

Year 11

Preliminary Assessment Task 3

Mathematics

2012

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

Note: Any time you have remaining should be spent revising your answers.

Total marks – 85

Section A

10 marks

- Attempt Questions 1 – 10

Section B

75 marks

- Attempt Questions 11 – 15
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Section A

10 marks

Attempt Questions 1–10

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☐ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐
 correct
 ↙

1 What is the value of $\sqrt{\frac{(3.059)^4}{16.38-11.063}}$ correct to three significant figures?

- (A) 1.759
- (B) 1.76
- (C) 4.058
- (D) 4.06

2 What are the values of a and b if $\sqrt{50} - \sqrt{72} = a\sqrt{b}$?

- (A) $a = -11$ and $b = 2$
- (B) $a = -1$ and $b = 2$
- (C) $a = -11$ and $b = 22$
- (D) $a = -1$ and $b = 22$

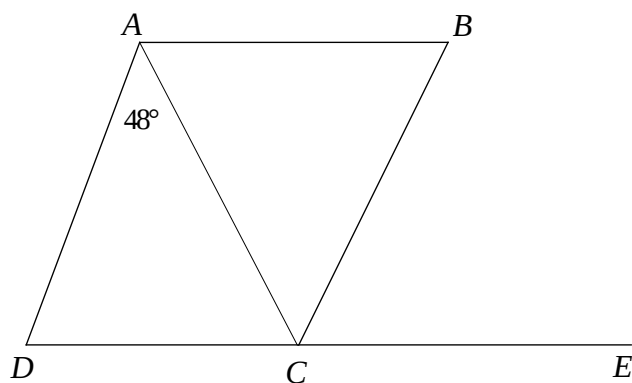
3 Which type of function is $f(x) = 5x^2 - x$?

- (A) Odd
- (B) Even
- (C) Neither odd or even
- (D) Zero

4 What is the exact value of $\sec 45^\circ$?

- (A) $\frac{1}{2}$
- (B) $\frac{1}{\sqrt{2}}$
- (C) 1
- (D) $\sqrt{2}$

5



Not to scale

In the diagram above, $ABCD$ is a rhombus where $\angle CAD = 48^\circ$ and DC is produced to E . AC is a diagonal.

What is the value of $\angle BCE$?

- (A) 48°
- (B) 72°
- (C) 84°
- (D) 96°

6 What is the midpoint of $(-3, 5)$ and $(2, -3)$?

- (A) $(-\frac{1}{2}, 1)$
- (B) $(\frac{1}{2}, -1)$
- (C) $(0, 1)$
- (D) $(2\frac{1}{2}, 4)$

7 Simplify $\frac{4a^2 - ab}{16a^2 - b^2}$.

- (A) $\frac{a}{(4a - b)}$
- (B) $\frac{4a - b}{(16a + b)}$
- (C) $\frac{4a - b}{(16a - b)}$
- (D) $\frac{a}{(4a + b)}$

8 What is the gradient of the line perpendicular to the line $2x + y - 2 = 0$?

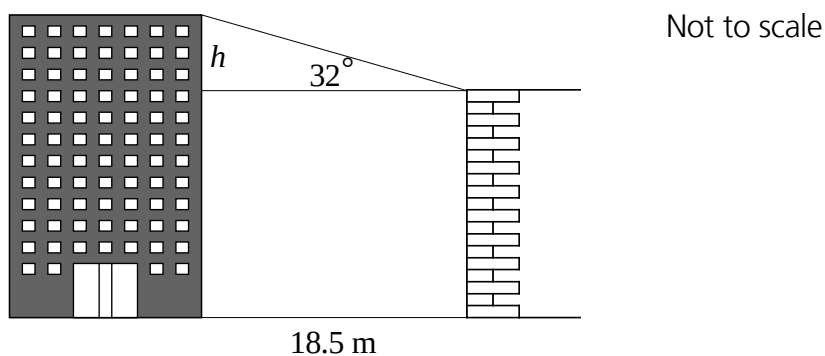
(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

9 The two buildings below are standing on level ground. The horizontal distance between the buildings is 18.5 metres and the angle of elevation between the buildings is 32° .



