Name ______________________

Class ____________

YEAR 11

ASSESSMENT TASK 3

2019

MATHEMATICS ADVANCED

General Instructions	• **Reading time – 5 minutes** • **Working time – 2 hours** • **Write using black pen** • **Calculators approved by NESA may be used** • **A separate reference sheet is provided** • **For questions in Section II, show relevant mathematical reasoning and/or calculations**

Total marks: 100	**Section I – 5 marks** • **Attempt Questions 1–5** • **Allow about 7 minutes for this section** **Section II – 80 marks** • **Attempt Questions 6-9** • **Allow about 1 hours and 53 minutes for this section**

The answers to all questions are to be returned in separate *stapled* bundles, clearly marked with Question 6, Question 7, etc., with your student number.

Section I - Multiple Choice. Answer on the Multiple Choice Sheet provided.

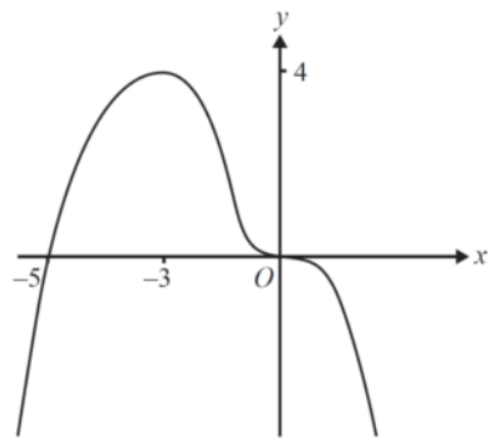
Question 1

What is the domain of $f(x) = \log_e[(x-2)(x-1)]$?

A. $1 \le x \le 2$
B. $1 < x < 2$
C. $x \le 1$ or $x \ge 2$
D. $x < 1$ or $x > 2$

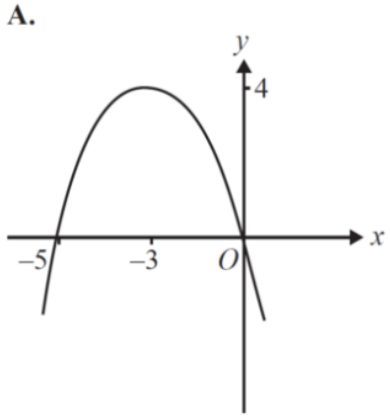
Question 2

The graph of the function $y = f(x)$ is shown below.

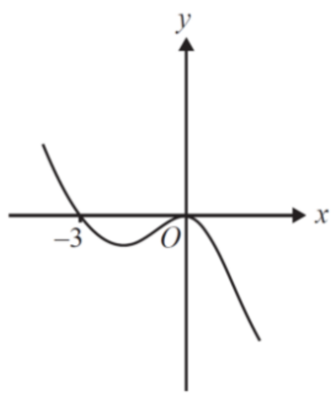


Which if the following could be the graph of the derivative function $y = f'(x)$?

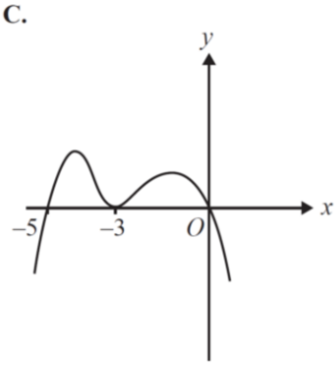
A.



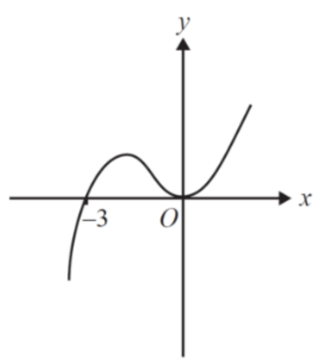
B.



C.



D.



Question 3

In a raffle, 30 tickets are sold and there is one prize to be won. What is the probability that someone buying 6 tickets wins the prize?

A. $\frac{1}{30}$ B. $\frac{1}{6}$ C. $\frac{1}{5}$ D. $\frac{1}{4}$

Question 4

What is the solution of $5^x = 4$?

A. $x = \frac{\log_e 4}{\log_e 5}$ B. $x = \frac{4}{\log_e 5}$ C. $x = \frac{\log_e 4}{5}$ D. $x = \log_e\left(\frac{4}{5}\right)$

Question 5

What are the solutions of $\sqrt{3}\tan x = -1$ for $0 \le x \le 2\pi$?

A. $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

B. $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$

C. $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

D. $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

Section II – Start each question on a new page.

Question 6 (20 Marks)	**START A NEW PAGE**	**Marks**

(a) Solve $\cos(2x - 10°) = -0.4$, for $0^0 \le x \le 180^0$, to the nearest minute. **3**

(b) Solve $3\sin^2\theta - 2\cos^2\theta = 1$ for $0^0 \le \theta \le 360^0$, to the nearest minute. **3**

(c) Simplify

(i) $\log_{10} x^5 + 3\log_{10} x^4$ **1**

(ii) $\log_a 1 - \log_a a^b$ **1**

(d) Solve $\left|\frac{x}{3} - 2\right| \le 4$ **2**

(d) Differentiate the following with respect to x

(i) $y = 3x^5 - x^3 + 2$ **1**

(ii) $\frac{3}{\sqrt[3]{x}}$ **1**

(iii) $y = e^{3x} + e^{-2x}$ **1**

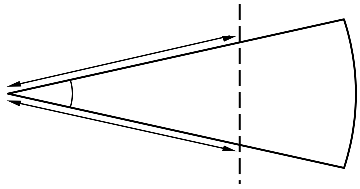
(iv) $f(x) = x^3\sqrt{6x^2 - 3x}$ **3**

(e) A curve C has equation $y = \frac{3}{(5-3x)^2}$, $x \ne \frac{5}{3}$. The point P on C has x-coordinate 2. **4**

Find the equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

Question 7 (20 Marks) **START A NEW PAGE** **Marks**

(a) Charles has a slice of cake; its cross-section is a sector of a circle, as shown in Figure 1. The radius is r cm and the sector angle is $\frac{\pi}{6}$ radians. He wants to give half of the slice to Jan. He makes a cut across the sector as shown.



