



KAMBALA

Student Number: _____

**Yearly Examination
September 2019**

Preliminary Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A Reference Sheet is provided for use in this task
- Answer questions 1 – 10 on the answer sheet provided
- Answer questions 11 through 26 on the paper provided
- For questions 11 through 26, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt Questions 11 through 26
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt questions 1-10.

Allow about 15 minutes for this section.

Use the answer sheet for Questions 1 – 10.

1. If $\sqrt{a} + \sqrt{a} = 16$, what is the value of a ?

(A) $2\sqrt{2}$

(B) 8

(C) 16

(D) 64

2. What is the simplest form of $\frac{x^6}{y^6} \div x^2y^{-3}$?

(A) $\frac{x^3}{y^2}$

(B) $\frac{x^4}{y^2}$

(C) $\frac{x^4}{y^3}$

(D) $\frac{x^4}{y^9}$

3. What are the coordinates of the vertex of the parabola with equation $y = x^2 + 4x + 5$?

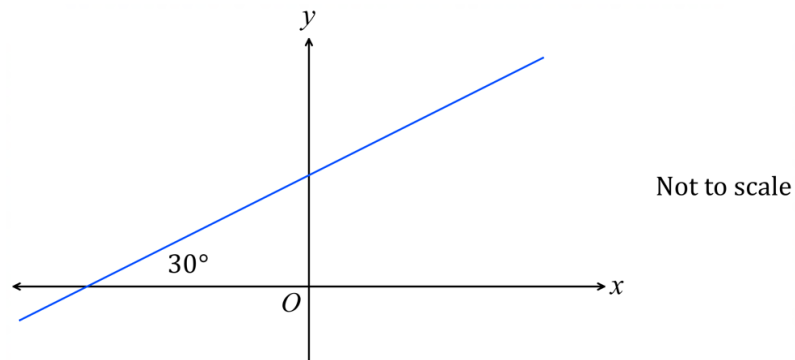
(A) $(2, -1)$

(B) $(2, 1)$

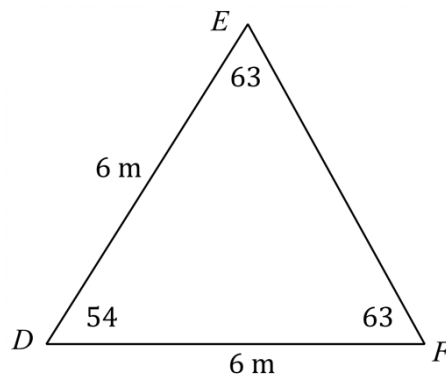
(C) $(-2, -1)$

(D) $(-2, 1)$

4. A line makes an angle of 30° with the positive direction of the x -axis. What is the gradient of the line?



- (A) $\frac{1}{\sqrt{3}}$
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{2}$
(D) $\frac{\sqrt{3}}{2}$
5. What is the area of $\triangle DEF$?