What the difference in height (h) between the buildings?

(A) 9.8 m

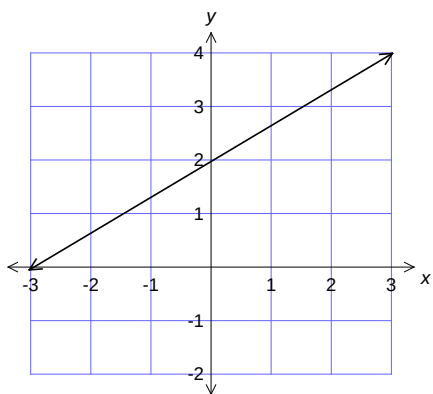
(B) 11.6 m

(C) 15.7 m

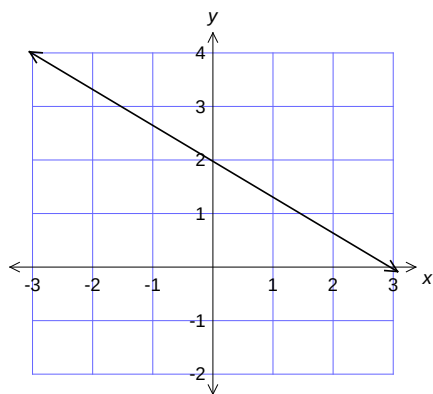
(D) 29.6 m

10 Which of the following could be the graph of $y = \frac{2}{3}x - 2$?

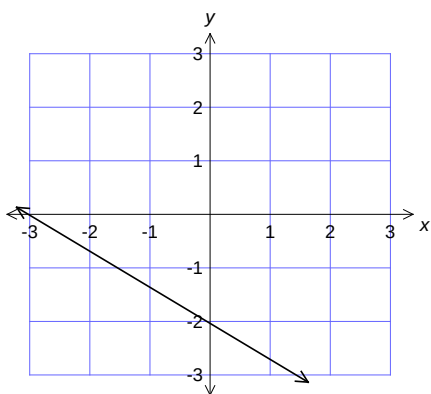
(A)



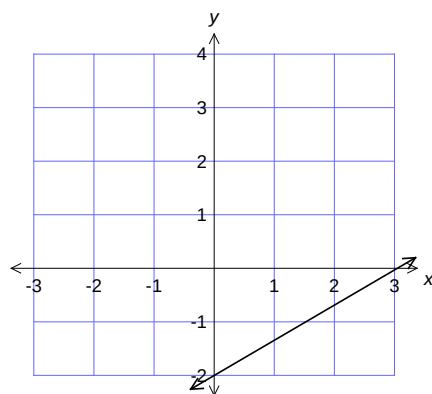
(B)



(C)



(D)



END OF SECTION A

Section B

Marks – 75

Attempt Questions 11–15

All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11 (15 marks)

Marks

- (a) Evaluate $\sqrt[3]{\frac{451}{3\pi}}$ correct to four significant figures. 2
- (b) Express $0.\dot{5}\dot{6}$ as a fraction in its simplest form. 2
- (c) Solve $2^{3x+1} = 16$ 2
- (d) Solve $|2x - 4| \leq 2$ 2
- (e) By rationalising the denominator, express $\frac{12}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$. 2
- (f) Factorise completely $2z^2 + 6zy + xz + 3xy$ 1
- (g) Simplify $\frac{x^2 - 4}{2x - 4} \div \frac{x + 2}{2}$. 2
- (h) Solve $\frac{x-5}{3} - \frac{x+1}{4} = 2$ 2