Show that when they each have half the slice, $a = r\sqrt{\frac{\pi}{6}}$ **4**

(b) Given that a and b are positive constants, solve the simultaneous equations: **3**

$$a = 3b$$

$$\log_3 a + \log_3 b = 2$$

Express a and b in exact form.

(c) The value, in dollars ($) of Bob's car can be calculated from the formula:

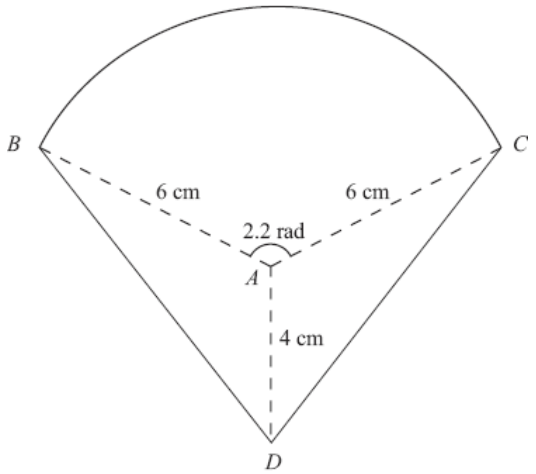
$$V = 17\,000\, e^{-0.25t} + 2000e^{-0.5t} + 500$$

where t is the age of the car in years.

(i) Find the value of the car when $t = 0$. **1**

(ii) How long will it take for Bob's car to be valued at $9500? **4**

(d) The shape BCD shown below is a design for a logo.



The straight lines DB and DC are equal in length. The curve BC is an arc of a circle with centre A and radius 6 cm. The size of $\angle BAC$ is 2.2 radians and $AD = 4$ cm.

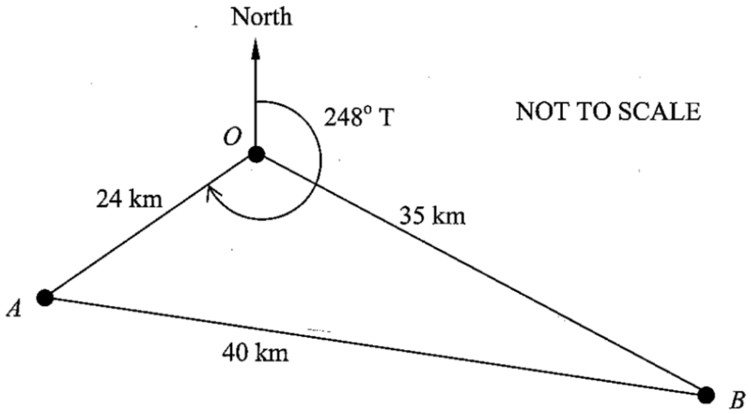
Find

(i) the length of the arc BC to 1 decimal place. **1**

(ii) the size of $\angle DAC$, in radians to 3 significant figures, **1**

(iii) the perimeter of the logo design, to the nearest cm **2**

(e) A section of rainforest is to be designated for a species count. The shape is shown below. The bearing of landmark A from landmark O is $248^0\ T$ and is 24 km in distance. The distance from landmark A to B is 40 km and from landmark B to O is 35 km.



(i) Find the size of $\angle AOB$ to the nearest degree. **1**

(ii) Hence, calculate the area of this section of the rainforest to the nearest square kilometre. **1**

(iii) Copy the diagram and hence find the size of the bearing of landmark O from landmark B? **2**

Question 8 (20 Marks) **START A NEW PAGE** **Marks**

(a) (i) On the same system axes sketch the graphs of **3**

$$y = x(4 - x)$$

and $$y = x^2(7 - x)$$

(ii) Find the point(s) of intersection, which contain coordinates that are entirely positive. Leave your answer(s) in exact form. **3**

(b) Consider the function $f(x) = log_e x$.

(i) Sketch on the same number plane the graph of the curves $y = f(x)$ and $y = -f(x)$. **2**

(ii) State the domain of the function $y = f(-x)$, giving your answer in interval notation. **1**

(c) The velocity v m/s of a particle moving along the x axis at the time t seconds is given by

$$v = \frac{\sqrt{t}}{1 + t^2}$$

(i) Find an expression for the acceleration of the particle. **2**

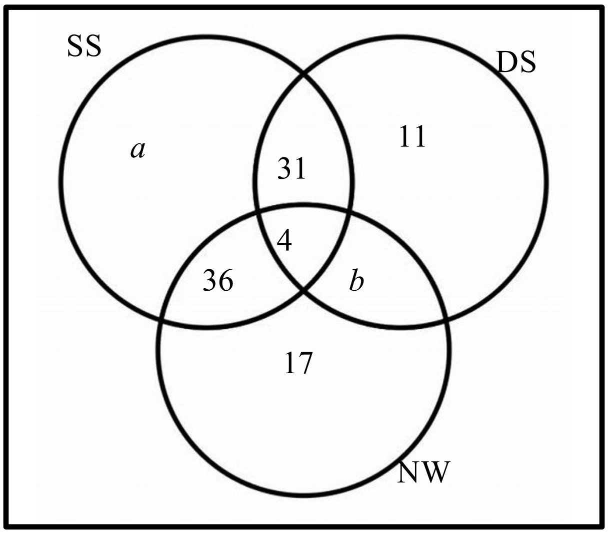
(ii) Find when the acceleration of the particle is zero. **2**

(d) A packet of lollies contains 5 red lollies and 14 green lollies. **2**
Two lollies are selected at random without replacement.
What is the probability that the two lollies are of different colours?

