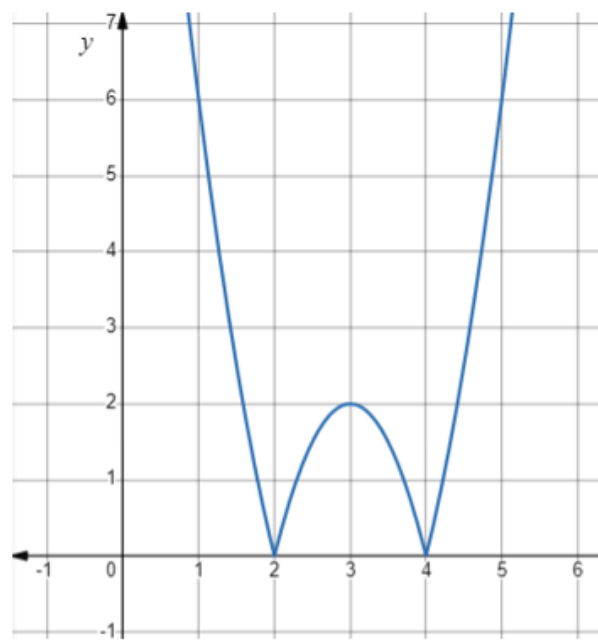
12 marks
Attempt Questions 11 – 16
Allow about 21 minutes on this section.

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.
Answer each question in the spaces provided.

2024 Mathematics Advanced Yearly Examination Section II Marks

Question 11

The graph of a function $y = f(x)$ is shown below. 1



State the range for the function $y = f(-x)$.

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Section II continued on next page ...

Question 12

A rectangle has an area of 12 cm^2 and length of $2 + \sqrt{7}\text{ cm}$.

2

Find its width in the form $a + b\sqrt{7}$, where a and b are integers.

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Question 13

Determine if the function $f(x) = x^2 + 4x + 1$ is odd, even or neither. Justify your answer.

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Section II continued on next page ...

Question 14

A function is defined as follows:

$$f(x) = \begin{cases} x^2 - x + 2 & \text{when } x < 0 \\ x^3 & \text{when } x \geq 0 \end{cases}$$

- a) Evaluate $f(-2) + f(5)$ 1

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- b) Evaluate $f(-n^2)$. 1

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Question 15

A piece of wire is 10 cm long. The wire is cut into two pieces, then each piece is bent into a square. 2
Let A be the sum of the total area of both squares and x be the length of one of the pieces. What is the expression for A in terms of x ?

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Section II continued on next page ...

Question 16

A function is given by $f(x) = x^2 + (k + 3)x + k$, where k is a real constant.

- a) Show that the discriminant can be expressed in the form $(k + m)^2 + n$, where m and n are integers. 2

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- b) Show that for all values of k , the equation $f(x) = 0$ has real roots. 1

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~ End of Section II ~

Section II: Extra Writing Space

If you use this space, clearly indicate the question you are answering.



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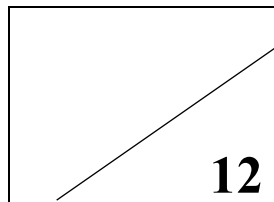
ASSESSMENT TASK 3
YEARLY EXAMINATION

Mathematics Advanced

Section III

Instructions

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SECTION III

12 marks
Attempt Questions 17 – 21
Allow about 21 minutes on this section.

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.
Answer each question in the spaces provided.

2024 Mathematics Advanced Yearly Examination Section III Marks

Question 17

How many solutions for x are there in the equation $\cos x = 1$ in the interval $[0^\circ, 720^\circ]$? 1

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Question 18

It is given that $\sin \theta = -\frac{20}{29}$ and $\cos \theta > 0$. Find the exact value of $\tan \theta$. 2

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Section III continued on next page ...

Question 19

Prove that:

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$$\frac{\sin^2 x}{1 + \cos x} + \frac{\sin^2 x}{1 - \cos x} = 2$$

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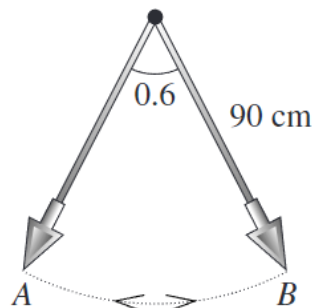
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Section III continued on next page ...

Section III continued ...

Question 20

A pendulum is 90 cm long and swings through an angle of 0.6 radians. The extreme positions of the pendulum are indicated by the points A and B in the diagram.



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- a) Find the length of the arc AB . 1

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- b) Find the straight-line distance between the extreme positions of the pendulum. 2

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- c) Find the area of the sector swept out by the pendulum. 1

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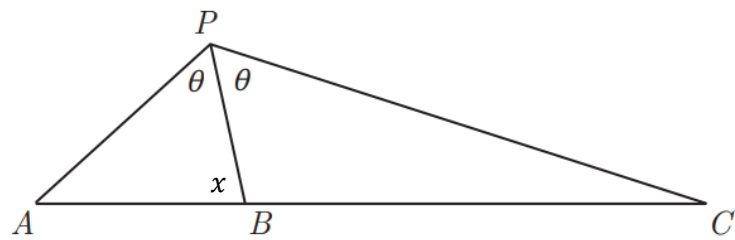
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Section III continued on next page ...