End Of Question 11

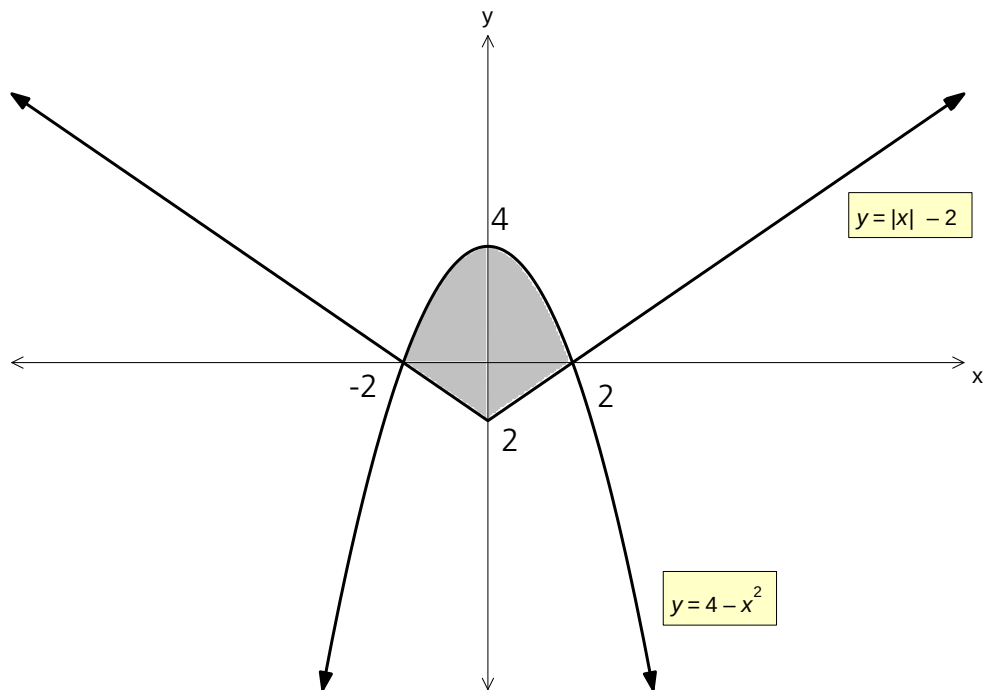
Question 12 (15 marks)**Marks**

(a) The function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} -3 & \text{for } x \leq -1 \\ x+1 & \text{for } -1 < x < 3 \\ x^2+1 & \text{for } x \geq 3 \end{cases}$$

- (i) Evaluate $f(-1) + f(3)$ 1
- (ii) Write an expression for $f(n^2 + 3)$ 2
- (b) State the domain and range of the function $f(x) = \sqrt{4 - x^2}$. 2
- (c) Make neat sketches of the following equations on separate sets of axes. Mark clearly the essential features of each graph.
- (i) $y = x^2 + 4$ 1
- (ii) $x^2 + y^2 = 9$ 1
- (iii) $y = \frac{1}{x-2}$ 1
- (d) If $f(x) = 3x^2 - 5x + 4$ and $g(x) = 2x + 10$ find the value(s) of x where $f(x)$ and $g(x)$ intersect. 3
- (e) Show that $f(x) = 1 - \frac{1}{x^2}$ is an even function. 2

- (f) The diagram shows the graphs of $y = |x| - 2$ and $y = 4 - x^2$.



Write down inequalities that together describe the shaded region.

