Name: ______________________________

2023

YEAR 11
ASSESSMENT TASK 3

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black or blue pen
- Diagrams are not to scale
- Calculators may be used
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 64

Section I – 10 marks (page 2-4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 54 marks (pages 5–20)

- Attempt Questions 11–13
- Allow about 1 hour 45 minutes for this section

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Section I: Multiple Choice Questions **10 marks**

Circle the correct answer.

1. What is the derivative of $\dfrac{2}{x^3}$?

   A. $-\dfrac{3}{x^4}$

   B. $\dfrac{6}{x^4}$

   C. $\dfrac{3}{x^4}$

   D. $-\dfrac{6}{x^4}$

2. What is the value of $\tan x$ given that $\sin x = \dfrac{2}{5}$ for $\dfrac{\pi}{2} \leq x \leq \pi$

   A. $-\dfrac{\sqrt{21}}{2}$

   B. $-\dfrac{5}{\sqrt{21}}$

   C. $\dfrac{2}{\sqrt{21}}$

   D. $-\dfrac{2}{\sqrt{21}}$

3. Which of the following is the equation of the line that passes through $(2, -1)$ and $(5, 3)$?

   A. $4x + 3y - 11 = 0$

   B. $3x - 4y + 11 = 0$

   C. $4x - 3y - 11 = 0$

   D. $3x - 4y - 11 = 0$

4. Which of the following is equivalent to $7x^2 - 56x + 105$?

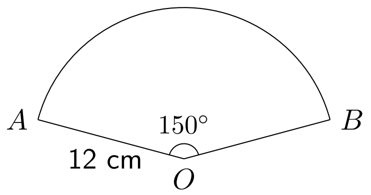
   A. $(x - 8)(x + 7)$

   B. $7(x - 3)(x - 5)$

   C. $(x - 3)(x - 5)$

   D. $7(x - 8)(x + 7)$

5. What is the length of arc AB in the diagram below?



   A. 10π cm

   B. 5 cm

   C. 5π cm

   D. 10 cm

6. The domain for the function $f(x) = \sqrt{2-x}$ is:
   - A. $[0, \infty)$
   - B. $(-\infty, 2]$
   - C. $[2, \infty)$
   - D. $(-\infty, \infty)$

7. Solve $\tan x = 1$ for $0 \leq x \leq 2\pi$.
   - A. $x = \frac{\pi}{4}, \frac{5\pi}{4}$
   - B. $x = \frac{\pi}{4}$
   - C. $x = \frac{\pi}{4}, \frac{3\pi}{4}$
   - D. $x = \frac{\pi}{4}, \frac{7\pi}{4}$

8. Megan tries to differentiate the function $y = \dfrac{3x}{(10-x)^2}$ with respect to x.

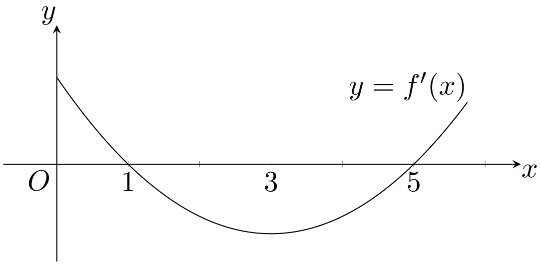
   Her working out is shown below.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{3(10-x)^2 + 3x \times 2(10-x)}{(10-x)^4} && \text{Line (1)} \\
&= \frac{3(10-x)^2 - 6x(10-x)}{(10-x)^4} && \text{Line (2)} \\
&= \frac{(10-x)[3(10-x)+6x]}{(10-x)^4} && \text{Line (3)} \\
&= \frac{30+3x}{(10-x)^3} && \text{Line (4)} \\
&= \frac{10+x}{(10-x)^3} && \text{Line (5)}
\end{aligned}$$

   Where did Megan make mistakes?
   - A. Line 1 & Line 5
   - B. Line 3 & Line 4
   - C. Line 1 & Line 3
   - D. Line 3 & Line 5

9. The height of a ball thrown in the air is given by the formula $h = 10t - 3t^2$ in metres, where t is measured in seconds. When the ball is thrown, what is the average rate of change of the height in the first second?
   - A. 4 metres per second
   - B. 7 metres
   - C. $10 - 6t$
   - D. 7 metres per second

10. The diagram below shows the graph of the gradient function of a curve.



For what value(s) of x does $f(x)$ have a local minimum?

A. $x = 1$

B. $x = 3$

C. $x = 1, x = 5$

D. $x = 5$

End of Section I

Section II

54 marks

Answer the questions in the spaces provided.
Show all necessary working.

Question 11 (1 marks) **1**

Show that $f(x) = x^3 + 3x$ is an odd function.

Question 12 (2 marks) **2**

What is the centre and radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$?

Question 13 (2 marks) **2**

Express $\dfrac{4}{4-\sqrt{13}}$ in the form $a + \sqrt{b}$, where a and b are integers.

Question 14 (7 marks)

Differentiate the following with respect to x.

(a) $y = 3x^5 - 2x^3 + 7$ **1**

(b) $y = (x^2 - 2)^3(x + 3)^2$ **2**

(c) $y = \sqrt[4]{4x + 3}$ **2**

(d) $y = \dfrac{2x}{5 - x}$ **2**

Question 15 (3 marks) **3**

Find the equation of the normal to the curve $y = 2x^2 - 5x + 1$ at the point $(2, -1)$

Question 16 (2 marks) **2**

Differentiate $f(x) = 2x + 3$ from first principles to show that $f'(x) = 2$.