- (A) 11.71 m^2
(B) 13.33 m^2
(C) 14.56 m^2
(D) 16.04 m^2

6. Which of the following functions is not one-to-one?

(A) $f(x) = 4 - x^2, x \geq 0$

(B) $f(x) = \sqrt{4 - x^2}$

(C) $f(x) = 1 - 4x$

(D) $f(x) = \frac{4}{x}$

7. What is the equation of the asymptote of the graph with equation $y = 4 \log_2(5x) + 2$?

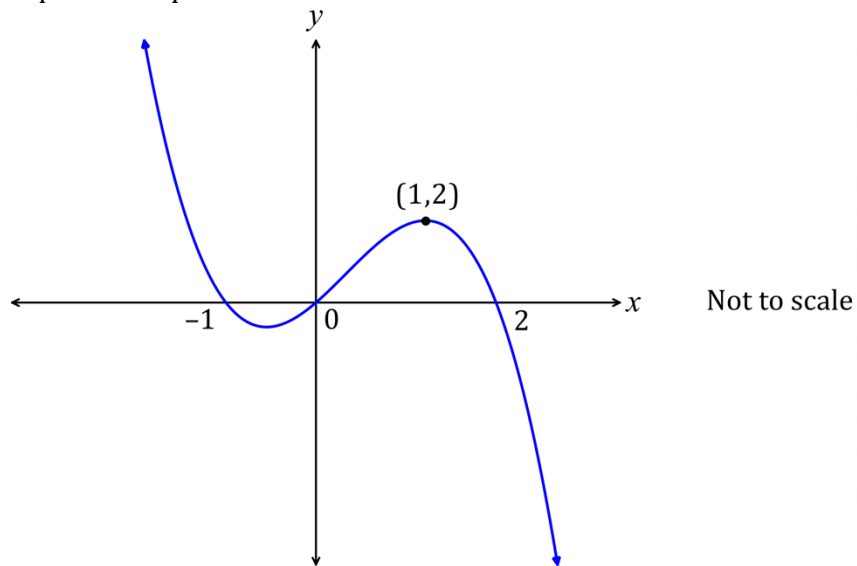
(A) $x = 0$

(B) $x = 2$

(C) $x = 4$

(D) $x = 5$

8. What is a possible equation of the cubic function below?



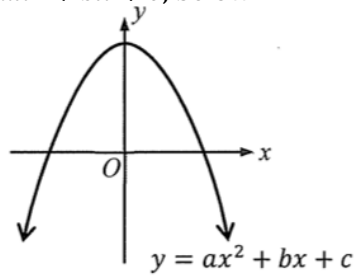
(A) $y = -x(x + 1)(x - 2)$

(B) $y = -x(x - 1)(x + 2)$

(C) $y = x(x + 1)(x - 2)$

(D) $y = x(x - 1)(x + 2)$

9. Consider the parabola $y = ax^2 + bx + c$, below.



Which of the following is true?

- (A) $b^2 - 4ac > 0$ and $a > 0$
 - (B) $b^2 - 4ac < 0$ and $a > 0$
 - (C) $b^2 - 4ac > 0$ and $a < 0$
 - (D) $b^2 - 4ac < 0$ and $a < 0$
10. What are the solutions of the equation $|x - 1| - 2 = 0$?
- (A) $x = -3$ or $x = 1$
 - (B) $x = 3$ or $x = -1$
 - (C) $x = -3$ or $x = -1$
 - (D) $x = 3$ or $x = 1$

Section II

60 marks

Attempt all questions.

Allow about 1 hour and 45 minutes for this section.

Answer the questions in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of the examination paper.

- 11.** Rationalise the denominator of $\frac{2}{\sqrt{3}+1}$. **2**

- 12.** Find the exact value of $\cos \theta$ if $\sin \theta = \frac{2}{5}$ and $90^\circ < \theta < 180^\circ$. **2**

- 13.** Differentiate the following expressions with respect to x .

- (a)** $(3x^2 - 5x)^5$ **2**

(b) $2x^2 + 5 - \frac{7}{x}$

2

[illegible]

(c) $\frac{4x^3 + 5}{2x - 9}$

3

[illegible]

14. Simplify:

(a) $\frac{1}{2}\log_x 16 + 2\log_x 5$

2

.....

(b) $\frac{x + 2y}{3} - \frac{2x - y}{4}$ 2

(c) $\frac{(2x^4)^3}{5x^{-1}}$ 2

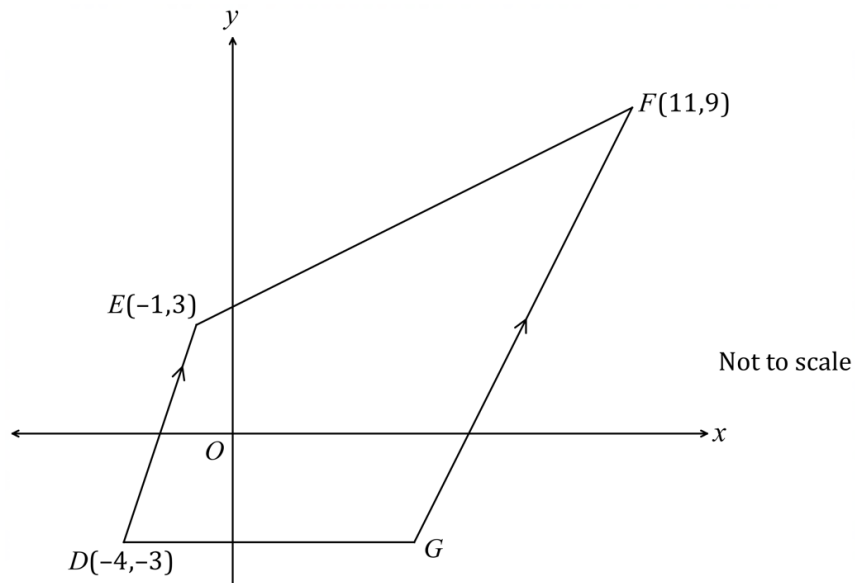
15. Given that $f(x) = x^2 - x$ and $g(x) = 2x + 1$, find:

(a) $f(-2)$ 1

(b) $f(g(x))$ 2

(c) x where $g(x) = 0$. 1

16.



