Preliminary
Examination
August 2021

2021

Mathematics Advanced

(Remote Assessment)

General Instructions	•	Reading time – 10 minutes
	•	Working time – 2 hours
	•	Write using black pen
	•	NESA approved calculators may be used
	•	Show relevant mathematical reasoning and/or calculations
	•	Reference Sheet is provided via an alternate link in Sundial

Total marks: **70 marks**

- Booklet 1: Attempt Questions1 – 6
- Booklet 2: Attempt Questions 7 – 12
- Booklet 3: Attempt Questions 13 – 16
- Booklet 4: Attempt Questions 17 – 20

Year 11 Mathematics Advanced

Answer Booklet 1

17 marks
Attempt Questions 1 – 6

Instructions

- Write using a **black pen** only including for all diagrams.
- You will need **A4 loose leaf writing paper** to write your solutions with title "Booklet 1."
- Number all your pages.
- Responses should include relevant mathematical reasoning and/or calculations.

Question 1 (3 marks)

ΔABC has vertices $A(-3,1)$, $B(5,-3)$ and $C(4,5)$. Point D is the midpoint of AB.

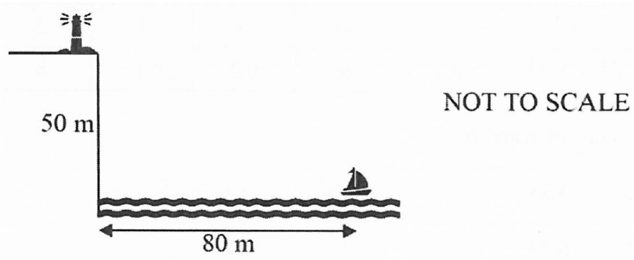
(a) Determine the coordinates of the midpoint D. **1**

(b) Show that $CD \perp AB$. **1**

(c) Find the equation of the line CD. **1**

Question 2 (2 marks)

A lighthouse sites on top of a 50 metre high cliff. A yacht is 80 metres out to sea below the lighthouse. What is the angle of depression of the yacht from the base of the lighthouse to the nearest degree? **2**



Question 3 (5 marks)

Find $f'(x)$:

(a) $f(x) = x^2 + \frac{3}{x}$ **1**

(b) $f(x) = 3e^{7x}$ **2**

(c) $f(x) = x^2\sqrt{2x-1}$ **2**

Question 4 (3 marks)

Determine the equation of the tangent to the curve $f(x) = \frac{x-1}{2x+3}$ at the point where $x = -1$. **3**

Question 5 (2 marks)

Sketch the function $y = (x-2)(x+3)^2$. Clearly label the x- and y-intercepts. **2**

Question 6 (2 marks)

Solve $|3x-4| = 2$ **2**

Year 11 Mathematics Advanced

Answer Booklet 2

18 marks
Attempt Questions 7 – 12

Instructions	• Write using a **black pen** only including for all diagrams. • You will need **A4 loose leaf writing paper** to write your solutions with title "Booklet 2." • Number all your pages. • Responses should include relevant mathematical reasoning and/or calculations.

Question 7 (5 marks)

Solve:

(a) $2^x = 30$, correct to two decimal places. **2**

(b) $\ln(2x + 8) = 2\ln x$ **3**

Question 8 (3 marks)

Given $f(x) = x^2 - 2$ and $g(x) = \sqrt{x}$

(a) Find $g\big(f(x)\big)$. **1**

(b) Determine if $g\big(f(x)\big)$ is odd, even or neither. Use appropriate working to justify your answer. **2**

Question 9 (4 marks)

Consider the function $f(x) = \sqrt{3 - 2x}$.