Question 21

In $\triangle APC$, B is the point on AC such that $\angle APB = \angle BPC = \theta$. Let $\angle PBA = x$. 3



Using the sine rule in $\triangle APB$ and $\triangle BPC$ or otherwise, show that $AB \times PC = AP \times BC$.

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~ End of Section III ~

Section III: Extra Writing Space

If you use this space, clearly indicate the question you are answering.



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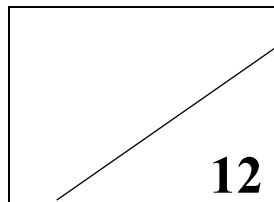
ASSESSMENT TASK 3
YEARLY EXAMINATION

Mathematics Advanced

Section IV

Instructions

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SECTION IV

12 marks
Attempt Questions 22 – 26
Allow about 21 minutes on this section.

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2024 Mathematics Advanced Yearly Examination Section IV

Marks

Question 22

Find the derivative of $f(x) = \frac{x}{3x+1}$, giving your answer in simplest form.

2

Question 23

Given $f(x) = (2x^3 - 5)^4$, evaluate $f'(-1)$.

2

Section IV continued on next page ...

Question 24

Find the derivative of $f(x) = 1 + 2x - x^2$ using differentiation via first principles. 3

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Question 25

The curve $y = x^3 + 4$ has a tangent to the curve with a gradient of 16. 2

Find the value(s) of x that satisfy this condition.

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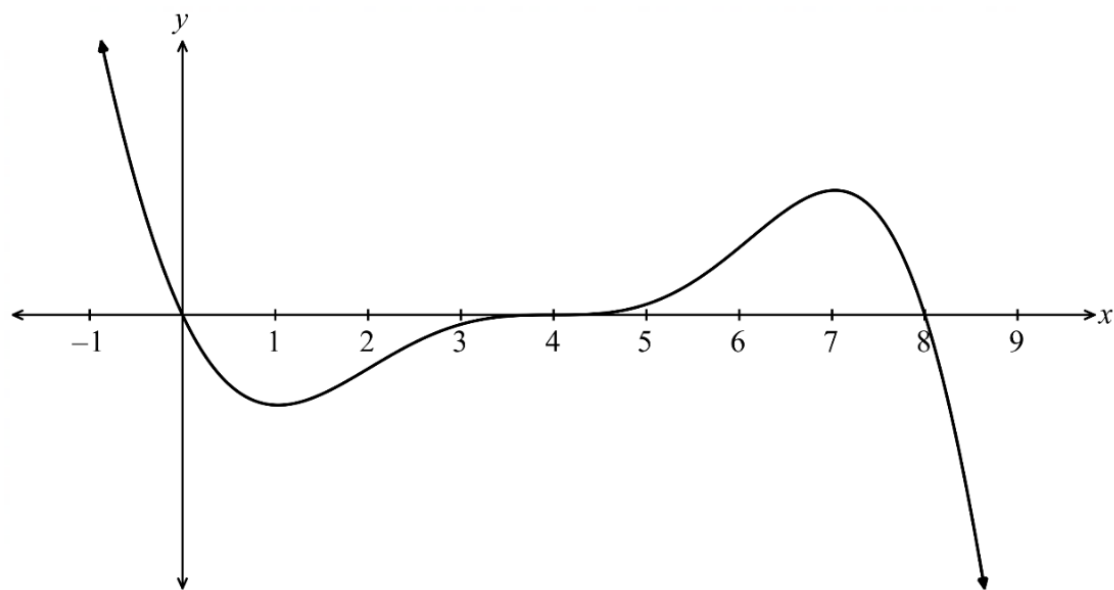
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Section IV continued on next page ...

Question 26

The function $y = f(x)$ is shown below.

