International Grammar School

Year 11
Mathematics
Yearly Examination
2011

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

Note: Any time you have remaining should be spent revising your answers.

Total marks – 84

- Attempt Questions 1 – 7
- All questions are of equal value
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Total Marks – 84
Attempt Questions 1–7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) **Marks**

(a) Evaluate $\sqrt{\frac{86.54\times3.16}{2.6^3}}$ correct to 2 significant figures. 2

(b) Express $\frac{3\sqrt{3}}{2\sqrt{3}-1}$ with a rational denominator. 2

(c) Factorise completely

i) $x^2 - 25$ 1

ii) $4x^2 - 8x + 3$ 1

(d) Solve $|x - 8| = 6$ 2

(e) Simplify $\frac{(3xy^3)^3}{3x^2y^4}$ 2

(f) Convert $0.\dot{2}\dot{8}$ to a fraction. 2

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $\lim_{x\to 3}\frac{x^2-9}{x-3}$ 2

(b) Consider the function

$$y = x^2 - 2x - 3$$

i) Find its y and x intercepts. 2

ii) Find the axis of symmetry and vertex. 2

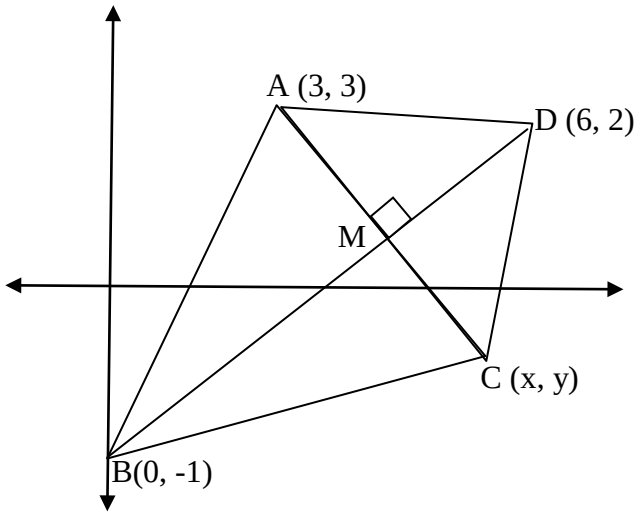
iii) Sketch this function showing all important features. 2

iv) State its Domain and Range. 2

(c) Show that $f(x) = x^3 - 25x$ is an odd function. 2

Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

In the quadrilateral ABCD, the points A, B and D are (3, 3), (0, -1) and (6, 2) respectively. The line BD bisects the line AC at right angles at the point M.



NOT TO SCALE

(a) Find the exact distance BD. 1

(b) Show that the gradient of BD is $\frac{1}{2}$. 1

(c) Show that the equation of the line BD is $x - 2y - 2 = 0$. 2

(d) Show that the equation of AC is $2x + y - 9 = 0$. 2

(e) Show that the coordinates of M are (4, 1) 1

(f) Show that the coordinates of C are (5, -1) 2

(g) Find the area of the quadrilateral $ABCD$. 3

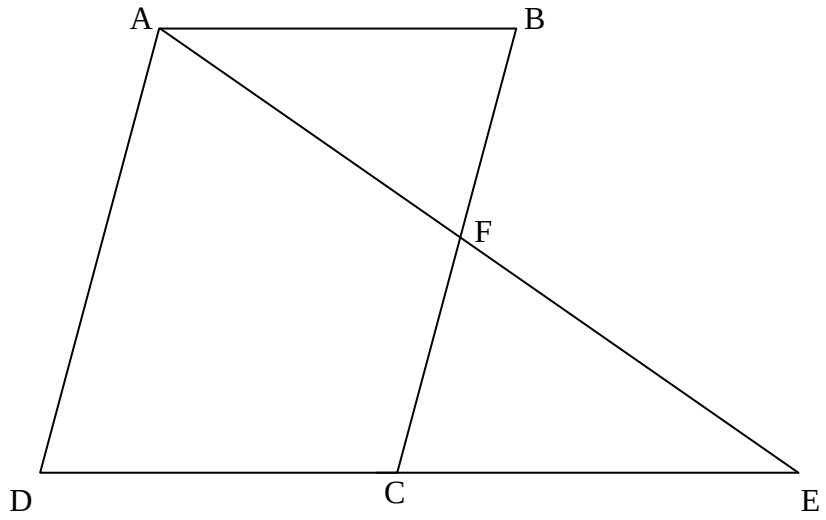
Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the acute angle which the line $2x - y + 3 = 0$ makes with the x axis to the nearest degree. 2