On a number plane the points D , E and F have coordinates $(-4, -3)$, $(-1, 3)$ and $(11, 9)$ respectively. DE is parallel to GF and DG is parallel to the x -axis.

(a) What is the gradient of DE ? 1

(b) What is the midpoint of DE ? 1

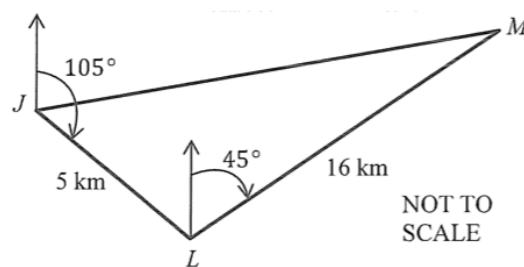
(c) Find the equation of the line FG . 2

- 18.** Solve the equation $2 \sin x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$.

2

100%

19. Mia swims from the point J on a bearing of 105° for 5 km to the point L . She then changes direction and swims on a bearing of 045° for a further 16 km to the point M .



- (a) Find $\angle JLM$.

1

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- (b) Find the distance JM that Mia will swim back to the point J . **2**

20. Solve the equation $\log_2 \frac{x+1}{x-1} = 2$ 2

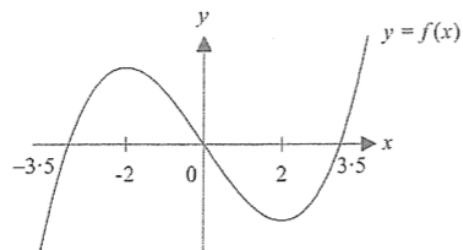
[illegible]

- 21.** Use the formula $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x)$ when $f(x) = x^2 + 3$. **2**

[illegible]

- 22.** Show that $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x} = 2\operatorname{cosec} x$.

3



2

Sketch the curve $y = f'(x)$ showing the shape of the curve and the x intercepts.

24. Sketch the curve $y = \sqrt{x - 2} + 3$ and state its domain and range. 4

25. A water tank is being filled at a variable rate. The height of the water, h cm, at any time, t minutes can be described by $h(t) = 2t^2 - 4t + 50$. Find:

- (a) The initial height of the water in the tank. 1

- (b) The instantaneous rate at which the height is changing at 4 minutes. 2

- (c) The average rate at which the height has changed over the first 4 minutes. **2**

[illegible]

- 26.** The number of bats, N , in the park at time t years, is given by the formula $N = 40 \times 3^{0.327t}$.

- (a) How many bats were in the park initially? 1

[illegible]

- (b) Make t the subject of the formula. 2

[illegible]

- (c) How many years would it take for the number of bats to reach 80? Answer to the nearest year. 2

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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 196. **Figure 188**
 197. **Figure 189**
 198. **Figure 190**
 199. **Figure 191**
 200. **Figure 192**
 201. **Figure 193**
 202. **Figure 194**
 203. **Figure 195**
 204. **Figure 196**
 205. **Figure 197**
 206. **Figure 198**
 207. **Figure 199**
 208. **Figure 200**
 209. **Figure 201**
 210. **Figure 202**
 211. **Figure 203**
 212. **Figure 204**
 213. **Figure 205**
 214. **Figure 206**
 215. **Figure 207**
 216. **Figure 208**
 217. **Figure 209**

Student Number: _____

Preliminary Mathematics Advanced

**Yearly Examination
September 2019**

Section I

Multiple-Choice Answer Sheet
Circle the answer of your choice.

