International Grammar School

Year 11
Mathematics
Yearly Examination
2014

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11- 14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II

60 marks

- Attempt Questions 11-14
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover
- Allow about 1 hour 45 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Section I
10 Marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

1 What is the midpoint of the points (3, 8) and (–5, –2)?

(A) (–1, 3)

(B) (4, 5)

(C) (–2, 6)

(D) (2, 10)

2 Solve $\cos\theta = -\frac{\sqrt{3}}{2}, 0 \le \theta \le 360^\circ$.

(A) $\theta = 210^\circ, 330^\circ$

(B) $\theta = 150^\circ, 330^\circ$

(C) $\theta = 150^\circ, 210^\circ$

(D) $\theta = 150^\circ, 210^\circ, 330^\circ$

3 The solutions to $|x+1| = 4$ are

(A) $x = 4$

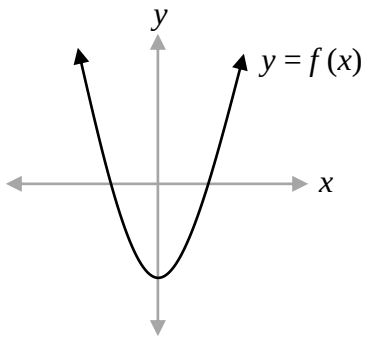
(B) $x = 3 \text{ and } x = -5$

(C) $x = 5 \text{ and } x = -4$

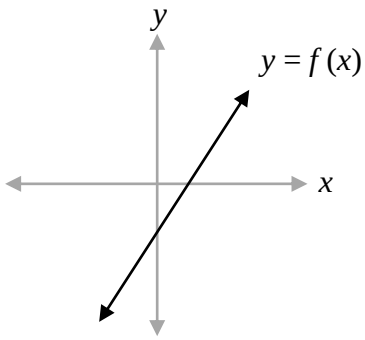
(D) $x = -5$

4 Which of the following graphs is NOT a function?

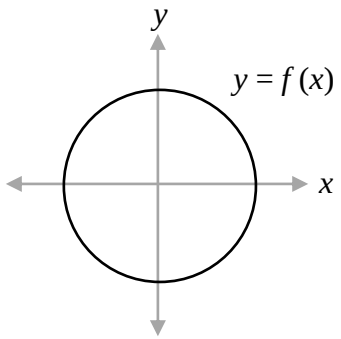
(A)



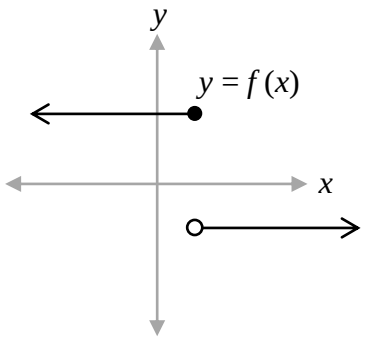
(B)



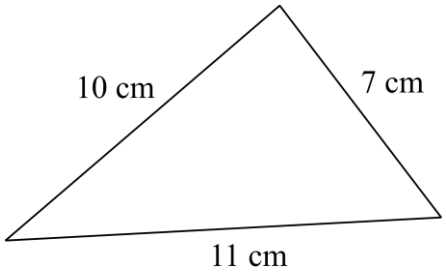
(C)



(D)



5 The following triangle has sides 7 cm, 10 cm and 11 cm.



Not to scale

Let A is the smallest angle.
Which of the following expressions is correct for angle A?