(b) i) Find the perpendicular distance from the line $3x - 4y + 10 = 0$ to the point (0,0) 2

ii) Hence explain why the line $3x - 4y + 10 = 0$ is a tangent to the circle $x^2 + y^2 = 4$ 2

(c) ABCD is a parallelogram. DC is produced to E. AE cuts BC at F. *AD=16cm*, *CE=9cm* and *BF=10cm*.



i) Prove that ΔABF is similar to ΔECF. 2

ii) Find AB. 2

(d) Using the method of first principals, find $f'(x)$ if $f(x) = 2x^2 - 1$ 2

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Differentiate each of the following functions.

i) $3x^4 - 2x + 1.$ 1

ii) $(2x^2 - 3)^4$ 1

iii) $(2x + 1)(5x^2 - 1)$ 2

iv) $\frac{2x}{x+3}$ 2

(b) For the function $y = 3x^2 + 4$ find the equation of the normal at $x = 2.$ 3

(c) Find the equation of the line that makes an angle of 60° in the positive direction with the x axis and passes through the point (2, -1) 3

Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the exact value of

$$cot\ 45° + sec\ 45°$$ 1

(b) Solve each of the following trigonometric equations.

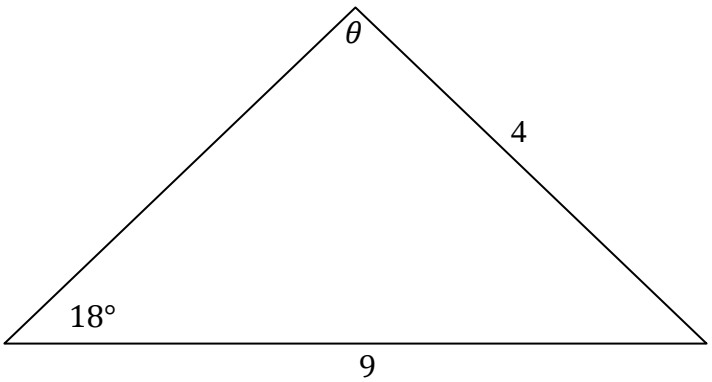
i. $2\sin\theta = 1 \quad \text{for } 0° \leq \theta \leq 360°$ 2

ii. $\tan 2\theta = -1\ for\ \ 0° \leq \theta \leq 360°$ 3

iii. $cos\frac{\theta}{2} = \frac{1}{\sqrt{2}} \quad for\ -180° \leq \theta \leq 180°$ 2

iv. $2sin^2\theta = 1 \quad for\ 0° \leq \theta \leq 360°$ 2

(c) Find the Value of θ (correct to the nearest degree) 2

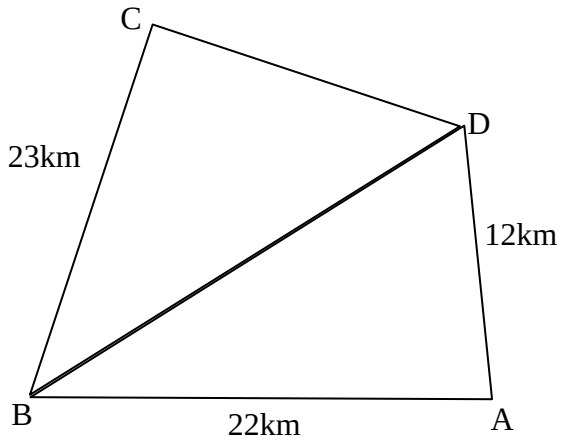


Question 7 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) If $sin\theta = -\frac{4}{11}$ and $tan\theta > 0$, find the exact value of $cos\,\theta$. 2

(b) Prove that $3cos^2\theta - 1 = 2 - 3sin^2\theta$ 3

(c) The diagram represents a cattle station ABCD in the shape of a quadrilateral. A is 22 km due east of B and D is 12 km from A at a bearing of 030°. C is 23 km from B at a bearing of 030° and at a bearing of 300° from D.