- | | | | | |
|-----|---|---|---|---|
| 1. | A | B | C | D |
| 2. | A | B | C | D |
| 3. | A | B | C | D |
| 4. | A | B | C | D |
| 5. | A | B | C | D |
| 6. | A | B | C | D |
| 7. | A | B | C | D |
| 8. | A | B | C | D |
| 9. | A | B | C | D |
| 10. | A | B | C | D |



KAMBALA

Student Number: Solutions

**Yearly Examination
September 2019**

Preliminary Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A Reference Sheet is provided for use in this task
- Answer questions 1 – 10 on the answer sheet provided
- Answer questions 11 through 26 on the paper provided
- For questions 11 through 26, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt Questions 11 through 26
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt questions 1-10.

Allow about 15 minutes for this section.

Use the answer sheet for Questions 1 – 10.

1. If $\sqrt{a} + \sqrt{a} = 16$, what is the value of a ?

(A) $2\sqrt{2}$

(B) 8

(C) 16

(D) 64

$$2\sqrt{a} = 16$$

$$\sqrt{4a} = 16$$

$$4a = 256$$

$$a = 64$$

2. What is the simplest form of $\frac{x^6}{y^6} \div x^2y^{-3}$?

(A) $\frac{x^3}{y^2}$

(B) $\frac{x^4}{y^2}$

(C) $\frac{x^4}{y^3}$

(D) $\frac{x^4}{y^9}$

$$\frac{x^6}{y^6} \times \frac{y^3}{x^2} = \frac{x^4}{y^3}$$

3. What are the coordinates of the vertex of the parabola with equation $y = x^2 + 4x + 5$?

(A) (2, -1)

(B) (2, 1)

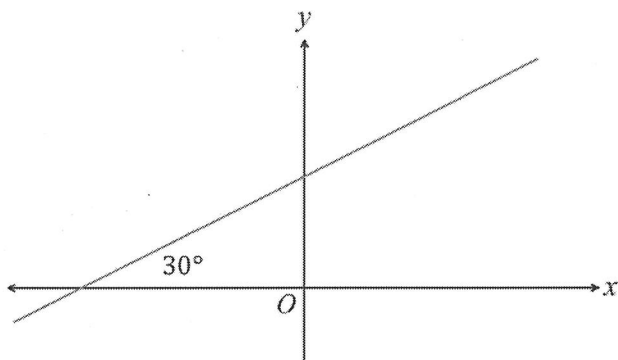
(C) (-2, -1)

(D) (-2, 1)

$$x = -\frac{4}{2} = -2$$

$$y = 4 - 8 + 5 = 1$$

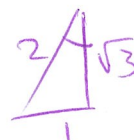
4. A line makes an angle of 30° with the positive direction of the x -axis. What is the gradient of the line?



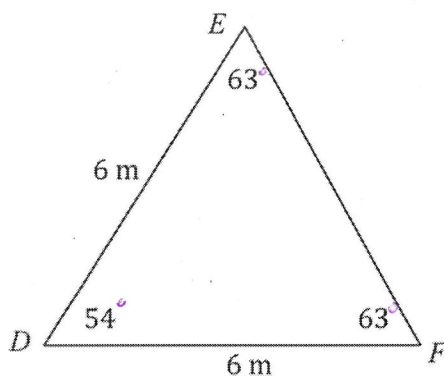
Not to scale

- (A) $\frac{1}{\sqrt{3}}$
 (B) $\frac{1}{\sqrt{2}}$
 (C) $\frac{1}{2}$
 (D) $\frac{\sqrt{3}}{2}$

$$\begin{aligned}\tan \theta &= m \\ m &= \tan 30^\circ \\ &= \frac{1}{\sqrt{3}}\end{aligned}$$



5. What is the area of $\triangle DEF$?



- (A) 11.71 m^2
 (B) 13.33 m^2
 (C) 14.56 m^2
 (D) 16.04 m^2

$$\begin{aligned}A &= \frac{1}{2} \times 6 \times 6 \times \sin 54^\circ \\ &= 14.56\end{aligned}$$