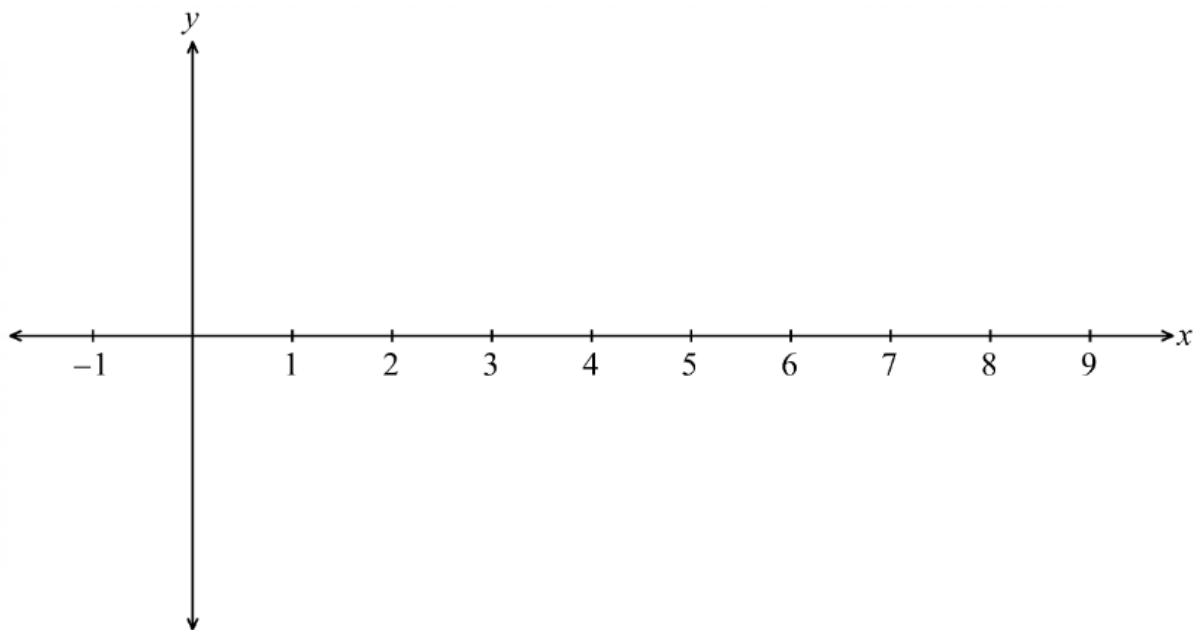


- a) For what values of x is $f'(x) > 0$? 1

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- b) Sketch the gradient function $y = f'(x)$ on the new set of axes below. 2



~ End of Section IV ~

Section IV: Extra Writing Space

If you use this space, clearly indicate the question you are answering.

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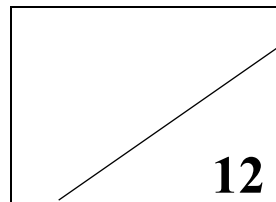
ASSESSMENT TASK 3
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Mathematics Advanced

Section V

Instructions

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SECTION V

12 marks

Attempt Questions 27 – 30

Allow about 21 minutes on this section.

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.

Answer each question in the spaces provided.

2024 Mathematics Advanced Yearly Examination Section V	Marks
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Question 27

If $\log_3 2 = a$ and $\log_3 13 = b$, express $\log_3 52$ in terms of a and b .

2

[illegible]

Section V continued on next page ...

Question 28

Solve $(\sqrt{3})^x = 9^{2x}$, leaving x in exact form.

2

Section V continued on next page ...

Question 29

3

Section V continued on next page

Question 30

The amount of electrical energy $E(t)$, stored in a battery after charging for t minutes is given by the equation $E(t) = 100 - Ae^{-0.1t}$.

- a) When $t = 0$, there is 10 units of electrical energy stored in the battery. 1

Show that $A = 90$.

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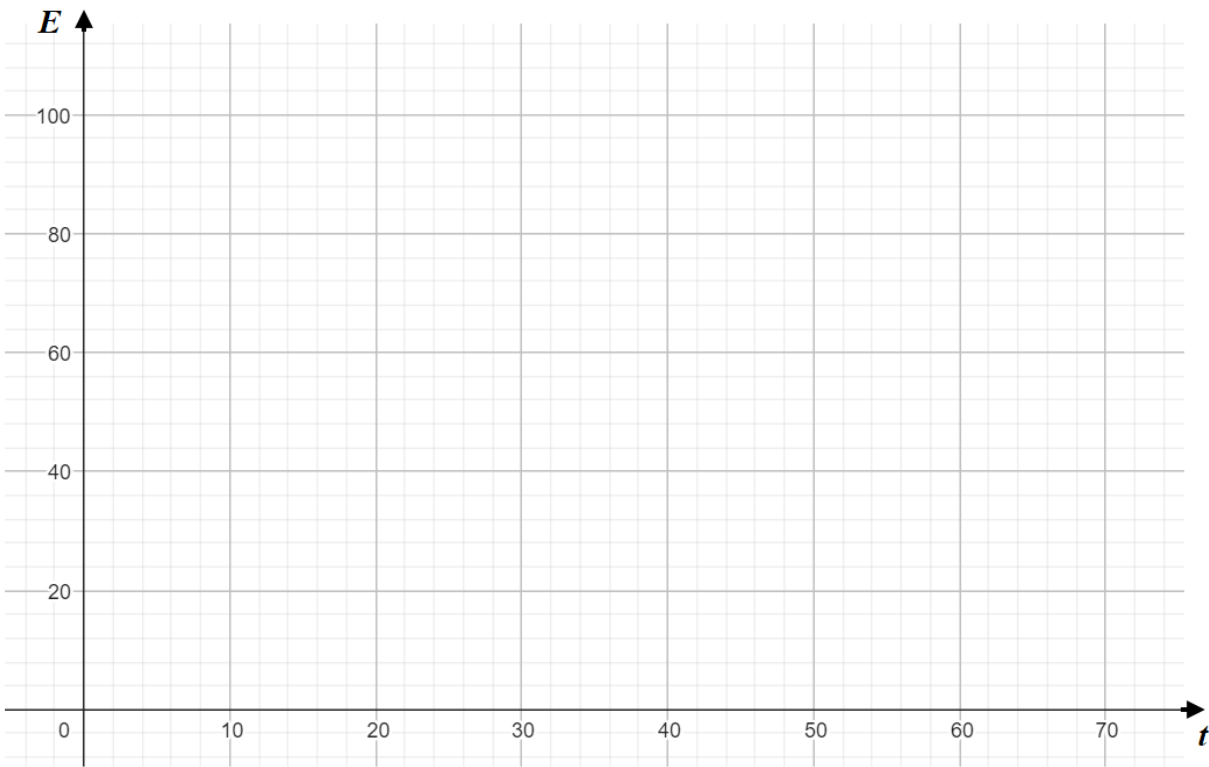
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- b) Fill in the missing value of $E(t)$ in the table below to 1 decimal place. 1

