International Grammar School

Year 11
Mathematics
Assessment Task 3
2017

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- In Questions 11 - 14, show relevant mathematical reasoning and/or calculations
- A reference sheet will be provided.

Total marks – 70

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II

60 marks

- Attempt Questions 11-14
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover
- Allow about 1 hour 45 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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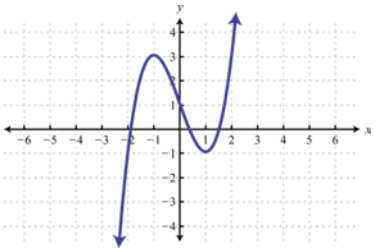
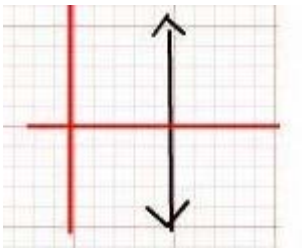
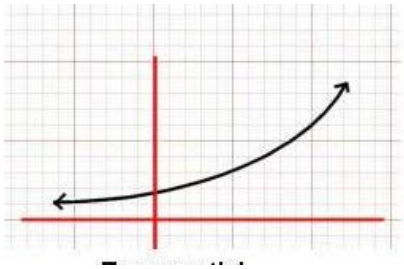
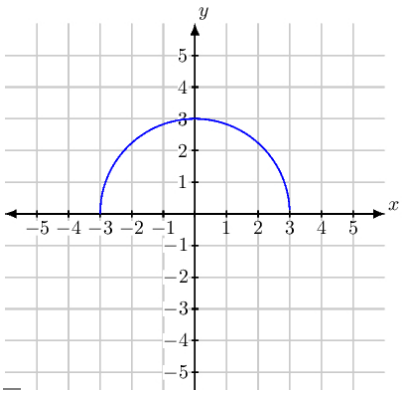
1 Which of the following is **not** a function

(A) 

(B) 

(C) 

(D) 