6. Which of the following functions is not one-to-one?

(A) $f(x) = 4 - x^2, x \geq 0$

(B) $f(x) = \sqrt{4 - x^2}$

(C) $f(x) = 1 - 4x$

(D) $f(x) = \frac{4}{x}$

7. What is the equation of the asymptote of the graph with equation $y = 4 \log_2(5x) + 2$?

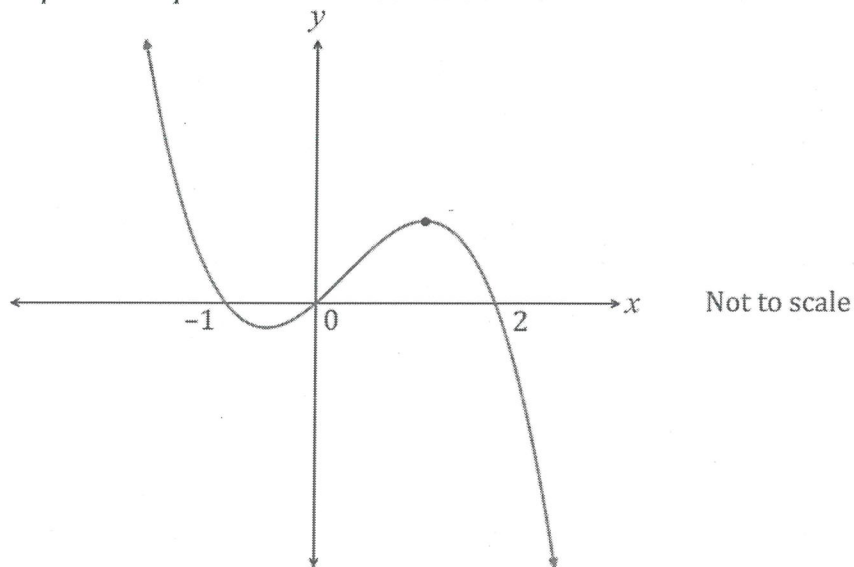
(A) $x = 0$

(B) $x = 2$

(C) $x = 4$

(D) $x = 5$

8. What is a possible equation of the cubic function below?



(A) $y = -x(x + 1)(x - 2)$

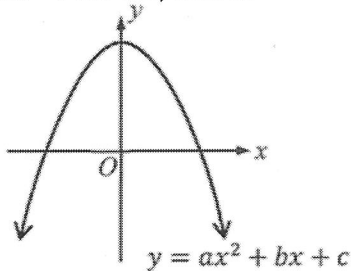
(B) $y = -x(x - 1)(x + 2)$

(C) $y = x(x + 1)(x - 2)$

(D) $y = x(x - 1)(x + 2)$

$-x(x+1)(x-2)$

9. Consider the parabola $y = ax^2 + bx + c$, below.



Which of the following is true?

- (A) $b^2 - 4ac > 0$ and $a > 0$
(B) $b^2 - 4ac < 0$ and $a > 0$
(C) $b^2 - 4ac > 0$ and $a < 0$
(D) $b^2 - 4ac < 0$ and $a < 0$

10. What are the solutions of the equation $|x - 1| - 2 = 0$?

- (A) $x = -3$ or $x = 1$
(B) $x = 3$ or $x = -1$
(C) $x = -3$ or $x = -1$
(D) $x = 3$ or $x = 1$

$$\begin{aligned} |x-1| &= 2 \\ x-1 &= 2 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} x-1 &= -2 \\ x &= -1 \end{aligned}$$

Section II

60 marks

Attempt all questions.

Allow about 1 hour and 45 minutes for this section.

Answer the questions in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

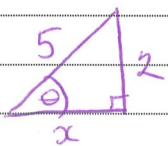
Extra writing space is provided at the back of the examination paper.

11. Rationalise the denominator of $\frac{2}{\sqrt{3}+1}$. 2

$$\frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{2\sqrt{3}-2}{2}$$

$$= \sqrt{3}-1$$

12. Find the exact value of $\cos \theta$ if $\sin \theta = \frac{2}{5}$ and $90^\circ < \theta < 180^\circ$. 2



$$\cos \theta = -\frac{\sqrt{21}}{5} \quad \text{as } 90^\circ < \theta < 180^\circ$$

