

Year 11

Mathematics

Yearly Examination

2013

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11- 14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II

60 marks

- Attempt Questions 11-14
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and “N/A” on the front cover
- Allow about 1 hour 45 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

BLANK PAGE

Section I

10 Marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

1 Find the value, correct to 3 significant figures of:

$$\sqrt[3]{\frac{64.9 \times 13.4}{8.62 + 4.69}}$$

- A 4.02
- B 4.027
- C 4.028
- D 4.03

2 What is $\frac{2}{2+\sqrt{3}}$ as a fraction with a rational denominator?

- A $\frac{4-2\sqrt{3}}{7}$
- B $\frac{4+2\sqrt{3}}{7}$
- C $4-2\sqrt{3}$
- D $4+2\sqrt{3}$

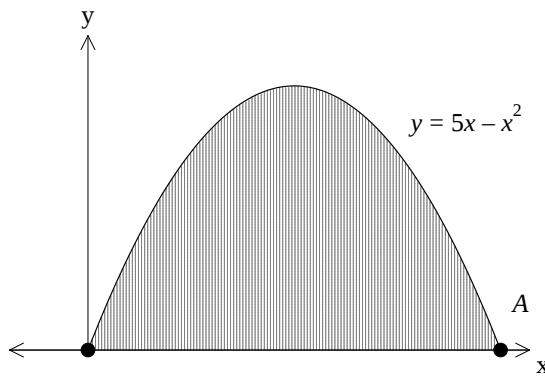
3 What is the solution to the equation $\frac{7x-3}{2} = 4$?

- A $x = \frac{7}{11}$
- B $x = \frac{5}{7}$
- C $x = 1\frac{2}{7}$
- D $x = 1\frac{4}{7}$

4 What is the value of $f(-1)$ if $f(x) = x^3 - x^2$?

- A -2
- B -1
- C 0
- D 2

5 The diagram shows the graph of the function $y = 5x - x^2$.



What pair of inequalities specify the shaded region?

- A $y \leq 5x - x^2$ and $y \leq 0$.
- B $y \leq 5x - x^2$ and $y \geq 0$.
- C $y \geq 5x - x^2$ and $y \leq 0$.
- D $y \geq 5x - x^2$ and $y \geq 0$.

6 What is the gradient of the line perpendicular to the line $2x + y - 2 = 0$?

A -2

B $-\frac{1}{2}$

C $\frac{1}{2}$

D 2

7 What is solution to the equation $2\cos\beta = -\sqrt{3}$ for $0^\circ \leq \beta \leq 360^\circ$?

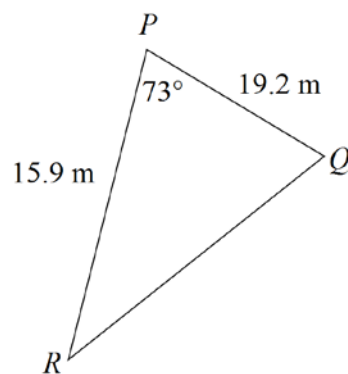
A $\beta = 30^\circ$ and 330°

B $\beta = 60^\circ$ and 300°

C $\beta = 150^\circ$ and 210°

D $\beta = 120^\circ$ and 240°

8 What is the area of triangle PQR to the nearest square metre?



A 141 m^2

B 146 m^2

C 283 m^2

D 296 m^2

- 9 What is the value of $f'(3)$ if $f(x) = 3x - x^3$?
- A $f'(3) = -24$
 - B $f'(3) = -18$
 - C $f'(3) = 0$
 - D $f'(3) = 9$
- 10 The sum of the interior angles of a regular polygon is 2520° . What is the size of each interior angle?
- A $140^\circ 30'$
 - B $157^\circ 30'$
 - C 210°
 - D 315°

Section II

60 marks

Attempt Questions 11 - 14

Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet.

