

Year 11

Mathematics

Assessment Task 3

2016

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11 - 14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II

60 marks

- Attempt Questions 11-14
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover
- Allow about 1 hour 45 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

1 Which of the following is the derivative of $y = x^4 + 5x^{-1}$?

(A) $\frac{dy}{dx} = 4x^3 + 5x^{-2}$

(B) $\frac{dy}{dx} = 4x^3 - 5x^{-2}$

(C) $\frac{dy}{dx} = 4x^3 + 5$

(D) $\frac{dy}{dx} = 4x^3 - 5$

2 Which of the following functions is neither odd nor even?

(A) $f(x) = \frac{1}{x}$

(B) $f(x) = 6x^4 + 8x^2 + 5$

(C) $f(x) = 2x^3 - 4x$

(D) $f(x) = 3x^3 - 5x^2$

3 The line $4x - ky = 3$ passes through the point $(1, 2)$. What is the value of k ?

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

$$4x - ky = 3$$

$$4(1) - k(2) = 3$$

$$4 - 2k = 3$$

$$2k = 1$$

$$k = \frac{1}{2}$$

4 What is the domain of the function $f(x) = \frac{1}{(x-3)(1-x)}$?

(A) $x \neq -1$ or $x \neq 3$

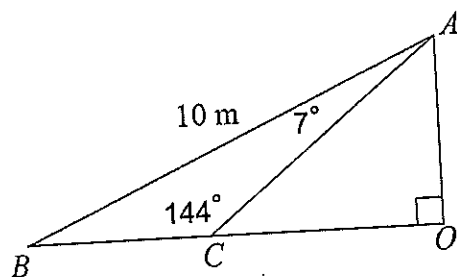
(B) $x \neq 1$ or $x \neq 3$

(C) $x \neq 1$ or $x \neq -3$

(D) $x \neq -1$ or $x \neq -3$

The triangle AOB is shown below. $\angle BAC = 7^\circ$, $\angle ACB = 144^\circ$ and $AB = 10\text{m}$.

5



Not to scale

Which trigonometric formula would be most useful in calculating the length of side BC ?

(A) $c^2 = a^2 + b^2 - 2ab \cos C$

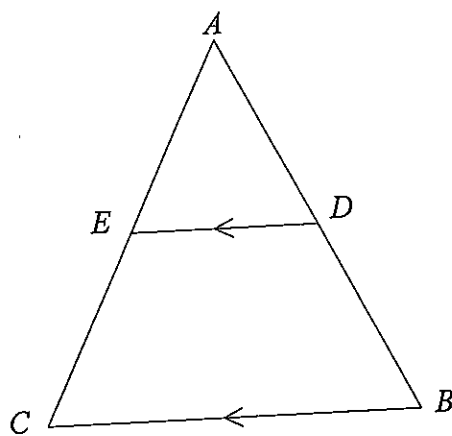
(B) $A = \frac{1}{2}ab \sin C$

(C) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

(D) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

In the diagram below, ABC is a triangle and DE is parallel to BC .

6



Given that $AD = 2\text{ cm}$, $BD = 5\text{ cm}$ and $DE = 1.5\text{ cm}$, what is the value of BC ?

(A) 3.75 cm

(B) 5.25 cm

(C) 6.67 cm

(D) 9.33 cm

$$\frac{BC}{BA} = \frac{DE}{DA}$$

$$\frac{BC}{7} = \frac{1.5}{2}$$

$$BC = \frac{1.5}{2} \times 7$$

$$= 5.25$$

7 Which of the following lines is parallel to $y = \frac{2}{3}x - 1$

(A) $2x + 3y - 7 = 0$

(B) $3x + 2y + 6 = 0$

(C) $2x - 3y + 5 = 0$

(D) $3x - 2y + 11 = 0$

8 What is the solution to the equation $\sin(20^\circ + 6x) = \cos(8x)$

(A) 5°

(B) 10°

(C) 20°

(D) 70°

$$\sin(20^\circ + 6x) = \cos(8x)$$

$$20 + 6x = 90 - 8x$$

$$14x = 70$$

$$x = 5$$

9 Solve for $|2x - 4| + 5 < 7$.

(A) $x < 3$

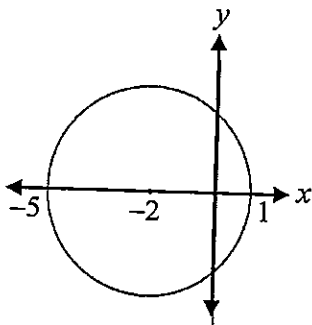
(B) $1 < x < 3$

(C) $-2 < x < 3$

(D) $x < 1$ or $x > 3$

What is the equation of the circle shown in the diagram below?