$$x = \sqrt{25-4}$$

$$= \sqrt{21}$$

13. Differentiate the following expressions with respect to x .

- (a) $(3x^2 - 5x)^5$ 2

$$y' = 5(3x^2 - 5x)^4 (6x - 5)$$

(b) $2x^2 + 5 - \frac{7}{x} = 2x^2 + 5 - 7x^{-1}$ 2

$$y' = 4x + 7x^{-2} \quad \text{or} \quad y' = 4x + \frac{7}{x^2}$$

(c) $\frac{4x^3 + 5}{2x - 9}$ 3

$$u = 4x^3 + 5 \quad v = 2x - 9$$

$$u' = 12x^2 \quad v' = 2$$

$$y' = \frac{12x^2(2x-9) - 2(4x^3+5)}{(2x-9)^2} = \frac{24x^3 - 108x^2 - 8x^3 - 10}{(2x-9)^2}$$

$$= \frac{16x^3 - 108x^2 - 10}{(2x-9)^2}$$

14. Simplify:

(a) $\frac{1}{2} \log_x 16 + 2 \log_x 5$ 2

$$\log_x \sqrt{16} + \log_x 5^2 = \log_x 4 \times 25 = \log_x 100$$

(b) $\frac{x+2y}{3} - \frac{2x-y}{4}$ 2

$$= \frac{4(x+2y) - 3(2x-y)}{12}$$

$$= \frac{4x+8y-6x+3y}{12}$$

$$= \frac{-2x+11y}{12}$$

(c) $\frac{(2x^4)^3}{5x^{-1}}$ 2

$$\frac{8x^{12}}{5x^{-1}} = \frac{8}{5}x^{13}$$

15. Given that $f(x) = x^2 - x$ and $g(x) = 2x + 1$, find:

(a) $f(-2) = (-2)^2 - (-2)$ 1

$$= 4 + 2$$

$$= 6$$

(b) $f(g(x)) = f(2x+1)$ 2

$$= (2x+1)^2 - (2x+1)$$

$$= 4x^2 + 4x + 1 - 2x - 1$$

$$= 4x^2 + 2x$$

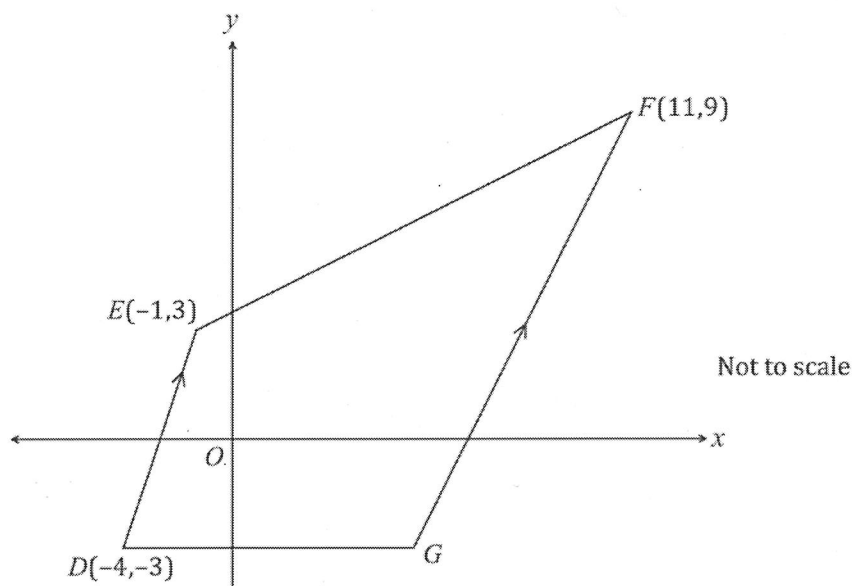
(c) x where $g(x) = 0$. 1

$$2x+1=0$$

$$2x=-1$$

$$x=-\frac{1}{2}$$

16.



On a number plane the points D , E and F have coordinates $(-4, -3)$, $(-1, 3)$ and $(11, 9)$ respectively. DE is parallel to GF and DG is parallel to the x -axis.

(a) What is the gradient of DE ?

1

$$m = \frac{3 - (-3)}{-1 - (-4)} = \frac{6}{3} = 2$$

(b) What is the midpoint of DE ?

1

$$x = \frac{-1 - 4}{2} = -\frac{5}{2} \quad y = \frac{3 - 3}{2} = 0$$

$$\left(-\frac{5}{2}, 0\right)$$

(c) Find the equation of the line FG .