(e) There are 180 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.

112 take systems support (SS),
70 take developing software (DS),
81 take networking (NW),
35 take developing software (DS) and systems support (SS),
28 take networking (NW) and developing software (DS),
40 take systems support (SS) and networking (NW),
4 take all three extra options.

The distribution of student choices is shown in the Venn diagram below.



(i) Use the information above to find the values of a and b as given in the Venn diagram. **2**

A student from the course is chosen at random.

(ii) Find the probability that this student takes none of the three options. **1**

(iii) Find the probability that this student takes networking only given that the student takes either networking or systems support but not both. **2**

Question 9 (20 Marks) **START A NEW PAGE** **Marks**

(a) Prove that $\sec^2 x + \sec x \tan x = \frac{1}{1-\sin x}$ **3**

(b) A radioactive isotope of Curium has a half-life of 163 days. Initially there are 10 mg of Curium in a container.
The mass $M(t)$ in milligrams of Curium, after t days, is given by $M(t) = Ae^{-kt}$, where A and k are constants.

(i) State the value of A. **1**

(ii) Given that after 163 days only half of the initial Curium remains, find the value of k. **2**

(c) Let $f(x) = \log_e x$ and $g(x) = x^2 + 1$.

(i) Find the rule for h, where $h(x) = f(g(x))$ **1**

(ii) Show that $h(x) + h(-x) = f\left(\left(g(x)\right)^2\right)$ **2**

(d) Let $f(x) = \frac{2}{x}$

(i) Sketch the graph of $y = f(x)$ showing all important features. **2**

(ii) If (p, q) is any point on the graph of $y = f(x)$ show that the equation of the tangent to $y = f(x)$ at this point can be written as $p(py - 2) = -2x + 2p$. **2**

(iii) Hence, or otherwise, find the coordinates of the points on the graph of $y = f(x)$ such that the tangents to the graph at these points intersect at $\left(\frac{8}{3}, \frac{2}{3}\right)$. **3**

(e) An egg marketing company buys its eggs from farm *A* and farm *B*. Let p be the proportion of eggs that the company buys from farm *A*. The rest of the company's eggs come from farm *B*. Each day, the eggs from both farms are taken to the company's warehouse.
Assume that $\frac{3}{5}$ of all eggs from farm *A* have white eggshells and 15 of all eggs from farm *B* have white eggshells.

(i) An egg is selected at random from the set of all eggs at the warehouse. **1**

Find, in terms of p, the probability that the egg has a white eggshell.

Another egg is selected at random from the set of all eggs at the warehouse.

(ii) Given that the egg has a white eggshell, find, in terms of p, the probability that it came from farm *B*. **2**

(iii) If the probability that this egg came from farm *B* is 0.3, find the value of p. **1**

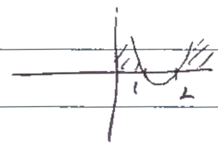
END OF EXAMINATION

Question	MCQ	6		7		8		9		Total
Functions										
Combinatorics										
Trigonometric Functions										
Calculus										
MCQ										
Total	**/5**		**/20**		**/20**		**/20**		**/20**	**/85**

Solutions 2019 YEAR 11 PRELIMS

Q1. $(x-2)(x-1) > 0$

$x<1$ or $x>2$



D

Q2. B

Q3 C

Q4 $x = \log_5 4$

$= \frac{\ln 4}{\ln 5}$ A

Q5. $\tan x = -\frac{1}{\sqrt{3}}$ $\frac{\checkmark 1}{\checkmark}$

$x =$ D

MULTIPLE CHOICE D B C A D

MATHEMATICS: Question 6 . a,b.

Suggested Solutions	Marks	Marker's Comments
a) $\cos(2x-10°) = -0.4 \quad 0° \leq x \leq 180°$		
$0 \leq 2x \leq 360°$ $-10° \leq 2x-10° \leq 350°$		1 for taking the inverse cos of -0.4
$\therefore 2x-10° = 113°35'$ or $246°25'$ $2x = 123°35'$ or $256°25'$		1 Correct values for both quadrants 2nd & 3rd
$\therefore x = 61°47'$ or $128°13'$		1 rounding
b) $3\sin^2\theta - 2\cos^2\theta = 1$		
$3\sin^2\theta - 2(1-\sin^2\theta) = 1$ $3\sin^2\theta - 2 + 2\sin^2\theta = 1$ $5\sin^2\theta = 3$ $\sin^2\theta = \frac{3}{5}$		1 for using Pythagorean Identity
$\sin\theta = \pm\sqrt{\frac{3}{5}}$		1
$\therefore \sin\theta = 50°46'$ or $129°14'$ or $230°46'$ or $309°14'$		1
		/6