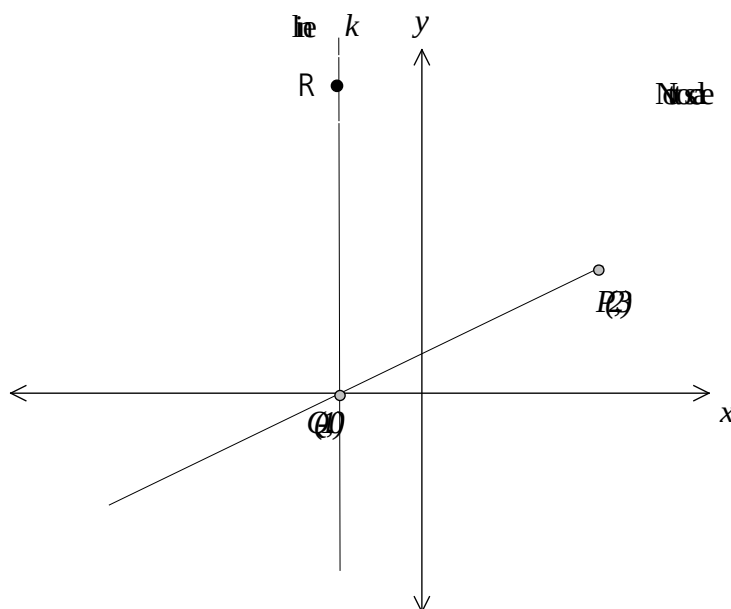
2

End Of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

Marks

- (a) The diagram shows the points $P(2,3)$, $Q(-1,0)$ and the line k , passing through Q , parallel to y -axis.



- | | | |
|-------|--|---|
| (i) | Calculate the length of the interval PQ . | 1 |
| (ii) | Find the gradient of the line PQ . | 1 |
| (iii) | What is the size of the acute angle between the line PQ and the line k ie; $\angle RQP$ | 2 |
| (iv) | Show that the equation of the line PQ is $x - y + 1 = 0$. | 1 |
| (v) | Copy the diagram and shade the region defined by $x - y + 1 \leq 0$. | 1 |
| (vi) | What is the equation of the line k ? | 1 |
| (vii) | The point R is on the line k such that PR is perpendicular to PQ . Find the coordinates of R . | 2 |

(b) Find the exact value of $\frac{2 - \tan 60}{\sec^2 45}$ 2

(c) Simplify $\frac{2\sec^2 A - 2}{4\tan A}$. 2

(d) Prove that $\sin \theta \tan \theta = \frac{1 - \cos^2 \theta}{\cos \theta}$ 2

End Of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

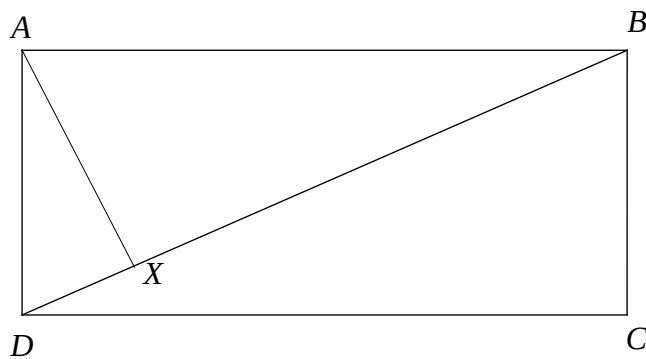
Marks

(a) Solve the following for x

(i) $\tan x = \frac{1}{\sqrt{3}}$ where $0^\circ \leq x \leq 360^\circ$ 2

(ii) $\sin 2x = \frac{1}{2}$ where $-180^\circ \leq x \leq 180^\circ$ 3

(b)



Not to scale

In the diagram above, $ABCD$ is a rectangle with $AB = 16$ cm and $AD = 12$ cm. AX is perpendicular to BD .

(i) Find the length of BD . 1

(ii) Prove that $\triangle ABX$ is similar to $\triangle DBA$. 2

(iii) Hence find the length of BX . 2

Question 14 continued on next page

- (c) Ashleigh and Bianca set out on a 2 hour bike ride from point O at the same time. Ashleigh travels at 20 km/h along a straight road at a bearing of 085° . Bianca travels at 25 km/h along another straight road at a bearing of 340° .
- (i) Draw a diagram to represent Ashleigh and Bianca's journey at the end of 2 hours. Show all angles and distances given in the question. 2
- (ii) Show that $\angle AOB$ is 105° where $\angle AOB$ is the angle between the directions taken by Ashleigh and Bianca. 1
- (iii) Find the distance Ashleigh and Bianca are apart to the nearest kilometre after two hours. 2

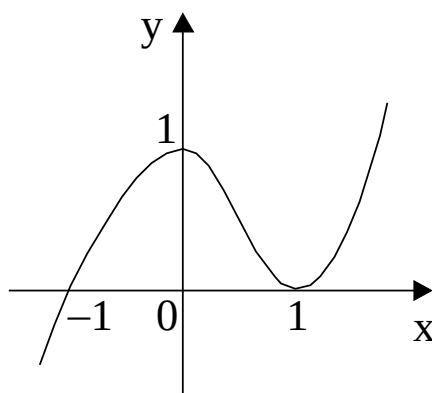
End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet

Marks

- (a) Copy the graph of the function given below in your answer booklet and sketch the graph of the **gradient function** on the same axes.