NOT TO SCALE

i) Copy or trace the diagram into your writing booklet and mark the lengths and the bearings on it. 1

ii) Explain why $\angle BAD = 120°$ and $\angle BCD$ is a right angle. 2

iii) Find the length of BD, correct to one decimal place. 2

iv) Find the area of the cattle station correct to the nearest square km. 2

End of Examination

2011 2U AT3 Prelim

1a) $\sqrt{\frac{86.54 \times 3.16}{2.63}} = 3.9445\ldots$ ✓

$= 3.9$ 2SF ✓

b) $\frac{3\sqrt{3}}{2\sqrt{3}-1} = \frac{3\sqrt{3}\times(2\sqrt{3}+1)}{(2\sqrt{3}-1)(2\sqrt{3}+1)}$ ✓

$= \frac{6\times 3 + 3\sqrt{3}}{4\times 3 - 1}$

$= \frac{18+3\sqrt{3}}{11}$ ✓

c) i) $x^2-25 = (x+5)(x-5)$ ✓

ii) $4x^2-8x+3$ P: 12
S: -8
F: -2, -6.

$= 4x^2-2x-6x+3$

$= 2x(2x-1)-3(2x-1)$

$= (2x-1)(2x-3)$ ✓

d) $|x-8| = 6$.

$x-8=6$ $\quad -(x-8)=6$

$x=14$ $\quad -x+8=6$

$-x=-2$

$x=2$

$\therefore x = 2, 14$ ✓✓

e) $\frac{(3xy^3)^3}{3x^2y^4} = \frac{27x^3y^9}{3x^2y^4}$ ✓

$= 9xy^5$ ✓

f) $x = 0.\dot{2}\dot{8}$

$= 0.282828\ldots$

$100x = 28.2828\ldots$

$99x = 28$

$x = \frac{28}{99}$ ✓✓

2a) $\lim_{x\to 3} \frac{x^2-9}{x-3}$

$= \lim_{x\to 3} \frac{(x+3)(x-3)}{(x-3)}$

$= \lim_{x\to 3} x+3$ ✓

$= 6.$ ✓

b) $y = x^2-2x-3$

$= (x-3)(x+1).$

i) y int: $y=-3$ ✓

x int: $x=-1, 3$ ✓

ii) axis sym:

$x = -\frac{b}{2a} = \frac{2}{2}$

$= 1.$

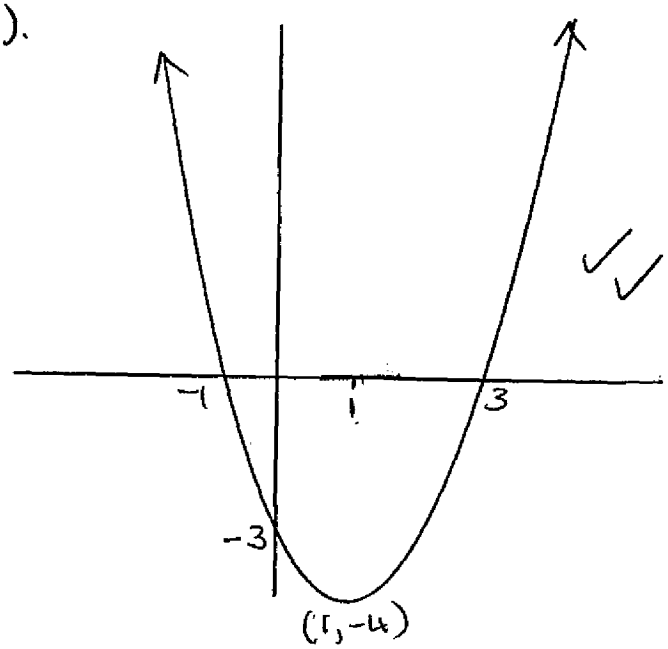
vertex:

$y = 1-2-3 = -4$

$\therefore$ axis: $x=1$ ✓

vertex: $(1,-4)$ ✓

iii).



✓✓

iv) D: all $x \in \mathbb{R}$. ✓

R: $y \geq -4$. ✓

g) If odd then $f(-x) = -f(x)$.

$f(x) = x^3 - 25x$

$f(-x) = (-x)^3 - 25(-x)$

$= -x^3 + 25x$ ✓

$= -(x^3 - 25x)$

$= -f(x)$ ✓

∴ odd since $f(-x) = -f(x)$

Q3.

A (3,3) C(x,y)

B (0,−1) D(6,2).

a) $BD = \sqrt{6^2 + 3^2}$

$= \sqrt{45} = 3\sqrt{5}$ u. ✓

b) $m_{BD} = \frac{2-(-1)}{6-0}$

$= \frac{3}{6} = \frac{1}{2}$ ✓

c) BD: $y+1 = \frac{1}{2}(x-0)$ ✓

$y+1 = \frac{1}{2}x$

$2y+2 = x$

$x - 2y - 2 = 0$ ✓

d) AC: $AC \perp BD$

so $m_{AC} = -2$. ✓

$y - 3 = -2(x-3)$

$= -2x + 6$

$2x + y - 9 = 0$ ✓

e) M is point of intersection BD and AC.

