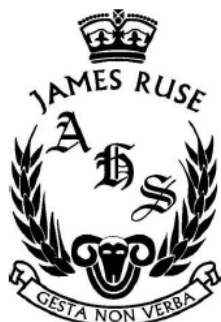


Name: _____

Class: _____



2018

Year 11

Preliminary Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 120 minutes
- Write using black pen
- NESA approved calculators may be used
- In Questions 6–9, show relevant mathematical reasoning and/or calculations

Total marks – 85

Section I – 5 marks (page 2)

- Attempt Questions 1–5

Section II – 80 marks (pages 3–6)

- Attempt Questions 6 –9

Multiple Choice (1 mark each)

1. At what point on the curve $y = (x - 2)(x + 3)$ is the tangent perpendicular to $x - y + 5 = 0$

- (A) $(-1, -6)$
- (B) $(0, -6)$
- (C) $(1, -4)$
- (D) $(0, 6)$

2. State the domain for $y = \sqrt{4 - x^2}$

- (A) $-4 \leq x \leq 4$
- (B) $x \leq -2$ or $x \geq 2$
- (C) $-2 \leq x \leq 2$
- (D) $-2 < x < 2$

3. Find x if $2^x \times 4^{x+1} = \frac{1}{2}$

- (A) $x = -1$
- (B) $x = 1$
- (C) $x = 2$
- (D) $x = -2$

4. Solve $|2x + 5| = x - 1$

- (A) $x = -6$
- (B) $x = -\frac{4}{3}$
- (C) $x = 0$
- (D) *no solution*

5. Evaluate $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

- (A) -5
- (B) 0
- (C) 5
- (D) 10

Question 6 (20 marks)

(a) Calculate $\frac{3}{2.78} + \frac{1}{\sqrt{3}}$ correct to 4 significant figures 1

(b) If $\sqrt{27} + \sqrt{12} = \sqrt{A}$, find A . 2

(c) Solve $t^{\frac{2}{3}} = 16$ 2

(d) Factorise the following expressions completely

(i) $3x^2 - 12y^2$ 1

(ii) $8 - 27d^3$ 1

(e) Given that $3 \log_x 2 - \log_x 128 = 4$, find the value of x . 3

(f) Find the EXACT value of

(i) $\sin 225^\circ$ 1

(ii) $\sec 120^\circ$ 1

(g) The point $P(x,y)$ moves so that its distance PA is always twice its distance PB . Given the points $A(1,5)$ and $B(4,-1)$.

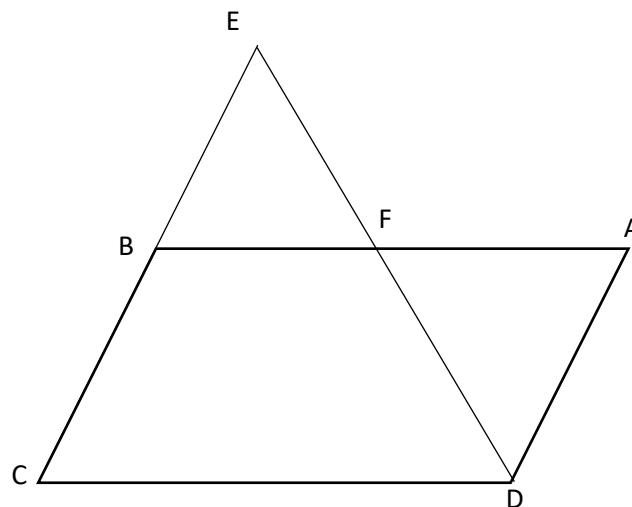
(i) Show that the equation of the locus of P is $x^2 + y^2 - 10x + 6y + 14 = 0$ 2

(ii) Find the centre and radius of this locus. 1

(h) $ABCD$ is a parallelogram and F is the midpoint of AB . DF and CB are extended to meet at E .

(i) Prove that $\triangle BEF \equiv \triangle ADF$ 3

(ii) Prove that B is the midpoint of CE 2



Question 7 (20 marks)

(a) Differentiate with respect to x :

(i) $x^3 + \sqrt{x}$ **2**

(ii) $\frac{x^2-2}{4x}$ **2**

(iii) $(3x + 1)^5$ **1**

(b) Find the equation of the curve, given the gradient function is $4x - 6$ and it passes through the point $(2,4)$ **2**

(c) Solve $3 + x - 2x^2 \geq 0$ **2**

(d) Find the value of k for which the quadratic expression $(k + 6)x^2 - 8x + k$ is positive definite. **2**

(e) Show the region of the number plane where the following hold simultaneously: **3**

$$(x - 1)^2 + y^2 \leq 9$$

$$y \leq x + 1$$

(f) Solve $2\cos^2\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$ **2**

(g) Two helicopters leave Airport C at the same time. Helicopter A flies 7.8 km on a bearing of 043° . Helicopter B flies 5.3 km due west.