1



- (b) Evaluate $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$

1

- (c) Using the method of first principles, find $f'(x)$ if $f(x) = x^2 + 2x$

2

- (d) Differentiate each of the following functions:

(i) $f(x) = (3x^2 - 1)^4$

1

(ii) $f(x) = (2x - 3)(x + 9)$

2

(iii) $f(x) = \frac{3x + 1}{x - 5}$

2

(iv) $f(x) = \frac{1}{\sqrt{x^3}}$

1

- (e) Find the equation of the tangent to the curve of $y = x^2 - 5x + 4$ where the tangent is parallel to the line $y = 3x - 1$

3

- (f) If $s = 5t^2 + 12t$, find the value of t when $\frac{ds}{dt} = 0$

2

End of Examination

Preliminary Mathematics

Section A Multiple-Choice Answer Sheet

- | | | | | |
|----|-------------------------|-------------------------|-------------------------|-------------------------|
| 1 | A ☐ | B ☐ | C ☐ | D ☐ |
| 2 | A ☐ | B ☐ | C ☐ | D ☐ |
| 3 | A ☐ | B ☐ | C ☐ | D ☐ |
| 4 | A ☐ | B ☐ | C ☐ | D ☐ |
| 5 | A ☐ | B ☐ | C ☐ | D ☐ |
| 6 | A ☐ | B ☐ | C ☐ | D ☐ |
| 7 | A ☐ | B ☐ | C ☐ | D ☐ |
| 8 | A ☐ | B ☐ | C ☐ | D ☐ |
| 9 | A ☐ | B ☐ | C ☐ | D ☐ |
| 10 | A ☐ | B ☐ | C ☐ | D ☐ |

Preliminary Mathematics

Section A Multiple-Choice Answer Sheet

- | | | | | |
|----|------------------------------------|------------------------------------|------------------------------------|------------------------------------|
| 1 | A ☐ | B ☐ | C ☐ | D ☒ |
| 2 | A ☐ | B ☒ | C ☐ | D ☐ |
| 3 | A ☐ | B ☐ | C ☒ | D ☐ |
| 4 | A ☐ | B ☐ | C ☐ | D ☒ |
| 5 | A ☐ | B ☐ | C ☒ | D ☐ |
| 6 | A ☒ | B ☐ | C ☐ | D ☐ |
| 7 | A ☐ | B ☐ | C ☐ | D ☒ |
| 8 | A ☐ | B ☐ | C ☒ | D ☐ |
| 9 | A ☐ | B ☒ | C ☐ | D ☐ |
| 10 | A ☐ | B ☐ | C ☐ | D ☒ |

Question 11

a) $3.6305 \dots \checkmark$
 3.631 (4 sig figs) $\checkmark$

b) $x = 0.5656 \dots$
 $100x = 56.5656$
 $99x = 56 \checkmark$
 $x = \frac{56}{99} \checkmark$

c) $3x + 1 = 4 \checkmark$
 $3x = 3$
 $x = 1 \checkmark$

d) $2x - 4 \leq 2$ $2x - 4 \geq -2$
 $2x \leq 6$ $2x \geq 2$
 $x \leq 3 \checkmark$ $x \geq 1 \checkmark$
 or
 $1 \leq x \leq 3$

e) $\frac{12}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{36-12\sqrt{5}}{4} \checkmark$
 $= 9-3\sqrt{5} \checkmark$