10



(A) $(x+2)^2 + y^2 = 9$

(B) $(x-2)^2 + y^2 = 3$

(C) $(x-2)^2 + (y-1)^2 = 3$

(D) $(x-2)^2 + (y-1)^2 = 9$

Question 11		
(a)	Simplify $\frac{a^5 \times a^3}{a^6}$	
	$\frac{a^5 \times a^3}{a^6} = \frac{a^8}{a^6} = a^2$	1 Correct answer
(b)	A pair of jeans were discounted by 15% to a selling price of \$63.75. Find the original marked price of the jeans before the discount was applied	
	$x = 63.75$ $0.85x = 63.75$ $x = \frac{63.75}{0.85} = 75$	1 Correct answer
(c)	Simplify $\frac{x^3 - 1}{x^2 - 1}$	
	$\frac{x^3 - 1}{x^2 - 1} \times = \frac{(x-1)(x^2 + x + 1)}{(x-1)(x+1)}$ $= \frac{x^2 + x + 1}{x+1}$	2 Complete solution or 1 correct factorisation
(d)	Express in simplest surd form : $\sqrt{45} - \sqrt{98} + 3\sqrt{80}$	
	$\sqrt{45} - \sqrt{98} + 3\sqrt{80} = 3\sqrt{5} - 7\sqrt{2} + 12\sqrt{5}$ $= 15\sqrt{5} - 7\sqrt{2}$	2 Complete solution or 1 correct simplification of surds
(e)	Rationalise the denominator $\frac{5}{3 - \sqrt{2}}$	
	$\frac{5}{3 - \sqrt{2}} \times \frac{3 + \sqrt{2}}{3 + \sqrt{2}} = \frac{5(3 + \sqrt{2})}{7}$	2 Complete solution or 1 correct conjugate
(d)	Find the x values of the points of intersection of the parabola $y = x^2 + 9x + 9$ and the line $y = 2x + 3$.	
	$x^2 + 9x + 9 = 2x + 3$ $y = 2x + 3$ $y = 2x + 3$ $x^2 + 7x + 6 = 0$ $= 2(-1) + 3$ $= 2(-6) + 3$ $(x+1)(x+6) = 0$ $= -2 + 3$ $= -12 + 3$ $x = -1, -6$ $= 1$ $= -9$ $(-1, 1)$ and $(-6, -9)$	2 Complete solution or 1 solves quadratic equation
(e)	Find the point on the curve $y = x^2 + 8x - 11$ where $\frac{dy}{dx} = 0$.	

	$\frac{dy}{dx} = 2x + 8$ $2x + 8 = 0$ $2x = -8$ $x = -4$	$y = (-4)^2 + 8(-4) - 11$ $= 16 - 32 - 11$ $= -27$ $(-4, -27)$	2 Complete solution or 1 solves quadratic equation
(f)	The function $y = f(x)$ is defined as follows: $f(x) = \begin{cases} x^2 + 1 & \text{for } x < 0 \\ 1 & \text{for } 0 \leq x \leq 3 \\ x & \text{for } x > 3 \end{cases}$		
	(i)	Find the value of $f(-3)$.	
		$f(-3) = 10$	1 Correct answer
	(ii)	Sketch the function $y = f(x)$.	
			2 Complete graph Or 1 Two correct regions

Question 12

a.

i.

$$\begin{aligned}d_{BC} &= \sqrt{(0+2)^2 + (3+2)^2} \\&= \sqrt{4+25} \\&= \sqrt{29}\end{aligned}$$

$$\begin{aligned}\text{ii. } m_{BC} &= \frac{3+2}{0+2} \\&= \frac{5}{2}\end{aligned}$$

$$y-3 = \frac{5}{2}(x-0)$$

$$2y-6 = 5x$$

$$5x-2y+6=0$$

iii.

$$d = \left| \frac{5(2) - 2(-1) + 6}{\sqrt{5^2 + (-2)^2}} \right|$$

$$= \left| \frac{18}{\sqrt{29}} \right|$$

$$= \frac{18}{\sqrt{29}} \text{ units.}$$

iv.

$$\begin{aligned}A &= \frac{1}{2} \times \sqrt{29} \times \frac{18}{\sqrt{29}} \\&= 9 \text{ units}^2\end{aligned}$$

$$\text{v. } D(4,4)$$

Question 12

b.