(a) State the domain of the function using interval notation. **2**

(b) Sketch the function, labelling *x*- and *y*-intercepts. **2**

Question 10 (2 marks)

A swimming pool is being dug by a team of labourers who remove *V* cubic metres of soil in *t* minutes, where $V = 10t - \frac{t^2}{20}$. What is the average rate at which soil is being removed in the first 5 minutes? **2**

Question 11 (2 marks)

Solve the equation $\log_2 x + \log_2 x^2 = 6$. **2**

Question 12 (2 marks)

Solve $\frac{81}{4\sqrt{3}-\sqrt{27}} = 3^x$. **2**

Year 11 Mathematics Advanced

Answer Booklet 3

18 marks
Attempt Questions 13 – 16

Instructions

- Write using a **black pen** only including for all diagrams.
- You will need **A4 loose leaf writing paper** to write your solutions with title "Booklet 3."
- Number all your pages.
- Responses should include relevant mathematical reasoning and/or calculations.

Question 13 (5 marks)

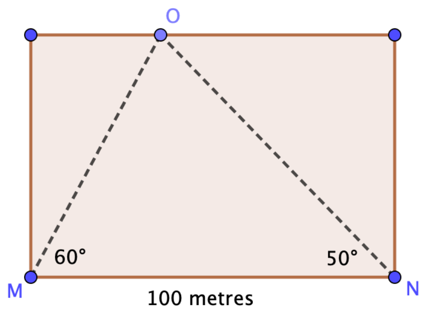
Solve:

(a) $2\cos\theta = -\sqrt{3}$ for $[0, 2\pi]$ **2**

(b) $\sin^2 x = \frac{1}{4}$ for $-180° \leq x \leq 180°$ **3**

Question 14 (4 marks)

Mel, Nina and Oren are positioned at points M, N, and O respectively on a rectangular playground as shown in the diagram below. The length of the playground is 100 metres, $\angle MNO = 50^{\circ}$ and $\angle NMO = 60^{\circ}$.



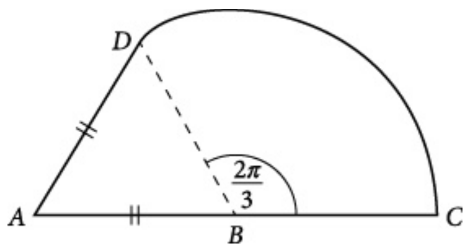
NOT TO SCALE

(a) Calculate the distance from Nina to Oren. Give your answer correct to the nearest metre. **2**

(b) Calculate the width of the playground. Give your answer correct to the nearest metre. **2**

Question 15 (4 marks)

In the diagram, $ABCD$ represents a garden. DBC is a sector of a circle with centre B and $\angle DBC = \frac{2\pi}{3}$. The points A, B and C are in a straight line and $AB = AD = 4$ metres.



(a) Show that $\angle DAB = \frac{\pi}{3}$. **1**

(b) Find the length of BC. **1**

(c) Calculate the area of the garden $ABCD$. Give your answer correct to one decimal place. **2**

Question 16 (5 marks)

(a) Prove that $\frac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta = \tan\theta$ **3**

(b) Hence, solve $\frac{\sin^3 2\theta}{\cos 2\theta} + \sin 2\theta\cos 2\theta = \frac{1}{4}$ for $0° \le \theta \le 180°$, to the nearest degree. **2**

Year 11 Mathematics Advanced

Answer Booklet 4

17 marks
Attempt Questions 17 – 20

Instructions

- Write using a **black pen** only including for all diagrams.
- You will need **A4 loose leaf writing paper** to write your solutions with title "Booklet 4."
- Number all your pages.
- Responses should include relevant mathematical reasoning and/or calculations.