2

$$y - y_1 = m(x - x_1)$$

$$y - 9 = 2(x - 11)$$

$$y - 9 = 2x - 22$$

$$y = 2x - 13$$

- (d) Show that the coordinates of the point
- G
- are
- $(5, -3)$
- .

2

 G is on the line FG

$$\therefore -3 = 2 \times 5 - 13$$

$$-3 = 10 - 13 \text{ correct } \therefore G(5, -3)$$

- (e) Find the distance
- EF
- .

1

$$\begin{aligned} EF &= \sqrt{(11+1)^2 + (9-3)^2} \\ &= \sqrt{121+36} \\ &= \sqrt{157} \quad \text{or } 12.5 \end{aligned}$$

- (f) Find the equation of a circle with the centre at
- F
- and the radius
- EF
- .

1

$$(x-11)^2 + (y-9)^2 = 157$$

17. Find the equation of the tangent to the curve
- $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$
- at the point
- $P(3, m)$
- .

3

$$x=3 \quad y = \frac{1}{3} \times 27 - 4 \times 9 + 8 \times 3 - 3 = 0$$

$$P(3, 0)$$

$$y' = \frac{1}{3} \times 3x^2 - 8x + 8$$

$$\text{when } x=3 \quad m_t = 9 - 8 \times 3 + 8 = -7$$

$$y = -7(x-3)$$

$$y = -7x + 21$$

18. Solve the equation $2 \sin x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$.

2

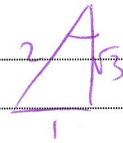
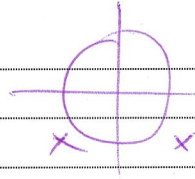
$$2 \sin x = -1$$

$$\sin x = -\frac{1}{2}$$

related angle = 30°

$$x = 180^\circ + 30^\circ, 360^\circ - 30^\circ$$

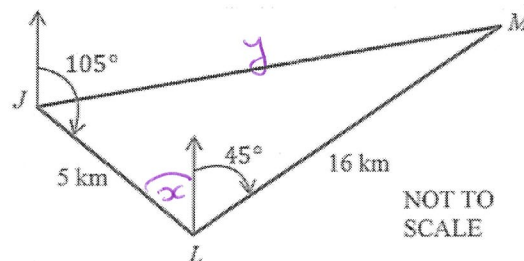
$$x = 210^\circ, 330^\circ$$



$$\sin x = \frac{1}{2}$$

$$x = 30^\circ$$

19. Mia swims from the point J on a bearing of 105° for 5 km to the point L . She then changes direction and swims on a bearing of 045° for a further 16 km to the point M .



- (a) Find $\angle JLM$.

1

$$x = 180^\circ - 105^\circ \text{ (co-interior angles on parallel lines)}$$

$$x = 75^\circ$$

$$\angle JLM = 75^\circ + 45^\circ$$

$$= 120^\circ$$

- (b) Find the distance
- JM
- that Mia will swim back to the point
- J
- .

2

$$y^2 = 5^2 + 16^2 - 2 \times 5 \times 16 \cos 120^\circ$$

$$y = \sqrt{25 + 256 - 160 \cos 120^\circ}$$

$$y = 19 \text{ km}$$

20. Solve the equation
- $\log_2 \frac{x+1}{x-1} = 2$

2

$$\frac{x+1}{x-1} = 4$$

$$x+1 = 4(x-1)$$

$$x+1 = 4x-4$$

$$3x = 5$$

$$x = \frac{5}{3}$$

21. Use the formula
- $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
- to find
- $f'(x)$
- when
- $f(x) = x^2 + 3$
- .

2

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 3 - x^2 - 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x + h)}{h}$$

$$= 2x$$

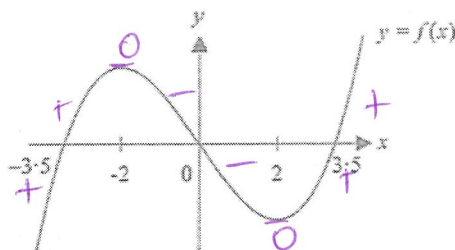
22. Show that $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x} = 2\operatorname{cosec} x$.