MATHEMATICS: Question 6 c, d

Suggested Solutions	Marks	Marker's Comments

(c) (i) $\log_{10} x^5 + 3\log_{10} x^4$

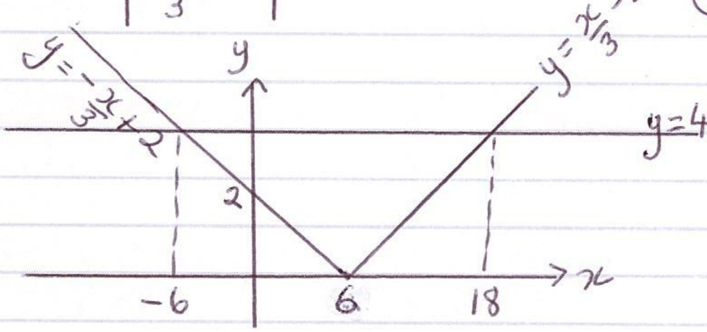
$= 5\log_{10} x + 12\log_{10} x$

$= 17\log_{10} x$

(ii) $\log_a 1 - \log_a a^b = 0 - b\log_a a$

$= -b$ ——— ①

(d) $\left|\frac{x}{3} - 2\right| \leq 4$ Method 1



$\frac{x}{3} - 2 = 4$ $\qquad$ $-\frac{x}{3} + 2 = 4$

$\frac{x}{3} = 6$ $\qquad$ $\frac{x}{3} = -2$

$x = 18$ $\qquad$ $x = -6$

$\therefore \left|\frac{x}{3} - 2\right| \leq 4$

when $-6 \leq x \leq 18$

OR {Method 2} Distance from 0.

$-4 \leq \frac{x}{3} - 2 \leq 4$

$-2 \leq \frac{x}{3} \leq 6$

$-6 \leq x \leq 18$

Marker's Comments:

① $17\log_{10} x$ or $\log_{10} x^{17}$

①

/2

① showing an understanding of absolute value

① For expressing answer correctly

Note: $18 \geq x \geq -6$, $x \leq 18$ or $x \geq -6$, $x \leq 18,\ x \geq -6$ were marked <u>incorrect</u>

The only correct answer was

$-6 \leq x \leq 18$

/2

d) (i) $l = r\theta$

$= 6 \times 2.2$

$= 13.2$ cm

(ii) $\angle DAC = \dfrac{2\pi - 2.2}{2}$

$= 2.04$

(iii) $CD^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos 2.04$

$CD = 8.59$

Perimeter $= 13.2 + 2 \times 8.59$

$= 30$ cm to nearest cm

e) (i) $\cos \angle AOB = \dfrac{24^2 + 35^2 - 40^2}{2 \times 24 \times 35}$

$\angle AOB = 83^\circ$

(ii) area $AOB = \frac{1}{2} \times 24 \times 35 \times \sin 83^\circ$

$= 416.9$ km^2

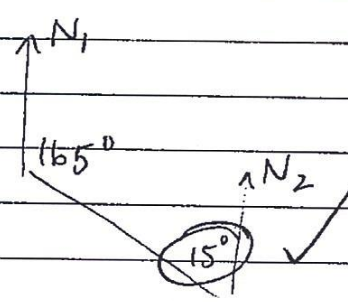
$\doteqdot 417$ km^2

(iii) $248^\circ - 83^\circ = 165^\circ = \angle NOB$

$\angle OBN_2 = 15^\circ$

$\therefore$ bearing $= 360^\circ - 15^\circ$

$= 345^\circ$



Both True and compass bearing were accepted

N 15° W

MATHEMATICS: Question 6e

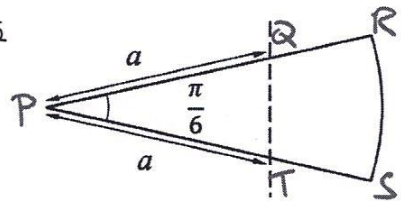
Suggested Solutions	Marks	Marker's Comments
(e) (i) $y = 3x^5 - x^3 + 2$ $\frac{dy}{dx} = 15x^4 - 3x^2$		1
(ii) $y = \frac{3}{\sqrt[3]{x}}$ $= 3x^{-\frac{1}{3}}$ $\frac{dy}{dx} = -x^{-\frac{4}{3}}$ $= -\frac{1}{\sqrt[3]{x^4}}$		1
(iii) $y = e^{3x} + e^{-2x}$ $\frac{dy}{dx} = 3e^{3x} - 2e^{-2x}$		1
(iv) $f(x) = x^3\sqrt{6x^2-3x}$ $= x^3(6x^2-3x)^{\frac{1}{2}}$ Let $u = x^3$ $\quad v = (6x^2-3x)^{\frac{1}{2}}$ $\frac{du}{dx} = 3x^2$ $\quad \frac{dv}{dx} = \frac{1}{2}(6x^2-3x)^{-\frac{1}{2}}(12x-3)$ $= \frac{12x-3}{2\sqrt{6x^2-3x}}$		1 realising it needed to be broken down to u & V (but it could have been put straight into product rule).
Using product rule $f'(x) = (6x^2-3x)^{\frac{1}{2}} \cdot 3x^2 + \frac{x^3(12x-3)}{2\sqrt{6x^2-3x}}$		1 Using Product rule
$= \frac{6x^2(6x^2-3x) + x^3(12x-3)}{2\sqrt{6x^2-3x}}$ $= \frac{36x^4 - 18x^3 + 12x^4 - 3x^3}{2\sqrt{6x^2-3x}}$		1 Simplified Answer.