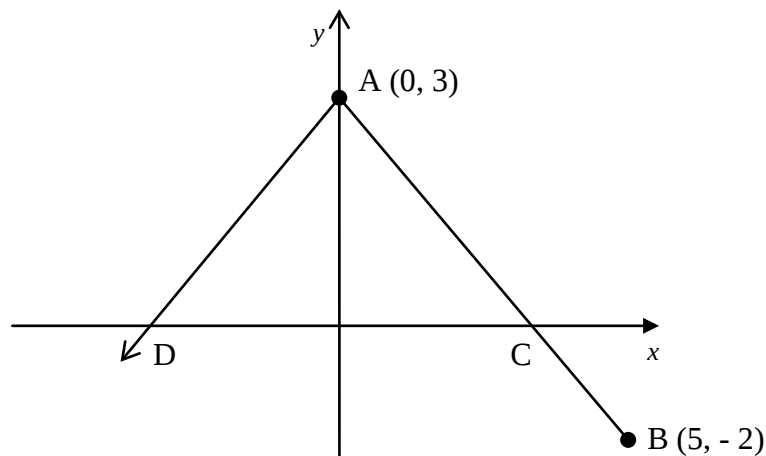
All necessary working should be shown in every question

	Marks
Question 11 (15 marks). Use a SEPARATE writing booklet.	
a) Factorise fully and simplify your answer where possible:	
(i) $8p^3 - 27$	2
(ii) $(x - 2)^2 - 16$	2
b) Solve:	
(i) $ 7 - 3x = 3$	2
(ii) $2x^2 + 7x - 4 = 0$	2
c) Simplify fully:	
(i) $\frac{5m^2 \times 4m^6}{10m^3}$	2
(ii) $\frac{x}{x^2 - 4} + \frac{2}{x - 2}$	2
d) Make neat sketches of the following equations on separate sets of axes. Mark clearly the essential features of each graph.	
(i) $y = x^2 + 4$	1
(ii) $x^2 + (y + 3)^2 = 36$	2

Marks

Question 12 (15 marks). Use a SEPARATE writing booklet.

- a) B is the point (5,-2) on the number plane.
 A (0, 3) is the point of intersection of the line AB with the y axis and C is the point of intersection of the line AB with the x axis.
 A line is drawn through A, perpendicular to AC which cuts the x axis at D.

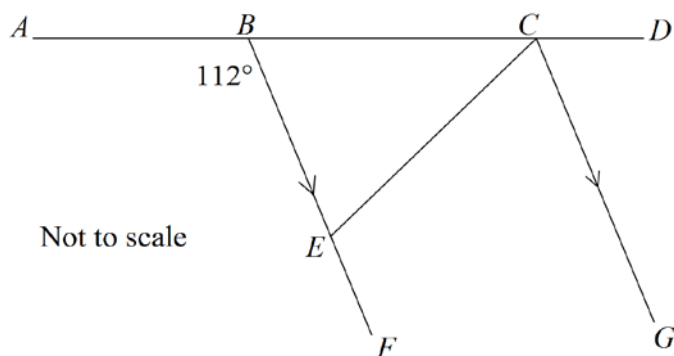


- | | | |
|-------|---|---|
| (i) | Show that the equation of AB is $x + y - 3 = 0$. | 2 |
| (ii) | Find the size of $\angle DCA$. | 1 |
| (iii) | Show that the equation of AD is $x - y + 3 = 0$. | 2 |
| (iv) | Find the coordinates of the points C and D. | 1 |
| (v) | Find the exact area of $\triangle ADC$. | 2 |
| (vi) | Show that $\triangle ADC$ is isosceles. | 1 |
- b) Find the exact shortest distance between the line $3x + 2y = 5$ and the point (5,3).
- 2

QUESTION 12 CONTINUES ON THE NEXT PAGE

Marks

- (c) In the diagram below BF is parallel to CG , $BC = EC$ and $\angle ABE = 112^\circ$.



- | | | |
|------|---|---|
| (i) | Show that $\angle BEC = 68^\circ$. | 2 |
| (ii) | Hence or otherwise, show that CG bisects $\angle DCE$. | 2 |

Marks

Question 13 (15 marks). Use a SEPARATE writing booklet.