t	0	10	20	30	40
$E(t)$	10		87.8	95.5	98.4

- c) Hence, draw the graph of $E(t)$ below for $0 \leq t \leq 4$. 2



Question 28 continued on next page ...

Question 30 continued...

d) What value will the electrical energy approach as t increases? 1

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~ End of Section V ~

Section V: Extra Writing Space

If you use this space, clearly indicate the question you are answering.

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HURLSTONE
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SCHOOL

STUDENT'S NAME: _____

TEACHER'S NAME: _____

2024

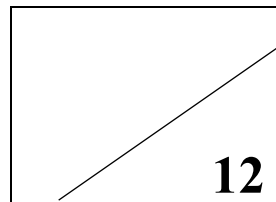
ASSESSMENT TASK 3
YEARLY EXAMINATION

Mathematics Advanced

Section VI

Instructions

- Follow the instructions provided on the examination paper
- Write using black or blue pen
- There is extra writing space at the end of the section
- You may not remove any booklets, used or unused, from the examination room.



SECTION VI

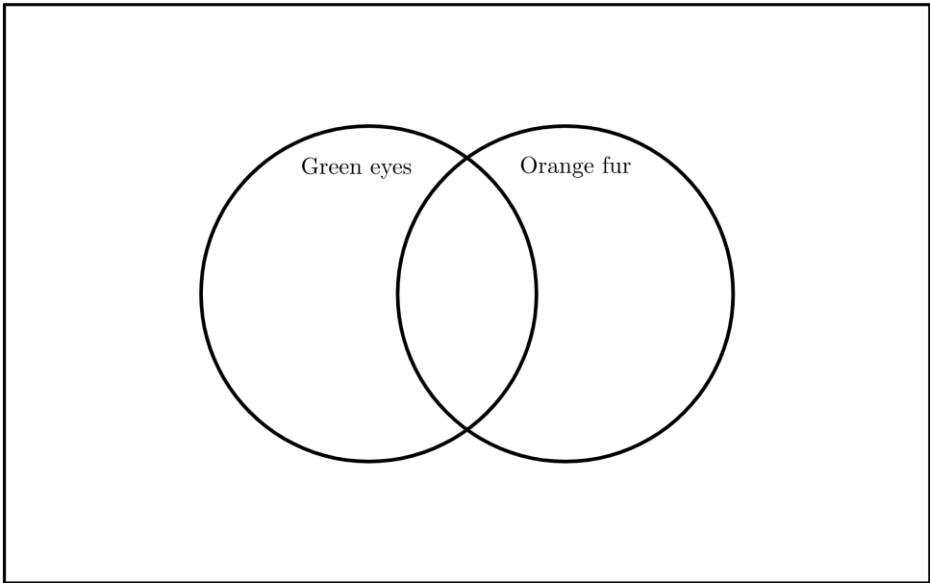
12 marks
Attempt Questions 31 – 33
Allow about 21 minutes on this section.

For questions in this section, your responses should include relevant mathematical reasoning and/or calculations.
Answer each question in the spaces provided.

Question 31

Mrs Tat has several pet cats.

- $\frac{2}{3}$ of her cats have green eyes.
 - $\frac{1}{3}$ of her cats have orange fur.
 - $\frac{1}{6}$ of her cats have green eyes and orange fur.
- a) Fill in the Venn diagram below with the correct probabilities.2



- b) Mrs Tat picks up a cat at random. Given that she picked up a cat with green eyes, what is the probability that it does not have orange fur?1

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Section VI continued on next page ...

Question 32

A tenpin bowler has a 28% chance of scoring a strike in bowling each time they bowl.

- a) The bowler bowls twice. By using a tree diagram or otherwise, calculate the probability that the bowler does not get a strike in both bowls. 2

[illegible]

- b) What is the minimum number of bowls required for the bowler to have a 90% chance of bowling at least one strike? 2

[illegible]

Section VI continued on next page ...

Question 33

X represents the amount of money won or lost per game in dollars on a poker machine.

The probability distribution table is shown below.

x	b	2	3	4	5
$P(X = x)$	0.35	0.20	a	0.17	0.09

a)Calculate the value of a .