$x - 2y - 2 = 0$ — ①

$2x + y - 9 = 0$ — ②.

① × 2.

$2x - 4y - 4 = 0$ ③.

② − ③

$5y - 5 = 0$ ✓

$y = 1$

$x - 2 - 2 = 0 \quad x = 4$ ∴ M(4,1)

f) M is midpt AC. C(x,y)

so $4 = \frac{3+x}{2} \quad 1 = \frac{3+y}{2}$ ✓

$8 = 3 + x \quad 2 = 3 + y$

$x = 5 \quad y = -1$

∴ C(5,−1) ✓

g) Area = 2 × area △BAD

$= 2 \times \frac{1}{2} \times BD \times AM$

$AM = \sqrt{(4-3)^2 + (1-3)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$ u ✓

so/ Area $= BD \times AM$

$= 3\sqrt{5} \times \sqrt{5}$

$= 15\ u^2$. ✓✓

4.

a) $2x - y + 3 = 0$

$y = 2x + 3$

$M = 2$. ✓

∴ $\tan\theta = 2$

$\theta = 63°26'$

$= 63°$ ✓

b) $3x - 4y + 10 = 0$

(0,0)

$d = \left|\frac{0 - 0 + 10}{\sqrt{3^2+4^2}}\right|$ ✓

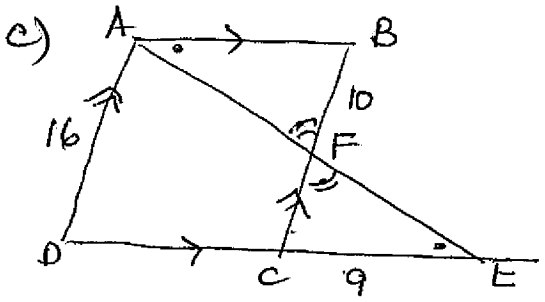
$= \frac{10}{5}$

$= 2$ u. ✓

4) $x^2+y^2=4$ is a circle centre $(0,0)$ $r=2$.

Since the $\perp$ distance b/w $3x-4y+10=0$ and the centre $(0,0)$ is = radius, the line must be a tangent ✓

c)



1) $\triangle ABF$ $\triangle ECF$ ✓

$\angle BAF = \angle CEF$ alternate angles AB ∥ CE

$\angle AFB = \angle EFC$ vertically opposite angles ✓

$\therefore \triangle ABF \,|||\, \triangle ECF$ equiangular

ii) $\frac{AB}{CE} = \frac{BF}{CF}$ corresponding sides similar $\triangle$s same proportion ✓

$\frac{AB}{9} = \frac{10}{6}$

$AB = \frac{90}{6}$

$= 15$ ✓

d) $f(x) = 2x^2-1$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{2(x+h)^2-1-(2x^2-1)}{h}$ ✓

$= \lim_{h\to 0} \frac{2x^2+4xh+2h^2-\cancel{1}-2x^2+\cancel{1}}{h}$

$= \lim_{h\to 0} \frac{4xh+2h^2}{h}$

$= \lim_{h\to 0} \frac{2\cancel{h}(2x+h)}{\cancel{h}}$

$= \lim_{h\to 0} 4x+h$

$= 4x.$ ✓

5. a)

i) $y = 3x^4-2x+1$

$y' = 12x^3-2$ ✓

ii) $y = (2x^2-3)^4$

$y' = 4 \cdot 4x(2x^2-3)^3$

$= 16x(2x^2-3)^3$ ✓

iii) $y = (2x+1)(5x^2-1)$

$u = 2x+1 \quad v = 5x^2-1$

$u' = 2 \quad v' = 10x$

$y' = 10x(2x+1) + 2(5x^2-1)$ ✓

$= 20x^2+10x+10x^2-2$

$= 30x^2+10x-2$ ✓

iv) $y = \frac{2x}{x+3}$

$u = 2x \quad v = x+3$

$u' = 2 \quad v' = 1$

$y' = \frac{2(x+3)-2x}{(x+3)^2}$ ✓

$= \frac{2x+6-2x}{(x+3)^2}$

$= \frac{6}{(x+3)^2}$ ✓

b) $y = 3x^2 + 4$.

$y' = 6x$. ✓

at $x = 2$ $m_T = 12$. $m_N = -\frac{1}{12}$

$x = 2$ $y = 3(4) + 4$

$= 16$.