Hint: Use the formula $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

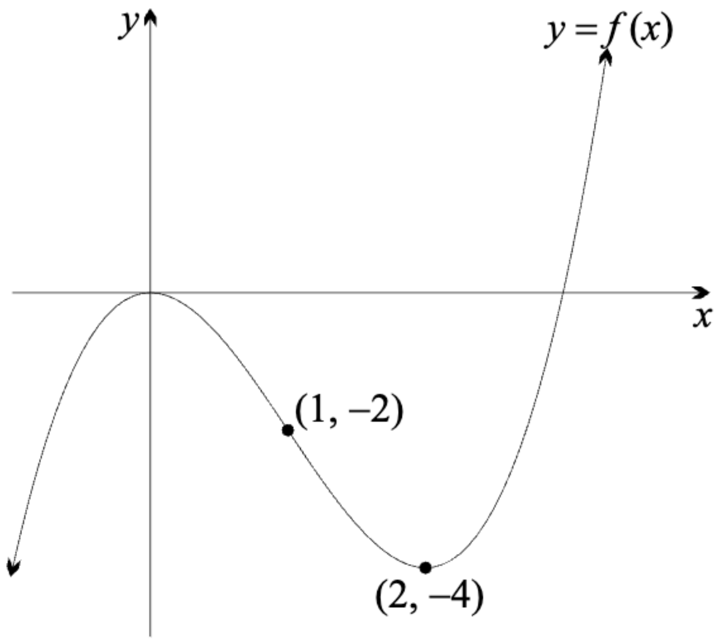
Question 17 (2 marks) **2**

Let $f(x) = 2x$ and $g(x) = \dfrac{x}{x+1}$. Solve $f\left(g\left(x\right)\right) = 3$.

Question 18 (2 marks) **2**

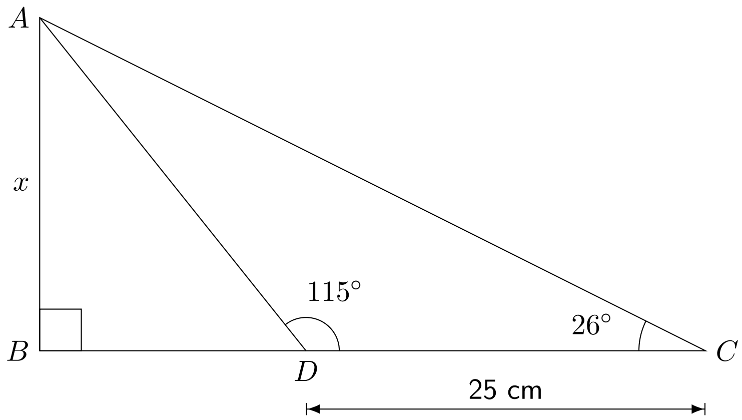
The function $y = f(x)$ is graphed in the diagram below. The points $(2, -4)$ and the origin are stationary points and the point $(1, -2)$ is a point of inflexion.

Sketch the graph of the curve $y = f'(x)$ on the same cartesian plane, showing the shape of the curve and any x intercepts.



Question 19 (3 marks) **3**

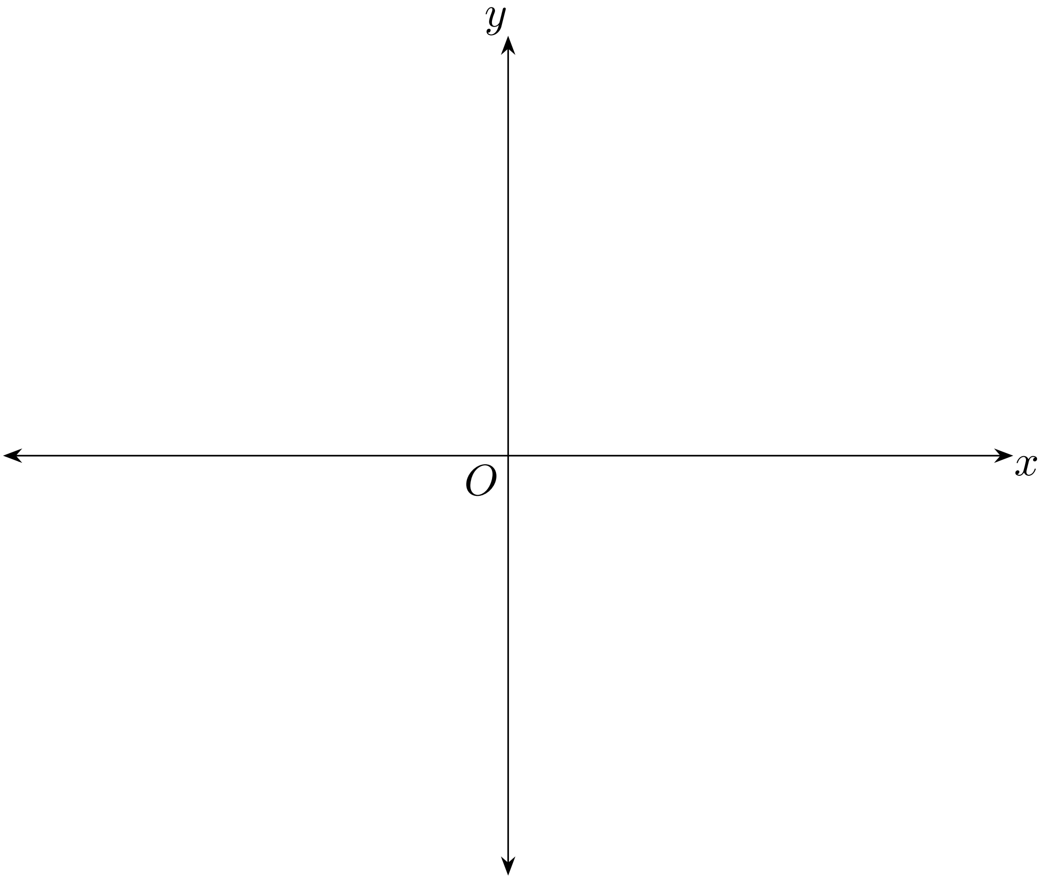
Find the value of x, correct to 2 decimal places.



