



KAMBALA

Student Number: _____

**Yearly Examination
September 2020**

Preliminary Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A Reference Sheet is provided for use in this task
- Answer questions 1 – 10 on the answer sheet provided
- Answer questions 11 through 26 on the paper provided
- For questions 11 through 26, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt Questions 11 through 26
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

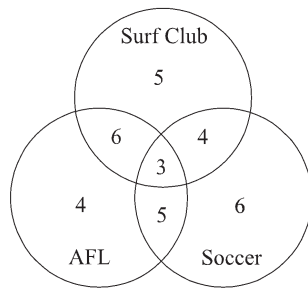
Attempt questions 1-10.

Allow about 15 minutes for this section.

Use the answer sheet for Questions 1 – 10.

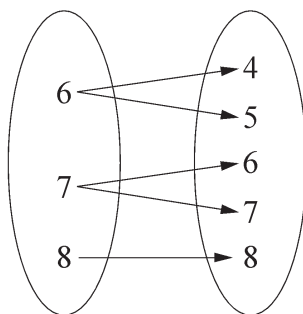
1. What is the value of $\sqrt{48} - \sqrt{12}$?
 - (A) 2
 - (B) $2\sqrt{3}$
 - (C) $2\sqrt{6}$
 - (D) 6
2. What are the x intercepts of the parabola $y = x^2 - 2x - 3$?
 - (A) -3 and -1
 - (B) -3 and 1
 - (C) -1 and 3
 - (D) 1 and 3
3. What are the solutions of the equation $|2x + 1| - 3 = 0$?
 - (A) $x = 2$ or $x = -1$
 - (B) $x = -2$ or $x = -1$
 - (C) $x = -2$ or $x = 1$
 - (D) $x = 2$ or $x = 1$
4. What is the perimeter of a sector of a circle of radius 6cm which contains a sector angle of $\frac{\pi}{6}$ radians at the centre of the circle?
 - (A) π cm
 - (B) 3π cm
 - (C) $(12 + \pi)$ cm
 - (D) $(12 + 3\pi)$ cm

5. In a classroom, students are asked what sports club they are members of and the results are shown in the Venn diagram.



A student who is a member of the soccer club is chosen at random. What is the probability that he/she is also a member of a surf club?

- (A) $\frac{2}{5}$
- (B) $\frac{4}{11}$
- (C) $\frac{2}{9}$
- (D) $\frac{7}{18}$
6. What type of relation is this?



- (A) Many-to-many
- (B) One-to-many
- (C) One-to-one
- (D) Many-to-one

7. If $y = \sqrt{2x + 1}$ what is the expression for $\frac{dy}{dx}$?

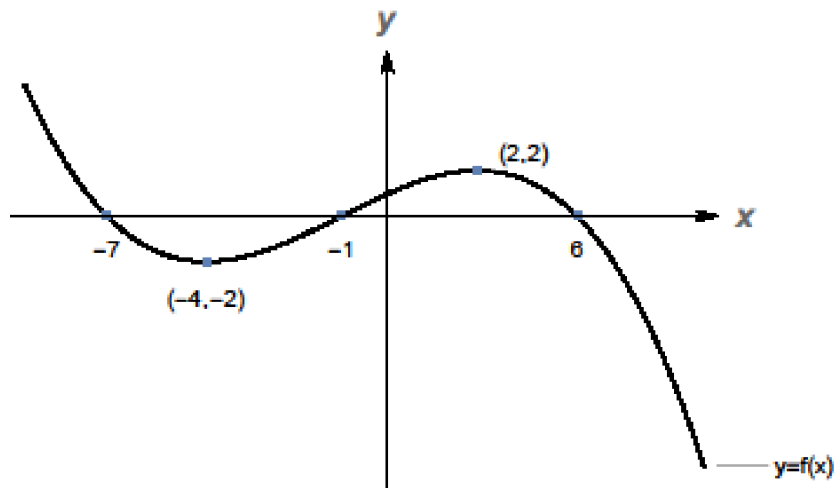
(A) $\frac{dy}{dx} = 2\sqrt{2x + 1}$

(B) $\frac{dy}{dx} = \frac{1}{2\sqrt{(2x+1)}}$

(C) $\frac{dy}{dx} = \frac{1}{\sqrt{(2x+1)}}$

(D) $\frac{dy}{dx} = \frac{2}{\sqrt{(2x+1)}}$

8.



For the graph $y = f(x)$ shown above, $f'(x)$ is negative when

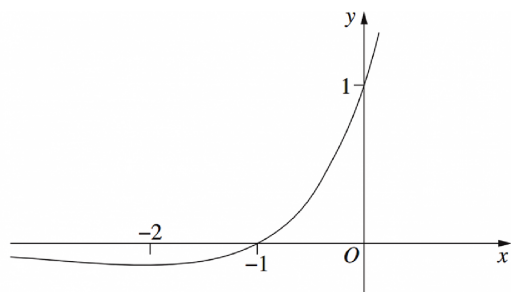
(A) $-7 < x < -1$ or $x > 6$

(B) $x < -4$ or $x > 2$

(C) $-4 < x < 2$

(D) $-2 < x < 2$

9. The diagram shows the graph of $y = e^x(1 + x)$ below.



How many solutions to $y = e^x(1 + x) = 1 - x^2$?

- (A) 0
 - (B) 1
 - (C) 2
 - (D) 3
10. If $a = e^x$ what is the correct expression for $\log_e \frac{1}{a^2}$?

- (A) $-x^2$
- (B) $-2x$
- (C) $2x$
- (D) x^2

Section II

60 marks

Attempt all questions.