1

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b)Hence, if you are expected to lose \$1.40 per game, calculate the value of b .

2

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c)Hence, calculate the variance.

2

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~ End of Section VI ~

Section VI: Extra Writing Space

If you use this space, clearly indicate the question you are answering.

HAHS Year 11 Mathematics Advanced Task 3 2024

Marking Guidelines

Section I – Multiple Choice Questions

Question 1 (MA11-1)

$$|x - 7| = 6$$

$$x - 7 = 6 \text{ or } x - 7 = -6$$

$$x = 13 \text{ or } x = 1$$

∴ Answer: B

Question 2 (MA11-2)

In the set of coordinates, there are repeating x and y values. Hence, the relation is many-to-many.

∴ Answer: D

Question 3 (MA11-4)

$$\begin{aligned}\sec 240^\circ &= \frac{1}{\cos 240^\circ} \\ &= \frac{1}{-\cos 60^\circ} \\ &= \frac{1}{-\frac{1}{2}} \\ &= -2\end{aligned}$$

∴ Answer: B

Question 4 (MA11-4)

$$2 \sin x + \sqrt{3} = 0$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$\text{Reference angle} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

$\sin$ is negative in the 3rd and 4th quadrants

$$\begin{aligned}\text{In the third quadrant, } x &= \pi + \frac{\pi}{3} \\ &= \frac{4\pi}{3} \\ &= -\frac{2\pi}{3} \quad (-\pi \leq x \leq \pi)\end{aligned}$$

$$\begin{aligned}\text{In the fourth quadrant, } x &= 2\pi - \frac{\pi}{3} \\ &= \frac{5\pi}{3} \\ &= -\frac{\pi}{3} \quad (-\pi \leq x \leq \pi)\end{aligned}$$

$$\therefore x = -\frac{2\pi}{3} \text{ or } -\frac{\pi}{3}$$

$\therefore$ Answer: D

Question 5 (MA11-5)

$$f(x) = -3x^4 + 6x - 1$$

$$f'(x) = -12x^3 + 6$$

$$\begin{aligned}f'(1) &= -12(1)^3 + 6 \\ &= -6\end{aligned}$$

$\therefore$ Answer: B

Question 6 (MA11-5)

$f(x)$ is increasing when $f'(x) > 0$ i.e. when the gradient of the tangent to the graph is positive. Note that the gradient at $x = a$ cannot be determined as the number of tangents possible is infinite, hence $x < b$ is not a valid answer.

$\therefore$ Answer: A

Question 7 (MA11-6)

I is not true, the correct logarithm law is $\log_a xy = \log_a x + \log_a y$.

II is not true, the correct logarithm law is $\log_a \frac{x}{y} = \log_a x - \log_a y$.

$\therefore$ Answer: C

Question 8 (MA11-6)

$$4^{2x+1} = 8$$

$$2^{2(2x+1)} = 2^3$$

$$2(2x+1) = 3$$

$$4x + 2 = 3$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\therefore 2^x = 2^{\frac{1}{4}} = \sqrt[4]{2}$$

$\therefore$ Answer: A

Question 9 (MA11-7)

$$\frac{24}{30} \times 960 = 768$$

∴ Answer: B

Question 10 (MA11-7)

If A and B are independent, then $P(A) = P(A|B)$.

$$\therefore P(A) = 0.6$$

∴ Answer: D

Year 11	Mathematics Advanced	Task 3 2024
Section II	Solutions and Marking Guidelines	
Outcomes Addressed in this Section		
MA11-1: uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems		
MA11-2: use the concepts of functions and relations to model, analyse and solve practical problems		
Outcome	Solutions	Marking Guidelines
MA11-2	Question 11 Range: $[0, \infty)$ or $0 \leq y < \infty$ or $y \geq 0$	Award 1 ~ correct answer.
MA11-1	Question 12 $A = lw$ $12 = (2 + \sqrt{7})w$ $w = \frac{12}{2 + \sqrt{7}}$ $= \frac{12(2 - \sqrt{7})}{(2 + \sqrt{7})(2 - \sqrt{7})}$ $= \frac{12(2 - \sqrt{7})}{4 - 7}$ $= \frac{12(2 - \sqrt{7})}{-3}$ $= -4(2 - \sqrt{7})$ $= -8 + 4\sqrt{7}$	Award 2 ~ correct solution. Award 1 ~ significant progress towards solution.
MA11-2	Question 13 $f(x) = x^2 + 4x + 1$ $f(-x) = (-x)^2 + 4(-x) + 1$ $= x^2 - 4x + 1$ $-f(x) = -(x^2 + 4x + 1)$ $= -x^2 - 4x - 1$ $f(x) \neq f(-x) \therefore \text{not an even function}$ $-f(x) \neq f(-x) \therefore \text{not an odd function}$ $\therefore f(x) \text{ is neither an odd nor an even function.}$	Award 2 ~ correct solution. Award 1 ~ significant progress towards solution.
MA11-2	Question 14 (a) $f(-2) + f(5) = (-2)^2 - (-2) + 2 + 5^3$ $= 133$	Award 1 ~ correct answer.