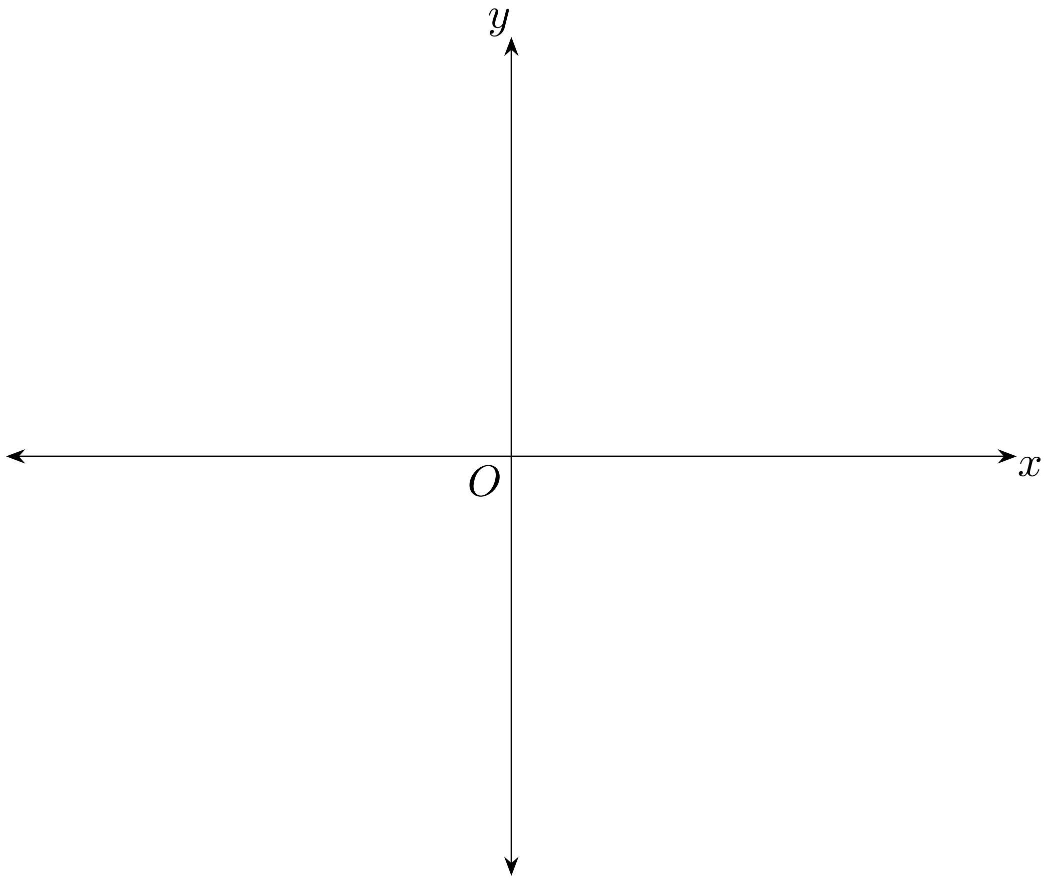
Question 20 (4 marks)

Sketch the graphs of the following, showing any asymptotes and any intercepts with the axes.

(a) $y = \dfrac{2}{2x - 5}$ **2**



(b) $y = |4x + 5|$ **2**



Question 21 (3 marks) **3**

Prove the identity $(\sec\theta + \tan\theta)^2 = \dfrac{1+\sin\theta}{1-\sin\theta}$.

Question 22 (3 marks)

Consider the function $x^4 - 12x^2 + 32 = 0$.