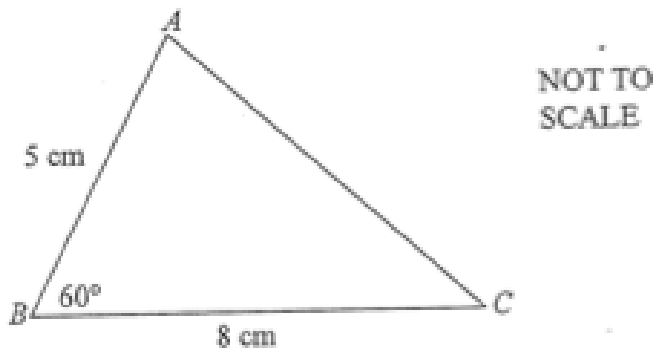
Allow about 1 hour and 45 minutes for this section.

Answer the questions in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of the examination paper.

Question 11 (3 marks)



ABC is a triangle in which $AB = 5\text{cm}$, $BC = 8\text{cm}$ and $\angle ABC = 60^\circ$

(a) Find the length of AC.

2

(b) Find in simplest exact form the area of $\triangle ABC$

1

Question 12 (2 marks)

(a) Solve for x

$$\frac{x-2}{4} + \frac{7-x}{6} = 1$$

2

.....

.....

.....

.....

.....

Question 13 (2 marks)

2 A(− 2, 1), B(2, − 1) and C (4, 3) are the vertices of a triangle. Show that $AB \perp BC$.

.....

.....

.....

.....

.....

.....

Question 14 (2 marks)

A fair die is rolled once. A is the event *the score is even* and B is the event *a multiple of 3*. Explain whether events A and B are independent. **2**

.....

.....

.....

.....

.....

.....

Question 15 (3 marks)

- (a) If $f(x) = 4 - x^2$ and $g(x) = \sqrt{x}$ find the composite function $f(g(x))$. **1**

- (b) Sketch the graph of this composite function showing the x and y intercepts. **2**

Question 16 (5 marks)

- (a) Differentiate **1**

$$x^4 + 2x^2 + 10$$

- (b) Differentiate **2**

$$e^x(x^2 + 1)$$

(c) Differentiate

2

$$\frac{4x^2}{1-x}$$

Question 17 (5 marks)

There are twelve chocolates in a box. Four of the chocolates have mint centres and eight have caramel centres. Ali randomly selects two chocolates and eats them.

(a) Draw a tree diagram

2

(b) What is the probability that the two chocolates have mint centres?

1

(c) What is the probability that the two chocolates have the different centres?

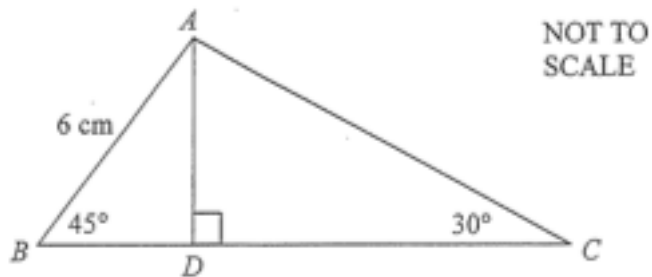
2

Question 18 (2 marks)

Sketch the graph of the function $f(x) = 1 - \log_e x$ showing the x intercept and the equation of the asymptote.

2

Question 19 (4 marks)



In the diagram, ABC is a triangle in which $\angle ABC = 45^\circ$, $\angle ACB = 30^\circ$ and $AB = 6 \text{ cm}$.
 ADS is the perpendicular from A to BC.

- (a) Find in simplest exact form the length of AD. 2

.....

.....

.....

.....

.....

- (b) Find in simplest exact form the length of DC. 2

.....

.....

.....

.....

.....

Question 20 (2 marks)

What is the domain of the function $f(x) = \frac{1}{x^2 - 4}$ 2

.....

.....

.....

.....

.....

Question 21 (2 marks)

Use the formula $S = V_0(1 - r)^n$ to find correct to 2 decimal places the time (n years) **2**
 it takes for an item with initial value $V_0 = \$2000$ to depreciate to the value $S = \$100$ at
 an annual rate of depreciation $r = 10\%$.

Question 22 (2 marks)

(a) Solve x for the domain $0 \leq x \leq 2\pi$ **2**

$$2\sin x + \sqrt{3} = 0$$

Question 23 (3 marks)

Find the coordinates of the point where the gradient of the tangent is zero, on the curve

$$y = (x - 1)(x - 2)^3$$
 3

Question 24 (2 marks)

Use the formula $f'(x) = \frac{f(x+h)-f(x)}{h}$ to find $f'(x)$ when $f(x) = x^2 - 2x$.

2

Question 25 (3 marks)

Show that $\frac{1}{1+\sin x} + \frac{1}{1-\sin x} = 2\sec^2 x$

3

.....

.....

.....

Question 26 (3 marks)

- (a) Show that the function $f(x) = |x| - 1$ is even. **1**

.....

.....

.....

- (b) Sketch the graph of the curve $f(x) = |x| - 1$ showing the intercepts on the axes. **2**

Question 27 (4 marks)

- (a) Find the equation of the tangent to the curve $y = \frac{8}{3x-1}$ at the point where $x = 1$. **3**

.....

.....

.....

.....

.....

.....