i) Draw a diagram and mark the relevant information on it. **1**

ii) Find the distance between Helicopter A and Helicopter B to 1 decimal place. **1**

iii) Hence or otherwise, find the true bearing of Helicopter A from Helicopter B **2**

Question 8 (20 marks)

- (a) Consider the curve $y = 2x^3 - 3x^2 - 12x + 2$
- (i) Find any stationary points and determine their nature **3**
 - (ii) Find any point(s) of inflexion **2**
 - (iii) Draw a neat sketch for the domain $-2 \leq x \leq 4$ **1**
 - (iv) What is the maximum value of the curve in this domain? **1**
- (b) Prove that $\tan \theta + \cot \theta = \sec \theta \operatorname{cosec} \theta$ **2**
- (c) The points A(-1,-1), B(4,2), C(3,6) and D (x_1, y_1) are the 4 vertices of a quadrilateral ABCD.
- (i) Draw a number plane and mark A, B and C on it. **1**
 - (ii) Find the gradient of line AB. **1**
 - (iii) Show that the line l parallel to AB and going through C
is $3x - 5y + 21 = 0$ **2**
 - (iv) If the equation of the line k through point A parallel to BC is
 $4x + y + 5 = 0$, find the point D (x_1, y_1) , the intersection of lines l and k .
Mark this point on your diagram. **2**
 - (v) Find the acute angle (to the nearest degree) that the line AB makes with the
positive x -axis. **1**
 - (vi) Find the perpendicular distance between point B and line l as an exact value. **2**
 - (vii) Find the exact area of ABCD. **2**

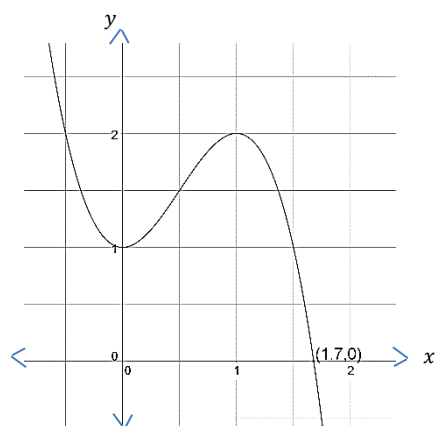
Question 9 (20 marks)

- (a) A parabola has its focus at $(2, 4)$ and directrix at $y=0$. Find the equation of the parabola 2

- (b) Sketch the graph $(y - 3)^2 = -12(x + 1)$
Label the focus, vertex, directrix 3

- (c) Sketch the graph $y = \frac{2x+1}{x}$. State its domain and range. 3

- (d) The function $f(x)$ is shown below:



- Sketch the graph of the derivative, $f'(x)$ 1

- (e) Let $y = 2x^2 - 2x$. Show that
 $x^2y'' - 2xy' + 2y = 0$ 2

- (f) Find by first principles, the derivative of $y = 2\sqrt{x}$ 2

- (g) Find the gradient of the curve $y = \frac{x}{x^2 + 1}$ at the origin and hence find the equation of the tangent to this curve at the origin. 2

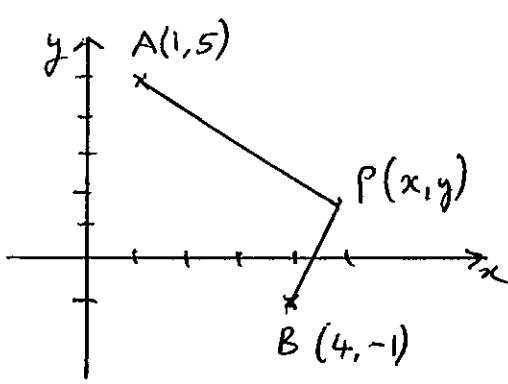
- (h) Kerry is on a kayak 500 metres from the nearest point P of a straight stretch of beach.
Her destination is 2 kilometres along the beach from P.
Kerry can paddle her kayak at 1.5 km/hour and she can walk along the sand at 2km/hour.
Let x be her landing point on the beach.