2 Rationalise $\dfrac{\sqrt{5}}{1+\sqrt{3}}$

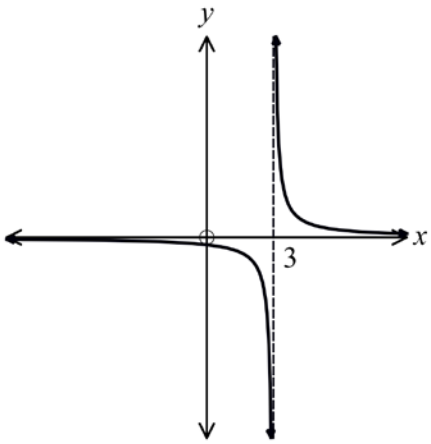
(A) $\dfrac{\sqrt{3}}{3}$

(B) $\dfrac{\sqrt{15}-\sqrt{5}}{2}$

(C) $\sqrt{5}$

(D) $\dfrac{\sqrt{5}+\sqrt{3}}{4+2\sqrt{3}}$

3 Which equation could describe the curve shown below



(A) $y = \frac{1}{x-9}$

(B) $y = \frac{1}{x-3}$

(C) $y = \frac{1}{x+3}$

(D) $y = \frac{1}{x+9}$

4 Solve $\sin\theta = \frac{1}{\sqrt{2}}, 0 \le 0 \le 360$

(A) $\theta = 45^o, 135^o$

(B) $\theta = 135^o, 225^o$

(C) $\theta = 225^o, 315^o$

(D) $\theta = 45^o, 315^o$

5 Factorise $4m^2 - 49$

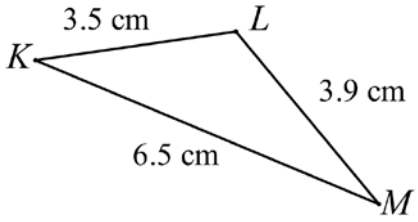
(A) $(2m+7)(2m-7)$

(B) $2(m+7)(m-7)$

(C) $(2m-7)^2$

(D) $4(m-7)(m+7)$

6 Find the size of $\angle LKM$ to the nearest degree.



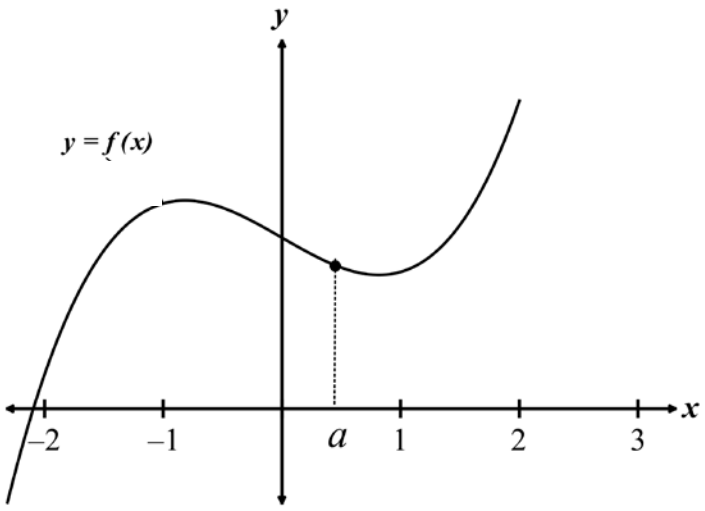
(A) 28^o

(B) 30^o

(C) 35^o

(D) 59^o

7 The diagram shows part of the graph $y = f(x)$



Which of the following is a true statement?

(A) $f(a)$ is positive and $f'(a)$ is positive

(B) $f(a)$ is negative and $f'(a)$ is positive

(C) $f(a)$ is positve and $f'(a)$ is negative

(D) $f(a)$ is negative and $f'(a)$ is negative

8 Which of the following is the domain of the function $y = \sqrt{x+4}$

(A) $y \geq -0$

(B) $y > 0$

(C) $x \geq -4$

(D) $x > -4$

9 $A(3, y)$ and $B(7, 2)$ are points on the number plane. The gradient of AB is $m = -3$

What is the value of y ?

(A) 14

(B) $\frac{1}{14}$

(C) -10

(D) $-\frac{1}{10}$

10 Which of the following is equivalent to $\sqrt{3} + \sqrt{27} - \sqrt{18}$?

(A) $\sqrt{3}$

(B) $\sqrt{12}$

(C) $4\sqrt{3} - 3\sqrt{2}$

(D) $10\sqrt{3} - 9\sqrt{2}$

End of Section I

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Solve each of the following equations

(i) $3x^2 + 2x - 5 = 0$ **2**

(ii) $|2x - 5| - 1 = 0$ **2**

(iii) $x^2 + 6x - 10 = 0$ **2**

(b) Solve $3\tan^2\theta = 1$ for $-180 \le \theta \le 180$ **2**

(c) Prove each of the following identities:

(i) $\dfrac{1-\sin^2\theta}{\sin^2\theta+\cos^2\theta} = \cos^2\theta$ **2**

(ii) $\tan A \sin A + \cos A = \sec A$ **2**

(d) Sapphire cycles 15 km due North, then 12 km due east and finally 20 km due south.

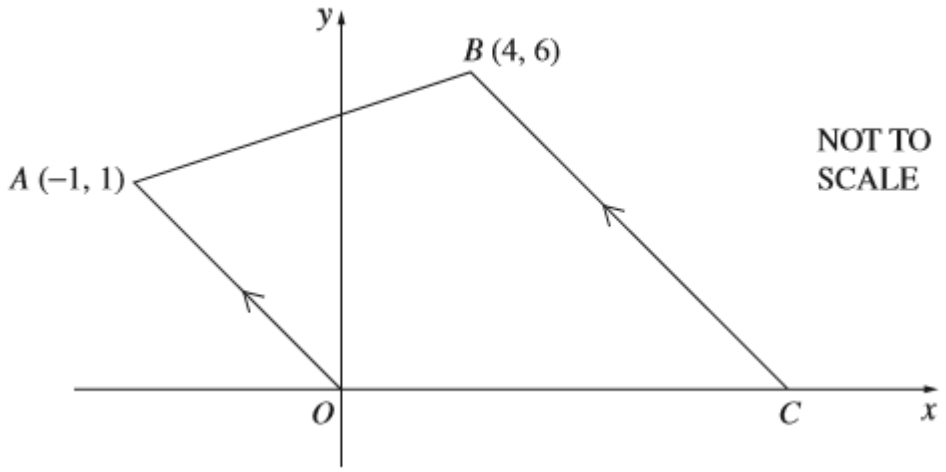
(i) Draw a diagram representing this information. **1**

(ii) Determine her bearing to (**to the nearest degree**) and distance from her original position. **2**

End of question 11

Question 12 (15 marks) **Marks**

(a) In the diagram $OABC$ is a trapezium with with $OA \| CB$. The coordinates of O, A and B are (0, 0), (-1, 1) and (4, 6) respectivley.