- a) Differentiate with respect to x and fully simplify your answer where possible:
- | | |
|-------------------------|---|
| (i) $4x^2 - x + 1$ | 1 |
| (ii) $4x\sqrt{x}$ | 2 |
| (iii) $\sqrt{3x^2 - 3}$ | 2 |
| (iv) $\frac{x}{5 - 3x}$ | 2 |
| (v) $(3x + 2)(x + 1)^2$ | 2 |
- b) Find the equations of the tangent and normal to the curve $y = (2x - 1)^4$ at the point where $x = 1$. 4
- c) Differentiate $f(x) = x^2 - x$ from first principles. 2

Marks

Question 14 (15 marks). Use a SEPARATE writing booklet.

a) Solve for $0^\circ \leq x \leq 360^\circ$

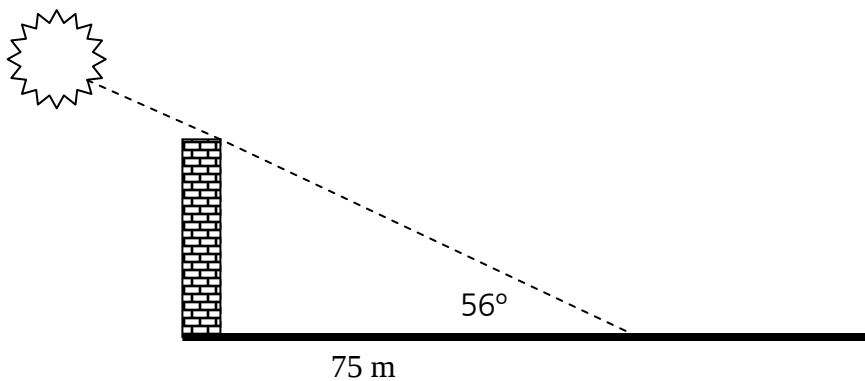
(i) $\cos x = \frac{1}{\sqrt{2}}$ 1

(ii) $2 \sin x + 1 = 0$ 2

(iii) $3 \tan^2 x = 1$ 3

b) Show that $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = 2 \sec^2 \theta$ 2

c) A building on level ground, casts a shadow of length 75 metres when the angle of elevation of the sun is 56° . 2

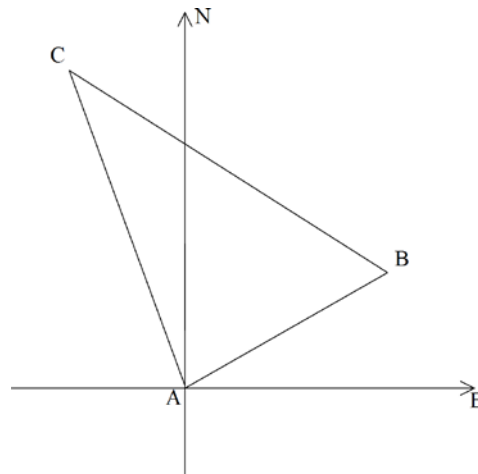


Calculate the height of the building. Answer correct to the nearest one-tenth of a metre.

QUESTION 14 CONTINUES ON THE NEXT PAGE

Marks

- d) A ship sails 50 km from Port A to Port B on a bearing of 63° , then sails 130 km from Port B to Port C, on a bearing of 296° .



- | | | |
|-------|--|---|
| (i) | Copy the diagram and mark on it all the given information. | 1 |
| (ii) | Show that $\angle ABC = 53^\circ$. | 1 |
| (iii) | Find, to the nearest km, the distance of Port A from Port C. | 1 |
| (iv) | Find the bearing of Port A from Port C, correct to the nearest degree. | 2 |

END OF EXAMINATION

2013 Year 11 Yearly Examination

Student Name:

Preliminary Mathematics

Section A Multiple-Choice Answer Sheet

- | | | | | |
|-----------|-------------------------|-------------------------|-------------------------|-------------------------|
| 1 | A ☐ | B ☐ | C ☐ | D ☐ |
| 2 | A ☐ | B ☐ | C ☐ | D ☐ |
| 3 | A ☐ | B ☐ | C ☐ | D ☐ |
| 4 | A ☐ | B ☐ | C ☐ | D ☐ |
| 5 | A ☐ | B ☐ | C ☐ | D ☐ |
| 6 | A ☐ | B ☐ | C ☐ | D ☐ |
| 7 | A ☐ | B ☐ | C ☐ | D ☐ |
| 8 | A ☐ | B ☐ | C ☐ | D ☐ |
| 9 | A ☐ | B ☐ | C ☐ | D ☐ |
| 10 | A ☐ | B ☐ | C ☐ | D ☐ |

2013 Year 11 Yearly Examination

Student Name: SOLS.....