3

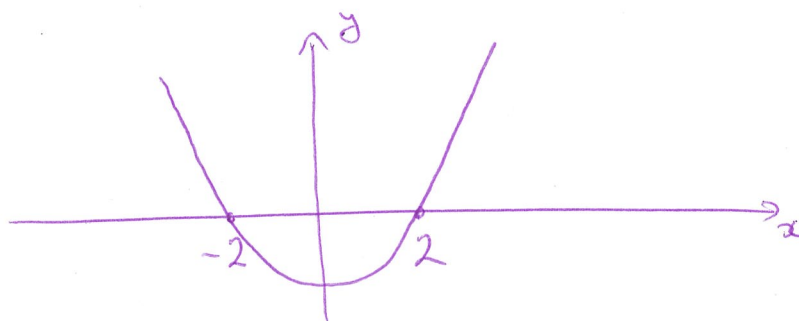
$$\begin{aligned}
 \text{LHS} &= \frac{(1+\cos x)^2 + \sin^2 x}{\sin x(1+\cos x)} \\
 &= \frac{1+2\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)} \\
 &= \frac{2+2\cos x}{\sin x(1+\cos x)} \\
 &= \frac{2(1+\cos x)}{\sin x(1+\cos x)} \\
 &= \frac{2}{\sin x} \\
 &= 2\operatorname{cosec} x \\
 &= \text{RHS}
 \end{aligned}$$

- 23.

2

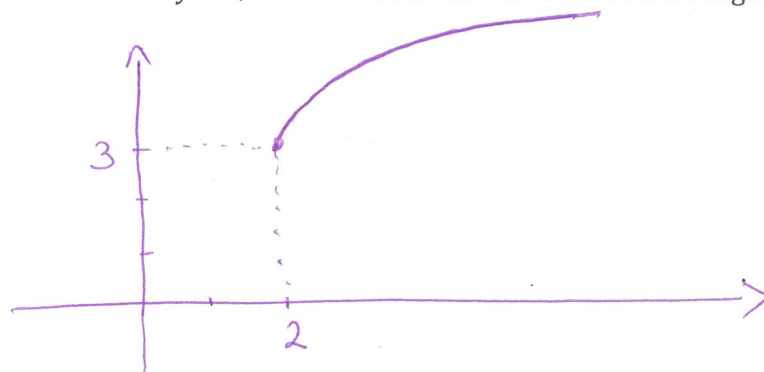


Sketch the curve $y = f'(x)$ showing the shape of the curve and the x intercepts.



24. Sketch the curve $y = \sqrt{x-2} + 3$ and state its domain and range.

4



$$x-2 \geq 0$$

$$x \geq 2$$

Domain: $x \geq 2$
Range: $y \geq 3$

25. A water tank is being filled at a variable rate. The height of the water, h cm, at any time, t minutes can be described by $h(t) = 2t^2 - 4t + 50$. Find:

- (a) The initial height of the water in the tank.

1

$$t=0 \quad h=50\text{cm}$$

- (b) The instantaneous rate at which the height is changing at 4 minutes.

2

$$\frac{dh}{dt} = 4t - 4$$

$$t=4 \quad \frac{dh}{dt} = 16 - 4 = 12\text{ cm/min}$$

- (c) The average rate at which the height has changed over the first 4 minutes.

2

$$t=4 \quad h = 2 \times 16 - 4 \times 4 + 50 \\ = 66 \text{ cm}$$

$$\text{average rate} = \frac{66-50}{4} \\ = 4 \text{ cm/min}$$

26. The number of bats, N , in the park at time t years, is given by the formula
 $N = 40 \times 3^{0.327t}$.

- (a) How many bats were in the park initially?

1

$$t=0 \\ N = 40 \times 3^0 \\ = 40 \text{ bats}$$

- (b) Make t the subject of the formula.

2

$$N = 40 \times 3^{0.327t}$$

$$3^{0.327t} = \frac{N}{40}$$

$$0.327t = \log_3 \frac{N}{40}$$

$$t = \frac{\log_3 \frac{N}{40}}{0.327}$$

- (c) How many years would it take for the number of bats to reach 80? Answer to the nearest year.

2

$$t = \frac{\log_3 \frac{80}{40}}{0.327}$$

$$= \log_3 2 \times \frac{1}{0.327}$$

$$= \frac{\log 2}{\log 3} \times \frac{1}{0.327}$$

$$= 1.93$$

$$\approx 2 \text{ years}$$

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Handwriting practice lines consisting of 20 horizontal dotted lines.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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