(a) Solve for x by first letting $u = x^2$. **2**

(b) If $x = \sin\theta$, explain why there are no solutions for θ. **1**

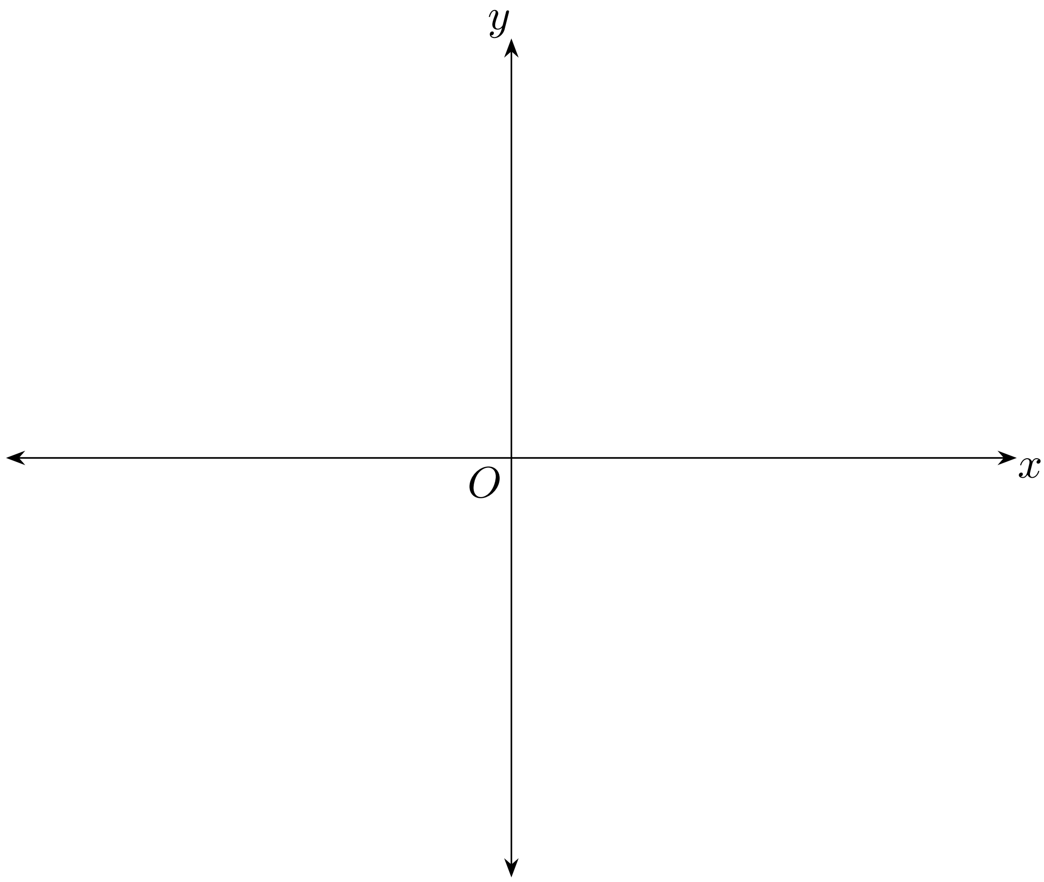
Question 23 (8 marks)

Consider the curve $f(x) = x^3 - 6x^2 + 9x - 4$.

(a) Find any stationary points and determine the nature. **3**

(b) Find the co-ordinates of the point of inflexion. **2**

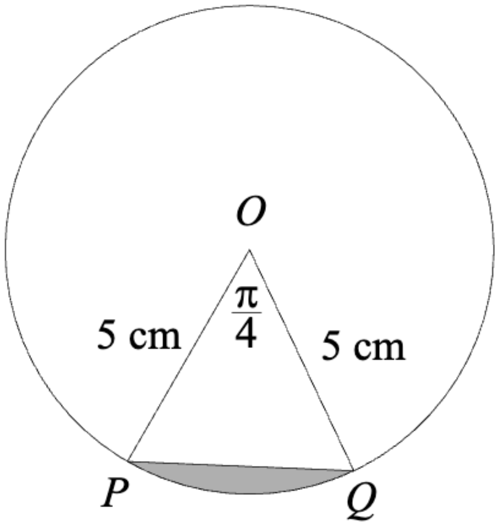
(c) Hence, sketch the curve, showing all turning points, the y-intercept and the point of inflexion. **3**



Question 24 (4 marks) **4**

Solve $2\cos^2 x + 3\sin x = 0$ for $0 \leq x \leq 2\pi$.

Question 25 (3 marks)



The diagram above shows a circle with centre O, radius 5 cm and $\angle POQ = \frac{\pi}{4}$. The points P and Q lie on the circumference of the circle.

(a) Find the exact area of the sector OPQ. **1**

(b) Find the exact area of $\triangle OPQ$ **1**

(c) Hence find the exact area of the minor shaded segment. **1**

Question 26 (5 marks)

Farmer Anneka wants to make a rectangular paddock with an area of 4000m^2.

However, fencing costs are sky-high so she wants the paddock to have a minimum perimeter.

(a) Show that the perimeter is given by the equation $P = 2x + \dfrac{8000}{x}$. **2**

(b) Find the dimensions of the rectangle that will give the minimum perimeter. **3**
Give your answer correct to 1 decimal place.

End of Task

Name: ______________________________

2023 YEAR 11
ASSESSMENT TASK 3

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Section I: Multiple Choice Questions **10 marks**

Circle the correct answer.