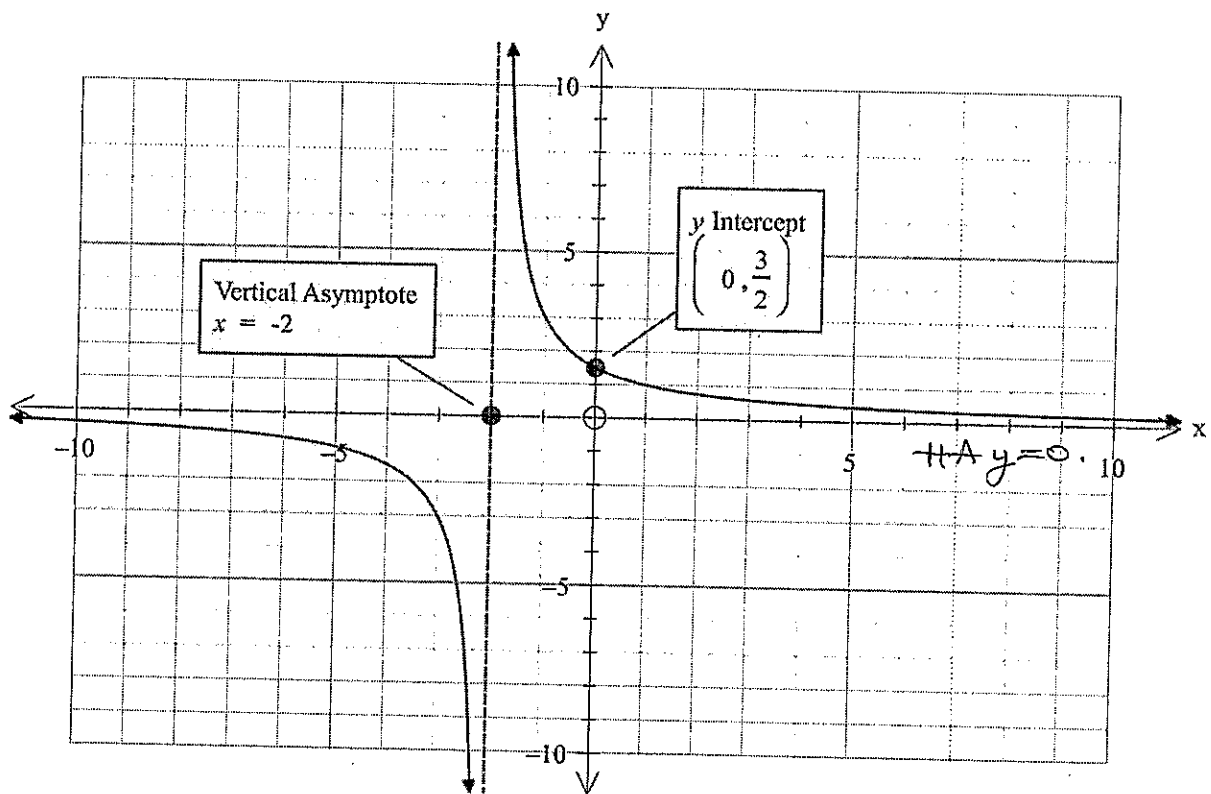
$$f(x) = \frac{3}{x+2}$$

i. Equation of VA: $x = -2$

ii. Domain: $\{x: x \neq -2\}$

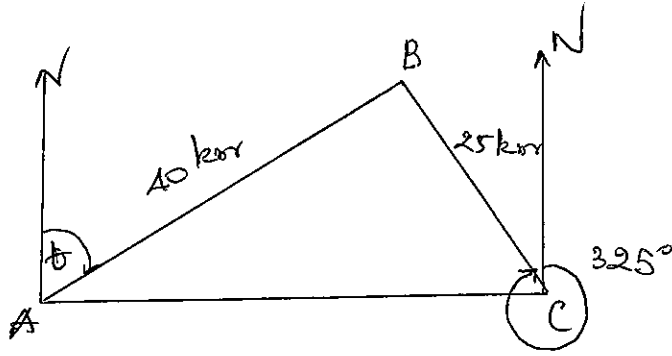
Range: $\{y: y \neq 0\}$

iii.