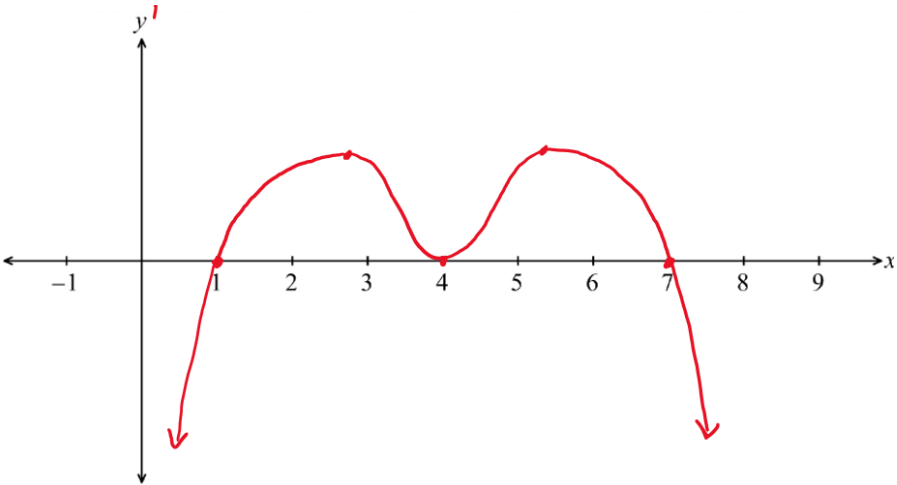
(b)	$f(-n^2) = (-n^2)^2 - (-n^2) + 2 \quad n \neq 0$ $= n^4 + n^2 + 2$ N.B. If $n = 0$ then $f(-n^2) = 0$	Award 1 ~ correct answer.
MA11-1	Question 15 The two pieces of wire will be x cm and $(10 - x)$ cm. These are the perimeters of the two squares. The side lengths of the two squares will be $\frac{x}{4}$ and $\frac{10-x}{4}$ cm. The area of one square is $\left(\frac{x}{4}\right)^2$ and the other is $\left(\frac{10-x}{4}\right)^2$ cm². $A = \left(\frac{x}{4}\right)^2 + \left(\frac{10-x}{4}\right)^2$ $= \frac{100 - 20x + 2x^2}{16}$ $= \frac{50 - 10x + x^2}{8}$	Award 2 ~ correct solution. Award 1 ~ significant progress towards solution.
MA11-2	Question 16 (a) $\Delta = b^2 - 4ac$ $= (k + 3)^2 - 4k$ $= k^2 + 6k + 9 - 4k$ $= k^2 + 2k + 9$ $= (k + 1)^2 + 8 \quad (\text{after completing the square})$ (b) For real roots, $\Delta \geq 0$ Since $\Delta = (k + 1)^2 + 8$, $(k + 1)^2 \geq 0$ for all k. Then $(k + 1)^2 + 8 \geq 8 > 0$ for all k. Thus $f(x)$ has real roots.	Award 2 ~ correct solution. Award 1 ~ significant progress towards solution. Award 1 ~ correct solution.

Year 11	Mathematics Advanced	Task 3 2024
Section III	Solutions and Marking Guidelines	
Outcomes Addressed in this Section		
MA11-3: uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes		
MA11-4: uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities		
Outcome	Solutions	Marking Guidelines
MA11-4	Question 17	1 mark Correct solution
	For $\cos x = 1$ there are 3 solutions in the interval $[0^\circ, 720^\circ]$.	
MA11-3	Question 18	2 marks Obtains correct expression for $\tan \theta$
	$\sin \theta = -\frac{20}{29}$ $\sin^2 \theta + \cos^2 \theta = 1$ $\left(-\frac{20}{29}\right)^2 + \cos^2 \theta = 1$ $\frac{400}{841} + \cos^2 \theta = 1$ $\cos^2 \theta = 1 - \frac{400}{841}$ $\cos^2 \theta = \frac{441}{841}$ $\cos \theta = \pm \sqrt{\frac{441}{841}}$ $\cos \theta = \pm \frac{21}{29}$ Since $\sin \theta < 0$ and $\cos \theta > 0$ our solution must lie in the fourth quadrant. $\therefore \cos \theta = \frac{21}{29}$ $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{20}{29}}{\frac{21}{29}}$ $\tan \theta = -\frac{20}{21}$	1 mark Obtains correct expression of either $\cos \theta$ OR $\tan \theta$ OR makes progress towards the correct solution.