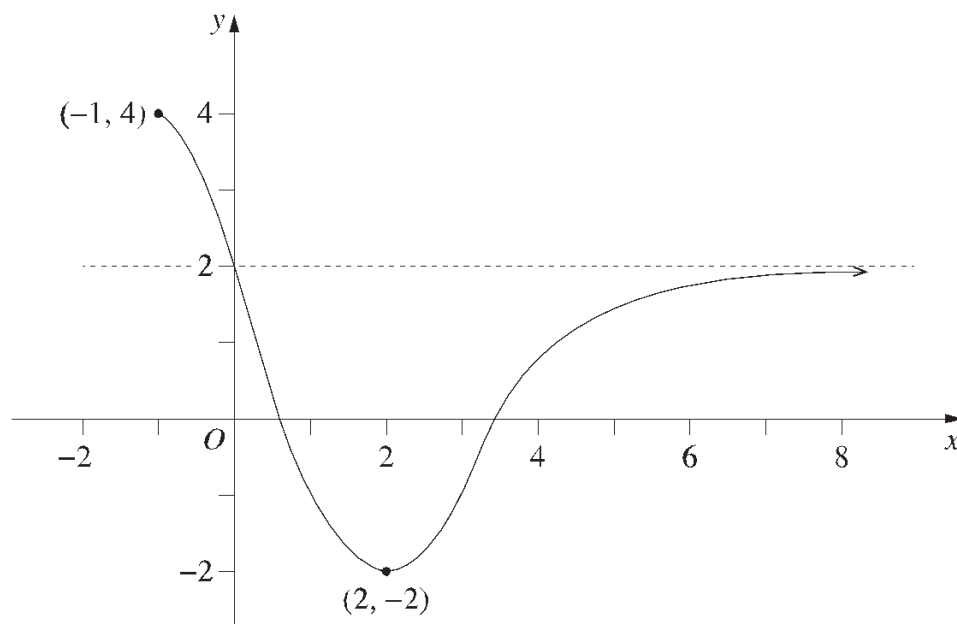
.....

(b) Find the angle of inclination of this tangent correct to the nearest degree. 1

Question 28 (2 marks)

2

The graph of a function $f(x)$ is shown. It has an asymptote at $y = 2$.



State the domain and range of $f(x)$

Question 29 (4 marks)

For a certain brand of medicine, the amount M present in the blood after t hours is given by

$$M = 3t^2 - t^3, \text{ for } 0 \leq t \leq 3$$

- (a) Find the average rate of change between $t = 0$ and $t = 3$ **2**

.....

.....

.....

.....

- (b) Find the instantaneous rate of change $\frac{dM}{dt}$ at $t = 1$ **2**

.....

.....

.....

.....

Question 30 (2 marks)

Solve for x $\log_2(x + 1) + \log_2(x - 1) = 3$.

2

.....

.....

.....

.....

.....

Question 31 (3 marks)

The energy, E Megajoules, radiated by an earthquake is given by the formula

$E = k \times 10^{1.5M}$ where M is the magnitude of the earthquake on the Richter scale

And k is a constant. An earthquake of magnitude 1 radiates 2 Megajoules of energy.

(a) Show that $\log_{10} E = \log_{10} 2 + 1.5(M - 1)$.

2

(b) Find the magnitude of an earthquake which radiates 2×10^6 Megajoules of energy.

1

.....

.....

.....

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[illegible]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Student Number: _____

Preliminary Mathematics Advanced

**Yearly Examination
September 2020**

Section I

Multiple-Choice Answer Sheet
Circle the answer of your choice.

- | | | | | |
|-----------|---|---|---|---|
| 1. | A | B | C | D |
| 2. | A | B | C | D |
| 3. | A | B | C | D |

- | | | | | |
|-----|---|---|---|---|
| 4. | A | B | C | D |
| 5. | A | B | C | D |
| 6. | A | B | C | D |
| 7. | A | B | C | D |
| 8. | A | B | C | D |
| 9. | A | B | C | D |
| 10. | A | B | C | D |



KAMBALA

AS - MC & Q 11
PS - Q12 - 18
KH - Q 19 = 25
SU - Q26 - 31

Student Number: _____

*Final with Solutions
7-9-20*

**Yearly Examination
September 2020**

Preliminary Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A Reference Sheet is provided for use in this task
- Answer questions 1 – 10 on the answer sheet provided
- Answer questions 11 through 26 on the paper provided
- For questions 11 through 26, show relevant mathematical reasoning and/or calculations

Total marks – ~~70~~ 70

Section I 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II 60 marks

- Attempt Questions 11 through 26
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt questions 1-10.

Allow about 15 minutes for this section.

Use the answer sheet for Questions 1 – 10.

1. What is the value of $\sqrt{48} - \sqrt{12}$?

(A) 2

(B) $\sqrt{12}$

(C) $\sqrt{24}$

(D) 6

$$4\sqrt{3} - 2\sqrt{3} = 2\sqrt{3} = \sqrt{12}$$

2. What are the x intercepts of the parabola $y = x^2 - 2x - 3$?

(A) -3 and -1

(B) -3 and 1

(C) -1 and 3

(D) 1 and 3

$$\begin{aligned} x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ x &= 3 \text{ and } -1 \end{aligned}$$

3. What are the solutions of the equation $|2x + 1| - 3 = 0$?

(A) $x = 2$ or $x = -1$

(B) $x = -2$ or $x = -1$

(C) $x = -2$ or $x = 1$

(D) $x = 2$ or $x = 1$

$$\begin{aligned} |2x+1| &= 3 \\ 2x+1 &= 3 \quad \text{or} \quad 2x+1 = -3 \\ 2x &= 2 \quad \quad \quad 2x = -4 \\ x &= 1 \quad \quad \quad x = -2 \end{aligned}$$

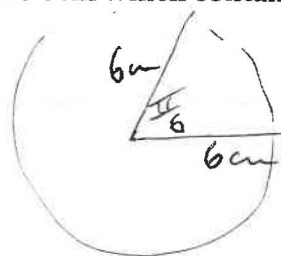
4. What is the perimeter of a sector of a circle of radius 6cm which contains a sector angle of $\frac{\pi}{6}$ radians at the centre of the circle?