(A) $\cos A = \dfrac{7^2 + 11^2 - 10^2}{2 \times 7 \times 11}$

(B) $\cos A = \dfrac{10^2 + 7^2 - 11^2}{2 \times 7 \times 10}$

(C) $\cos A = \dfrac{10^2 + 11^2 - 7^2}{2 \times 10 \times 11}$

(D) $\cos A = \dfrac{10^2 + 7^2 - 11^2}{2 \times 10 \times 11}$

6 Simplify $\dfrac{\cos(90^\circ - \theta)}{\sin(90^\circ - \theta)}$:

(A) 1

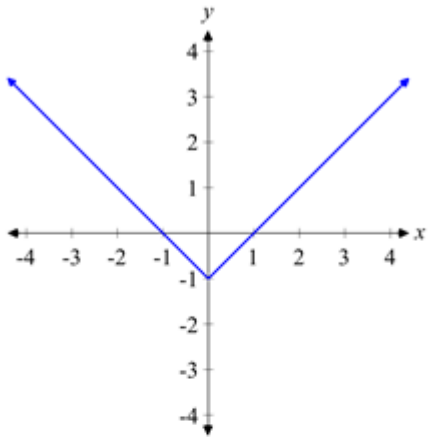
(B) $\cot\theta$

(C) $\tan\theta$

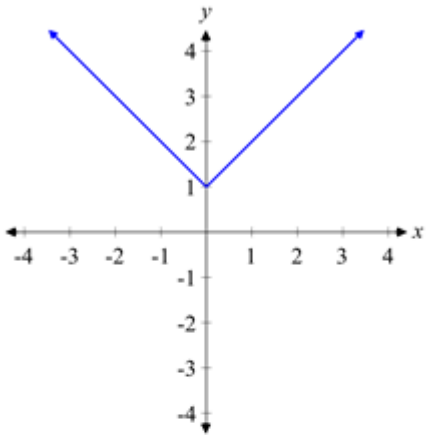
(D) $-\tan\theta$

7 Which graph best represents $y = |x| - 1$?

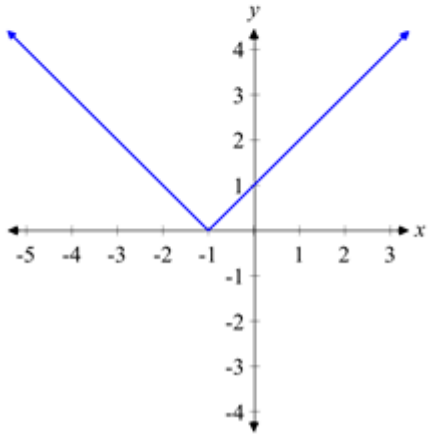
(A)



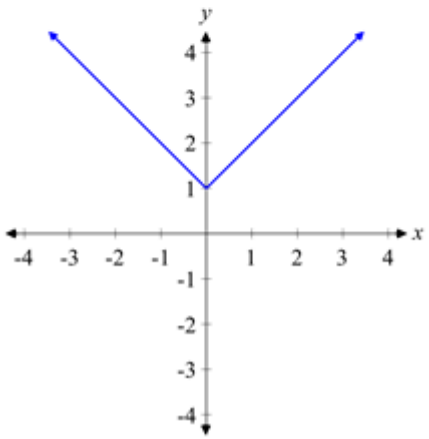
(B)



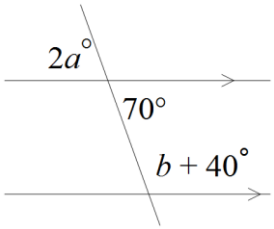
(C)



(D)



8 What are the values of a and b?



(A) $a=35^\circ, b=30^\circ$

(B) $a=35^\circ, b=70^\circ$

(C) $a=70^\circ, b=30^\circ$

(D) $a=70^\circ, b=70^\circ$

9 Two points A and B lie on opposite sides of the line l. A and B are equidistant from the line l. Which statement is always true?

(A) The line through A and B is perpendicular to the line l.

(B) The line through A and B is parallel to the line l.

(C) The midpoint of AB lies on the line l.

(D) If a line passing through A is parallel to a line passing through B, both these lines are parallel to the line l.

10 The line $6x-ky=2$ passes through the point $(3,2)$. What is the value of k?

(A) $\frac{10}{3}$

(B) $-\frac{10}{3}$

(C) -8

(D) 8

END OF SECTION I

Section II

60 marks
Attempt Questions 11 - 14
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet.
All necessary working should be shown in every question

Marks

Question 11 (15 marks). Use a SEPARATE writing booklet.

a) Factorise completely:

(i) $a^2 - 4a + 4$ 1

(ii) $ab - 2bc + 5a - 10c$ 2

b) Solve:

(i) $8(x+2) = 9 - x$ 2

(ii) $\frac{5y}{2} - \frac{2y}{3} = 3$ 2

c) Expand and simplify $3(4x - 7)^2 - (12x - 8x^2)$. 2

d) Simplify $\frac{4k^3 \times 6k^4}{12k^5}$ 2

e) State the domain and range of $y = \sqrt{x-5}$ 2

f) Solve these simultaneous equations. 2

$2x + y = 1$

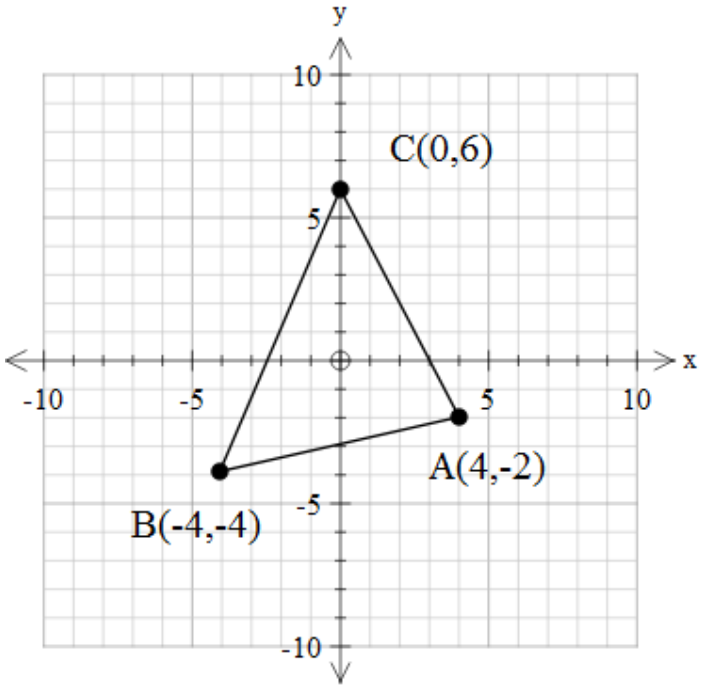
$6x - 4y = 10$

END OF QUESTION 11

Marks

Question 12 (15 marks). Use a SEPARATE writing booklet.

a) $A(4,-2), B(-4,-4), C(0,6)$ are the vertices of a triangle ABC



(i) Find the midpoint of AC. 1

(ii) Find the exact length of BC. 1

(iii) Show the equation of the line BC is $5x-2y+12=0$ 2

(iv) Calculate the exact perpendicular distance of A from the line BC 2

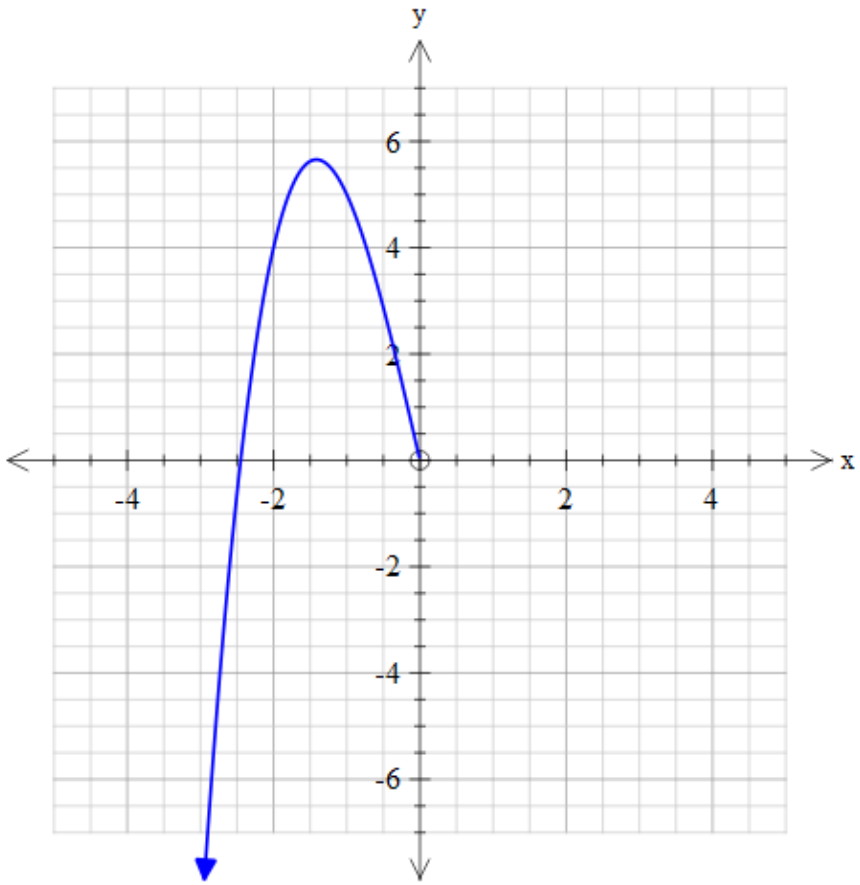
(v) Find the area of the triangle ABC. 1

(vi) Find the coordinates of D such that $ABCD$ is a parallelogram. 2

QUESTION 12 CONTINUES ON THE NEXT PAGE

b) (i) Show that $f(x) = x^3 - 6x$ is an odd function 2

(ii) Below is an incompleted graph of the function $f(x) = x^3 - 6x$. 1
Copy or trace the diagram into your writing booklet
and then **complete** the graph.



c) Find the equation of the **normal** to the curve $f(x) = x^2 - 4x + 1$ 3
at the point where $x = 1$.