f) $2z^2 + 6zy + xz + 3xy$
 $= 2z(z+3y) + x(z+3y)$
 $= (2z+x)(z+3y) \checkmark$

g) $\frac{(x-2)(x+2)}{2(x-2)} \times \frac{2}{x+2} \checkmark$
 $= \frac{2}{2} \checkmark$
 $= 1 \checkmark$

h) $\frac{x-5}{3} - \frac{x+1}{4} = 2$

$\frac{4(x-5) - 3(x+1)}{12} = 2 \checkmark$

$4x - 20 - 3x - 3 = 24$

$x - 23 = 24$

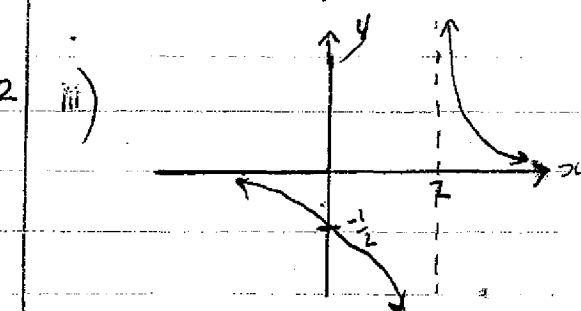
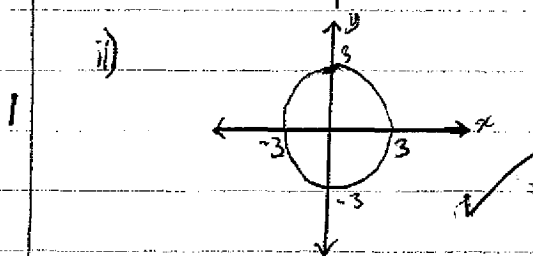
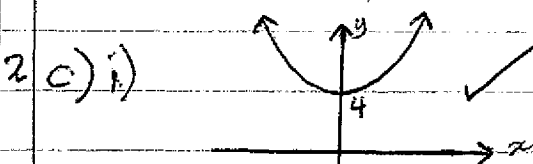
$x = 47 \checkmark$

Question 12

a) i) $f(-1) = -3$, $f(3) = 8^2 + 1 = 10$
 $\therefore f(-1) + f(3) = -3 + 10$
 $= 7 \checkmark$

ii) $f(n^2+3) = (n^2+3)^2 + 1 \checkmark$
 $= n^4 + 6n^2 + 9 + 1$
 $= n^4 + 6n^2 + 10 \checkmark$

b) D: $-2 \leq x \leq 2 \checkmark$
 R: $2 \geq y \geq 0 \checkmark$



$$d) 3x^2 - 5x + 4 = 2x + 10 \quad \checkmark$$

$$3x^2 - 7x - 6 = 0$$

$$3x^2 + 2x - 9x - 6 = 0$$

$$x(3x+2) - 3(3x+2) = 0$$

$$(3x+2) = 0 \quad x-3 = 0$$

$$\boxed{x = -\frac{2}{3}} \quad \checkmark \quad \boxed{x = 3} \quad \checkmark$$

e) function is even if $f(x) = f(-x)$

$$f(x) = 1 - \frac{1}{x^2}$$

$$f(-x) = 1 - \frac{1}{(-x)^2}$$

$$= 1 - \frac{1}{x^2}$$

$$= f(x) \quad \checkmark$$

$\therefore$ function is even

$$f) y \geq |x| - 2 \quad y \leq 4 - x^2$$

Question 13

$$a) i) d_{pq} = \sqrt{(2-(-1))^2 + (3-0)^2}$$

$$= \sqrt{9+9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2} \text{ units. } \checkmark$$