(A) π cm

(B) 3π cm

(C) $(12 + \pi)$ cm

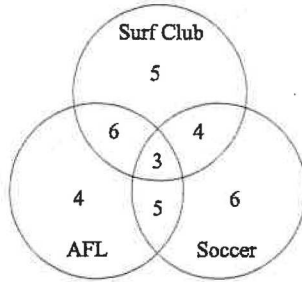
(D) $(12 + 3\pi)$ cm



$$l = r\theta = 6 \times \frac{\pi}{6} = \pi \text{ cm}$$

$$\begin{aligned} P &= 6 + 6 + \pi \\ &= 12 + \pi \end{aligned}$$

5. In a classroom, students are asked what sports club they are members of and the results are shown in the Venn diagram.



A student who is a member of the soccer club is chosen at random. What is the probability that he/she is also a member of a surf club?

(A) $\frac{2}{5}$

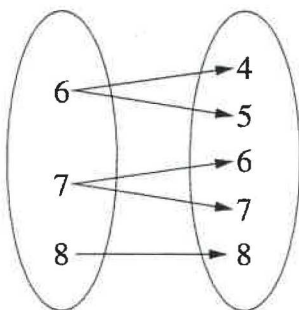
(B) $\frac{4}{11}$

(C) $\frac{2}{9}$

(D) $\frac{7}{18}$

$\frac{7}{18}$ $\frac{\text{surf \& soccer}}{\text{soccer}}$

6. What type of relation is this?



(A) Many-to-many

(B) One-to-many

(C) One-to-one

(D) Many-to-one

7. If $y = \sqrt{2x + 1}$ what is the expression for $\frac{dy}{dx}$?

(A) $\frac{dy}{dx} = 2\sqrt{2x + 1}$

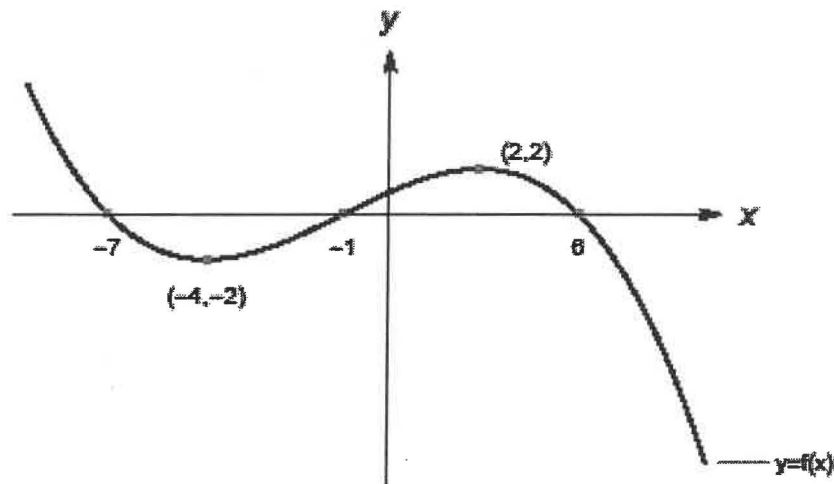
(B) $\frac{dy}{dx} = \frac{1}{2\sqrt{2x+1}}$

(C) $\frac{dy}{dx} = \frac{1}{\sqrt{2x+1}}$

(D) $\frac{dy}{dx} = \frac{2}{\sqrt{2x+1}}$

$$\frac{dy}{dx} = \frac{1}{2} \times 2(2x+1)^{-\frac{1}{2}}$$

8.



For the graph $y = f(x)$ shown above, $f'(x)$ is negative when

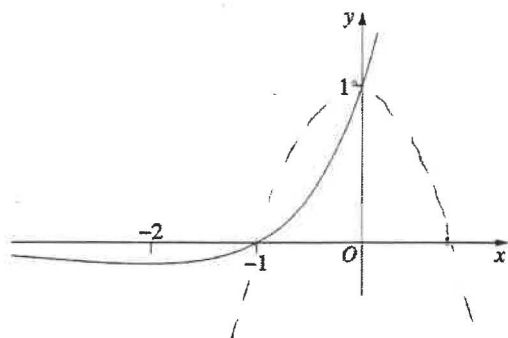
(A) $-7 < x < -1$ or $x > 6$

(B) $x < -4$ or $x > 2$

(C) $-4 < x < 2$

(D) $-2 < x < 2$

9. The diagram shows the graph of $y = e^x(1+x)$ below.



How many solutions to $y = e^x(1+x) = 1 - x^2$?

(A) 0

(B) 1

(C) 2

(D) 3

$$1 - x^2 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

10. If $a = e^x$ what is the correct expression for $\log_e \frac{1}{a^2}$?

(A) $-x^2$

(B) $-2x$

(C) $2x$

(D) x^2

$$\log_e \left(\frac{1}{(e^x)^2} \right) = \log_e e^{-2x} = -2x$$

Section II

60 marks

Attempt all questions.

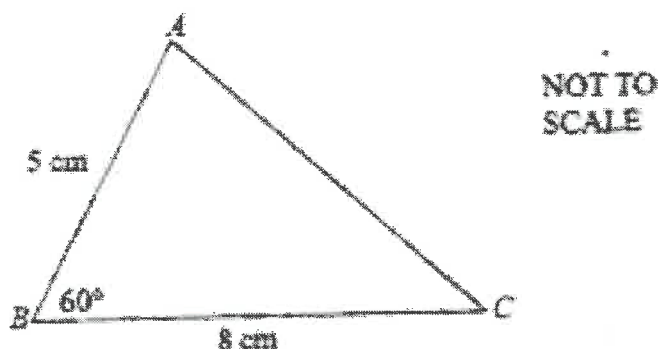
Allow about 1 hour and 45 minutes for this section.

Answer the questions in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of the examination paper.

Question 11 (3 marks)



ABC is a triangle in which $AB = 5\text{cm}$, $BC = 8\text{cm}$ and $\angle ABC = 60^\circ$

(a) Find the length of AC.