END OF QUESTION 12

Marks

Question 13 (15 marks). Use a SEPARATE writing booklet.

a) Differentiate with respect to x and fully simplify your answer where possible:

(i) $3x^4 + 7x^2 + 1$ 1

(ii) $\sqrt{x^2 + 3}$ 2

(iii) $\dfrac{2}{\sqrt{x}}$ 2

(iv) $\dfrac{3x+1}{3-4x}$ 2

(v) $(2x-1)(x-2)^4$ 2

b) Given the curve $f(x) = x^2 - 4x$

(i) Show that $f(x+h) = x^2 - 2xh + h^2 - 4x - 4h$ 1

(ii) Hence, differentiate $f(x) = x^2 - 4x$ from first principles. 2

c) The function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} 7 \ for \ x < 0 \\ x^2 + 3 \ for \ x \geq 0 \end{cases}$$

(i) Find the value of $f(-1)$. 1

(ii) Sketch the function $y = f(x)$. 2

END OF QUESTION 13

Marks

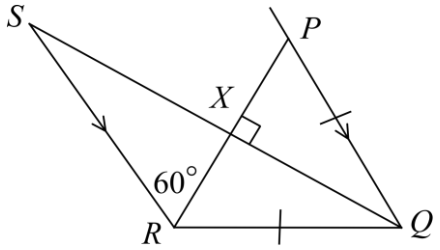
Question 14 (15 marks). Use a SEPARATE writing booklet.

a) Sketch the graph and clearly label the vertex, asymptotes (if any) and the points where the graph cuts the x and y-axes.

(i) $y = x^2 - x - 6$ 3

(ii) $y = \dfrac{4}{x-2}$ 3

b) In the diagram below, PQ is parallel to SR, $PQ = QR$, $\angle XRS = 60^\circ$ and lines PR and SQ are perpendicular.



Copy or trace the diagram into your writing booklet

(i) Find the size of $\angle RQS$. Give reasons. 2

(ii) Prove that ΔQRS is an isosceles triangle. 2

c) Prove $\sin\theta\cos\theta\tan\theta = 1 - \cos^2\theta$ 2

d) Draw a sketch showing the region on the number plane where the inequalities $x^2 + y^2 \leq 4$ and $x > 1$ hold simultaneously. 3

END OF EXAMINATION

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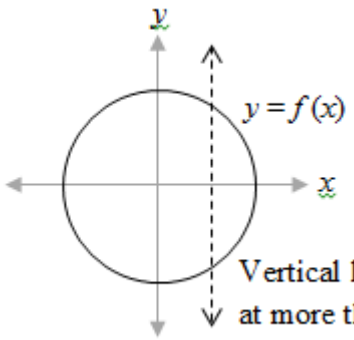
2014 Year 11 Yearly Examination **Student Name: . . . . . .**

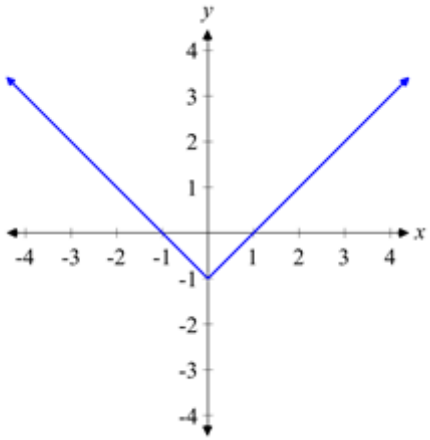
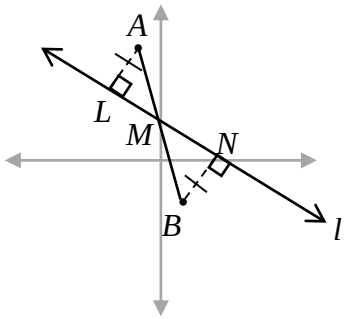
Preliminary Mathematics

Section A Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

SECTION I

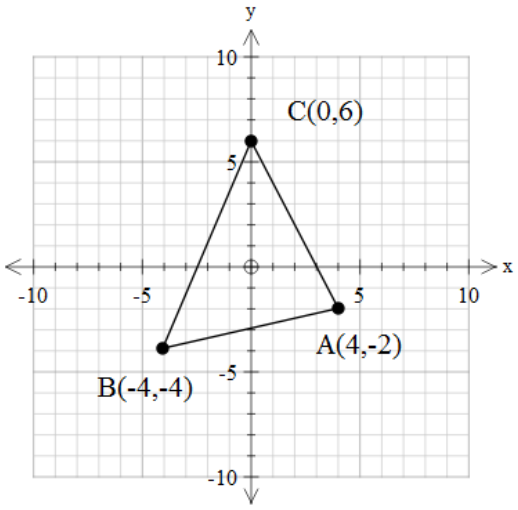
1	$(x,y)=\left(\frac{3+(-5)}{2},\frac{8+(-2)}{2}\right)$ $=(-1,3)$	A
2	$\cos\theta=-\frac{\sqrt{3}}{2}$ $\cos 30^\circ=\frac{\sqrt{3}}{2}$ cos is negative in the 2nd and 3rd quadrants. $\therefore\theta=180^\circ-30^\circ,180^\circ+30^\circ$ $=150^\circ,210^\circ$	C
3	$\lvert x+1\rvert=4\therefore x+1=4\therefore x=3$ and $x+1=-4\therefore x=-5$	B
4	 Vertical line cuts the graph at more than one point	C
5	Smallest angle is opposite the smallest side. $\cos A=\frac{10^2+11^2-7^2}{2\times10\times11}$	C
6	$\frac{\cos(90^\circ-\theta)}{\sin(90^\circ-\theta)}=\frac{\sin\theta}{\cos\theta}=\tan\theta$	C