$$= \frac{48x^4 - 21x^3}{2\sqrt{6x^2-3x}}$$

6

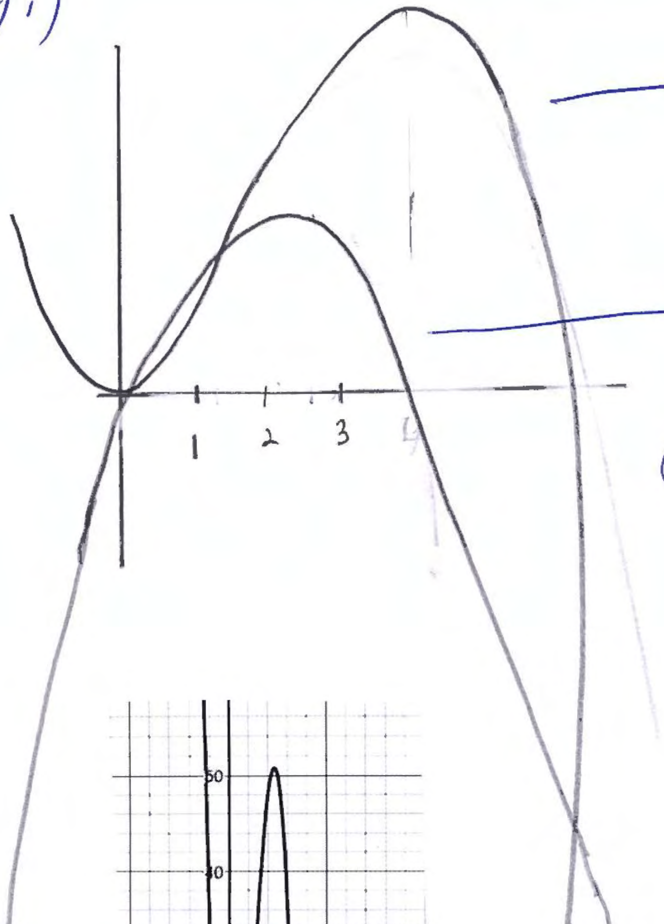
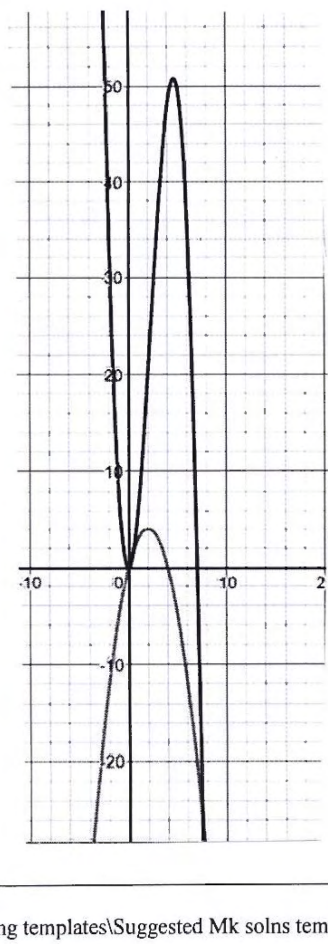
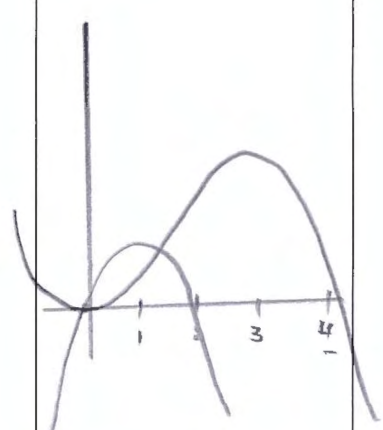
MATHEMATICS: Question 6f.		
Suggested Solutions	**Marks**	**Marker's Comments**
(f) $y = 3(5-3x)^{-2}$ $y' = -6(5-3x)^{-3} \cdot -3$ $y' = \dfrac{18}{(5-3x)^3}$		1
When $x=2$ $\quad y = \dfrac{3}{(5-6)^2}$ $= 3$ $\therefore P = (2,3)$		1
When $x=2$ $\quad y' = \dfrac{18}{(-1)^3} = -18$ $\therefore$ gradient of tangent $= -18$ Gradient of normal $= +\dfrac{1}{18}$		1
Equation of normal; $y - 3 = \frac{1}{18}(x-2)$ $18y - 54 = x - 2$ $x - 18y + 52 = 0$	$\frac{4}{}$	1 Must be expressed $ax+by+c=0$ where a, b, c are integers.

(Note e of given as a total $/10$)

MATHEMATICS Advanced: Question 7		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
(a) Half of the slice (volume) corresponds to half of the top-face surface area: $\frac{A_{\text{triangle}}}{A_{\text{sector}}} = \frac{\frac{1}{2}a^2\cancel{\sin\left(\frac{\pi}{6}\right)}^{\frac{1}{2}}}{\frac{1}{2}r^2\left(\frac{\pi}{6}\right)}$  $= \frac{3a^2}{\pi r^2}$ Require $\frac{A_{\text{trianlge}}}{A_{\text{sector}}} = \frac{1}{2}$, so $\frac{1}{2} = \frac{3a^2}{\pi r^2} \Rightarrow \frac{a}{r} = \sqrt{\frac{\pi}{6}}$ $= \frac{\sqrt{6\pi}}{6}$ or $a:r = \sqrt{\pi/6} : 1, \qquad a:r = \sqrt{\pi} : \sqrt{6}$ or anything equivalent, expressed as a **ratio** of a to r.	1 for correct area of sector. 1 for correct area of triangle. 1 for correct setup and working. 1 for correct answer, expressed as a ratio.	Question not handled well. Some common errors: Evaluating area of triangle using area of sector formula (i.e. stating area of triangle as $\frac{1}{2}a^2\left(\frac{\pi}{6}\right)$). Using area of segment formula to evaluate area $QRST$ (which isn't a segment here) and equating with area of triangle PQT. The idea of equating area was not the problem, using incorrect area was. Many students failed to express as a ratio in the form $\frac{a}{r}$ or $a:r$. Pay attention to the instructions in a question.
(b) Solve $a = 3b \quad \ldots(1)$ $\log_3 a + \log_3 b = 2 \quad \ldots(2)$ Given $\log_3 a + \log_3 b = \log_3 ab$ from (2) $\log_3 ab = 2 \quad \ldots(3)$ Multiplying (1) by b gives $ab = 3b^2 \quad \ldots(4)$ Sub. (4) into (3): $\log_3 3b^2 = 2 \Rightarrow \log_3 3 + 2\log_3 b = 2$ $1 + 2\log_3 b = 2$ $\log_3 b = \frac{1}{2}$	1 mark for reasonable working. 1 mark for correct value of b.	Part (b) handled generally well. Common errors were: Solving the resulting equations and failing to reject negative values for a, b. Failing to recognise the scope of the exponent in $\log_3 3b^2$ is b, not $3b$, and incorrectly 'dropping' the power as $2\log_3 3b$ when it would only occur when the scope