2

$$\begin{aligned} b^2 &= 5^2 + 8^2 - 2(5)(8)\cos 60^\circ \\ &= 25 + 64 - 80 \times \frac{1}{2} \\ &= 49 \\ b &= 7 \\ AC &= 7 \text{ cm} \end{aligned}$$

(b) Find in simplest exact form the area of $\triangle ABC$

1

$$\begin{aligned} \text{Area} &= \frac{1}{2}(5)(8)\sin 60^\circ \\ &= 20 \times \frac{\sqrt{3}}{2} \\ &= 10\sqrt{3} \text{ cm}^2 \end{aligned}$$

Question 12 (2 marks)

(a) Solve for x

$$\frac{x-2}{4} + \frac{7-x}{6} = 1$$

$$\begin{aligned} 3(x-2) + 2(7-x) &= 12 \\ 3x - 6 + 14 - 2x &= 12 \\ x + 8 &= 12 \\ x &= 4 \end{aligned}$$

Question 13 (2 marks)

$A(-2,1)$, $B(2,-1)$ and $C(4,3)$ are the vertices of a triangle. Show that $AB \perp BC$.

$$m_1 = \text{Gradient } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{2 - (-2)} = \frac{-2}{4} = -\frac{1}{2}$$

$$m_2 = \text{Gradient } BC = \frac{3 - (-1)}{4 - 2} = \frac{4}{2} = 2$$

$$m_1 m_2 = -\frac{1}{2} \times 2 = -1 \therefore AB \perp BC$$

$$\begin{aligned} P(\text{even}) &= \frac{1}{2} \\ P(\text{multiple of } 3) &= \frac{1}{3} \end{aligned}$$

Question 14 (2 marks)

A fair die is rolled once. A is the event the score is even and B is the event a multiple of 3. Explain whether events A and B are independent.

$$A = \{2, 4, 6\} \quad P(A) = \frac{3}{6} = \frac{1}{2}$$

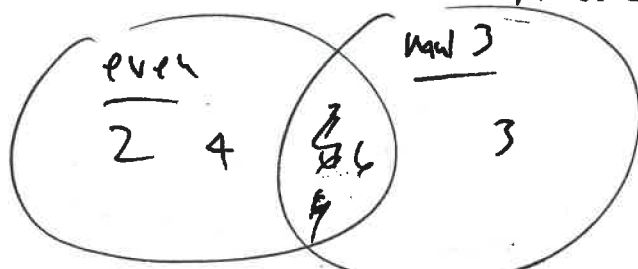
$$B = \{3, 6\} \quad P(B) = \frac{2}{6} = \frac{1}{3}$$

$$A \cap B = \{6\} \quad P(A \cap B) = \frac{1}{6}$$

$$P(A \cap B) = P(A) \times P(B)$$

$$\frac{1}{6} = \frac{1}{2} \times \frac{1}{3}$$

Yes A and B are independent events



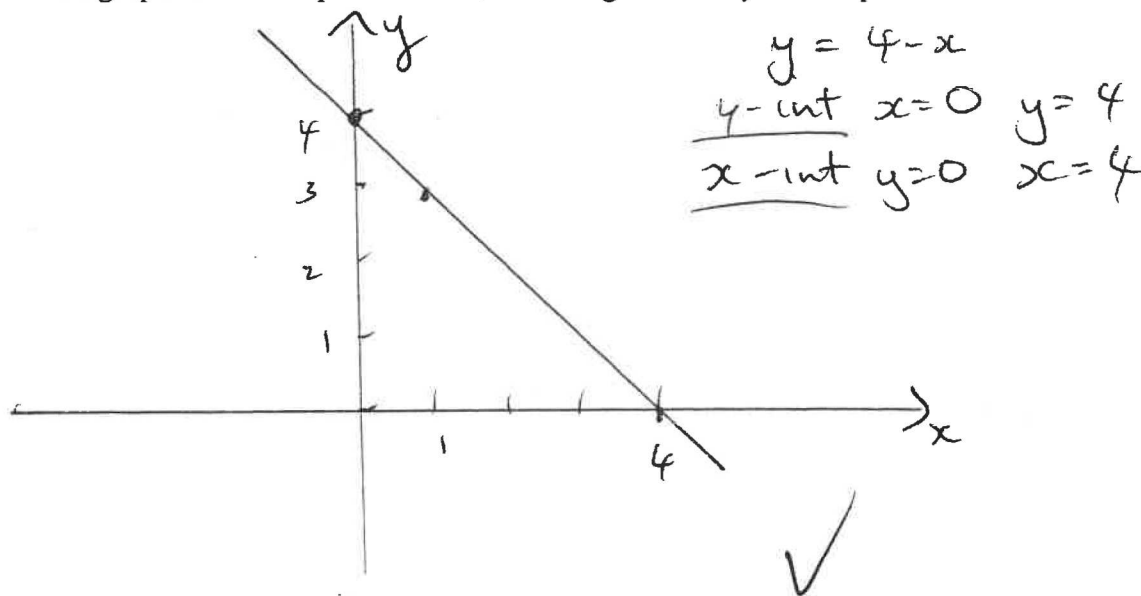
$$\begin{aligned} P(A \text{ even} | \text{multiple of } 3) &= \frac{1}{2} \\ P(\text{multiple of } 3 | \text{even}) &= \frac{1}{3} \end{aligned}$$

Question 15 (3 marks)

- (a) If
- $f(x) = 4 - x^2$
- and
- $g(x) = \sqrt{x}$
- find the composite function
- $f(g(x))$
- .
- 1

$$f(g(x)) = 4 - (\sqrt{x})^2 = 4 - x$$

- (b) Sketch the graph of this composite function showing the
- x
- and
- y
- intercepts.
- 2