1. What is the derivative of $\frac{2}{x^3}$?

   A. $-\frac{3}{x^4}$

   B. $\frac{6}{x^4}$

   C. $\frac{3}{x^4}$

   D. $-\frac{6}{x^4}$

$y = 2x^{-3}$

$y' = -6x^{-4}$

$= -\frac{6}{x^4}$

2. What is the value of $\tan x$ given that $\sin x = \frac{2}{5}$ for $\frac{\pi}{2} \leq x \leq \pi$

   A. $-\frac{\sqrt{21}}{2}$

   B. $-\frac{5}{\sqrt{21}}$

   C. $\frac{2}{\sqrt{21}}$

   D. $-\frac{2}{\sqrt{21}}$

3. Which of the following is the equation of the line that passes through $(2, -1)$ and $(5, 3)$?

   $x_1\ y_1$ $x_2\ y_2$

   A. $4x + 3y - 11 = 0$

   B. $3x - 4y + 11 = 0$

   C. $4x - 3y - 11 = 0$

   D. $3x - 4y - 11 = 0$

$m = \frac{3+1}{3}$

$= \frac{4}{3}$

$y+1 = \frac{4}{3}(x-2)$

$3y+3 = 4x-8$

$0 = 4x - 3y - 11$

4. Which of the following is equivalent to $7x^2 - 56x + 105$?

   A. $(x-8)(x+7)$

   B. $7(x-3)(x-5)$

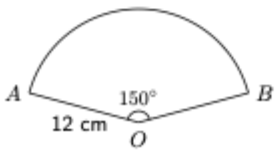
   C. $(x-3)(x-5)$

   D. $7(x-8)(x+7)$

$7(x^2 - 8x + 15)$

$= 7(x-3)(x-5)$

5. What is the length of arc AB in the diagram below?



   A. 10π cm

   B. 5 cm

   C. 5π cm

   D. 10 cm

$\frac{150\pi}{180} = \frac{5\pi}{6}$

$l = r\theta$

$l = 12 \times \frac{5\pi}{6}$

6. The domain for the function $f(x) = \sqrt{2-x}$ is:
   A. $[0, \infty)$
   B. $(-\infty, 2]$
   C. $[2, \infty)$
   D. $(-\infty, \infty)$

$2 - x \geq 0$
$-x \geq -2$
$x \leq 2$

7. Solve $\tan x = 1$ for $0 \leq x \leq 2\pi$.
   A. $x = \frac{\pi}{4}, \frac{5\pi}{4}$
   B. $x = \frac{\pi}{4}$
   C. $x = \frac{\pi}{4}, \frac{3\pi}{4}$
   D. $x = \frac{\pi}{4}, \frac{7\pi}{4}$

8. Megan tries to differentiate the function $y = \dfrac{3x}{(10-x)^2}$ with respect to x.

   Her working out is shown below.

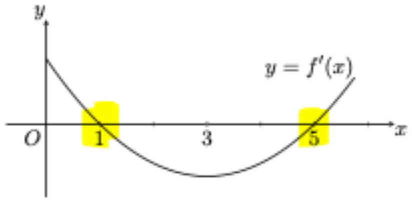
ambiguous

$$\frac{dy}{dx} = \frac{3(10-x)^2 + 3x \times 2(10-x)}{(10-x)^4} \quad \text{Line (1)}$$
$$= \frac{3(10-x)^2 - 6x(10-x)}{(10-x)^4} \quad \text{Line (2)}$$
$$= \frac{(10-x)[3(10-x) + 6x]}{(10-x)^4} \quad \text{Line (3)}$$
$$= \frac{30 + 3x}{(10-x)^3} \quad \text{Line (4)}$$
$$= \frac{10 + x}{(10-x)^3} \quad \text{Line (5)}$$

   Where did Megan make mistakes?
   A. Line 1 & Line 5
   B. Line 3 & Line 4
   C. Line 1 & Line 3
   D. Line 3 & Line 5

9. The height of a ball thrown in the air is given by the formula $h = 10t - 3t^2$ in metres, where t is measured in seconds. When the ball is thrown, what is the average rate of change of the height in the first second?
   A. 4 metres per second
   B. 7 metres
   C. $10 - 6t$
   D. 7 metres per second

10. The diagram below shows the graph of the gradient function of a curve.



$f'(x)$		0	
	\	—	/

For what value(s) of x does $f(x)$ have a local minimum?

A. $x = 1$

B. $x = 3$

C. $x = 1, x = 5$

D. $x = 5$

End of Section I

Section II

54 marks

Answer the questions in the spaces provided.
Show all necessary working.

Question 11 (1 marks) **1**

Show that $f(x) = x^3 + 3x$ is an odd function.

$f(-x) = (-x)^3 + 3(-x)$
$= -x^3 - 3x$
$= -(x^3 + 3x)$
$\therefore f(-x) = -f(x)$
$\therefore$ odd

① Mark for correct answer

Question 12 (2 marks) **2**

What is the centre and radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$?

$x^2 - 6x + y^2 + 4y = 12$
$x^2 - 6x + 9 + y^2 + 4y + 4 = 12 + 9 + 4$
$(x-3)^2 + (y+2)^2 = 25$
$\therefore$ centre: $(3, -2)$ $r = 5$

① Mark for correctly completing the square.
① Mark for correct centre and radius

Question 13 (2 marks) **2**

Rationalise the denominator of $\dfrac{\sqrt{2}+4}{2\sqrt{2}-1}$. Write your answer in the form $a + b\sqrt{2}$, where a and b are rational.

$\dfrac{\sqrt{2}+4}{2\sqrt{2}-1} \times \dfrac{2\sqrt{2}+1}{2\sqrt{2}+1}$

$\dfrac{(\sqrt{2}+4)(2\sqrt{2}+1)}{7} = \dfrac{4+\sqrt{2}+8\sqrt{2}+4}{7}$

$= \dfrac{8+9\sqrt{2}}{7}$

$\therefore a = \dfrac{8}{7}$ $b = \dfrac{9}{7}$

① Mark for multiplying with conjugate
① Mark for correct solution.