- (α) Draw a diagram to represent this situation. 1

- (β) Show the total time taken to reach her destination is $\frac{\sqrt{x^2 + 0.5^2}}{1.5} + \frac{2-x}{2}$ 2

- (γ) Towards what point on the beach should she row to reach her destination in the least possible time? 2

Suggested Solutions	Marks	Marker's Comments
a) $\frac{3}{2.78} + \frac{1}{\sqrt{3}} = 1.65648, \dots$ $\quad \quad \quad = \underline{\underline{1.656}} \text{ (4 s.f.)}$	1	Must be exactly right.
b) $\sqrt{27} + \sqrt{12} = 3\sqrt{3} + 2\sqrt{3}$ $\quad \quad \quad = 5\sqrt{3}$ $\quad \quad \quad = \sqrt{75}$ $\therefore \underline{\underline{A = 75}}$	1	
c) $\pm^{2/3} = 16$ $\pm^{1/3} = \pm 4$ $\underline{\underline{\pm = \pm 64}}$	1 1 1	Many missed the -ve sign
d) i) $3x^2 - 12y^2 = 3(x^2 - 4y^2)$ $\quad \quad \quad = \underline{\underline{3(x-2y)(x+2y)}}$	1	No half marks
ii) $8 - 27d^3 = 2^3 - (3d)^3$ $\quad \quad \quad = \underline{\underline{(2-3d)(4+6d+9d^2)}}$	1	
e) $\log_x 2^3 - \log_x 2^7 = 4$ $\log_x \frac{2^3}{2^7} = 4$ $\log_x 2^{-4} = 4$ $-4 \log_x 2 = 4$ $\log_x 2 = -1$ $\therefore 2 = x^{-1}$ $\therefore \underline{\underline{x = 1/2}}$	1 1 1 1 1	Several ways of approach. 9 th -ve sign appeared in logic it had to be explained away.
f) i) $\sin 225^\circ = \underline{\underline{-1/\sqrt{2}}} = \underline{\underline{-\sqrt{2}/2}}$	1	No half marks
ii) $\sec 120^\circ = \underline{\underline{-2}}$	1	

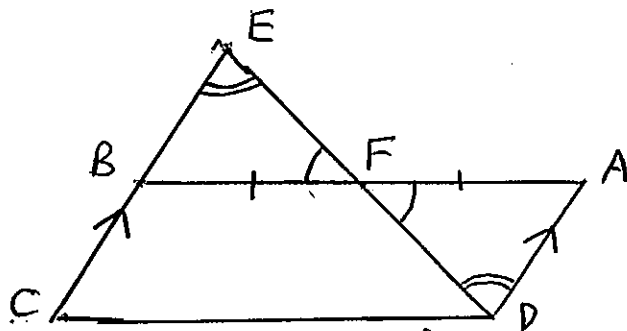
Suggested Solutions	Marks	Marker's Comments
g) i)  Let P be (x, y) $PA = 2 PB$ $(PA)^2 = 4(PB)^2$ $(x-1)^2 + (y-5)^2 = 4\{(x-4)^2 + (y+1)^2\}$ $x^2 - 2x + 1 + y^2 - 10y + 25 = 4x^2 - 32x + 64 + 4y^2 + 8y + 4$ $3x^2 + 3y^2 - 30x + 18y + 42 = 0$ $x^2 + y^2 - 10x + 6y + 14 = 0$ ii) $(x-5)^2 - 25 + (y+3)^2 - 9 + 14 = 0$ $(x-5)^2 + (y+3)^2 = 20$ $\therefore$ Centre <u>$(5, -3)$</u> Radius = <u>$\sqrt{20} = 2\sqrt{5}$ units</u>	1 1 1	or the square root equivalent. All parts had to be correct.

Suggested Solutions

Marks

Marker's Comments

h) i)

In $\Delta BEF, ADF$ $BF = AF$ (F given midpoint) $\angle AFD = \angle BFE$ (Vertically opposite angles are equal) $\angle ADF = \angle BEF$ (Alternating angles on parallels AD, BE) $\therefore \underline{\underline{\Delta BEF \equiv \Delta ADF \text{ (AAS)}}}$ ii) $BE = AD$ (Corresponding sides in congruent triangles) $AD = BC$ (Opposite sides of a parallelogram are equal) $\therefore BE = BC$ i.e. B is the midpoint of CE.

There were several approaches. Test needed to be included.

Most people got this out. There were other options.

MATHEMATICS: Question...7

Suggested Solutions	Marks	Marker's Comments
i) $y = x^3 + \sqrt{x}$ $= x^3 + x^{\frac{1}{2}}$ $y' = 3x^2 + \frac{1}{2}x^{-\frac{1}{2}}$ $= 3x^2 + \frac{1}{2\sqrt{x}}$	2	1 mark for differentiating each term correctly
ii) $y = \frac{x^2 - 2}{4x}$ $= \frac{x^2}{4x} - \frac{2}{4x}$ $= \frac{x}{4} - \frac{1}{2x}$ $= \frac{x}{4} - \frac{1}{2}x^{-1}$ $y' = \frac{1}{4} + \frac{1}{2}x^{-2}$ $= \frac{1}{4} + \frac{1}{2x^2}$	2	1 mark for differentiating each term correctly
OR $y = \frac{x^2 - 2}{4x}$ $y' = \frac{2x(4x) - 4(x^2 - 2)}{(4x)^2}$ $= \frac{8x^2 - 4x^2 + 8}{16x^2}$ $= \frac{4x^2 + 8}{16x^2}$ $= \frac{x^2 + 2}{4x^2}$	2	Students lost marks for doing $\frac{uv' - u'v}{v^2}$ instead of $\frac{u'v - uv'}{v^2}$
iii) $y = (3x+1)^5$ $y' = 5(3x+1)^4(3)$ $= 15(3x+1)^4$	1	

MATHEMATICS: Question....7...