(i) Calculate the length of the interval OA **1**

(ii) Find the gradient of the line OA **1**

(iii) Find the equation of the line BC **1**

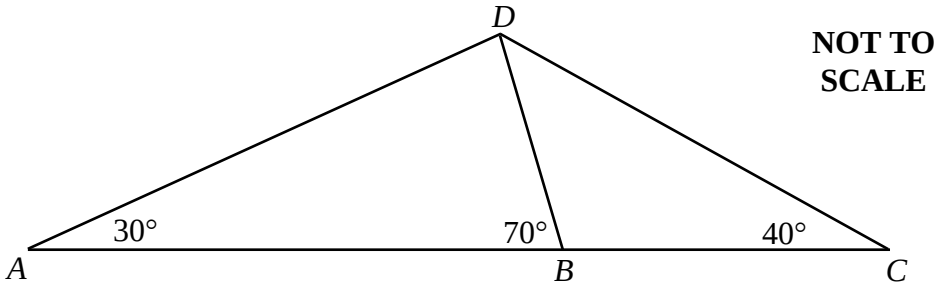
(iv) Hence, find the coordinates of the point C **1**

(v) Show that the perpendicular distance from O to the line BC is $5\sqrt{2}$ **2**

(vi) Calculate the area of the trapezium OABC **2**

b) Find the equation of the line that is parallel to $2x - y + 3 = 0$ and goes through the point $(2,10)$ **2**

c) The diagram shows ΔACD where $\angle DAC = 30°$ and $\angle DCA = 40°$ The point B lies on AC such that $\angle DBA = 70°$.



(i) Prove that ΔACD and ΔDCB are similar. **3**

Given that $CD = x$ cm, $AC = (x+3)$ cm and $BC = (x-2)$ cm, Using (i), or otherwise:

(ii) Find the value of x. **2**

End of question 12

Question 13 (15 marks) **Marks**

(a) Show that the function $f(x) = 3x^4 - 4x^2$ is even. **2**

(b) Find the equation of the circle with radius 3 units and centre $(0, -4)$. **1**

(c) State the domain and range of the graph of $y = \dfrac{1}{2x - 12}$. **2**

(d) (i) Sketch the graph of $y = x^2 - 9, \quad -4 \le x \le 3$ **2**

(ii) Hence determine its range **1**

(e) Sketch $y = |2x - 6|$ **2**

(f) Consider the functions $y = 6x - x^2$ and $y = 2x$

(i) Find their points of intersection. **2**

(ii) Shade the region defined by $y \le 6x - x^2$ and $y > 2x$. **3**

End of question 13

Question 14 (15 marks) **Marks**

(a) Find the derivative function for $f(x) = x^2 - 3x$ using first principles. **2**

(b) Differentiate each of the following

(i) $f(x) = x^2 - 10$ **1**

(ii) $f(x) = 3x^4 - 5x + 3$ **1**

(iii) $f(x) = \dfrac{x^3 - 6x^2}{x}$ **1**

(c) Consider the function $f(x) = \dfrac{1}{2}x^4 - 16x + 2$. Evaluate:

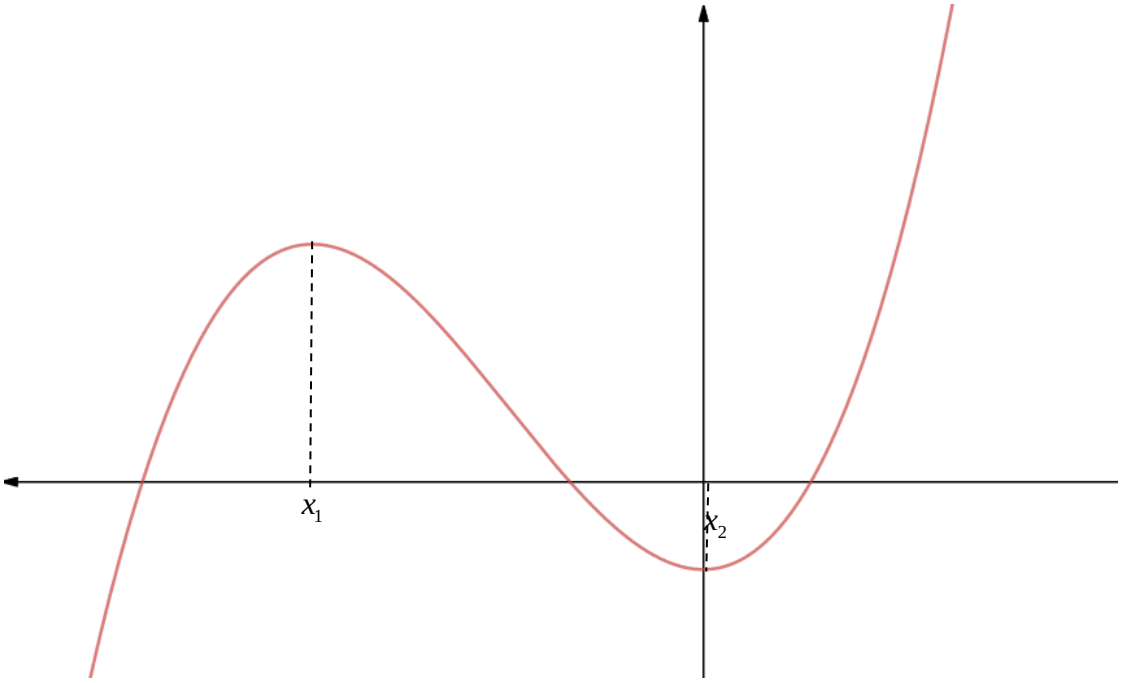
(i) $f(1)$ **1**

(ii) $f'(1)$ **2**

(iii) Determine any points on the curve at which the gradient is 0. **2**

(d) Copy or trace the diagram below into your answer booklet.

Sketch the gradient function for the following function **1**



(e) Consider $y = \dfrac{x^2 - 16}{x - 4}$

(i) Find the limit: **2**

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$

(ii) Hence sketch the curve $y = \dfrac{x^2 - 16}{x - 4}$, clearly showing the point of discontinuity. **2**

End of Examination

Student Name: .....

2017 Year 11 Assessment Task 3

Mathematics

Section 1 Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

Student Name: Solutions

2017 Year 11 Assessment Task 3

Mathematics

Section 1 Multiple-Choice Answer Sheet

1	A ○	B ●	C ○	D ○
2	A ○	B ●	C ○	D ○
3	A ○	B ●	C ○	D ○
4	A ●	B ○	C ○	D ○
5	A ●	B ○	C ○	D ○
6	A ○	B ●	C ○	D ○
7	A ○	B ○	C ●	D ○
8	A ○	B ○	C ●	D ○
9	A ●	B ○	C ○	D ○
10	A ○	B ○	C ●	D ○

Y11 AT3 2017 Solutions

Question 11

a) i) $3x^2+2x-5=0$ $P=-15$ $S=2$

$3x^2-3x+5x-5=0$

$3x(x-1)+5(x-1)=0$ ✓ progress

$(3x+5)(x-1)=0$

$3x+5=0$ $\quad x-1=0$

$x=\frac{-5}{3}$ $\quad x=1$ ✓ getting both

ii) $|2x-5|=1$

$2x-5=\pm 1$ ✓

$2x-5=1$ $\quad 2x-5=-1$

$2x=6$ $\quad 2x=4$

$x=3$ $\quad x=2$ ✓ both

iii) $x^2+6x-10=0$

use quadratic formula or completing the square

$x=\frac{-6\pm\sqrt{6^2-4\times1\times(-10)}}{2\times1}$ ✓ correct use of formula OR completing the square

$=\frac{-6\pm\sqrt{36+40}}{2}$

$=\frac{-6\pm2\sqrt{19}}{2}$

$x=-3\pm\sqrt{19}$ ✓

b) $\tan^2\theta=\frac{1}{3}$ $\quad \tan\theta=\pm\frac{1}{\sqrt{3}}$

$\theta=\pm30, \pm150°$

✓ ✓ ✓ ✓

✓ 2 angles

✓ all 4 angles

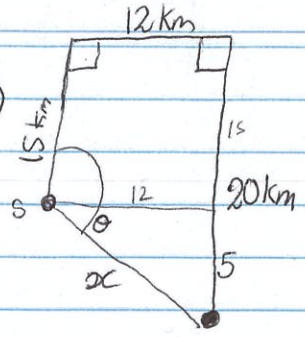
c) i) $\frac{1-\sin^2\theta}{\sin^2\theta+\cos^2\theta}=\frac{\cos^2\theta}{1}$ ✓ $=\cos^2\theta$ ✓ $=RHS$

ii) $\tan A\sin A+\cos A=\frac{\sin A}{\cos A}\times\sin A+\cos A$

$=\frac{\sin^2A+\cos^2A}{\cos A}$ ✓

$=\frac{1}{\cos A}=\sec A$ ✓

d) i)