$$ii) m_{pq} = \frac{3-0}{2-(-1)} = \frac{3}{3} = 1 \quad \checkmark$$

$$iii) \angle PQX \Rightarrow \tan \theta = 1 \quad \checkmark$$

$$\theta = 45^\circ$$

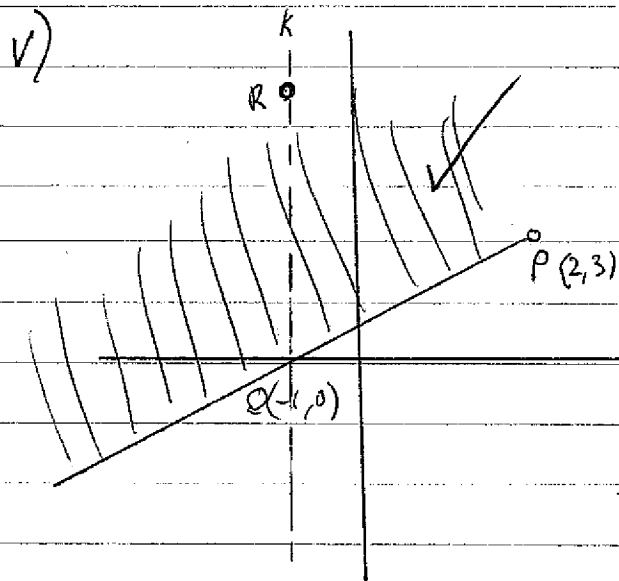
$$\angle RQP = 90 - 45$$

$$= 45^\circ \quad \checkmark$$

$$3 iv) y - 3 = 1(x - 2) \quad \checkmark$$

$$y - 3 = x - 2$$

$$\therefore x - y + 1 = 0$$



$$vi) x = -1 \quad \checkmark$$

$$vii) m_{pq} = -1$$

$$\frac{3-y}{2-(-1)} = -1$$

$$\frac{3-y}{3} = -1$$

$$3-y = -3$$

$$6 = y \quad \checkmark$$

$$\therefore R(-1, 6) \quad \checkmark$$

$$2 b) \frac{2-\sqrt{3}}{\sqrt{2}} \quad \checkmark$$

$$\frac{2-\sqrt{3}}{\sqrt{2}} \quad \checkmark \quad \text{or } \frac{2-\sqrt{3}}{\sqrt{2}} \quad \checkmark \quad \text{or } \frac{2\sqrt{2}-\sqrt{6}}{2}$$

$$c) \frac{2(\sec^2 A - 1)}{4 \tan A} = \frac{2 \tan^2 A}{4 \tan A} \checkmark$$

$$= \frac{\tan A}{2} \checkmark$$

$$d) \sin \theta \tan \theta = \sin \theta \times \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\sin^2 \theta}{\cos \theta} \checkmark$$

$$= \frac{1 - \cos^2 \theta}{\cos \theta}$$

$$= \text{RHS} \checkmark$$

a) Question 14

$$(i) x = 30, 180 + 30$$

$$x = 30^\circ, 210^\circ \checkmark$$

$$ii) 2x = 30, 180 - 30, -180 - 30, -360 + 30$$

$$= 30, 150, -210, -330$$

$$x = 15^\circ, 75^\circ, -105^\circ, -165^\circ \checkmark$$

$$b) i) BD^2 = 16^2 + 12^2$$

$$= 400$$

$$BD = 20 \text{ cm} \checkmark$$

$$ii) \triangle BAD \quad \angle DAB = \angle BXA \text{ (given)} \checkmark$$

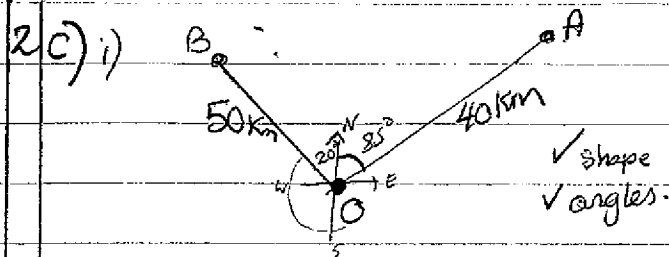
$$\angle ABD = \angle ABX \text{ (common)} \checkmark$$

$$\therefore \triangle ABX \text{ is similar to } \triangle DBA \text{ (AA)} \checkmark$$