$\Leftrightarrow \sqrt{3} = b$ By (1), $a = 3b = 3\sqrt{3}$ Note: if you solved differently, at some point having to solve $b^2 = 3$, then $b = \pm\sqrt{3}$, but only $b = \sqrt{3}$ can be taken as the domain of $\log_3 x$ is $\mathbb{R}^+$ (all real $x > 0$).	1 mark for correct value of a.	of the exponent is the entire argument of log: $\log_3(3b)^2 = 2\log_3 3b$
(c) The value V, in \$, is $V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ (i) When $t = 0$, the value of the car is $V_0 = 17000 + 2000 + 500$ $= \$19500$ (ii) We need t when $V = \$9500$. Noting $e^{-0.5t} = (e^{-0.25t})^2$, $V = 17000(e^{-.25t}) + 2000(e^{-0.25}\ )^2 + 500$ So, $2000(e^{-0.25t})^2 + 17000(e^{-0.5t}) + 500 - V = 0$ $\equiv 2000u^2 + 17000u + 500 - V = 0$ Solving for u (by factoring, quadratic equation or completing the square), noting that $V = \$9500$, $2000u^2 + 17000u + 500 - 9500 = 0$ $2u^2 + 17u - 9 = 0$ $(2u - 1)(u + 9) = 0$ So, $u = e^{-0.25} = \frac{1}{2}$ or $u = e^{-0.25t} = -9$ But $e^{-0.25} > 0$ for all $t \in \mathbb{R}$, so the only possible solution is $e^{-0.25t} = \frac{1}{2} \Rightarrow -0.25t = \log_e\left(\frac{1}{2}\right)$ i.e. $t = 4\log_e 2 \approx 2.77$ years.	1 mark for correct answer only, part (i). 1 mark for correct setup. 1 mark for recognition of quadratic. 1 mark for reasonable working. 1 mark for correct final answer, expressed in exact or approximate form.	Part (c) (i) was handled well. Part (c) (ii) was handled poorly. Common sources of error were: Failing to recognise the form of the equation in e to be quadratic in $e^{-0.25t}$. Incorrectly applying the logarithm sum to product rule: e.g. if $x = a + b$ then $\log x = \log a + \log b$ But this is not the case. The correct form would be $\log x = \log(a + b)$ You cannot go further without additional information. An example demonstrates the fault in the reasoning: if $4 = 1 + 3$, then $\log_{10} 4$ $= \log_{10} 1 + \log_{10} 3$ $= 0 + \log_{10} 3$ Hence $4 = 3$.

Prelim T3 Yr 11 2019.

MATHEMATICS	:: Question 8	1/4
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
a) i)  (1) (1) (1) – for at least 2 pts of intersection and $y=x^2(7-x)$ higher than $y=x(4-x)$  	shape shape	1  marked this correct only because pt ii should have been first.

MATHEMATICS	Question 8	2/4
Suggested Solutions	**Marks Awarded**	**Marker's Comments**

8a ii) $4x - x^2 = 7x^2 - x^3$

$x^3 - 8x^2 + 4x = 0$

$x(x^2 - 8x + 4) = 0$

$x = 0$ or $x = \dfrac{8 \pm \sqrt{64-16}}{2(1)}$

$= \dfrac{8 \pm \sqrt{48}}{2}$

$= \dfrac{8 \pm 4\sqrt{3}}{2}$

$= 4 \pm 2\sqrt{3}$

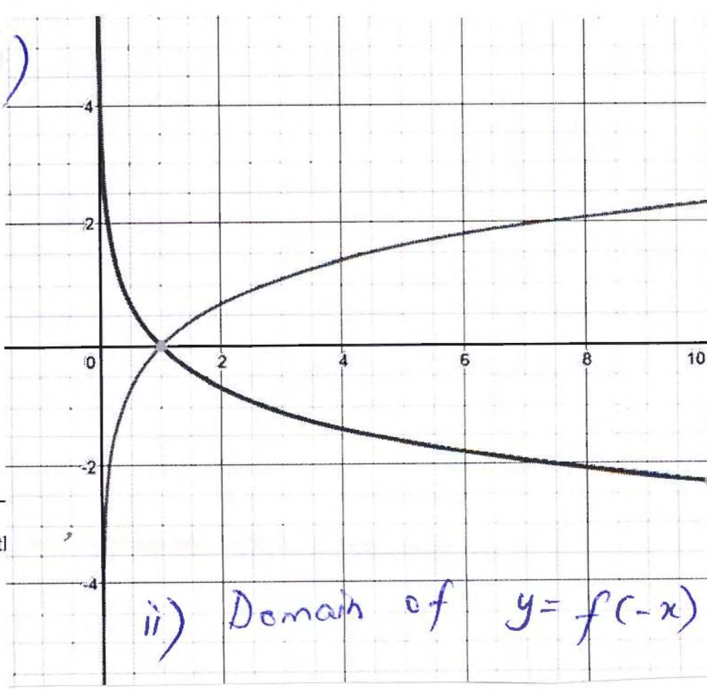
no marks deducted if

POI when $x = 0$ $y = 0$ — (1)