Suggested Solutions

Marks

Marker's Comments

b) $y' = 4x - 6$

$$y = \frac{4x^2}{2} - 6x + C$$

$$= 2x^2 - 6x + C$$

Passes through (2, 4)

$$\therefore 4 = 2(2)^2 - 6(2) + C$$

$$C = 8$$

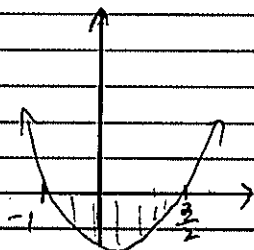
$$\therefore y = 2x^2 - 6x + 8$$

c) $3 + x - 2x^2 > 0$

$$2x^2 - x - 3 < 0$$

$$(2x - 3)(x + 1) < 0$$

$$-1 < x < \frac{3}{2}$$



d) for positive definite $a > 0$ & $\Delta < 0$

$$k + 6 > 0 \quad k > -6$$

$$\Delta = (-8)^2 - 4(k+6)k$$

$$= 64 - 4k^2 - 24k$$

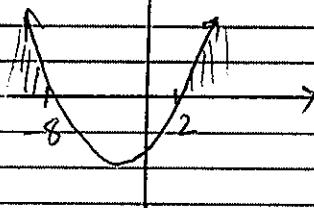
$$\therefore 64 - 4k^2 - 24k < 0$$

$$k^2 + 6k - 16 > 0$$

$$(k+8)(k-2) > 0$$

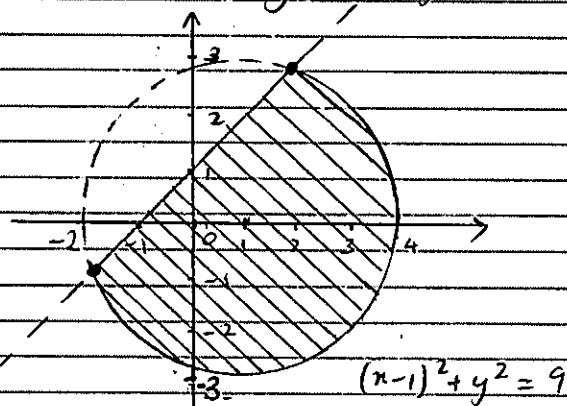
$$k < -8, \quad k > 2$$

$$\text{but } k > -6$$



$$\therefore k > 2 \text{ only. } \nearrow y = x + 1$$

e)



1

for integrating correctly

1

for substituting & getting the constant

Note! you lost a mark if you wrote $x > -1$ or $x \leq \frac{3}{2}$

1

1

1

1

1

1

1

for correct line

for correct circle

for correct region

Students lost marks on having uneven scales or not labelling scales at all. You must show intercepts when possible.

MATHEMATICS: Question.....

Suggested Solutions

Marks

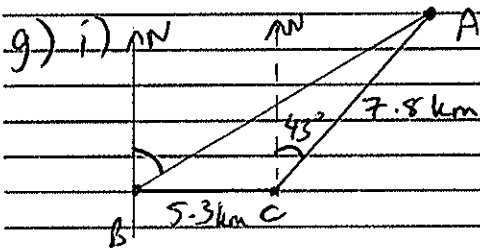
Marker's Comments

f) $2\cos^2 \theta = 1$

$\cos^2 \theta = \frac{1}{2}$

$\cos \theta = \pm \frac{1}{\sqrt{2}}$

$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$



ii) $\angle ACB = 43^\circ + 90^\circ$
 $= 133^\circ$

$AB^2 = AC^2 + BC^2 - 2 \times AC \times BC \cos 133$
 $= 7.8^2 + 5.3^2 - 2 \times 7.8 \times 5.3 \cos 133$

$AB = 12.054775 \dots$
 $AB = 12.1 \text{ km (1dp)}$

iii) let $\angle ABC = \alpha$

$\frac{\sin \alpha}{7.8} = \frac{\sin 133}{AB}$

$\sin \alpha = \frac{7.8 \sin 133}{AB}$

$\alpha = 28^\circ 15'$ using the exact value of AB

$\alpha = 28^\circ 8'$ using $AB = 12.1$

$\therefore$ bearing is $90 - \alpha$

so $90 - 28^\circ 15'$ or $90 - 28^\circ 8'$
 $= 61^\circ 45'$ $= 61^\circ 52'$
 $= 62^\circ$ $= 62^\circ$

OR

using cosine rule

$\cos \alpha = \frac{AB^2 + 5.3^2 - 7.8^2}{2 \times AB \times 5.3}$

$\alpha = 28^\circ 15'$ using the exact value of AB

$\alpha = 27^\circ 36'$ using $AB = 12.1$

2

for all 4 solutions.

only 1 mark if students forgot the $\pm$ & got only 2 solutions.