Question 17 (4 marks)

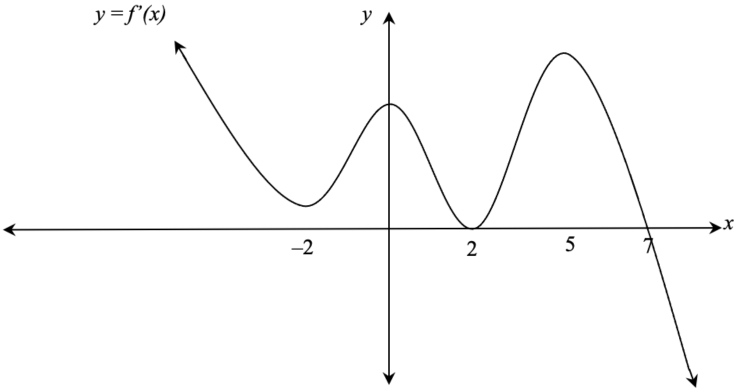
For what values of x do the graphs of $y = x^3 + x^2 + x - 2$ and $y = x^3$

(a) intersect? **2**

(b) have the same gradient? **2**

Question 18 (3 marks)

The diagram below is a sketch of $y = f'(x)$.



(a) Determine the x-values for which $y = f(x)$ is increasing. **1**

(b) Determine the x-values for which $y = f(x)$ is decreasing. **1**

(c) State the x-value(s) where $y = f(x)$ has a horizontal tangent line. **1**

Question 19 (6 marks)

A scientist is modelling the mouse plague of central west, New South Wales. The area affected by the mice is given by $A = 800e^{0.15n}$ hectares, where n is the number of weeks after the initial observation.

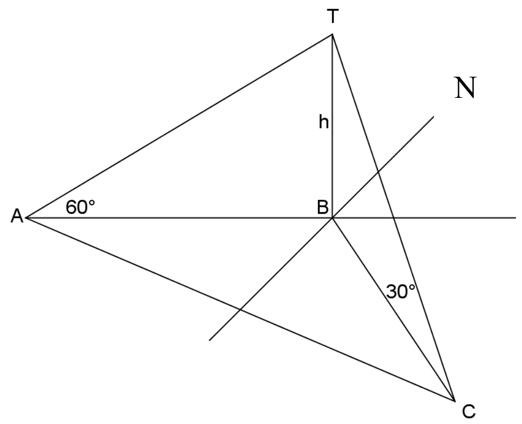
(a) Find the affected area after 7 weeks. Give your answer to the nearest hectare. **1**

(b) At what rate is the affected area increasing 7 weeks after the initial observation? Give your answer correct to the nearest whole number. **2**

(c) New South Wales is approximately 20 million hectares. Assuming the mouse plague continues to increase, how long after the initial observation will it take for the mice to infest all of NSW? **3**

Question 20 (4 marks)

Mount Kosciuszko, BT, is observed from point A, due west of B and the angle of elevation to the top of the mountain, T, is found to be $60°$. The observer then walks 5 km to point C, which is on bearing $150°$ from B and the angle of elevation is now observed to be $30°$.



(a) Show that $AB = \frac{h}{\sqrt{3}}$ and $CB = \sqrt{3}h$ **1**

(b) Determine the height of the mountain, h. Answer correct to the nearest metre. **3**

End of Examination

Marking Criteria in Red

Q1

(a) $A(-3,1)$ $B(5,-3)$ $C(4,5)$

midpoint D $(1,-1)$ (1 mark)

(b) $m_{CD} = 6/3 = 2$

$m_{AB} = -4/8 = -\frac{1}{2}$

$m_{CD} \times m_{AB} = 2 \times -\frac{1}{2} = -1$ (1 mark)

$\therefore CD \perp AB$

(c) line CD

$y - 5 = 2(x - 4)$ (1 mark)

$y = 2x - 3$ any form accepted

Q2

$\tan \theta = \frac{5}{8}$ (1 mark)

$\theta = 32^\circ$ (1 mark)

$\therefore$ angle of depression $= 32^\circ$

Q3

(a) $f(x) = x^2 + \frac{3}{x}$

$$f'(x) = 2x - \frac{3}{x^2}$$ (1 mark)