7		A
8	$2a = 70,\ a = 35$ $70 + b + 40 = 180, b = 70$	B
9	$\Delta ALM \equiv \Delta BNM$ (AAS) $\therefore AM = BM$ (corresponding sides of congruent triangles) $\therefore$ M is the midpoint of AB and it lies on the line l. 	C
10	$6x - ky = 2, (3,2)$ $6 \times 3 - 2k = 2$ $-2k = -16$ $k = 8$	D

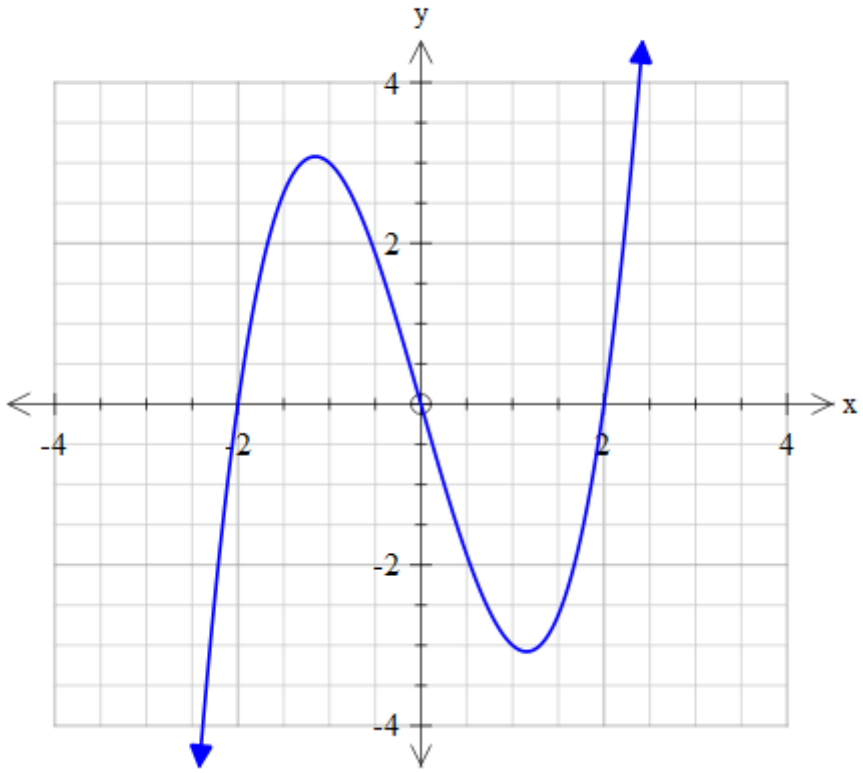
END OF SECTION I

SECTION II

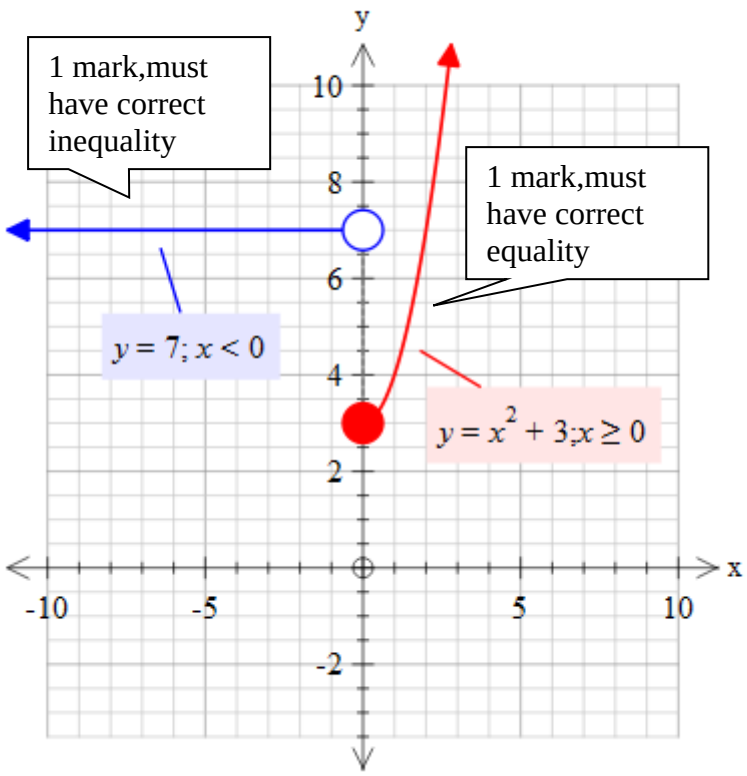
11		
a		
i	$a^2-4a+4=(a-2)^2$	1
ii	$ab-2bc+5a-10c=b(a-2c)+5(a-2c)$ $=(a-2c)(b+5)$	1 mark 1 mark
b		
i	$8(x+2)=9-x$ $8x+16=9-x$ $9x=-7$ $x=\frac{-7}{9}$	1 mark (correct expansion) 1 mark
ii	$6\times\left(\frac{5y}{2}-\frac{2y}{3}\right)=3\times 6$ $15y-4y=18$ $11y=18$ $y=\frac{18}{11}=1\frac{7}{11}$	1 mark 1 mark