MA11-4	Question 19 $ \begin{aligned} LHS &= \frac{\sin^2 x}{1 + \cos x} + \frac{\sin^2 x}{1 - \cos x} \\ &= \frac{\sin^2 x (1 - \cos x) + \sin^2 x (1 + \cos x)}{(1 - \cos^2 x)} \\ &= \frac{\sin^2 x - \sin^2 x \cos x + \sin^2 x + \sin^2 x \cos x}{\sin^2 x} \\ &= \frac{2\sin^2 x}{\sin^2 x} \\ &= 2 = RHS \end{aligned} $	2 marks Correctly proves the identity. MUST communicate solution in clear and structured manner. 1 mark Makes significant progress towards the correct solution.
MA11-3 (a) (b) (c)	Question 20 Arc length = 90×0.6 = 54 cm Using the cosine rule, line AB can be calculated as: $AB^2 = 90^2 + 90^2 - 2(90)(90) \cos 0.6$ $AB^2 \approx 2829.563 \dots$ $AB \approx 53.2 \text{ cm (1 d.p.)}$ $A = \frac{1}{2}(90)^2 \times 0.6$ $A = 2430 \text{ cm}^2$	1 mark Correct solution 2 marks Correct solution obtained AND correctly shows use of the cosine rule. 1 mark Some progress towards the correct solution. 1 mark Correct solution (units not required)

	Question 21 $\frac{AB}{\sin \theta} = \frac{AP}{\sin \angle ABP}$ $\frac{BC}{\sin \theta} = \frac{PC}{\sin \angle CBP}$ $\angle ABP + \angle CBP = 180^\circ$ Then, $\sin \angle ABP = \sin(180^\circ - \angle CBP)$ $= \sin \angle CBP$ $\frac{AB}{AP} = \frac{\sin \theta}{\sin \angle ABP}$ $\frac{BC}{PC} = \frac{\sin \theta}{\sin \angle CBP}$ $\therefore \frac{AB}{AP} = \frac{BC}{PC}$ $AB \times PC = BC \times AP$	3 marks Correctly communicates solution in a clear manner, outlining any relevant geometrical reasoning. 2 marks Shows significant progress towards the correct solution. 1 mark Makes some progress towards the correct solution.
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Year 11	Mathematics Advanced	Task 3 2024
Section IV	Solutions and Marking Guidelines	
Outcomes Addressed in this Section		
MA11-5: interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems		
Outcome	Solutions	Marking Guidelines
MA11-5	Question 22 $f(x) = \frac{x}{3x+1}$ $f'(x) = \frac{3x+1-3x}{(3x+1)^2}$ $= \frac{1}{(3x+1)^2}$	2 marks correct solution 1 mark clearly attempts a known differentiation rule
MA11-5	Question 23 $f(x) = (2x^3 - 5)^4$ $f'(x) = 3 \times 2x^2 \times 4(2x^3 - 5)^3$ $= 24x^2(2x^3 - 5)^3$ $f'(-1) = 24(-1)^2(2(-1)^3 - 5)^3$ $= -8232$	2 marks correct solution 1 mark correctly differentiates $f(x)$ or correctly finds $f'(-1)$ from a partially correct $f'(x)$
MA11-5	Question 24 $f(x) = 1 + 2x - x^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 + 2(x+h) - (x+h)^2 - (1 + 2x - x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 + 2x + 2h - x^2 - 2hx - h^2 - 1 - 2x + x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{2h - 2hx - h^2}{h}$ $= \lim_{h \rightarrow 0} 2 - 2x - h$ $= 2 - 2x$	3 marks correct solution 2 marks expanding correctly expressions in difference quotient 1 mark correctly substituting $f(x)$ into different quotient
MA11-5	Question 25 $y = x^3 + 4$ $y' = 3x^2$ If a tangent has a gradient of 16, then: $3x^2 = 16$ $x = \pm \frac{4}{\sqrt{3}}$	2 marks correct solution 1 mark links gradient of tangent to derivative of y

MA11-5	Question 26 (a) (1, 4) OR (4, 7) Note: students were not penalised for using AND. (b) 	1 mark correct answer 2 marks correct graph 1 mark sketches two stationary points out of three correctly and graph resembles parts of correct graph
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Year 11	Mathematics Advanced	Task 3 2024
Section V	Solutions and Marking Guidelines	
Outcomes Addressed in this Section		
MA11-6: manipulates and solves expressions using logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems		
Outcome	Solutions	Marking Guidelines
MA11-6	Question 27 $\begin{aligned}\log_3 52 &= \log_3 (4 \times 13) \\ &= \log_3 4 + \log_3 13 \\ &= 2 \log_3 2 + \log_3 13 \\ &= 2a + b\end{aligned}$	2 marks correct solution 1 mark substantial progress
MA11-6	Question 28 $\begin{aligned}(\sqrt{3})^x &= 9^{2x} \\ (3^{0.5})^x &= (3^2)^{2x} \\ 0.5x &= 4x\end{aligned}$	2 marks correct solution 1 mark substantial progress
MA11-6	Question 29 $\begin{aligned}y' &= e^{1-x} \\ \frac{dy}{dx} &= -e^{1-x}\end{aligned}$ At $(0, e)$, $\begin{aligned}m_{\text{tangent}} &= -e^{1-0} \\ &= -e \\ m_{\text{normal}} &= -e^{1-0} \\ &= \frac{1}{e}\end{aligned}$ The equation of the normal at $(0, e)$ is: $\begin{aligned}y - e &= \frac{1}{e}(x - 0) \\ ey - e^2 &= x \\ x - ey + e^2 &= 0 \text{ or } y = \frac{x}{e} + e\end{aligned}$	3 marks 1 mark for correct differentiation 1 mark for correct gradient of normal 1 mark for correct equation of normal in any form
MA11-6 (a)	Question 30 $\begin{aligned}E(0) &= 10 \\ 100 - Ae^{-0.1(0)} &= 10 \\ 100 - A \times 1 &= 10 \\ A &= 90\end{aligned}$	1 mark correct answer