1

correctly labelled diagram with all relevant information

1

1

for using sine or cosine rule to find α .

1

for final bearing - students were not penalised for not having the D in front of 62°

$\therefore$ bearing $061^\circ 45'$ or $062^\circ 24'$
 $= 062^\circ$ $= 062^\circ$

MATHEMATICS: Question..8....

Suggested Solutions	Marks	Marker's Comments																																		
(a) $y = 2x^3 - 3x^2 - 12x + 2$ (i) Find stationary points and determine their nature. $\frac{dy}{dx} = 6x^2 - 6x - 12$ $= 6(x^2 - x - 2)$ $6(x-2)(x+1)$ For SP to occur $\frac{dy}{dx} = 0$ $\therefore x-2 = 0 \quad \text{or} \quad x+1 = 0$ $x = 2 \quad \quad x = -1 \quad \checkmark$ when $x=2$; $y = 16 - 12 - 24 + 2 = -18$ $x = -1$; $y = -2 - 3 + 12 + 2 = 9$. $\therefore$ points are $(2, -18)$ and $(-1, 9)$ $\checkmark$ <u>Test Nature</u> <u>Method 1</u> Point $(2, -18)$ <table><tr><td>x</td><td>0</td><td>2</td><td>3</td></tr><tr><td>$\frac{dy}{dx}$</td><td>-12</td><td>-</td><td>24</td></tr></table> Concave up $\therefore$ Minimum TP Point $(-1, 9)$ <table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td></tr><tr><td>$\frac{dy}{dx}$</td><td>24</td><td>-</td><td>-12</td></tr></table> Concave down $\therefore$ Maximum TP $\therefore$ There is a minimum Turning Point at $(2, -18)$ and Maximum TP at $(-1, 9)$	x	0	2	3	$\frac{dy}{dx}$	-12	-	24	x	-2	-1	0	$\frac{dy}{dx}$	24	-	-12	3	$\checkmark$ 1 mark for putting $\frac{dy}{dx} = 0$ and getting $x = 2$ or -1 $\checkmark$ 1 mark for getting correct points $(-2, 18)$ and $(-1, 9)$ $\checkmark$ 1 mark for Testing the nature. students who used other values to test nature should have <table><tr><th>x</th><th>$\frac{dy}{dx}$</th></tr><tr><td>1.5</td><td>-7.5</td></tr><tr><td>1.9</td><td>-1.74</td></tr><tr><td>2.1</td><td>1.86</td></tr><tr><td>2.5</td><td>10.5</td></tr><tr><td>-0.5</td><td>-7.5</td></tr><tr><td>-0.9</td><td>-1.74</td></tr><tr><td>-1.1</td><td>1.86</td></tr><tr><td>-1.5</td><td>10.5</td></tr></table>	x	$\frac{dy}{dx}$	1.5	-7.5	1.9	-1.74	2.1	1.86	2.5	10.5	-0.5	-7.5	-0.9	-1.74	-1.1	1.86	-1.5	10.5
x	0	2	3																																	
$\frac{dy}{dx}$	-12	-	24																																	
x	-2	-1	0																																	
$\frac{dy}{dx}$	24	-	-12																																	
x	$\frac{dy}{dx}$																																			
1.5	-7.5																																			
1.9	-1.74																																			
2.1	1.86																																			
2.5	10.5																																			
-0.5	-7.5																																			
-0.9	-1.74																																			
-1.1	1.86																																			
-1.5	10.5																																			

MATHEMATICS: Question.....

Suggested Solutions	Marks	Marker's Comments								
Q(ii) Find point(s) of inflexion Points of inflexion occurs when $f''(x) = 0$ $\therefore 12x - 6 = 0$ $x = \frac{1}{2}$ when $x = \frac{1}{2}$ $y = 2\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 - 12\left(\frac{1}{2}\right) + 2$ $= -4\frac{1}{2}$ <u>Test for concavity</u> <table border="1"> <tr> <td>x</td><td>0</td><td>$\frac{1}{2}$</td><td>1</td></tr> <tr> <td>y''</td><td>-6</td><td>-</td><td>6</td></tr> </table> concavity changes $\therefore$ Point of inflexion occurs at $\left(\frac{1}{2}, -4\frac{1}{2}\right)$	x	0	$\frac{1}{2}$	1	y''	-6	-	6	2	✓ getting pt $\left(\frac{1}{2}, -4\frac{1}{2}\right)$ ✓ Testing concavity.
x	0	$\frac{1}{2}$	1							
y''	-6	-	6							
(iii) sketch	1	Must have the <u>correct</u> - end points $(-2, -2)$ and $(4, 34)$ - y-intercept is 2. Graphs poorly done - Scaling on axis - labelling points								