c	$\frac{4k^3\times 6k^4}{12k^5}$ $=\frac{24k^7}{12k^5}$ $=2k^2$	1 mark 1 mark
d	Domain: $x\geq 5$ Range: $y\geq 0$	1 mark each

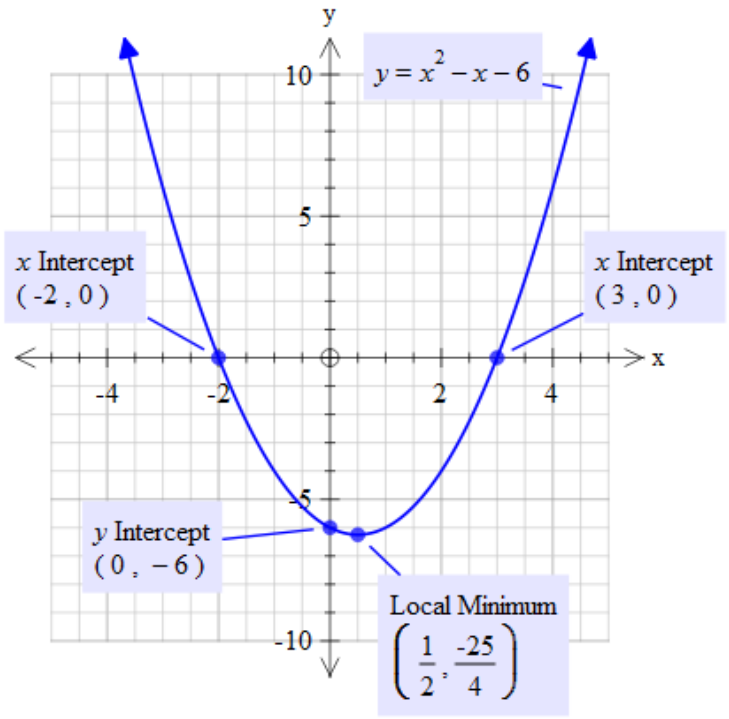
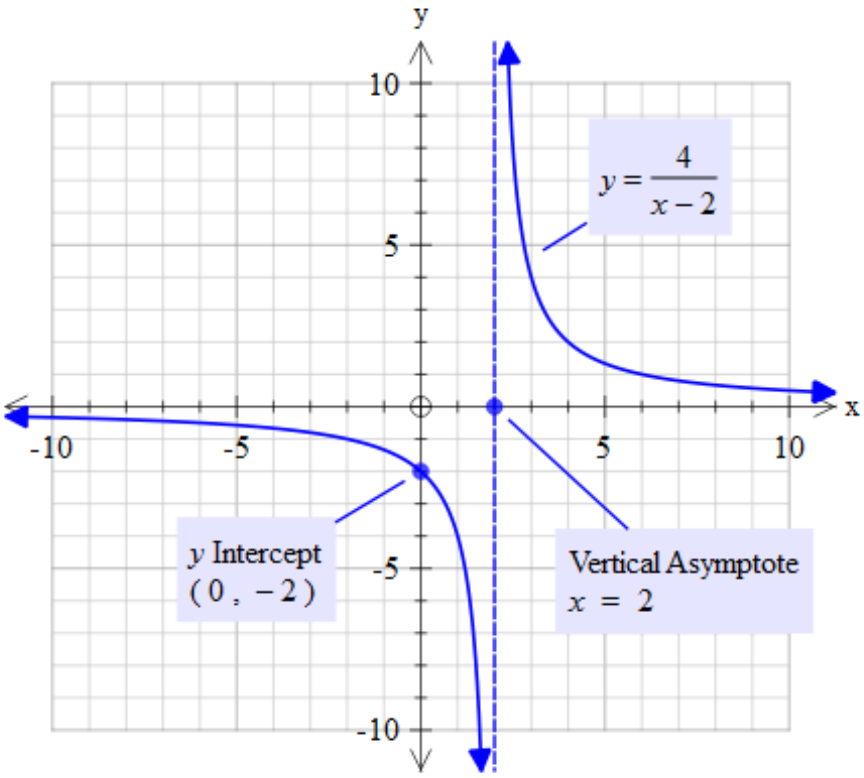
e	$\begin{cases} 2x+y=1 & (1)\times 3 \\ 6x-4y=10 & (2) \end{cases}$ $\begin{cases} 6x+3y=3 \\ 6x-4y=10 \end{cases}$ $7y=-7$ $y=-1$ $Sub\ y=-1 \text{ int } o\ (1)$ $2x-1=1$ $x=1$ $\therefore x=1; y=-1$	2 marks correct values for x and y
f	$f(x)=x^2-4x+1$ At $x=1$ $f(1)=1^2-4\times1+1=-2$ $f'(x)=2x-4$ At $x=1$ $f'(1)=2\times1-4=-2$ ie gradient of tangent is -2 Equation of the tangent at $(1,-2)$ $y=-2x+b$ $sub(1,-2)$ $-2=-2+b$ $b=0$ $y=-2x \text{ or } 2x+y=0$	1 mark gradient of tangent -2 1 mark correct line
12		
a		
		