Preliminary Mathematics

Section A Multiple-Choice Answer Sheet

- | | | | | |
|----|------------------------------------|------------------------------------|------------------------------------|------------------------------------|
| 1 | A ☐ | B ☐ | C ☐ | D ☒ |
| 2 | A ☐ | B ☐ | C ☒ | D ☐ |
| 3 | A ☐ | B ☐ | C ☐ | D ☒ |
| 4 | A ☒ | B ☐ | C ☐ | D ☐ |
| 5 | A ☐ | B ☒ | C ☐ | D ☐ |
| 6 | A ☐ | B ☐ | C ☒ | D ☐ |
| 7 | A ☐ | B ☐ | C ☒ | D ☐ |
| 8 | A ☐ | B ☒ | C ☐ | D ☐ |
| 9 | A ☒ | B ☒ | C ☐ | D ☐ |
| 10 | A ☐ | B ☒ | C ☐ | D ☐ |

Year 11 2 Unit Assessment Task 3 2013 ①

1) $4.0\dot{3}$ D

2) $\frac{2}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2(2-\sqrt{3})}{4-3}$
 $= 4-2\sqrt{3}$ C

3) $7x-3=8$
 $7x=11$
 $x=\frac{11}{7}=1\frac{4}{7}$ D

4) $f(-1)=(-1)^3-(-1)^2$
 $=-2$ A

5) $y \geq 0$ and
 $y \leq 5x-x^2$
 $0 \leq 5-1$ B

6) $y = -2x+2$
 $m_2 = \frac{1}{2}$ C

7) $\cos \beta = -\frac{\sqrt{3}}{2}$
 $\beta = 150^\circ, 210^\circ$ C

8) $A = \frac{1}{2} \times 19.2 \times 15.9 \times \sin 73^\circ$
 $= 145.9$ B

$$f'(u) = 3 - 3u^2$$

②

$$\begin{aligned} 9) \quad f'(3) &= 3 - 3(3)^2 \\ &= 3 - 27 \\ &= -24 \end{aligned}$$

A

$$10) \quad (n-2)180 = 2520$$

$$\begin{aligned} n-2 &= 14 \\ n &= 16 \end{aligned}$$

$$\begin{aligned} \theta &= 2520 \div 16 \\ &= 157.5^\circ \end{aligned}$$

B

(3)

$$11 \text{ a) i) } 8p^3 - 27 = (2p)^3 - 3^3 \quad \checkmark$$

$$= (2p-3)(4p^2+6p+9) \quad \checkmark$$

1 mark recognising correct cubes
1 mark correct

$$\text{ii) } (x-2)^2 - 16 = [(x-2)+4][(x-2)-4]$$

$$= (x+2)(x-6)$$

1 mark correct DOS
1 mark fully simplified

$$\text{b) i) } |7-3x| = 3$$

$$7-3x = 3$$

$$-3x = -4$$

$$x = \frac{4}{3} \quad \checkmark$$

$$-7+3x = 3$$

$$3x = 10$$

$$x = \frac{10}{3} \quad \checkmark$$

1 mark each solution

$$\text{ii) } 2x^2 + 7x - 4 = 0$$

$$P: -8$$

$$2x^2 + 8x - x - 4 = 0$$

$$S: +7$$

$$2x(x+4) - (x+4) = 0$$

$$F: +8, -1$$

$$(x+4)(2x-1) = 0$$

$$x = -4, \frac{1}{2} \quad \checkmark$$

1 mark correct working any method
1 mark both correct

$$\text{c) i) } \frac{5m^2 \times 4m^6}{10m^3} = \frac{20m^8}{10m^3} \quad \checkmark$$

$$= 2m^5 \quad \checkmark$$

1 mark correct indices

1 mark correct

$$\text{ii) } \frac{x}{x^2-4} + \frac{2}{x-2} = \frac{x+2(x+2)}{(x+2)(x-2)} \quad \checkmark$$