**Question 16** (5 marks)

- (a) Differentiate
- 1

$$x^4 + 2x^2 + 10 \quad \frac{dy}{dx} = 4x^3 + 4x$$

- (b) Differentiate
- 2

$$e^x(x^2 + 1) \quad \frac{dy}{dx} = e^x \times 2x \times (x^2 + 1) + e^x(x^2 + 1)^2$$

$$= e^x(x^2 + 1)(4x + x^2 + 1)$$

$$e^x(x^2 + 1) \quad \frac{dy}{dx} = e^x \times 2x + e^x \times (x^2 + 1)$$

$$= 2xe^x + e^x(x^2 + 1)$$

(c) Differentiate

2

$$\frac{4x^2}{1-x} = \frac{u}{v} \quad \frac{dy}{dx} = \frac{(1-x) \times 8x - 4x^2(-1)}{(1-x)^2}$$

$$= \frac{8x - 8x^2 + 4x^2}{(1-x)^2}$$

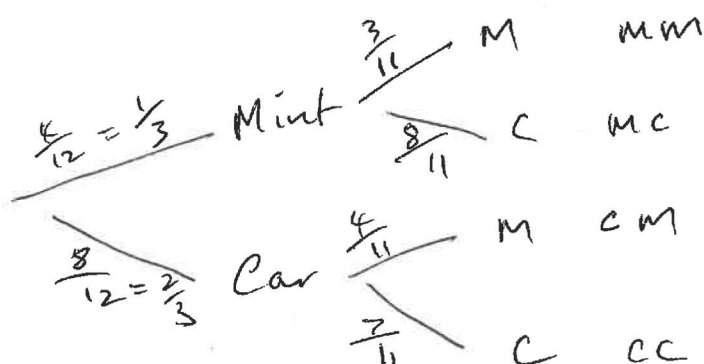
$$= \frac{8x - 4x^2}{(1-x)^2}$$

Question 17 (5 marks)

There are twelve chocolates in a box. Four of the chocolates have mint centres and ^{eight} ~~four~~ have caramel centres. Ali randomly selects two chocolates and eats them.

(a) Draw a tree diagram

2



(b) What is the probability that the two chocolates have mint centres?

1

$$P(MM) = \frac{1}{3} \times \frac{3}{11} = \frac{1}{11}$$

(c) What is the probability that the two chocolates have the different centres?

2

$$P(MC) + P(CM) = \frac{1}{3} \times \frac{8}{11} + \frac{2}{3} \times \frac{4}{11}$$

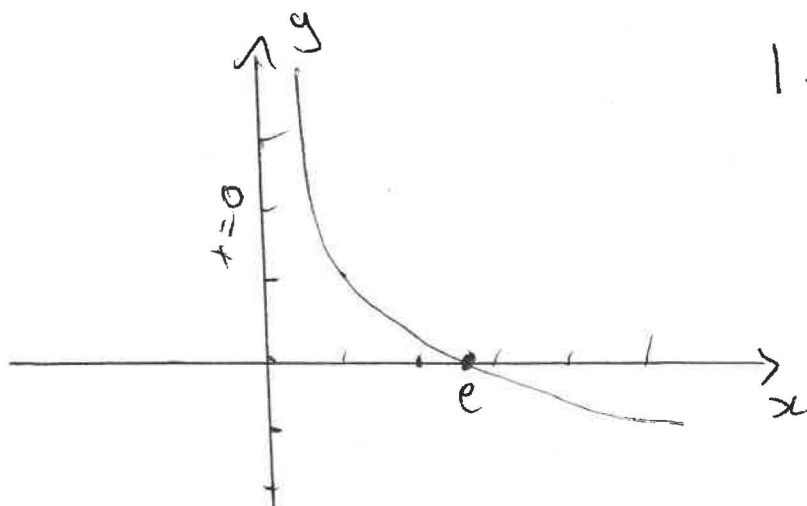
$$= \frac{8}{33} + \frac{8}{33}$$

$$= \frac{16}{33}$$

Question 18 (2 marks)

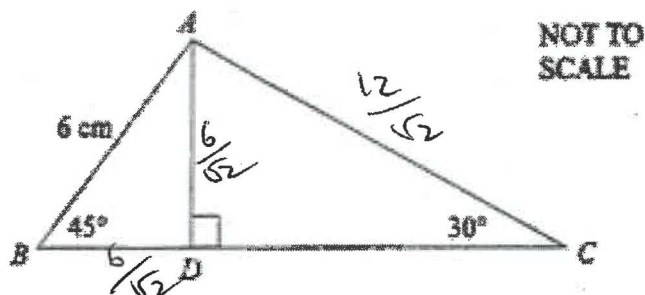
Sketch the graph of the function $f(x) = 1 - \log_e x$ showing the x intercept and the equation of the asymptote.

2



$$\begin{aligned} 1 - \log_e x &= 0 \\ \log_e x &= 1 \\ e^1 &= x \\ x &\approx 2.7 \dots \end{aligned}$$

Question 19 (4 marks)



In the diagram, ABC is a triangle in which $\angle ABC = 45^\circ$, $\angle ACB = 30^\circ$ and $AB = 6$ cm .
 ADS is the perpendicular from A to BC .

- (a) Find in simplest exact form the length of AD.

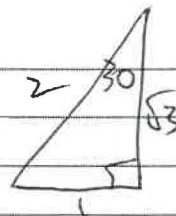
2

$$AD = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{2} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

- (b) Find in simplest exact form the length of DC.