✓

ii) $x^2=12^2+5^2$

$x=\sqrt{12^2+5^2}$

$x=13$km ✓

bearing $\rightarrow$ $\tan\theta=\frac{5}{12}$

$\theta=23°$ (nearest degree)

$\therefore$ her bearing is $90+23$

$=113°T$ ✓

Question 12

(i) $d=\sqrt{(0--1)^2+(0-1)^2}$

$=\sqrt{1+1}$

$=\sqrt{2}$ units ✓

(ii) $m_{OA}=\frac{1-0}{-1-0}=\frac{1}{-1}=-1$ ✓

iii) $m_{BC}=m_{OA}=-1$ $\quad B(4,6)$

$y-6=-1(x-4)$

$y-6=-x+4$

$y=-x+10$ ✓ OR $x+y-10=0$

iv) at $y=0 \rightsquigarrow -x+10=0$

$x=10$

$\therefore C(10,0)$ ✓

vi) $BC=\sqrt{6^2+6^2}=\sqrt{72}=6\sqrt{2}$ ✓

$A_{ABC}=\frac{1}{2}\times5\sqrt{2}\,(6\sqrt{2}+\sqrt{2})$

$=35$ units^2 ✓

b) $m=2$ ✓ $\quad y-10=2(x-2)$

$y-10=2x-4$ ✓ $\rightarrow$ $y=2x+6$

c) i) In $\triangle ACD$ and $\triangle DCB$

$\angle DBC = 180 - 70° = 110°$ ($\angle$'s on a straight line)

$\therefore \angle BDC = 180 - (40+110)$ ($\angle$ sum of $\triangle$)

$= 30°$

$\therefore \angle BDC = \angle DAC = 30°$ (Proved above) ✓

$\angle DBC = \angle DCA = 40°$ (given) ✓

$\therefore \angle DBC = \angle ADC$ ($\angle$ sum of $\triangle$) ✓

$\therefore \triangle ACD \,|||\, \triangle DCB$ (AAA)

ii) $\frac{x}{x-2} = \frac{x+3}{x}$ ✓

$x^2 = (x+3)(x-2)$

$x^2 = x^2 + x - 6$

$x - 6 = 0$

$\underline{x = 6}$ ✓

Question 13

a) function is even if $f(x) = f(-x)$ ✓

$f(x) = 3x^4 - 4x^2$

$f(-x) = 3(-x)^4 - 4(-x)^2$

$= 3x^4 - 4x^2$ ✓

$\therefore f(x) = f(-x)$

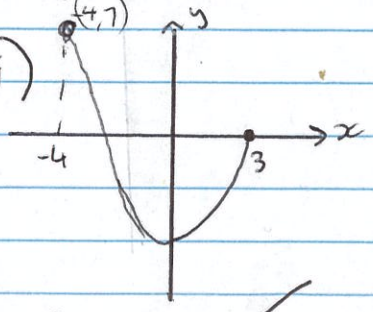
Hence function is even

b) $x^2 + (y+4)^2 = 9$ ✓

c) D: $x \neq 6$ ✓

R: $y \neq 0$ ✓

d) i)