Question 14 (7 marks)

Differentiate the following with respect to x.

(a) $y = 3x^5 - 2x^3 + 7$ **1**

$y' = 15x^4 - 6x^2$

① Mark for correct answer

(b) $y = (x^2 - 2)^3(x + 3)^2$ **2**

$u = (x^2-2)^3 \qquad v = (x+3)^2$

$u' = 6x(x^2-2)^2 \qquad v' = 2(x+3)$

$y' = 6x(x^2-2)^2(x+3)^2 + 2(x+3)(x^2-2)^3$

$y' = 2(x^2-2)^2(x+3)[3x(x+3) + x^2-2]$

$y' = 2(x^2-2)^2(x+3)(4x^2+9x-2)$

① mark for correct chain rule

① Mark for correct factorisation.

(c) $y = \sqrt[4]{4x + 3}$ **2**

$y = (4x+3)^{\frac{1}{4}}$

$y' = \frac{1}{4}(4x+3)^{-\frac{3}{4}} \times 4$

$y' = (4x+3)^{-3/4}$

$y' = \dfrac{1}{\sqrt[4]{(4x+3)^3}}$

① Mark for converting to fractional index.

① Mark for correct solution

(d) $y = \dfrac{2x}{5 - x}$ **2**

$u = 2x \qquad v = 5-x$

$u' = 2 \qquad v' = -1$

$y' = \dfrac{2(5-x)+2x}{(5-x)^2}$

$y' = \dfrac{10}{(5-x)^2}$

① Mark for recognising quotient rule

① Mark for correct solution

Question 15 (3 marks) **3**

Find the equation of the normal to the curve $y = 2x^2 - 5x + 1$ at the point $(2, -1)$

$y' = 4x - 5$

$m_1 = 4(2) - 5$

$m_1 = 3$

$m_2 = -\frac{1}{3}$ $(2, -1)$

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{1}{3}(x - 2)$

$y + 1 = -\frac{1}{3}x + \frac{2}{3}$

$y = -\frac{1}{3}x - \frac{1}{3}$

① Mark for differentiating

① Mark for finding gradient of normal

① Mark for correct equation.

Question 16 (2 marks) **2**

Differentiate $f(x) = 2x + 3$ from first principles to show that $f'(x) = 2$.

Hint: Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

$f(x) = 2x + 3$

$f(x+h) = 2(x+h) + 3$

$= 2x + 2h + 3$

$f'(x) = \lim_{h \to 0} \frac{2x + 2h + 3 - 2x - 3}{h}$

$= \lim_{h \to 0} \frac{2h}{h}$

$= 2$

① Mark for correctly substituting f(x) and f(x+h) into formula.

① Mark for correct simplification

Question 17 (2 marks) **2**

Let $f(x) = 2x$ and $g(x) = \dfrac{x}{x+1}$. Solve $f(g(x)) = 3$.

$$f\left(\frac{x}{x+1}\right) = 2\left(\frac{x}{x+1}\right) = 3$$

$$\frac{2x}{x+1} = 3$$

$$2x = 3x + 3$$

$$-3 = x$$

$$\therefore x = -3$$

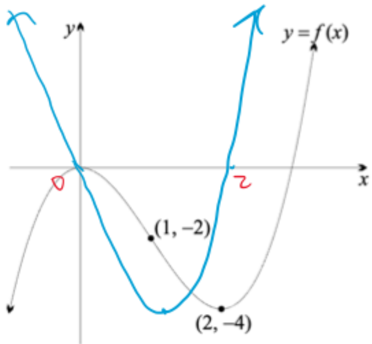
① Mark for correct substitution into correct function.

① Mark for correct value of x

Question 18 (2 marks) **2**

The function $y = f(x)$ is graphed in the diagram below. The points $(2, -4)$ and the origin are stationary points and the point $(1, -2)$ is a point of inflexion.

Sketch the graph of the curve $y = f'(x)$ on the same cartesian plane, showing the shape of the curve and any x intercepts.

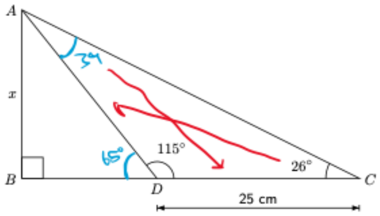


① Mark for correct intercepts.

① Mark for correct shape.

Question 19 (3 marks) **3**

Find the value of x, correct to 2 decimal places.



$\angle BDA = 180 - 115 = 65$

$\angle DAC = 180 - 115 - 26 = 39$

$\frac{AD}{\sin 26} = \frac{25}{\sin 39}$

$AD = \frac{25 \sin 26}{\sin 39}$

$AD = 17.41$

$\therefore \sin 65 = \frac{x}{17.41}$

$x = 15.78$ cm

① Mark for identifying $\angle BDA = 65°$

① Mark for correct use of sine rule

① Mark for correct use of trigonometry.

Question 20 (4 marks)

Sketch the graphs of the following, showing any asymptotes and any intercepts with the axes.