Question 12

c.



i. $\angle ACB = 325 - 270$
 $= 55^\circ$



ii. In $\triangle ABC$

$$\frac{\sin A}{25} = \frac{\sin 55}{40}$$



$$\sin A = \frac{25 \times \sin 55}{40}$$

$$A = 31^\circ$$



$$\text{Bearing} = 90^\circ - 31^\circ$$
$$= 59^\circ$$



Question 13

a.

i. $\tan 240 = \tan (180 + 60)$

$$= \tan 60$$

$$= \sqrt{3}$$



ii. $\sec 510 = \sec (360 + 150)$

$$= \sec (150)$$



$$= \sec (180 - 30)$$

$$= -\sec 30$$

$$= -\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= -\frac{2\sqrt{3}}{3}$$



b. $4 \sin^2 \theta = 3 \quad -180 \leq \theta \leq 180$

$$\sin^2 \theta = \frac{3}{4}$$

$$\sin \theta = \pm \frac{\sqrt{3}}{2}$$



$$\theta = -60^\circ, -120^\circ, 60^\circ, 120^\circ$$



c.

i. $y = \sqrt{x}$

$$= x^{\frac{1}{2}}$$

$$y' = \frac{1}{2} x^{\frac{1}{2} - 1}$$

$$= \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}}$$



Question 13

c.

ii. $y = x(x-1)^6$

$$u = x \rightarrow u' = 1$$

$$v = (x-1)^6 \rightarrow v' = 6(x-1)^5$$

$$y' = u'v + v'u$$

$$= (x-1)^6 + 6x(x-1)^5 \quad \checkmark$$

$$= (x-1)^5(x-1+6x)$$

$$= (x-1)^5(7x-1) \quad \checkmark$$

iii. $y = \frac{3x}{x-2}$

$$u = 3x \rightarrow u' = 3$$

$$v = x-2 \rightarrow v' = 1$$

$$y' = \frac{u'v - v'u}{v^2}$$

$$= \frac{3(x-2) - 3x}{(x-2)^2} \quad \checkmark$$

$$= \frac{3x - 6 - 3x}{(x-2)^2}$$

$$= \frac{-6}{(x-2)^2} \quad \checkmark$$

Question 13

d.

$$i. \sec \theta \sin \theta = \frac{1}{\cos \theta} \times \sin \theta$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta \quad \checkmark$$

$$ii. \tan \theta + \sec \theta = \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}$$

$$= \frac{\sin \theta + 1}{\cos \theta} \quad \checkmark$$

e.

$$i. y = x^2 - 6x$$

$$y' = 2x - 6 \quad \checkmark$$

$$\cdot \text{At } (-1, 7)$$

$$\cdot \text{tangent } y' = 2x - 1 - 6 \\ = -8$$

Equation of the tangent

$$y - 7 = -8(x + 1)$$

$$y - 7 = -8x - 8$$

$$8x + y + 1 = 0 \quad \checkmark$$

ii.

$$\cdot \text{Normal} = \frac{1}{8}$$

Equation of the normal

$$y - 7 = \frac{1}{8}(x + 1)$$

$$8y - 56 = x + 1$$

$$x - 8y + 57 = 0 \quad \checkmark$$

Question 14

a.

$$\begin{aligned} \text{i. Angle sum} &= 180^\circ \times (n-2) \\ &= 180^\circ \times (5-2) \\ &= 540^\circ \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{ii. } a + 2a + 2a + 2a + 2a &= 540^\circ \\ 9a &= 540^\circ \\ a &= 60^\circ \quad \checkmark \end{aligned}$$

Question 14

b.

i. In $\triangle ABC$ and $\triangle DEC$.