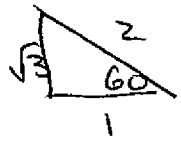
so, $m_N = -\frac{1}{12}$ $(2, 16)$

$y - 16 = -\frac{1}{12}(x - 2)$ ✓

$12y - 192 = -x + 2$.

$x + 12y - 194 = 0$. ✓

c) $\tan 60° = m$.



$\therefore m = \sqrt{3}$. ✓

$(2, -1)$.

$y + 1 = \sqrt{3}(x - 2)$ ✓

$= \sqrt{3}x - 2\sqrt{3}$

$\sqrt{3}x - y - 2\sqrt{3} - 1 = 0$. ✓

6a) $\cot 45° + \sec 45°$

$= 1 + \sqrt{2}$

$= 1 + \sqrt{2}$. ✓

b) i) $2\sin\theta = 1$ $0° \le \theta \le 360°$

$\sin\theta = \frac{1}{2}$ Q1, Q2, 30°

$\theta = 30°, 150°$ ✓✓

ii) $\tan 2\theta = -1$ $0° \le \theta \le 360°$,
$0° \le 2\theta \le 720°$

Q2, Q4, 45°

$2\theta = 135°, 315°$,
$495°, 675°$ ✓

$\theta = 67.5°, 157.5°$,
$247.5°, 337.5°$ ✓✓

iii) $\cos\frac{\theta}{2} = \frac{1}{\sqrt{2}}$ $-180° \le \theta \le 180°$

$-90° \le \frac{\theta}{2} \le 90°$

Q1, Q4, 45°.

-Q1, -Q4.

$\frac{\theta}{2}$ = ~~-Q4~~ / $-315°$, $-45°$, Q1 45°, ~~Q4~~ $315°$

not in domain

$\theta = -90°, 90°$. ✓✓

iv) $2\sin^2\theta = 1$ $0° \le \theta \le 360°$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$ all Q, 45°

$\theta = 45°, 135°, 225°, 315°$.

c) $\frac{\sin\theta}{9} = \frac{\sin 18°}{4}$

$\sin\theta = \frac{9\sin 18°}{4}$ ✓

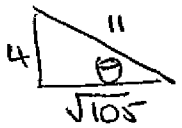
$= 0.6952\ldots$

$\theta = 44°3'$

$= 44°$ ✓

7(a) $\sin\theta = -\frac{4}{11}$ $\tan\theta > 0$

$\therefore$ Q3.



$\cos\theta = -\frac{\sqrt{105}}{4}$ ✓✓

b) $3\cos^2\theta - 1 = 2 - 3\sin^2\theta$

LHS $= 3(1 - \sin^2\theta) - 1$ ✓

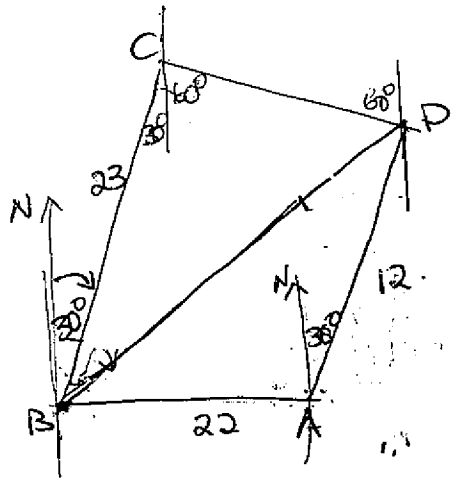
$= 3 - 3\sin^2\theta - 1$

$= 2 - 3\sin^2\theta$ ✓

$=$ RHS

c)

i)



✓

ii) A due east B

$\therefore \angle NBA = 90°$

So $\angle BAD = 90° + 30°$

$= 120°$ ✓

$\angle BCD = 30° + 60°$

$= 90°$ ✓

iii) $BD^2 = 12^2 + 22^2 - 2 \times 12 \times 22 \times \cos 120°$ ✓

$= 892$

$BD = 29.866..$

$= 29.9$ km. ✓

iv) Area = Area ΔBCD + Area BDA.

$CD = \sqrt{BD^2 - BC^2} = \sqrt{892 - 23^2} = 19.05$

$\Delta BCD = \frac{1}{2} \times 23 \times 19.05$

$= 219.075 km^2$

$\Delta BDA = \frac{1}{2} \times 22 \times 12 \times \sin 120°$

$= 114.32 km^2$ ✓

Total = 333.395..

$= 333 km^2$ ✓