(b)

t	0	10	20	30	40
$E(t) = 100 - 90e^{-0.1t}$	10	66.9	87.8	95.5	98.4

1 mark

correct answer

(c)

2 marks

correct graph

1 mark

some correct features

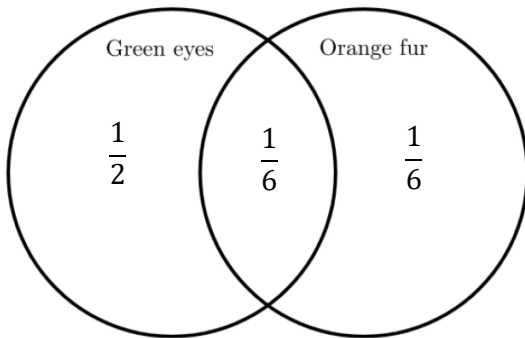
(d)

When $t \rightarrow \infty$, y approaches 100.
 Or when $t \rightarrow \infty$, $\lim_{t \rightarrow \infty} e^{-0.1t} = 0$

$$\therefore \text{The maximum amount of energy} = 100 - 90(0) \\ = 100$$

1 mark

correct answer

Year 11	Mathematics Advanced	Task 3 2024
Section VI	Solutions and Marking Guidelines	
Outcomes Addressed in this Section		
MA11-7: uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions		
Outcome	Solutions	Marking Guidelines
MA11-7	Question 31	
	(a) $P(\text{green eyes no orange fur}) = \frac{2}{3} - \frac{1}{6}$ $= \frac{1}{2}$ $P(\text{orange fur no green eyes}) = \frac{1}{3} - \frac{1}{6}$ $= \frac{1}{6}$ $P(\text{no green eyes no orange fur}) = 1 - \frac{1}{2} - \frac{1}{6} - \frac{1}{6}$ $= \frac{1}{6}$ 	2 marks correct solution 1 mark at least two numbers correct
	(b) $P(\text{no orange fur} \mid \text{green eyes}) = \frac{P(\text{no orange fur} \cap \text{green eyes})}{P(\text{green eyes})}$ $= \frac{\frac{1}{2}}{\frac{2}{3}}$ $= \frac{3}{4}$	1 mark correct solution
MA11-7	Question 32	
(a)	$P(\text{no strike in two bowls}) = 0.72 \times 0.72$ $= 0.5184$	2 marks correct solution

	Note: a tree diagram was not necessary to get full marks, however students should be in the habit of producing a probability tree diagram and listing out the sample space. The probability tree diagram allowed some students to receive partial marks for this question as an error was made when calculating the probability.	1 mark substantial progress towards solution
(b)	Let n = the number of bowls $P(\text{at least one strike}) = 1 - P(\text{no strike})$ $= 1 - 0.72^n$ $P(\text{at least one strike}) \geq 0.9$ $1 - 0.72^n \geq 0.9$ $-0.72^n \geq -0.1$ $0.72^n \leq 0.1$ $\log 0.72^n \leq \log 0.1$ $n \log 0.72 \leq \log 0.1$ $n \geq \frac{\log 0.1}{\log 0.72} \quad (\text{note when } 0 < x < 1, \log x < 0)$ $n \geq 7.009 \dots$ As n must be an integer that satisfies the inequality above, $\therefore n = 8$ A minimum of 8 bowls is required for a 90% chance of getting at least one strike.	2 marks correct solution involving the correct use of logarithm laws and inequality signs 1 mark substantial progress towards solution that does not involve guessing and checking
MA11-7	Question 33	
(a)	$0.35 + 0.20 + a + 0.17 + 0.09 = 1$ $a = 1 - 0.35 - 0.20 - 0.17 - 0.09$ $= 0.19$	1 mark correct solution
(b)	$E(X) = \sum xp(x) = -1.4$ $0.35b + 2 \times 0.20 + 3 \times 0.19 + 4 \times 0.17 + 5 \times 0.09 = -1.4$ $0.35b + 2.1 = -1.4$ $0.35b = -0.35$ $b = -10$	2 marks correct solution 1 mark substantial progress towards solution
(c)	$E(X^2) = \sum x^2p(x)$ $= (-10)^2 \times 0.35 + 2^2 \times 0.20 + 3^2 \times 0.19 + 4^2 \times 0.17 + 5^2 \times 0.09$ $= 42.48$ $\text{Var}(X) = E(X^2) - [E(X)]^2$ $= 42.48 - (-1.4)^2$ $= 42.48 - 1.96$ $= 40.52$	2 marks correct solution 1 mark substantial progress towards solution