$$\angle ACB = \angle DCE \text{ (vertically opposite angles)}$$

$$\angle BAC = \angle EDC \text{ (alternate angles, parallel lines)}$$

$\therefore \triangle ABC \sim \triangle DEC$ (Two pairs of corresponding angles are equal)

ii. $\frac{AC}{CD} = \frac{AB}{DE}$ (corresponding sides of similar triangles are proportional)

$$\frac{AC}{20} = \frac{25}{15}$$

$$AC = \frac{20 \times 25}{15}$$

$$= 33 \frac{1}{3} \text{ cm}$$

iii. $\angle BAC = \angle EDC$ (corresponding angles of similar $\triangle$ s)

$$\cos(\angle EDC) = \frac{15^2 + 20^2 - 12^2}{2 \times 15 \times 20}$$

$$= 0.801666$$

$$\angle EDC = 36.7^\circ$$

$$\therefore \angle BAC = 37^\circ$$

Question 14.

c.

i. In $\triangle BXY$ and $\triangle CYZ$

$$XB = CY \text{ (given)}$$

$$CB = CD \text{ (sides of a square)}$$

$$ZD = CY \text{ (given)}$$

$$\therefore ZC = YB$$

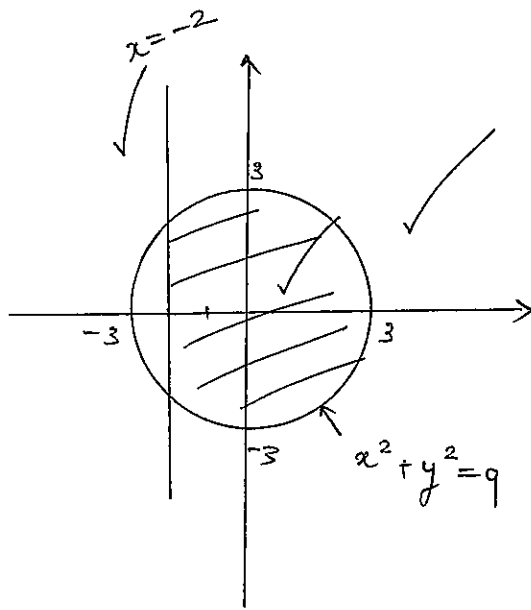
$$\angle ZCY = \angle YBX \text{ (angles of a square)}$$

$$\therefore \triangle BXY \equiv \triangle CYZ \text{ (SAS)}$$

ii. $\angle YZC = \angle XYB$ (corresponding angles in congruent $\triangle$'s)

Question 14

d.



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Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 Which of the following is the derivative of $y = x^4 + 5x^{-1}$?

(A) $\frac{dy}{dx} = 4x^3 + 5x^{-2}$

(B) $\frac{dy}{dx} = 4x^3 - 5x^{-2}$

(C) $\frac{dy}{dx} = 4x^3 + 5$

(D) $\frac{dy}{dx} = 4x^3 - 5$

2 Which of the following functions is neither odd nor even?

(A) $f(x) = \frac{1}{x}$

(B) $f(x) = 6x^4 + 8x^2 + 5$

(C) $f(x) = 2x^3 - 4x$

(D) $f(x) = 3x^3 - 5x^2$

3 The line $4x - ky = 3$ passes through the point $(1, 2)$. What is the value of k ?

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

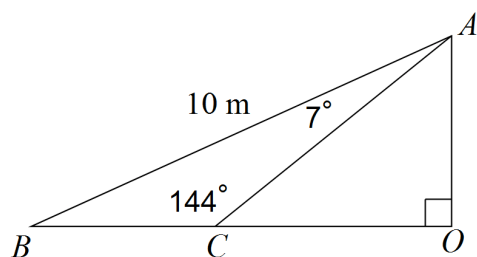
(D) 2

4 What is the domain of the function $f(x) = \frac{1}{(x-3)(1-x)}$?

(A) $x \neq -1$ or $x \neq 3$

- (B) $x \neq 1$ or $x \neq 3$
- (C) $x \neq 1$ or $x \neq -3$
- (D) $x \neq -1$ or $x \neq -3$

- 5 The triangle AOB is shown below. $\angle BAC = 7^\circ$, $\angle ACB = 144^\circ$ and $AB = 10\text{m}$.