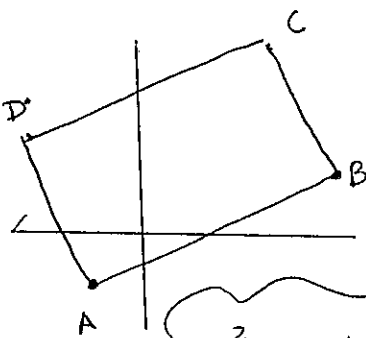
MATHEMATICS: Question...8...

Suggested Solutions	Marks	Marker's Comments
iv) Maximum value. is 34	1	needed only the y value • Points (4,34) not accepted.
⑥ Prove $\tan \theta + \cot \theta = \sec \theta \operatorname{cosec} \theta$ <u>Method 1</u> LHS $\tan \theta + \cot \theta$ $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$ $= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$ $= \frac{1}{\sin \theta \cos \theta}$ $= \sec \theta \operatorname{cosec} \theta$ $= \text{RHS.}$ <u>Method 2</u> LHS $\tan \theta + \cot \theta$ $\tan \theta + \frac{1}{\tan \theta}$ $= \frac{\tan^2 \theta + 1}{\tan \theta}$ $= \frac{\sec^2 \theta}{\tan \theta}$ $= \frac{1}{\cos^2 \theta} \times \frac{\cos \theta}{\sin \theta}$ $= \frac{1}{\cos \theta \sin \theta}$ $= \sec \theta \operatorname{cosec} \theta$	2	well done. Be careful with setting out.
⑦ (i).	1	Not all points correctly
(ii) Gradient of AB $M_{AB} = \frac{2 - (-1)}{4 - (-1)} = \frac{3}{5}$	1	Gradient of $\frac{3}{5}$

MATHEMATICS: Question. 8....

Suggested Solutions	Marks	Marker's Comments
(iii) Show equation of l - which is parallel to AB and passes through C is $3x - 5y + 21 = 0$ ✓ Gradient of parallel line is also $\frac{3}{5}$ and passes through $(3, 6)$ ✓ ∴ equation $y - y_1 = m(x - x_1)$ $y - 6 = \frac{3}{5}(x - 3)$ $5y - 30 = 3x - 9$ $0 = 3x - 5y + 21$	2	well done. 1 mark for getting $\frac{3}{5}$ as gradient and 1 mark for forming the equation with point C.
iv) Line k is $4x + y + 5 = 0$ line l is $3x - 5y + 21 = 0$ Intersection of k and l will give point D $3x - 5y + 21 = 0 \text{ --- (1)}$ $4x + y + 5 = 0 \text{ --- (2)}$ eq (2) $\times 5$ $20x + 5y + 25 = 0 \text{ --- (3)}$ eq (1) + eq (3) $23x + 46 = 0$ $x = -2 \text{ ✓}$ when $x = -2$ sub in eq (2) $4(-2) + y + 5 = 0$ $-8 + y + 5 = 0$ $y = 3$ ∴ point D is $(-2, 3)$ ✓. Also mark D on Number plane	2	1 mark for showing the process of finding point of Intersection and getting $x = -2$ 1 m. for getting the point $(-2, 3)$
(v) <u>acute angle</u> $\tan \theta = \text{gradient}$. $\therefore \tan \theta = \frac{3}{5}$ $\theta = 30.96375653$ $\therefore \theta = 31^\circ$	1	1 mark for rounded off value of θ (31°).

MATHEMATICS: Question. 5....

Suggested Solutions	Marks	Marker's Comments
(vi) Perpendicular distance from B. to line l. $d = \frac{ ax + by + c }{\sqrt{a^2 + b^2}}$ $= \frac{ 4(3) - 5(2) + 21 }{\sqrt{3^2 + (-5)^2}}$ $= \frac{ 23 }{\sqrt{34}}$ $\therefore$ perpendicular distance is $\frac{23}{\sqrt{34}}$ or $\frac{23\sqrt{34}}{34}$	2	Note B(4,2) and $l = 3x - 5y + 21$ $\therefore a = 3$ $b = -5$ $c = 21$ $x = 4$ $y = 2$ 1 mark for correct use of formula 1 mark for Answer
(v) Exact area of ABCD - Area of parallelogram = base $\times$ perpendicular height  $= AB \times \frac{23}{\sqrt{34}}$ $= \sqrt{34} \times \frac{23}{\sqrt{34}}$ $AB^2 = 2^2 + (-1)^2 + (4 - (-1))^2$ $= 3^2 + 5^2$ $AB = \sqrt{34} \checkmark$ $= 23 \text{ units} \checkmark$	2	1 mark for getting distance AB or DC 1 mark for calculating the area,