i	$mid_{AC}=(\frac{4+0}{2};\frac{-2+6}{2})=(2,2)$	1
ii	$d=\sqrt{(-4-0)^2+(-4-6)^2}$ $=\sqrt{16+100}=\sqrt{116}=2\sqrt{29}$	1

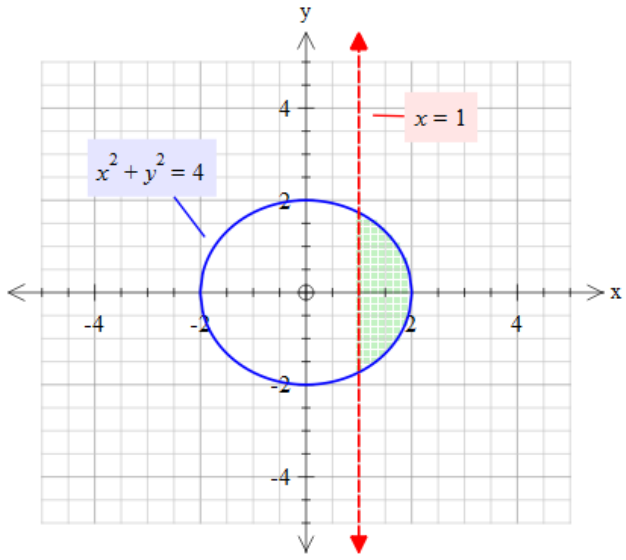
iii	$m = \frac{-4-6}{-4-0} = \frac{5}{2}$ $y-6 = \frac{5}{2}(x-0)$ $y-6 = \frac{5x}{2}$ $2y-12 = 5x$ $0 = 5x-2y+12$	1 mark gradient 1 mark line
iv	$d = \left\|\frac{Ax_1+By_1+C_1}{\sqrt{A^2+B^2}}\right\|$ $= \left\|\frac{5\times4-2\times-2+12}{\sqrt{25+4}}\right\|$ $= \frac{36}{\sqrt{29}}$	1 mark correct substitution correct formula 1 mark
v	$A = \frac{1}{2}\times 2\sqrt{29}\times\frac{36}{\sqrt{29}} = 36\,u^2$	1
vi	Midpoint CA is (2,2) $2 = \frac{x_D-4}{2} \rightarrow x_D = 8$ Therefore $2 = \frac{y_D-4}{2} \rightarrow y_D = 8$ $D = (8,8)$ Other correct methods accepted. Accept point in Q2 (-8,4) or (0,-12)	2 marks each correct coordinate OR 1 mark correct progress
b		
i	$f(x) = x^3-4x$ $f(-x) = (-x)^3-4(-x)$ $= -x^3+4x$ $= -(x^3-4x)$ $= -f(x)$ Function is odd since $f(-x) = -f(x)$	1 mark explicit sub of –x. 1 mark arriving at correct result