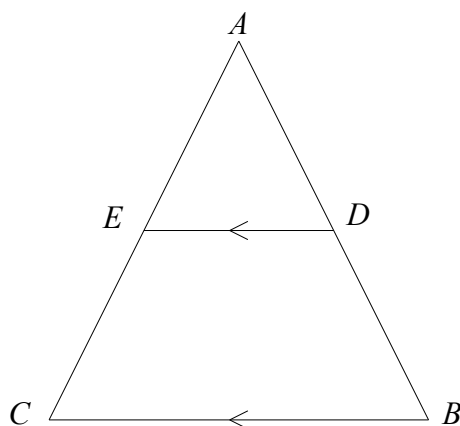


Not to scale

Which trigonometric rule can be used to find the length of side BC ?

- (A) $c^2 = a^2 + b^2 - 2ab \cos C$
- (B) $A = \frac{1}{2}ab \sin C$
- (C) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
- (D) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

- 6 In the diagram below, ABC is a triangle and DE is parallel to BC .



Given that $AD = 2$ cm, $BD = 5$ cm and $DE = 1.5$ cm, what is the value of BC (to 2 decimal places)?

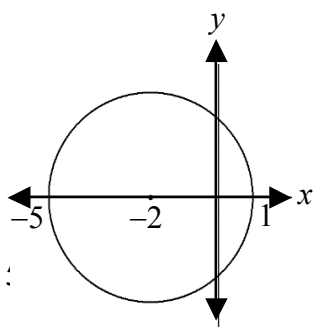
- (A) 3.75 cm
(B) 5.25 cm
(C) 6.67 cm
(D) 9.33 cm
- 7 Which of the following lines is parallel to $y = \frac{2}{3}x - 1$
- (A) $2x + 3y - 7 = 0$
(B) $3x + 2y + 6 = 0$
(C) $2x - 3y + 5 = 0$
(D) $3x - 2y + 11 = 0$
- 8 What is the solution to the equation $\sin(20^\circ + 6x) = \cos(8x)$

- (A) 5°
(B) 10°
(C) 20°
(D) 70°

9 Solve for $|2x - 4| + 5 < 7$.

- (A) $x < 3$
- (B) $1 < x < 3$
- (C) $-2 < x < 3$
- (D) $x < 1$ or $x > 3$

10 What is the equation of the circle shown in the diagram below?



- (A) $(x + 2)^2 + y^2 = 9$
- (B) $(x - 2)^2 + y^2 = 3$
- (C) $(x - 2)^2 + (y - 1)^2 = 3$
- (D) $(x - 2)^2 + (y - 1)^2 = 9$

End of Section I

Section II

60 marks

Attempt Questions 11–14

Allow about 1 hour 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

Marks

- | | |
|---|----------|
| (a) Simplify $\frac{a^5 \times a^3}{a^6}$ | 1 |
| (b) A pair of jeans were discounted by 15% to a selling price of \$63.75. Find the original marked price of the jeans before the discount was applied | 1 |
| (c) Simplify $\frac{x^3 - 1}{x^2 - 1}$. | 2 |
| (d) Express in simplest surd form : $\sqrt{45} - \sqrt{98} + 3\sqrt{80}$. | 2 |
| (e) Rationalise the denominator of $\frac{5}{3 - \sqrt{2}}$. | 2 |
| (d) Find the x values of the points of intersection of the parabola $y = x^2 + 9x + 9$ and the line $y = 2x + 3$. | 2 |
| (e) Find the point on the curve $y = x^2 + 8x - 11$ where $\frac{dy}{dx} = 0$. | 2 |

- (f) The function $y = f(x)$ is defined as follows:

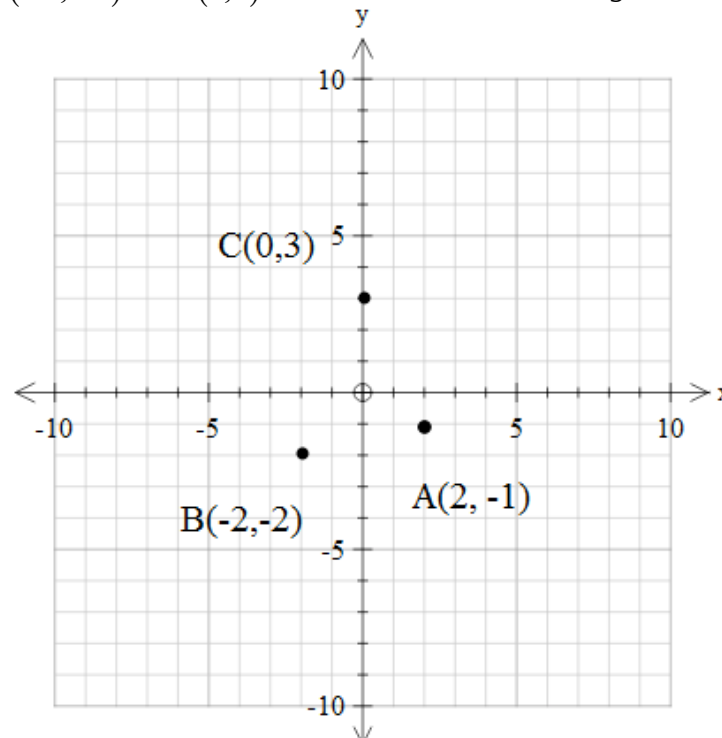
$$f(x) = \begin{cases} x^2 + 1 & \text{for } x < 0 \\ 1 & \text{for } 0 \leq x \leq 3 \\ 4 - x & \text{for } x > 3 \end{cases}$$

- (i) Find the value of $f(-3)$. **1**
- (ii) Sketch the function $y = f(x)$. **2**

End of question 11

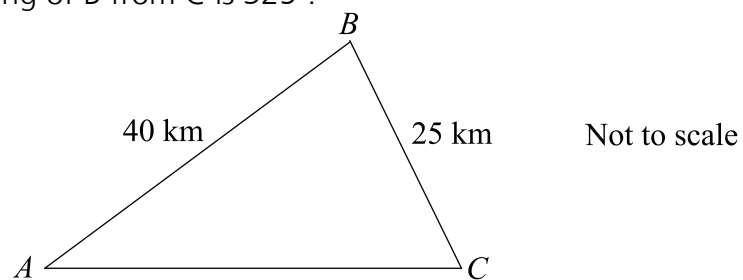
Question 12 (15 marks)**Marks**

- (a) $A(2, -1)$, $B(-2, -2)$ and $C(0, 3)$ are the vertices of a triangle ABC



- | | |
|--|----------|
| (i) Find the length of BC . | 1 |
| (ii) Show the equation of the line BC is $5x - 2y + 6 = 0$ | 2 |
| (iii) Show that the perpendicular distance of A from the line BC is $\frac{18}{\sqrt{29}}$ units | 2 |
| (iv) Find the area of the triangle ABC . | 1 |
| (v) Find the coordinates of D such that $ABCD$ is a parallelogram. | 1 |
- (b) For the function $f(x) = \frac{3}{x+2}$
- | | |
|---|----------|
| (i) Find the equation of vertical asymptote. | 1 |
| (ii) State the domain and range. | 1 |
| (iii) Sketch the graph and clearly label intercepts and asymptotes. | 2 |

- (c) Point C is due east of A. Point B is 40 km from A and 25 km from C. The bearing of B from C is 325° .



- (i) Show that $\angle ACB = 55^\circ$ **1**
- (ii) What is the bearing of B from A ? **3**

End of question 12

Question 13 (15 marks)**Marks**

(a) Find the exact value of:

(i) $\tan 240^\circ$ **1**

(ii) $\sec 510^\circ$ **2**

(b) Solve $4\sin^2\theta=3$ for $-180^\circ \leq \theta \leq 180^\circ$. **2**

(c) Differentiate with respect to x .