MATHEMATICS: Question 9

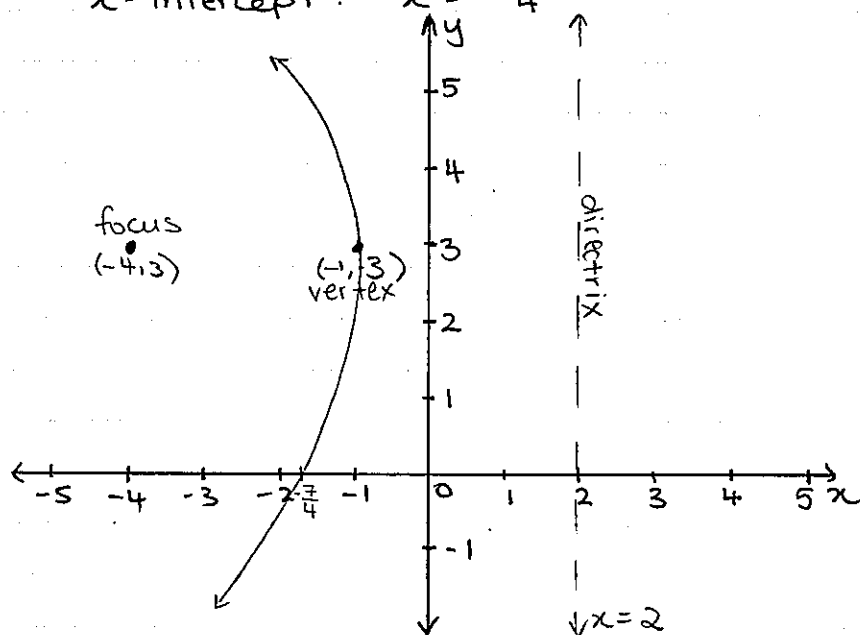
Suggested Solutions

Marks

Marker's Comments

a) $V = (2, 2)$ $a = 2$
 $(x-2)^2 = 8(y-2)$

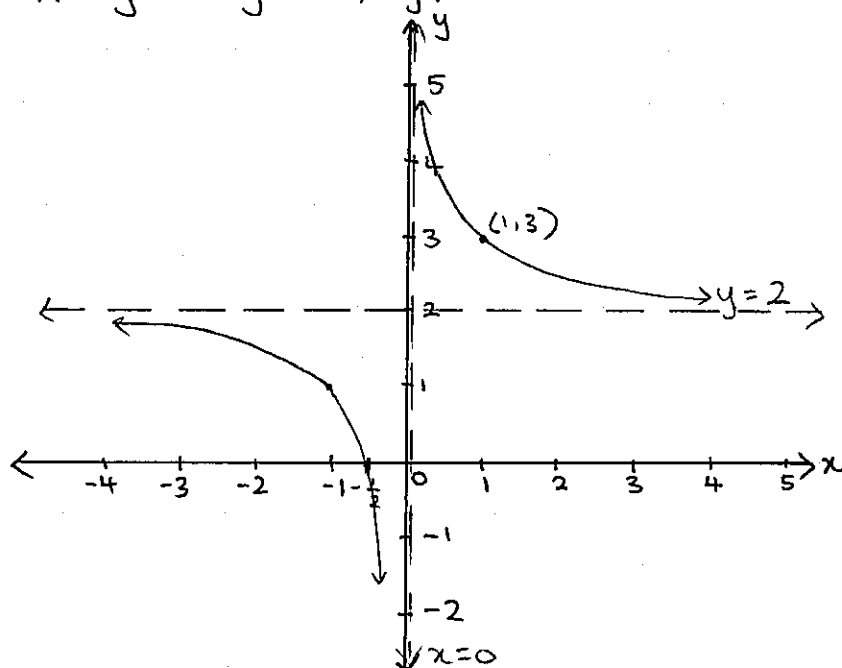
b) $V: (-1, 3)$ $a = 3$
 directrix: $x = 2$
 x-intercept: $x = -\frac{7}{4}$



c) $y = \frac{2x+1}{x}$

Domain: $x \in \mathbb{R}, x \neq 0$

Range: $y \in \mathbb{R}, y \neq 2$



1

many students did not write all real x/y .