(b) $$f(x) = 3e^{7x}$$
$$f'(x) = 21e^{7x}$$

1: recognise $\frac{d}{dx}[e^x]$
1: chain rule

(c) $$f(x) = x^2\sqrt{2x-1}$$
$$f'(x) = 2x(2x-1)^{1/2} + x^2\left[\tfrac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2\right]$$
$$f'(x) = 2x\sqrt{2x-1} + \frac{x^2}{\sqrt{2x-1}}$$
$$= \frac{2x(2x-1) + x^2}{\sqrt{2x-1}}$$
$$= \frac{5x^2 - 2x}{\sqrt{2x-1}}$$
$$= \frac{x(5x-2)}{\sqrt{2x-1}}$$

1: recognise need for product rule
1: correct derivative

Q4

$$f(-1) = \frac{-1-1}{2(-1)+3} = -2 \qquad (-1,-2)$$

$$f'(x) = \frac{(2x+3)(1) - (x-1)(2)}{(2x+3)^2}$$

1: correct derivative

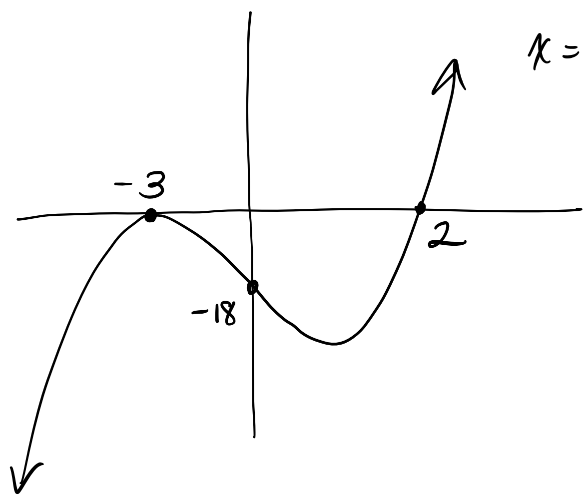
$$f'(-1) = \frac{1+4}{1} = 5 = m$$

1: gradient

$$y + 2 = 5(x + 1)$$
$$y = 5x + 3$$

1: equation of line

Q5



$x = 2, -3$

1: shape
1: intercepts

Q6

$$|3x - 4| = 2$$

$3x - 4 = 2$ or $3x - 4 = -2$

$x = 2$ $\qquad$ $x = 2/3$

1: only 1 correct solution
1: both solutions given/correct

Q7

(a) $\ln 2^x = \ln 30$

$$x \ln 2 = \ln 30$$

$$x = \frac{\ln 30}{\ln 2} = 4.91$$

1: use of logs to solve

1: correct solution

(b) $\ln(2x + 8) = 2 \ln x$

$e^{\ln(2x+8)} = e^{\ln x^2}$

1: make logical progress towards solution

$2x + 8 = x^2$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x = 4$ or $\cancel{-2}$

outside of domain

$\boxed{x = 4}$

1: correct solutions
1: eliminates $x = -2$ as outside domain

Q8

(a) $g(f(x)) = \sqrt{x^2 - 2}$ (1 mark)

(b)
$$g(f(-x)) = \sqrt{(-x)^2 - 2}$$
$$= \sqrt{x^2 - 2}$$
$$= g(f(x))$$

1: justification
1: answer

$\therefore$ $g(f(x))$ is even as $g(f(x)) = g(f(-x))$

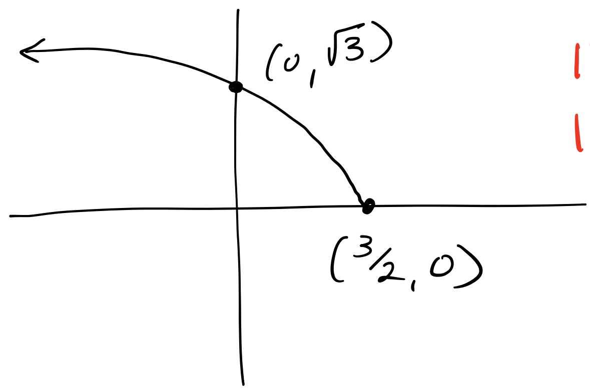
Q9

(a)
$$3 - 2x \geq 0$$
$$-2x \geq -3$$
$$x \leq 3/2$$

1: correct domain
1: correct use of

$x \in [-\infty, 3/2]$ interval notation

(b)