1 mark correct denom

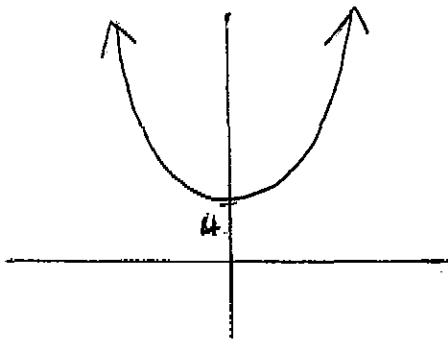
$$= \frac{x+2x+4}{(x+2)(x-2)}$$

$$= \frac{3x+4}{(x+2)(x-2)} \quad \checkmark$$

1 mark correct

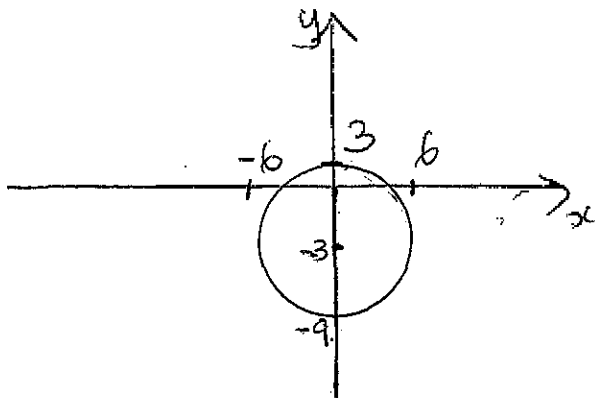
4

d). 1) $y = x^2 + 4$



✓ shape and vertex.

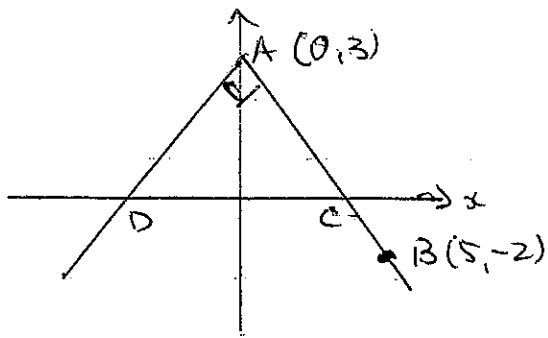
ii) $x^2 + (y+3)^2 = 36$
circle centre $(0, -3)$, $r=6$.



✓ circle with centre $(0, -3)$
✓ indication of correct radius

(5)

12.



i) $m_{AB} = \frac{-2-3}{5-0}$
 $= -1$ ✓

1 mark
gradient

eq AB: $y-3 = -x$
 $x+y-3=0$

1 mark
correct working

ii) $\tan \angle DCA = |-1|$
 $\angle DCA = 45^\circ$ ✓

1 mark

iii) AD, AC + AD
 $\therefore m_{AD} = 1$ ✓

1 mark
gradient

eq AD: $y-3 = x$
 $x-y+3=0$ ✓

1 mark
correct working

iv) at C $y=0: x=3$ C (3,0) ✓
 at D $y=0: x=-3$ D (-3,0) ✓

1 mark
both

v) $AC^2 = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$
 $AD^2 = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$

Area $\triangle ADC = \frac{1}{2} \times 3\sqrt{2} \times 3\sqrt{2}$
 $= 9u^2$ ✓

Alternate
 $A = \frac{1}{2} \times 6 \times 3$
 $A = 9u^2$

1 mark
correct area

vi) since $AC=AD$ (above)
 $\triangle ADC$ is isosceles

Alternate.
 $\tan \angle CDA = 1$
 $\therefore \angle CDA = 45^\circ$ ($\angle CDA = \angle DCA$)
 $\therefore \triangle ADC$ is isosceles