(a) $y = \frac{2}{2x-5}$ **2**

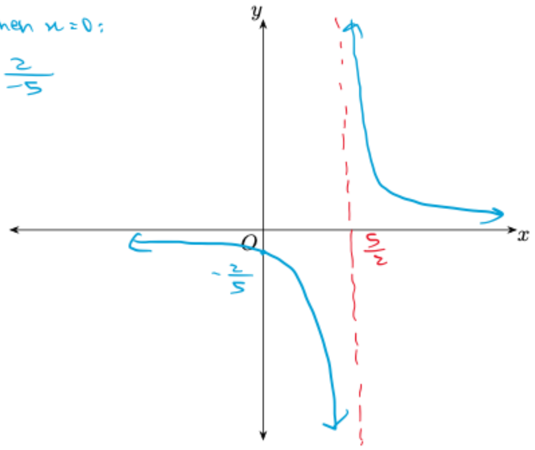
vertical asymptote: $2x-5 \neq 0$

$2x \neq 5$

$x \neq \frac{5}{2}$

y-int when $x=0$:

$y = \frac{2}{-5}$



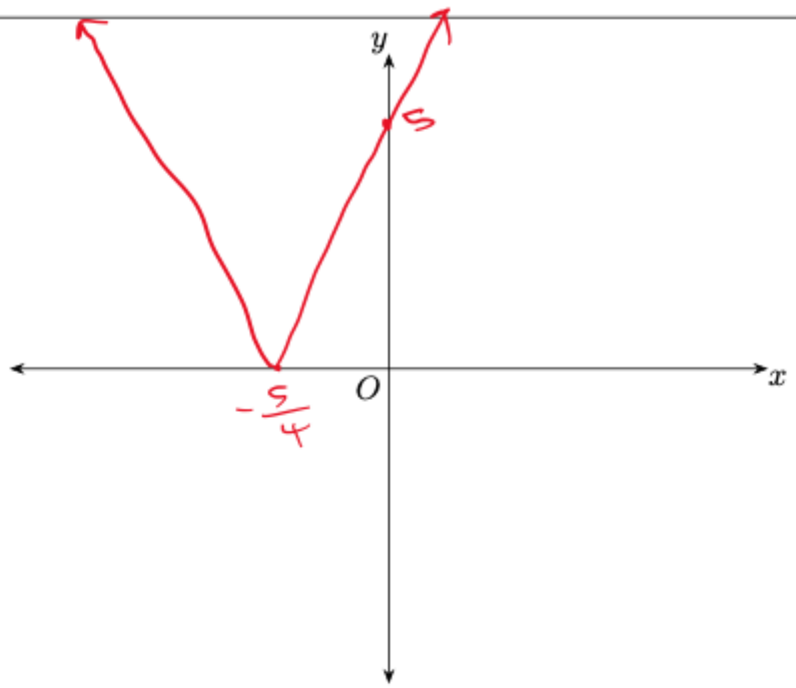
(1) Mark for calculating correct vertical asymptote and y-intercept.

(1) Mark for correct general shape of curve

(b) $y = |4x + 5|$ **2**

y-int when $x = 0$

y-int $= 5$



Question 21 (3 marks) **3**

Prove the identity $(\sec\theta + \tan\theta)^2 = \dfrac{1+\sin\theta}{1-\sin\theta}$.

L.H.S:

$(\sec\theta+\tan\theta)^2$

$= \sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta$

$= \dfrac{1}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} + 2\times\dfrac{1}{\cos\theta}\times\dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1+\sin^2\theta}{\cos^2\theta} + \dfrac{2\sin\theta}{\cos^2\theta}$

$= \dfrac{\sin^2\theta + 2\sin\theta + 1}{1-\sin^2\theta}$

$= \dfrac{(\sin\theta+1)^2}{1-\sin^2\theta}$

$= \dfrac{(1+\sin\theta)^{\cancel{2}}}{(1-\sin\theta)\cancel{(1+\sin\theta)}}$

$= \dfrac{1+\sin\theta}{1-\sin\theta} = R.H.S$

1 Mark for correct expansion.

1 Mark for combining fractions together

1 Mark for factorising difference of two squares.

Question 22 (3 marks)

Consider the function $x^4 - 12x^2 + 32 = 0$.

(a) Solve for x by first letting $u = x^2$. **2**

$u^2 - 12u + 32 = 0$

$(u-8)(u-4) = 0$

$u = 8 \qquad u = 4$

$x^2 = 8 \qquad x^2 = 4$

$x = \pm 2\sqrt{2} \qquad x = \pm 2$

① Mark for correct factorisation.

① Mark for finding correct solutions.

(b) If $x = \sin\theta$, explain why there are no solutions for θ. **1**

lies outside amplitude for the sine curve which is between 1 and −1.

① Mark for correct solutions.

Question 23 (8 marks)

Consider the curve $f(x) = x^3 - 6x^2 + 9x - 4$.

(a) Find any stationary points and determine the nature. **3**

$f'(x) = 3x^2 - 12x + 9$

let $f'(x) = 0$ to find st. pts

$0 = 3(x^2 - 4x + 3)$

$0 = (x-3)(x-1)$

$x = 1 \quad x = 3$

$y = 0 \quad y = -4$

$f''(x) = 6x - 12$

sub $x = 1$ for concavity:

$f''(1) = 6 - 12$

$= -6$

$\therefore$ concave down

$f''(3) = 6(3) - 12$

$= 6$

$\therefore$ concave up

① Mark for correct differentiation.