$$iii) \frac{BX}{BA} = \frac{AB}{DB} \rightarrow \frac{BX}{16} = \frac{16}{20} \checkmark$$

$$BX = 16 \times \frac{16}{20}$$

$$= 12.8 \text{ cm} \checkmark$$



$$ii) \angle AOB = \angle BON + \angle NOA$$

$$= 20 + 85 \checkmark$$

$$= 105^\circ$$

$$iii) x^2 = 50^2 + 40^2 - 2 \times 50 \times 40 \cos 105 \checkmark$$

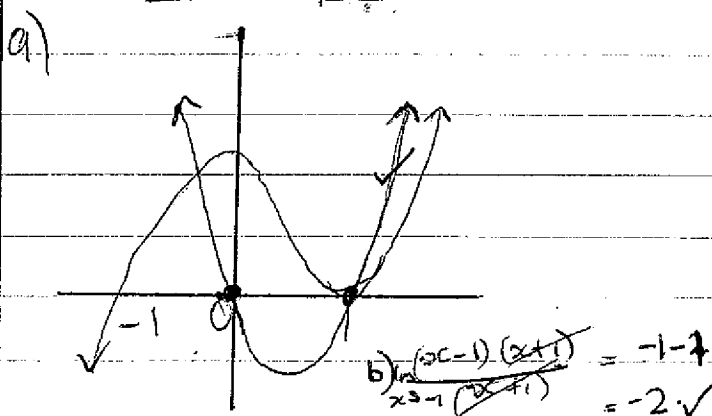
$$= 4100 - 4000 \cos 105$$

$$= 5135.276 \dots$$

$$x = 71.66084 \dots$$

$$= 72 \text{ km (to the nearest km)} \checkmark$$

Question 15



$$a) f(x+h) = (x+h)^2 + 2(x+h)$$

$$= x^2 + 2xh + h^2 + 2x + 2h$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h} \checkmark$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \rightarrow 0} (2x + h + 2)$$

$$= 2x + 0 + 2 \checkmark$$

$$= 2x + 2 \checkmark$$

$$d) i) f'(x) = 4x \cdot 6x (3x^2 - 1)^3 \\ = 24x^2 (3x^2 - 1)^3 \quad \checkmark$$

$$ii) u = 2x - 3 \quad v = x + 9 \\ du = 2 \quad dv = 1 \quad \checkmark$$

$$f'(x) = 2(5x+9) + 1(2x-3) \quad \checkmark \\ = 2x + 18 + 2x - 3 \quad \leftarrow \text{OR} \\ = 4x + 15 \quad \checkmark$$

$$iii) u = 3x + 1 \quad v = x - 5 \\ du = 3 \quad dv = \cancel{1} \quad 1 \quad \checkmark$$

$$f'(x) = \frac{3(x-5) - 1(3x+1)}{1^2} \quad \checkmark \\ = \frac{3x - 15 - 3x - 1}{1} \\ = -14 \quad \checkmark$$

$$iv) f(x) = x^{-\frac{3}{2}} \quad \checkmark$$

$$f'(x) = -\frac{3}{2} x^{-\frac{5}{2}} \quad \checkmark \\ = -\frac{3}{2\sqrt{x^5}} \quad \checkmark$$

$$e) m = 3$$

$$y' = 2x - 5$$

$$2x - 5 = 3 \quad \checkmark$$

$$2x = 8$$

$$\boxed{x = 4}$$

$$2 \quad y = 16 - 20 + 4 \\ = 0$$

$$(4, 0) \quad \checkmark$$

$$3 \quad y - 0 = 3(x - 4)$$

$$y = 3x - 12 \quad \checkmark \\ \text{OR}$$

$$3x - y - 12 = 0$$

$$f) \frac{ds}{dt} = 10t + 12 \quad \checkmark$$

$$10t + 12 = 0$$

$$10t = -12$$

$$t = \frac{-12}{10} \quad \checkmark \\ \text{OR}$$

$$= -\frac{6}{5}$$