$\left.\begin{array}{l} x = 4 + 2\sqrt{3} \quad y = -8\sqrt{3} - 12 \\ x = 4 - 2\sqrt{3} \quad y = 8\sqrt{3} - 12 \end{array}\right\}$ — (2)

Coordinates entirely positive in exact form is $(4 - 2\sqrt{3},\ 8\sqrt{3} - 12)$

b) i)



— $y = \ln x$ (1)

$y = -\ln x$ (1)

ii) Domain of $y = f(-x)$ in interval notation *

$D : (-\infty, 0)$ — (1)

MATHEMATICS Extension 1 : Question…….. 3/4		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**

c) i) $v = \sqrt{t}$ —— (1)

$\frac{\sqrt{t}}{1+t^2}$ —— (1) (squared)

$\frac{dv}{dt} = a = \frac{1+t^2 \cdot \frac{1}{2}t^{-\frac{1}{2}} - \sqrt{t} \cdot 2t}{(1+t^2)^2}$

because some students didnt simplify.

$= \frac{(1+t^2) - 4t^2}{2\sqrt{t}\,(1+t^2)^2}$

$= \frac{1-3t^2}{2\sqrt{t}(1+t^2)^2}$ or $\frac{\sqrt{t}(1-3t^2)}{(1+t^2)^2\, 2t}$

ii) acceleration of particle zero

$\frac{1-3t^2}{2\sqrt{t}(1+t^2)^2} = 0$ —— (1)

$\therefore 1-3t^2 = 0$

$t^2 = \frac{1}{3}$

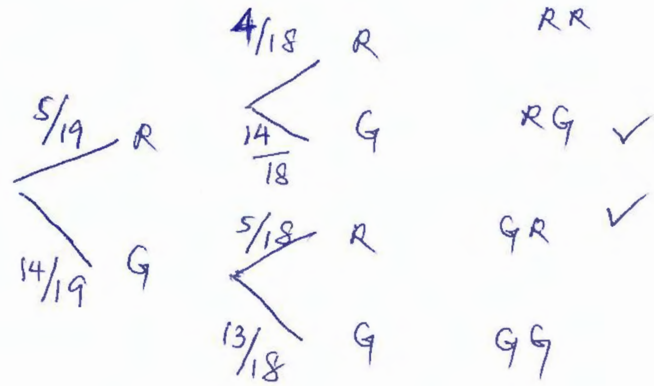
$\therefore t = \pm\sqrt{\frac{1}{3}}$ $t > 0$ —— (1).

$\therefore t = \frac{1}{\sqrt{3}}$ s.

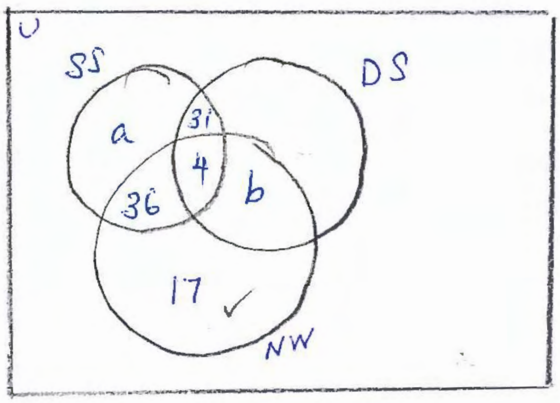
d) $P(\text{diff colours}) = \frac{5}{19} \times \frac{14}{18} + \frac{14}{19} \times \frac{5}{18} = \frac{70}{171}$

or $1 - \frac{5}{19} \times \frac{4}{18} - \frac{14}{19} \times \frac{13}{18} = \frac{70}{171}$

or .



e) i)



Given

$SS = 112$

$a = 112 - (31 + 4 + 36)$

$= 41$

$DS = 70$ (given)

$b = 70 - (11 + 31 + 4)$

$= 24$

P(none of the 3 options)

ii) $1 - \frac{164}{180} = \frac{16}{180} = \frac{4}{45}$

iii) $P(\text{networking only}) = \frac{17}{41+31+24+17} = \frac{17}{113}$ —— 1 —— 1

sample space

(NW or SS not both)

Page 1

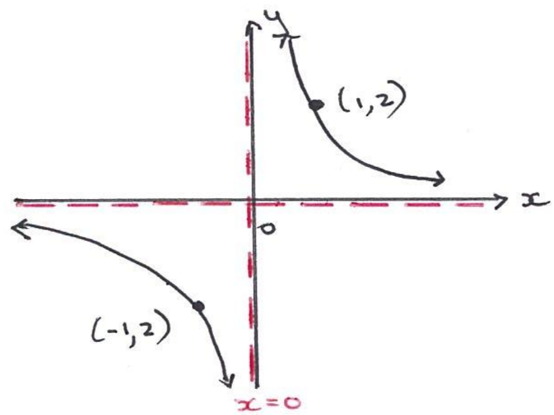
MATHEMATICS : Question 9

Suggested Solutions	Marks	Marker's Comments
(a) Prove $\sec^2 x + \sec x \tan x = \frac{1}{1-\sin x}$	3	
LHS $= \frac{1}{\cos^2 x} + \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$		→ 1 mark for simplifying $\sec^2 x$, $\sec x$ and $\tan x$.
$= \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$		
$= \frac{1+\sin x}{\cos^2 x}$		
$= \frac{1+\sin x}{1-\sin^2 x}$		
$= \frac{1+\sin x}{(1-\sin x)(1+\sin x)}$		→ 1 mark for $(1-\sin^2 x) = (1+\sin x)(1-\sin x)$
$= \frac{1}{1-\sin x}$		
$=$ RHS		→ 1 mark for working to prove LHS = RHS
Alternate Method		
replace $\sec^2 x$ with $(1+\tan^2 x)$		
or $\left(1+\frac{\sin^2 x}{\cos^2 x}\right)$		Note: A mark was not awarded if you did not show $(1-\sin^2 x)$ is equal to $(1+\sin x)(1-\sin x)$