2

$$DC = 3\sqrt{2} \times \sqrt{3} = 3\sqrt{6}$$



Question 20 (2 marks)

What is the domain of the function $f(x) = \frac{1}{x^2-4}$

2

All real except $x = \pm 2$

Question 21 (2 marks)

Use the formula $S = V_0(1 - r)^n$ to find correct to 2 decimal places the time (n years) it takes for an item with initial value $V_0 = \$2000$ to depreciate to the value $S = \$100$ at an annual rate of depreciation $r = 10\%$. 2

$$100 = 2000(1 - 0.1)^n$$

$$\frac{100}{2000} = 0.9^n$$

$$\log \frac{1}{20} = n \log 0.9$$

$$\frac{\log \frac{1}{20}}{\log 0.9} = n$$

$$n = 28.43 \text{ years}$$

Question 22 (2 marks)

(a) Solve x for the domain $0 \leq x \leq 2\pi$ 2

$$2 \sin x + \sqrt{3} = 0$$

$$2 \sin x = -\sqrt{3}$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$x = \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$= \frac{4\pi}{3}, \frac{5\pi}{3}$$

ref angle = $\frac{\pi}{3}$ or 60°

S	A
T	F
✓	✓

Question 23 (3 marks)

Find the coordinates of the point where the gradient of the tangent is zero, on the curve

$$y = (x - 1)(x - 2)^3$$

$$\frac{dy}{dx} = (x - 1) \times 3(x - 2)^2 + (x - 2)^3 \times 1$$

$$= (x - 2)^2 (3(x - 1) + (x - 2))$$

$$= (x - 2)^2 (3x - 3 + x - 2)$$

$$= (x - 2)^2 (4x - 5) = 0$$

$$4x - 5 = 0$$

$$x = 2 \text{ or } x = \frac{5}{4}$$

Question 24 (2 marks)

Use the formula $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x)$ when $f(x) = x^2 - 2x$.

2

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(2x + h - 2)}{h} \\
 &= \lim_{h \rightarrow 0} 2x + h - 2 \quad f'(x) = 2x - 2
 \end{aligned}$$

Question 25 (3 marks)

Show that $\frac{1}{1+\sin x} + \frac{1}{1-\sin x} = 2\sec^2 x$

3

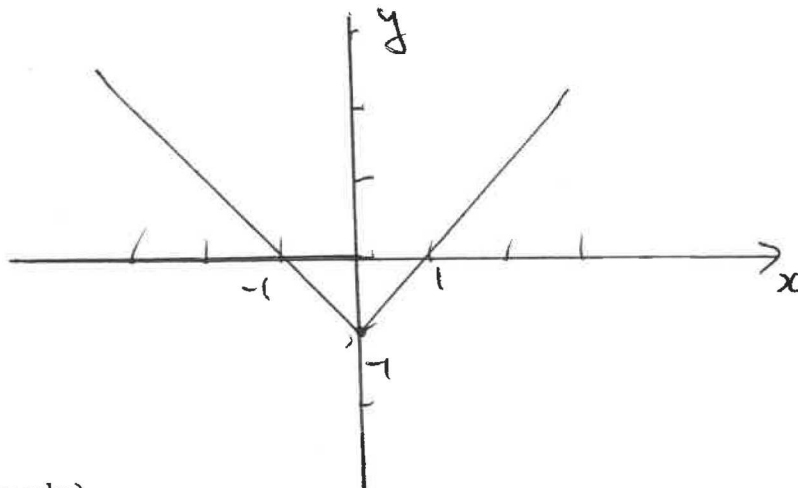
$$\begin{aligned}
 \text{LHS} &= \frac{1 - \sin x + 1 + \sin x}{(1 + \sin x)(1 - \sin x)} \\
 &= \frac{2}{1 - \sin^2 x} \\
 &= \frac{2}{\cos^2 x} \\
 &= 2 \sec^2 x \\
 &= \text{RHS}
 \end{aligned}$$

Question 26 (3 marks)

(a) Show that the function $f(x) = |x| - 1$ is even. 1

Even $f(-x) = f(x)$ $f(-x) = |-x| - 1$
 $= |x| - 1$
 $\therefore$ even function $= f(x)$

(b) Sketch the graph of the curve $f(x) = |x| - 1$ showing the intercepts on the axes. 2



Question 27 (4 marks)

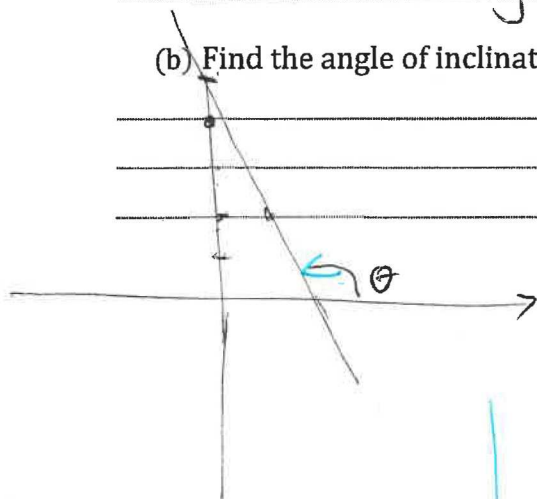
(a) Find the equation of the tangent to the curve $y = \frac{8}{3x-1}$ at the point where $x = 1$. 3

$y = 8(3x-1)^{-1}$
 $\frac{dy}{dx} = \frac{(3x-1) \times 0 - 8 \times 3(3x-1)^{-2}}{(3x-1)^2}$ $\frac{dy}{dx} = \frac{-24}{(3x-1)^2}$
 $= \frac{-24}{(3x-1)^2}$
 $f'(1) = \frac{-24}{(3 \times 1 - 1)^2} = \frac{-24}{4} = -6$

$x=1$ $y = \frac{8}{3-1} = \frac{8}{2} = 4$ $(1, 4)$ $y - 4 = -6(x - 1)$
 $y - 4 = -6x + 6$
 $y = -6x + 10$ $y = -6x + 10$

(b) Find the angle of inclination of this tangent correct to the nearest degree. 1