(6)

$$b) 3x + 2y = 5$$

$$3x + 2y - 5 = 0$$

$$3x + 2y - 5 = 0 \quad (5, 3)$$

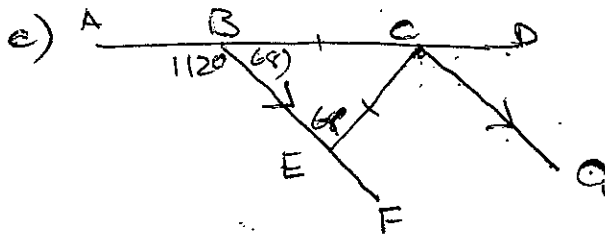
$$d = \left| \frac{3(5) + 2(3) - 5}{\sqrt{3^2 + 2^2}} \right| \checkmark$$

$$= \left| \frac{16}{\sqrt{13}} \right|$$

1 mark correct
sub into correct
formula.

$$\therefore d = \frac{16}{\sqrt{13}} \text{ u.} \quad \checkmark$$

1 mark correct
(only CTWE if
formula used correctly)



$$i) \angle CBE = 68^\circ \text{ (AD straight line)} \quad \checkmark$$

$$BC = EC \therefore \triangle BEC \text{ isosceles} \quad \checkmark$$

$$\text{so } \angle CBE = \angle BEC$$

$$= 68^\circ \text{ (base ls of isosceles } \triangle)$$

correct
reasoning

$$ii) \angle QCE = 68^\circ \text{ alternate angles } BF \parallel CQ \quad \checkmark$$

$$\angle DCQ = 68^\circ \text{ corresponding angles } BF \parallel CQ$$

$$\therefore \angle QCE = \angle DCQ$$

$$\& CQ \text{ bisects } \angle DCE.$$

correct
reasoning

13. a) i) $y = 4x^2 - x + 1$
 $y' = 8x - 1$ ✓

1 mark

ii) $y = 4x\sqrt{x}$
 $= 4x^{3/2}$ ✓

$y' = 4 \times \frac{3}{2} \times x^{1/2}$

$= 6\sqrt{x}$ or $6x^{1/2}$ ✓

1 mark
 under form

1 mark

iii) $y = \sqrt{3x^2 - 3}$
 $= (3x^2 - 3)^{1/2}$

$y' = \frac{1}{2} \times 6x (3x^2 - 3)^{-1/2}$ ✓

$= 3x (3x^2 - 3)^{-1/2}$

or $\frac{3x}{\sqrt{3x^2 - 3}}$ ✓

1 mark
 correct use chain rule

1 mark correct

iv) $y = \frac{x}{5 - 3x}$

$u = x$
 $u' = 1$

$v = 5 - 3x$
 $v' = -3$

$y' = \frac{(1)(5 - 3x) - x(-3)}{(5 - 3x)^2}$ ✓

$= \frac{5 - 3x + 3x}{(5 - 3x)^2}$

$= \frac{5}{(5 - 3x)^2}$ ✓

1 mark correct
 use product rule.

1 mark
 correct

v) $y = (3x + 2)^3 (x + 1)^2$

$u = (3x + 2)$

$v = (x + 1)^2$

$u' = 3$

$v' = 2(x + 1)$

$y' = 2(x + 1)(3x + 2) + 3(x + 1)^2$ ✓

$= (x + 1)[2(3x + 2) + 3(x + 1)]$

$= (x + 1)(6x + 4 + 3x + 3)$

$= (x + 1)(9x + 7)$ ✓

1 mark correct
 use product rule
 1 mark some
 simplification ✓

OR $9x^2 + 16x + 7$

8

b) $y = (2x-1)^4$
 $y' = 4 \cdot 2 (2x-1)^3$
 $= 8(2x-1)^3$ ✓

1 mark diff

at $x=1$, $m_T = 8(2-1)^3$
 $= 8$

$x=1$, $y = (2-1)^4$
 $= 1$

(1,1) ✓

1 mark coords

eq tang: $m_T = 8$ (1,1)

$y-1 = 8(x-1)$
 $= 8x-8$

$y = 8x-7$

$8x-y-7=0$

} ✓

1 mark

eq normal: $m_N = -\frac{1}{8}$ (1,1)