MATHEMATICS : Question 9

Suggested Solutions	Marks	Marker's Comments
(b) given isotope of curium has half-life of 163 days and initially there is 10 mg	1	
(i) Value of A $M(t) = Ae^{-kt}$ $10 = Ae^{0}$ $\therefore A = 10$		1 mark for getting $A=10$
(ii) Value of k $M(t) = 10e^{-kt}$ $5 = 10e^{-163k}$ $\frac{1}{2} = e^{-163k}$	2	1 mark for $\frac{1}{2} = e^{-163k}$
$\ln\frac{1}{2} = -163k$ $\therefore k = \frac{\ln(\frac{1}{2})}{-163}$ or $k = \frac{\ln 2}{163}$ or $k = 0.0042524$.		1 mark for getting the value of k
c)(i) $f(x) = \log_e x$ $g(x) = x^2+1$ $h(x) = f(g(x))$ $= \log_e(x^2+1)$	1	1 mark for correct expression.
(ii) Show $h(x)+h(-x) = f((g(x)^2)$ $h(-x) = \log_e((-x)^2+1) = \log_e(x^2+1)$ $\therefore h(x)+h(-x) = \log_e(x^2+1)+\log_e(x^2+1)$ $= \log_e(x^2+1)^2$	2	1 mark for simplifying to $\log_e(x^2+1)^2$
since $g(x) = x^2+1$: $= \log_e(g(x)^2)$ since $f(x) = \log_e x$: $= f(g(x)^2)$		1 mark for replacing (x^2+1) as $g(x)$ and $\log_e(\ )$ to $f(x)$

MATHEMATICS : Question 9

Suggested Solutions	Marks	Marker's Comments
d (i) Sketch $y = \frac{2}{x}$ 	2	For 2 marks: ✓ Both horizontal and vertical asymptotes are shown ✓ Showing 2 points on the graph.
(ii) show equation of tangent is $p(py-2)=-2x+2p$	2	
$y' = \frac{-2}{x^2}$		
$\therefore$ m of tangent at p is $m = \frac{-2}{p^2}$		1 mark for differentiating and getting the gradient $\left(\frac{-2}{p^2}\right)$
If $p = x$ value then $y = \frac{2}{p}$		
Equation of tangent $y - y_1 = m(x - x_1)$ $y - \frac{2}{p} = \frac{-2}{p^2}(x-p)$ $p^2\left(y - \frac{2}{p}\right) = -2(x-p)$ $p(py-2) = -2x+2p$		1 mark for getting $y = \frac{2}{p}$ and formulating the equation. A mark was not awarded if the y value of $\left(\frac{2}{p}\right)$ was not shown.

MATHEMATICS : Question 9 page 4 .

Suggested Solutions	Marks	Marker's Comments
d(iii) Coordinates of point on graph when tangents intersect at $\left(\frac{8}{3}, \frac{2}{3}\right)$. Note: $\left(\frac{8}{3}, \frac{2}{3}\right)$ lies on $p(py-2) = -2x+2p$ $p\left(\frac{2p}{3}-2\right) = \frac{-16}{3}+2p$ $2p^2-6p = -16+6p$ $2p^2-12p+16=0$ $p^2-6p+8=0$ $(p-4)(p-2)=0$ $\therefore p = 4$ or 2. $\therefore$ points are $(2,1)$ and $\left(4,\frac{1}{2}\right)$	3	1 mark for substituting $x=\frac{8}{3}$ and $y=\frac{2}{3}$ 1 mark for correctly working to get to $p=4$ and $p=2$ 1 mark for the points.
e(i) Let eggs from farm A be P and $\therefore$ eggs from farm B is $(1-P)$ $\therefore P(AW)+P(BW) = \frac{3}{5}A + \frac{1}{5}B$ $= \frac{3}{5}P + \frac{1}{5}(1-P)$ $= \frac{3}{5}P + \frac{1}{P} - \frac{P}{5}$ $= \frac{2P}{5}+\frac{1}{5}$ or $\frac{2P+1}{5}$	1	1 mark for correct answer.

MATHEMATICS : Question 9	Page 5	
Suggested Solutions	**Marks**	**Marker's Comments**
e (ii). p (that it came from farm B) $\therefore P(B\mid W) = \dfrac{P(B\cap W)}{P(W)}$ $= \dfrac{\frac{1-p}{5}}{\frac{2p+1}{5}}$ (from farm 5) (answer from part (i)) $= \dfrac{1-p}{2p+1}$	2	1 mark for using correct formula $P(B\cap W)$ ie $\frac{1}{5}(1-P)$ 1 mark for correctly dividing by answer from part (1)
e(iii) what is value of p given probability of eggs from farm B is 0.3 $P(B\mid W) = \frac{3}{10}$ $\dfrac{1-p}{2p+1} = \dfrac{3}{10}$ $10 - 10p = 6p + 3$ $7 = 16p$ $\therefore p = \frac{7}{16}$ or 0.4375	1	1 mark for taking answer from part (ii) and putting that equal to $\frac{3}{10}$ and correctly working out the answer.