1: shape
1: intercepts

Q10

$V(0) = 0$

$V(5) = 10(5) - \frac{5^2}{20} = 48.75$

1: establishes y-values

$$\text{average rate} = \frac{48.75 - 0}{5 - 0}$$

$$= 9.75 \ m^3/min$$

1: correct solution

Q11

$\log_2 x + \log_2 x^2 = 6$

$\log_2 x^3 = 6$

$x^3 = 2^6$

$x = \sqrt[3]{2^6}$

1: application of log laws

$x = 4$ 1: solution

Q12

$$\frac{81}{4\sqrt{3} - 3\sqrt{3}} = 3^x$$

1: make progress toward solution

$$\frac{81}{\sqrt{3}} = 3^x$$

1: correct solution

$$\frac{3^4}{3^{1/2}} = 3^x$$

$$3^{7/2} = 3^x$$

$$\boxed{x = 7/2}$$

Q13

$[0, 2\pi]$

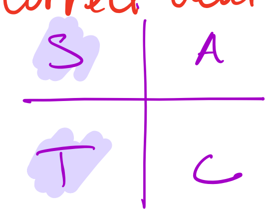
(a) $2\cos\theta = -\sqrt{3}$

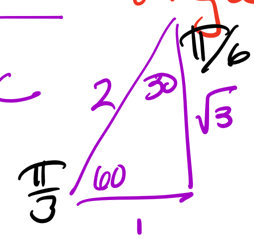
$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \pi - \frac{\pi}{6},\ \pi + \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6},\ \frac{7\pi}{6}$$

1: Quadrant and correct acute equivalent angle





1: correct solution

(b) $\sin^2 x = \frac{1}{4}$ $[-180, 180]$

$$\sin x = \pm\frac{1}{2}$$

1: solves for $\sin x$ including $\pm$

$$x = \pm 30,\ \pm 150$$

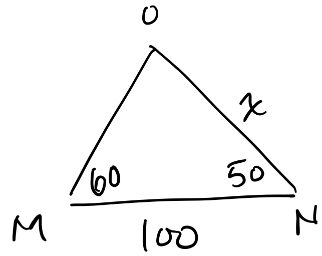
1: Quadrant, correct

Acute equivalent ∠

1: all solutions

Q14

(a)



$$\frac{x}{\sin 60} = \frac{100}{\sin 70}$$

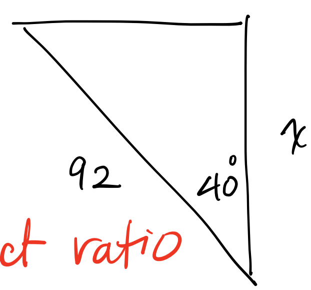
$$x = \frac{(\sin 60)(100)}{\sin 70}$$

$x = 92$ metres

1: uses sine rule

1: solution

(b)



$$\cos 40 = \frac{x}{92}$$

$92 \cos 40 = x$

70 metres $= x$

1: correct ratio

1: solution

Q15

(a) $\angle ABD = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

ΔABD is isosceles, $\therefore \angle BDA = \frac{\pi}{3}$

$\angle DAB = \pi - \pi/3 - \pi/3 = \pi/3$

as required. (1 mark)

(b) $BD = 4$ as ΔABD is equilateral

$BC = BD = 4$ radii of the circle.