(i) $y = \sqrt{x}$ **1**

(ii) $y = x(x-1)^6$ **2**

(iii) $y = \frac{3x}{x-2}$ **2**

(d) Prove that

(i) $\sec\theta \sin\theta = \tan\theta$ **1**

(ii) $\tan\theta + \sec\theta = \frac{1 + \sin\theta}{\cos\theta}$ **1**

(e) For the curve $y = x^2 - 6x$

(i) Find the equation of the tangent at $(-1, 7)$. **2**

(ii) Find the equation of the normal at the same point. **1**

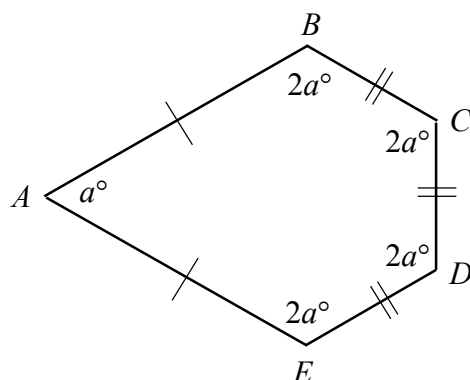
End of question 13**Question 14 (15 marks)****Marks**

(a) (i) Find the angle sum of a pentagon. **1**

- (ii) The diagram shows a pentagon $ABCDE$.

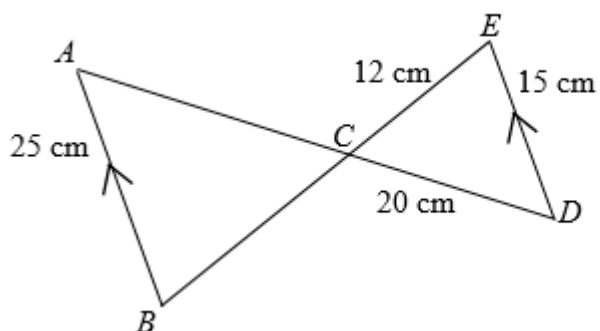
1

$$AB = AE \text{ and } BC = CD = DE.$$



Find the value of a .

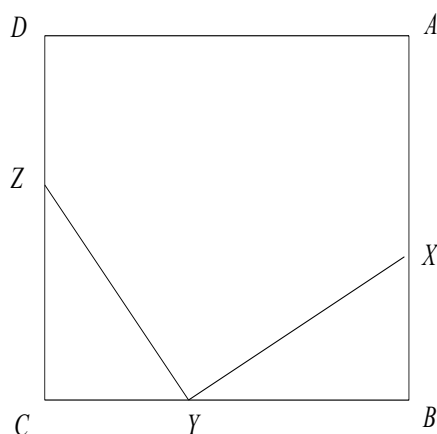
- (b) The diagram shows $\triangle ABC$ and $\triangle DEC$. AB is parallel to ED .



**NOT TO
SCALE**

- (i) Prove that $\triangle ABC$ is similar to $\triangle DEC$. 2
- (ii) $CD = 20$ cm, $CE = 12$ cm, $DE = 15$ cm and $AB = 25$ cm. 2
- Find the length of AC .
- (iii) Find the size of $\angle BAC$. 2

- (c) In the diagram below, $ABCD$ is a square. X , Y and Z are points on sides AB , BC and CD respectively such that $XB = YC = ZD$.



- (i) Prove that $\triangle BXY \cong \triangle CYZ$? **3**
- (ii) Hence prove that $\angle YZC = \angle XYB$ **1**
- (d) Draw a sketch showing the region on the number plane where the inequalities $x^2 + y^2 \leq 9$ and $x \geq -2$ hold simultaneously. **3**

End of Examination

Student Name:

2016 Year 11 Assessment Task 3

Mathematics

Section 1 Multiple-Choice Answer Sheet

- | | | | | |
|-----------|-------------------------|-------------------------|-------------------------|-------------------------|
| 1 | A ☐ | B ☐ | C ☐ | D ☐ |
| 2 | A ☐ | B ☐ | C ☐ | D ☐ |
| 3 | A ☐ | B ☐ | C ☐ | D ☐ |
| 4 | A ☐ | B ☐ | C ☐ | D ☐ |
| 5 | A ☐ | B ☐ | C ☐ | D ☐ |
| 6 | A ☐ | B ☐ | C ☐ | D ☐ |
| 7 | A ☐ | B ☐ | C ☐ | D ☐ |
| 8 | A ☐ | B ☐ | C ☐ | D ☐ |
| 9 | A ☐ | B ☐ | C ☐ | D ☐ |
| 10 | A ☐ | B ☐ | C ☐ | D ☐ |