$y-1 = -\frac{1}{8}(x-1)$

$8y-8 = -x+1$

$x+8y-9=0$ OR $y = \frac{1}{8}(9-x)$ ✓

1 mark

c) $f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - (x+h) - x^2 + x}{h}$

$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}$ ✓

1 mark
correct
expansion

$= \lim_{h \rightarrow 0} \frac{2xh + h^2 - h}{h}$

$= \lim_{h \rightarrow 0} \frac{h(2x+h-1)}{h}$

$= \lim_{h \rightarrow 0} 2x+h-1$ ✓

$= 2x-1$

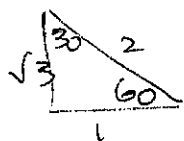
correct

14a) i) $\cos x = \frac{1}{\sqrt{2}} \quad x = 45^\circ, 315^\circ \quad \checkmark$

1 mark

ii) $2\sin x + 1 = 0 \quad 0^\circ \leq x \leq 360^\circ$

$\sin x = -\frac{1}{2} \quad Q3, Q4. 30^\circ$



$x = 210^\circ, 330^\circ \quad \checkmark \checkmark$

1 mark each

iii) $3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{1}{\sqrt{3}} \quad \checkmark$

all Quadrants
 30°

1 mark

$x = 30^\circ, 150^\circ, 210^\circ, 330^\circ \quad \checkmark \checkmark$

2 marks all correct
1 mark 2 correct

b) $\frac{1}{1+\sin \theta} + \frac{1}{1-\sin \theta} = 2\sec^2 \theta$

LHS = $\frac{1-\sin \theta + 1+\sin \theta}{(1-\sin^2 \theta)} \quad \checkmark$

1 mark

$= \frac{2}{\cos^2 \theta} \quad \checkmark$

1 mark

$= 2\sec^2 \theta$

$= \text{RHS}$

c) $\tan 43^\circ 37' = \frac{12}{h} \quad \checkmark$
 $h = \frac{12}{\tan 43^\circ 37'} \quad \checkmark$
 $= 12.5939$
 $= 12.6 \text{ cm} \quad \checkmark$

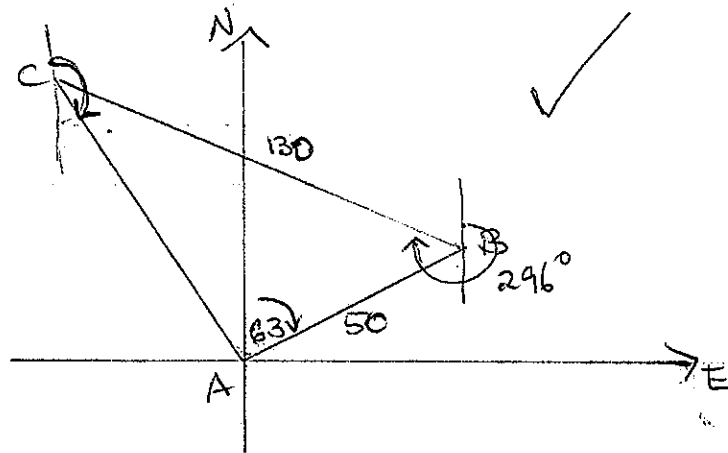
$\tan 56^\circ = \frac{h}{75} \quad \text{use of tan.}$

$h = 75 \tan 56^\circ$
 $= 111.2 \text{ cm}$

correct

(10)

d)



1) 1 mark all
info marked
correctly.

i) $\angle ABC = 53^\circ$

$$\angle ABC = 296^\circ - 180^\circ - 63^\circ$$

$$= 53^\circ$$

1 mark
correct reasoning

ii) $AC^2 = 50^2 + 130^2 - 2 \times 50 \times 130 \times \cos 53^\circ$ ✓

$$= 11576.4047$$

correct use
Cosine Rule

$$AC = 107.59..$$

$$= 108 \text{ km}$$

correct.

iii) Bearing A from C.

$$\frac{\sin C}{50} = \frac{\sin 53^\circ}{108}$$

$$\sin C = \frac{50 \times \sin 53^\circ}{108}$$

$$= 0.3697...$$

$$C = 21^\circ 42'$$

1 mark $\angle C$
any method.

$$\text{Bearing} = 22^\circ + (180^\circ - 116^\circ) \text{ * alternate angle from B.}$$

$$= 138^\circ$$

nb: $\cos C = \frac{130^2 + 108^2 - 50^2}{2 \times 130 \times 108}$

$$C = 22^\circ$$

1 mark
correct
bearing