(1 mark)

(c) Area $\Delta = \frac{1}{2}(4)(4)\sin\frac{\pi}{3}$

$= 8\left(\sqrt{3}/2\right)$ 1: areas

$= 4\sqrt{3}$

Area (sector) $= \frac{1}{2}(4)^2\left(\frac{2\pi}{3}\right)$

$= \frac{16\pi}{3}$

Area $= 4\sqrt{3} + \frac{16\pi}{3} = 23.7\ m^2$

1: total area

Q16

(a) LHS: $\frac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta$

1: common denominator $\frac{\sin^3\theta}{\cos\theta} + \frac{\sin\theta\cos^2\theta}{\cos\theta}$

$\frac{\sin^3\theta + \sin\theta\cos^2\theta}{\cos\theta}$

1: factors $\frac{\sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta}$

1: uses pythag identity

$\tan\theta$

LHS = RHS as required.

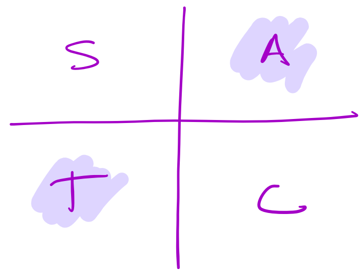
(b) by substitution from (a)

$$\tan 2\theta = \frac{1}{4} \qquad [0, 180]$$

$\times 2$

$$[0, 360]$$

$$2\theta = 14.03, \quad 180 + 14.03$$



$$\theta = 7^\circ, \ 97^\circ$$

1: Quadrant, correct equivalent ∠

1: solutions

Q17

(a) $x^3 + x^2 + x - 2 = x^3$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, \ 1$$

1: set 2 equations equal

1: solutions

(b) $y' = 3x^2 + 2x + 1, \quad y' = 3x^2$

$$3x^2 + 2x + 1 = 3x^2$$

$$2x + 1 = 0$$

$$x = -1/2$$

1: derivatives

1: solution

Q18

(a) $f(x)$ is increasing $x \in (-\infty, 2)(2, 7)$ (1 mark)

(b) $f(x)$ is decreasing $x \in (7, \infty)$ (1 mark)

(c) $f'(x) = 0$ @ (1 mark)
$x = 2$ or 7
$\therefore f(x)$ has a horizontal tangent @ $x = 2$ or 7.

Q19

(a) $A(7) = 800e^{(0.15\times 7)}$ (1 mark)
$= 2286$ hectares

(b) $A'(n) = 800 \times 0.15 e^{0.15n}$ 1: derivative
$= 120e^{0.15n}$
$A'(7) = 120e^{(0.15\times 7)}$ 1: Solution
$= 343$ hectares/ week

(c) $20{,}000{,}000 = 800e^{0.15n}$ 1: equation
$\ln 25000 = \ln e^{0.15n}$ 1: makes progress towards correct sol.
$\ln 25000 = 0.15n$
$n = \dfrac{\ln 25000}{0.15}$
$n = 67.51\ldots$ 1: Solution
between 67 and 68 weeks.

Q20

(a) $\tan 60 = \frac{h}{AB}$ $\quad$ $\tan 30 = \frac{h}{CB}$

$AB = \frac{h}{\tan 60}$ $\quad$ $CB = \frac{h}{\tan 30}$

$AB = \frac{h}{\sqrt{3}}$ $\quad$ $CB = \sqrt{3}\, h$

1: mark

(b) $5^2 = AB^2 + CB^2 - 2(AB)(CB)\cos 120°$

$25 = \left(\frac{h}{\sqrt{3}}\right)^2 + (\sqrt{3}\, h)^2 - 2\left(\frac{h}{\sqrt{3}}\right)\left(\sqrt{3}\, h\right)\cos 120$

$25 = \frac{1}{3}h^2 + 3h^2 - 2h^2 \cos 120$

$h^2 = \frac{25}{10/3 - 2\cos 120}$ $\quad$ 1: correct angle

$h = \sqrt{\frac{25}{10/3 - 2\cos 120}}$ $\quad$ 1: use of cosine

$h = 2.4019\ldots$ km $\quad$ 1: correct solution.

$h = 2402$ metres