ii		1 mark correct x intercept x=2 1 mark correct shape
c	$y = 2x^3 + ax + b$ $At\ (1,1)$ $1 = 2 + a + b$ $a + b = -1 \quad (1)$ $y' = 6x + a$ $x = 1, y' = 6 + a$ $Parallel\ to\ the\ line\ y = x, y' = 1$ $so\ 6 + a = 1, a = -5$ $Sub\ a = -5\ \text{int}\ o\ (1)\ b = 4$ $\therefore a = -5, b = 4$	1 mark correct progress to equating y' =1 1 mark correct values of a and b
13		
a		
i	$\frac{d}{dx}\left(3x^4 + 7x^2 + 1\right) = 12x^3 + 14x$ 1 mark	1
ii	$\frac{d}{dx}\sqrt{x^2+3} = \frac{d}{dx}\left(x^2+3\right)^{\frac{1}{2}} = \frac{1}{2}\left(x^2+3\right)^{\frac{-1}{2}} \times 2x = \frac{x}{\sqrt{x^2+3}}$ 1 mark	2
iii	$\frac{d}{dx}\frac{2}{\sqrt{x}} = \frac{d}{dx}2x^{\frac{-1}{2}} = 2\frac{-1}{2}x^{\frac{-3}{2}} = -\frac{1}{x\sqrt{x}}$ 1 mark 1 mark	2

iv	$\frac{d}{dx}\left(\frac{3x+1}{3-4x}\right)=\frac{3(3-4x)+4(3x+1)}{(3-4x)^2}$ $=\frac{9-12x+12x+4}{(3-4x)^2}$ $=\frac{13}{(3-4x)^2}$	1 mark correct use of quotient rule 1 mark correct
v	$\frac{d}{dx}(2x-1)(x-2)^4=2(x-2)^4+4(x-2)^3(2x-1)$ $=2(x-2)^3[(x-2)+2(2x-1)]$ $=2(x-2)^3(5x-4)$	1 mark correct use of product rule 1 mark correct
b		
i	$f(x)=x^2-4x$ $f(x+h)=(x+h)^2-4(x+h)$ $=x^2+2xh+h^2-4x-4h$	1
ii	$f(x)=x^2-4x$ $f'(x)=\lim_{h\to 0}\frac{[(x+h)^2-4(x+h)]-(x^2-4x)}{h}$ $=\lim_{h\to 0}\frac{x^2+2xh+h^2-4x-4h-x^2+4x}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2-4h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h-4)}{h}$ $=\lim_{h\to 0}2x+h-4$ $=2x-4$	1 mark NOTE $\lim_{h\to 0}$ must appear in every line until last 1 mark
c		
i	$f(-1)=7$	1

ii	 y 10 8 6 4 2 -2 x -10 -5 5 10 1 mark,must have correct inequality 1 mark,must have correct equality $y = 7; x < 0$ $y = x^2 + 3; x \geq 0$	2

14		
a		
i		1 mark all intercepts correct 1 mark cords of vertex correct 1 mark correct shape
ii		1 mark correct VA at x=2 1 mark correct y intercept 1 mark correct shape

b		
i	$\angle QPX = \angle XRS = 60^\circ$ (alternate angles are equal, PQ // SR) ΔPQR is an isosceles triangle (two equal sides, $PQ = QR$) $\angle QPX = \angle QRX = 60^\circ$ (base angles are equal, isosceles triangle) $\angle RXQ + \angle PXQ = 180^\circ$ (adjacent angles on a straight line) $\angle RXQ + 90^\circ = 180^\circ$ $\angle RXQ = 90^\circ$ In ΔQXR $\angle RQX + \angle RXQ + \angle QRX = 180^\circ$ $\angle RQX + 90^\circ + 60^\circ = 180^\circ$ $\angle RQX = 30^\circ$ or $\angle RQS = 30^\circ$	2.
ii	In ΔQRS $\angle QSR + \angle QRS + \angle RQS = 180^\circ$ (Angle sum of a triangle is 180˚) $\angle QSR + (60^\circ + 60^\circ) + 30^\circ = 180^\circ$ $\angle QSR = 30^\circ$ $\therefore \angle RQS = \angle QSR = 30^\circ$ ΔQRS is an isosceles triangle (two base angles are equal).	2
c	$\text{LHS} = \sin\theta\cos\theta\tan\theta$ $= \sin\theta\cos\theta \times \frac{\sin\theta}{\cos\theta}$ $= \sin^2\theta$ $= 1 - \cos^2\theta$ $= \text{RHS}$	1 mark for each correct substitution
d	$x^2 + y^2 = 4$ is a circle centre (0,0) and radius 2 $x = 1$ is a vertical line passing through (1,0) 	1 mark correct circle 1 mark correct x>1 1 mark correct region

End of examination