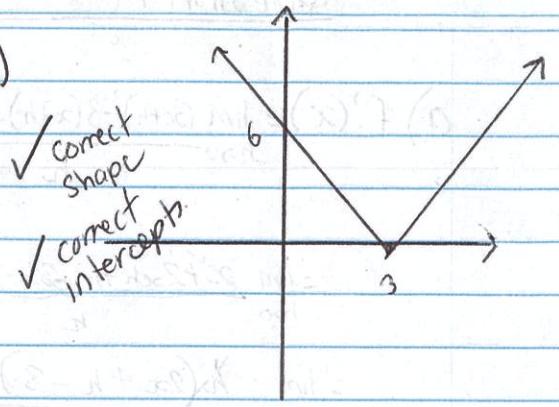


✓ correct intercepts

✓ correct domain

ii) $-9 \leq y \leq 7$ ✓

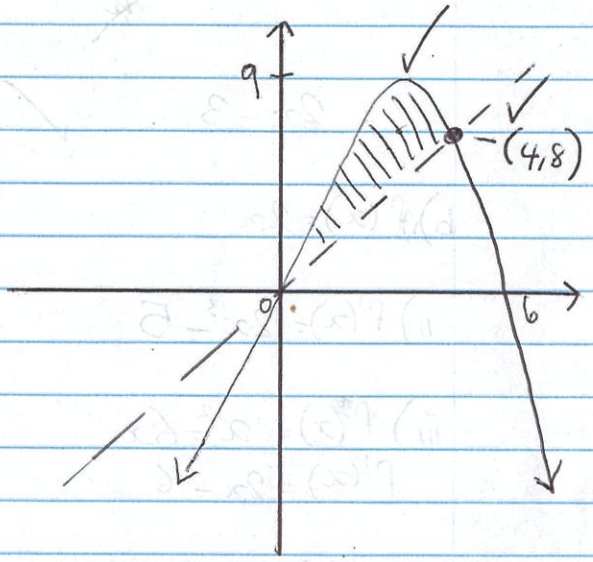
e)



✓ correct shape

✓ correct intercepts

f)



ii) $6x - x^2 = 2x$

$x^2 - 6x + 2x = 0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0 \quad x = 4$

$y = 0 \quad y = 8$

POI → (0,0) ✓ (4,8) ✓

iii) refer to diagram for shading ✓

Question 14

a) $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$ ✓

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$ ✓

$= \lim_{h \to 0} \frac{h(2x + h - 3)}{h}$

$= 2x - 3$ ✓

b) i) $f'(x) = 2x$

ii) $f'(x) = 12x^3 - 5$

iii) $f(x) = x^2 - 6x$

$f'(x) = 2x - 6$

c) $f'(x) = 2x^3 - 16$

i) $f(1) = \frac{1}{2}(1)^4 - 16(1) + 2$

$= -13\frac{1}{2}$ OR $-27/2$ ✓

ii) $f'(1) = 2(1)^3 - 16$ ✓ for differentiating for both parts i) & ii)

$= -14$ ✓

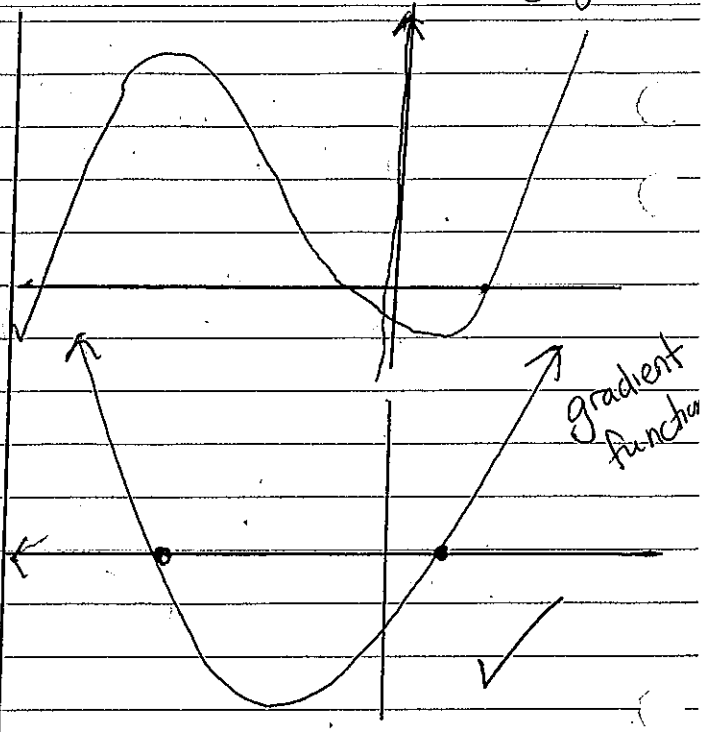
iii) $2x^3 - 16 = 0$ ✓

$2x^3 = 16$

$x^3 = 8$

$x = 2$ ✓

original



gradient function

e) i) $\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$

$= \lim_{x \to 4} \frac{(x-4)(x+4)}{x-4}$ ✓

$= 8$ ✓