1 asymptotes

1 intercept & shape

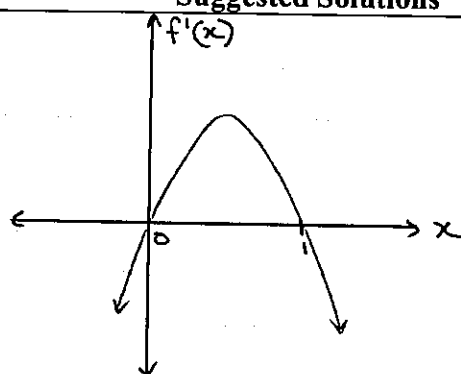
MATHEMATICS: Question 9

Suggested Solutions

Marks

Marker's Comments

d)



1

must pass through 0 and 1.

e) $y = 2x^2 - 2x$

$y' = 4x - 2$

$y'' = 4$

Show: $x^2 y'' - 2xy' + 2y = 0$

LHS = $x^2(4) - 2x(4x - 2) + 2(2x^2 - 2x)$

$= 4x^2 - 8x^2 + 4x + 4x^2 - 4x$

$= 0$

$= \text{RHS}$

$\therefore x^2 y'' - 2xy' + 2y = 0$

f) let $f(x) = 2\sqrt{x}$

$\therefore \frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \rightarrow 0} \frac{2\sqrt{x+h} - 2\sqrt{x}}{h}$

$= 2 \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$

$= 2 \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$

$= 2 \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$

$= 2 \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$

$= \frac{2}{\sqrt{x} + \sqrt{x}}$

$= \frac{2}{2\sqrt{x}}$

$= \frac{1}{\sqrt{x}}$

} 1

1

1

1

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g) $y = \frac{x}{x^2 + 1}$

$$y' = \frac{(x^2 + 1) - x(2x)}{(x^2 + 1)^2}$$

$$= \frac{1 - x^2}{(x^2 + 1)^2}$$

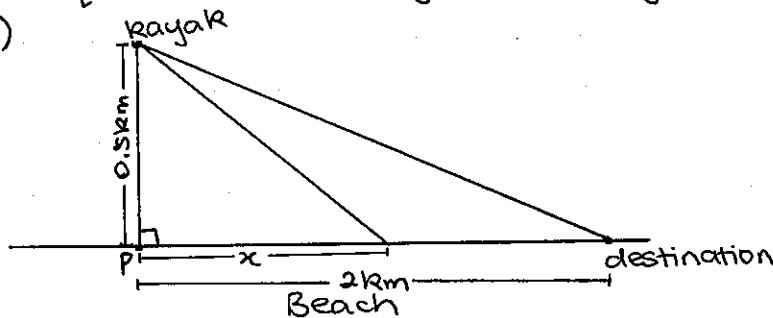
at origin, $x = 0$

$$y' = \frac{1 - (0)^2}{(0^2 + 1)^2}$$

$$= 1$$

$\therefore$ equation of tangent is: $y = x$

h) α



β) kayak:

$$\text{distance}^2 = x^2 + 0.5^2 \quad (\text{pythagoras theorem})$$

$$\text{distance} = \sqrt{x^2 + 0.5^2}$$

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{\sqrt{x^2 + 0.5^2}}{1.5}$$

beach:

$$\text{distance} = 2 - x$$

$$\text{time} = \frac{2 - x}{2}$$

$$\therefore \text{total time} = \frac{\sqrt{x^2 + 0.5^2}}{1.5} + \frac{2 - x}{2}$$

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$$8) t = \frac{\sqrt{x^2 + 0.5^2}}{1.5} + \frac{2-x}{2}$$

$$\frac{dt}{dx} = \frac{2}{3} \times \frac{1}{2} (x^2 + 0.5^2)^{-\frac{1}{2}} \times 2x - \frac{1}{2}$$

$$= \frac{2x}{3\sqrt{x^2 + 0.5^2}} - \frac{1}{2}$$

for stationary points, $\frac{dt}{dx} = 0$

$$\frac{2x}{3\sqrt{x^2 + 0.5^2}} - \frac{1}{2} = 0$$

$$\frac{2x}{3\sqrt{x^2 + 0.5^2}} = \frac{1}{2}$$

$$4x = 3\sqrt{x^2 + 0.5^2}$$

$$16x^2 = 9(x^2 + 0.5^2)$$

$$16x^2 = 9x^2 + \frac{9}{4}$$

$$7x^2 = \frac{9}{4}$$

$$x^2 = \frac{9}{28}$$

$$x = \pm \frac{3}{\sqrt{28}}$$

$x > 0$ for minimum.

$$\text{when } x = \frac{3}{\sqrt{28}}$$

x	$\frac{1}{2}$	$\frac{3}{\sqrt{28}}$	1
$\frac{dt}{dx}$	-0.03	0	0.096

$\therefore x = \frac{3}{\sqrt{28}}$ is a minimum

$\therefore$ she should row roughly 566.95 m along the beach from P towards her destination.

must test point to show minimum