$\tan \theta = \frac{-6}{1} = -6$
 $\theta = 99.46^\circ$
 $\theta = 99^\circ$

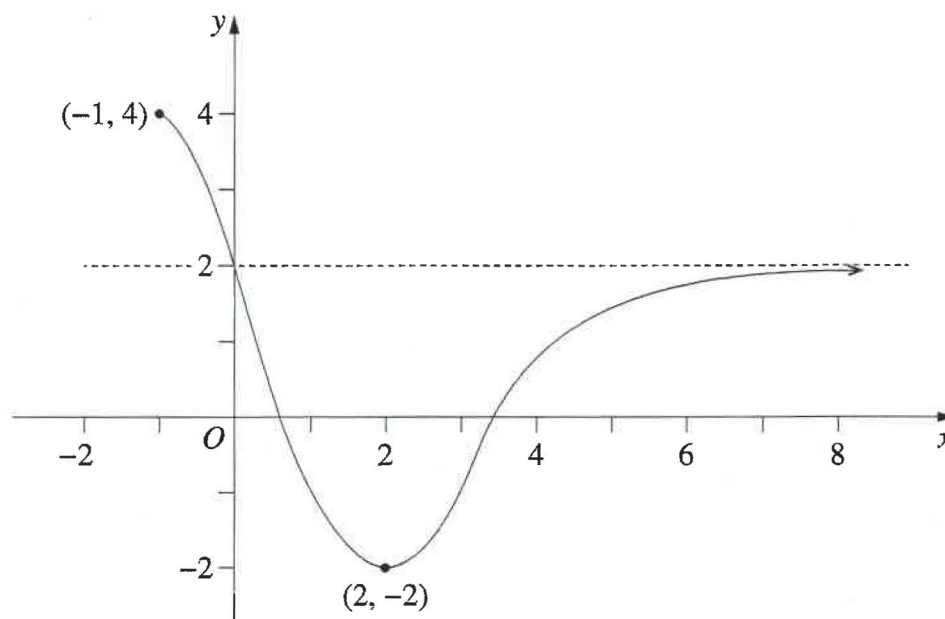


$\tan \theta = \frac{O}{A} = \frac{\text{rise}}{\text{run}} = \text{gradient}$

Question 28 (2 marks)

2

The graph of a function $f(x)$ is shown. It has an asymptote at $y = 2$.



State the domain and range of $f(x)$

$$D : x \geq -1$$

$$R : -2 \leq y \leq 4$$

Question 29 (4 marks)

For a certain brand of medicine, the amount M present in the blood after t hours is given by

$$M = 3t^2 - t^3, \text{ for } 0 \leq t \leq 3$$

- (a) Find the average rate of change between $t = 0$ and $t = 3$

$$f(3) = 27 - 27 = 0$$

$$f(0) = 0$$

$$\text{Average rate of change} = \frac{f(3) - f(0)}{3 - 0} = \frac{0 - 0}{3} = 0$$

- (b) Find the instantaneous rate of change $\frac{dM}{dt}$ at $t = 1$

2

$$\frac{dM}{dt} = 6t - 3t^2$$

$$f'(1) = 6 - 3 = 3$$

Question 30 (2 marks)

2

Solve for x $\log_2(x+1) + \log_2(x-1) = 3$.

$$\log_2(x^2 - 1) = 3$$

$$2^3 = x^2 - 1$$

$$8 + 1 = x^2$$

$$x^2 = 9 \quad x > 1$$

$$x = +3$$

45 min

Question 31 (3 marks)

The energy, E Megajoules, radiated by an earthquake is given by the formula $E = k \times 10^{1.5M}$ where M is the magnitude of the earthquake on the Richter scale. And k is a constant. An earthquake of magnitude 1 radiates 2 Megajoules of energy.

(a) Show that $\log_{10} E = \log_{10} 2 + 1.5(M - 1)$.

2

$$E = k \times 10^{1.5M}$$

$$M = 1 \quad E = 2$$

$$\frac{E}{k} = 10^{1.5M}$$

$$2 = k \times 10^{1.5}$$

$$\log_{10} \frac{E}{k} = 1.5M$$

$$k = \frac{2}{10^{1.5}} = 0.063$$

$$M = \frac{1}{1.5} \log_{10} \frac{E}{k}$$

$$M = \frac{1}{1.5} (\log_{10} E - \log_{10} k)$$

$$1.5M = \log_{10} E - \log_{10} \frac{2}{10^{1.5}}$$

$$= \log_{10} E - (\log_{10} 2 - \log_{10} 10^{1.5})$$

$$1.5M = \log_{10} E - \log_{10} 2 + \log_{10} 10^{1.5}$$

(b) Find the magnitude of an earthquake which radiates 2×10^6 Megajoules of energy.

1

$$\log_{10} E = \log_{10} 2 + 1.5M - 1.5$$

$$\log_{10} (2 \times 10^6) = \log_{10} 2 + 1.5M - 1.5$$

$$6 = 1.5M$$

$$M = 4$$

End of paper

$$\log_{10} E = \log_{10} 2 + 1.5M - 1.5$$

$$\log_{10} E = \log_{10} 2 + 1.5(M - 1)$$

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Student Number: _____

Preliminary Mathematics Advanced

Yearly Examination
September 2020

Section I

Multiple-Choice Answer Sheet
Circle the answer of your choice.

- | | | | | |
|-----|---|----------|----------|----------|
| 1. | A | <u>B</u> | C | D |
| 2. | A | B | <u>C</u> | D |
| 3. | A | B | <u>C</u> | D |
| 4. | A | B | <u>C</u> | D |
| 5. | A | B | C | <u>D</u> |
| 6. | A | <u>B</u> | C | D |
| 7. | A | B | <u>C</u> | D |
| 8. | A | <u>B</u> | C | D |
| 9. | A | B | <u>C</u> | D |
| 10. | A | <u>B</u> | C | D |

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There is no text or other markings on the paper.