① mark for finding stationary points

① mark for proving concavity

(b) Find the co-ordinates of the point of inflexion. **2**

let $f''(x) = 0$

$6x - 12 = 0$

$x = 2 \quad y = -2$

① Mark for finding x-coordinate

①

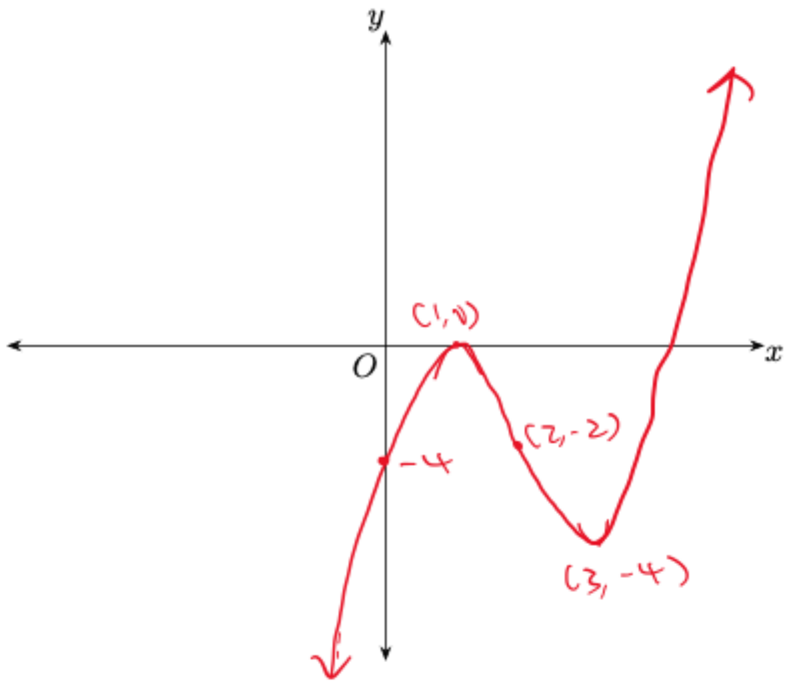
Must show change in concavity for second mark

by showing value not just sign

(c) Hence, sketch the curve, showing all turning points, the y-intercept and the point of inflexion. **3**

y-int : when $x = 0$

y-int $= -4$



Question 24 (4 marks) **4**

Solve $2\cos^2 x + 3\sin x = 0$ for $0 \le x \le 2\pi$.

$2(1-\sin^2 x)+3\sin x=0$

$2-2\sin^2 x+3\sin x=0$

$0=2\sin^2 x-3\sin x-2$

$0=2\sin^2 x-4\sin x+\sin x-2$

$0=2\sin x(\sin x-2)+(\sin x-2)$

$0=(\sin x-2)(2\sin x+1)$

$\sin x=2$ $\quad$ $\sin x=-\frac{1}{2}$

no sol

$\sin\alpha=\frac{1}{2}$

$\alpha=\frac{\pi}{6}$

$\therefore x=\pi+\frac{\pi}{6},\ 2\pi-\frac{\pi}{6}$

$x=\frac{7\pi}{6},\ \frac{11\pi}{6}$

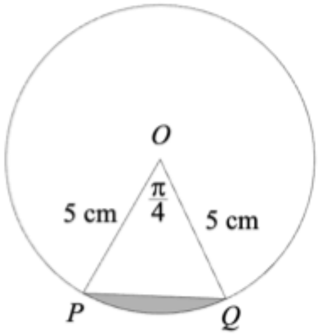
① Mark for substitution of $1-\sin^2 x$

① Mark for correct factorisation.

① Mark for finding $\sin x=2$ and $\sin x=-\frac{1}{2}$

① Mark for finding all correct solutions

Question 25 (3 marks)



The diagram above shows a circle with centre O, radius 5 cm and $\angle POQ = \frac{\pi}{4}$. The points P and Q lie on the circumference of the circle.

(a) Find the exact area of the sector OPQ. **1**

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2} \times 25 \times \frac{\pi}{4}$

$A = \frac{25\pi}{8}\text{ cm}^2$

① Mark for correct answer

(b) Find the exact area of $\triangle OPQ$ **1**

$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2} \times 25 \times \sin\frac{\pi}{4}$

$A = \frac{25\sqrt{2}}{4}\text{ cm}^2$

① Mark for correct answer

(c) Hence find the exact area of the minor shaded segment. **1**

$A = \frac{25\pi}{8} - \frac{25\sqrt{2}}{4}$

$A = \frac{25\pi - 50\sqrt{2}}{8}$

$A = \frac{25(\pi - 2\sqrt{2})}{8}\text{ cm}^2$

① Mark for correct answer,

Question 26 (5 marks)

Farmer Anneka wants to make a rectangular paddock with an area of 4000m^2.

However, fencing costs are sky-high so she wants the paddock to have a minimum perimeter.

(a) Show that the perimeter is given by the equation $P = 2x + \dfrac{8000}{x}$. **2**

$A = x \times y \qquad P = 2x + 2y$

$4000 = x \times y$

$P = 2x + 2\left(\dfrac{4000}{x}\right)$

$y = \dfrac{4000}{x}$

$P = 2x + \dfrac{8000}{x}$

① Mark for making y the subject.

① Mark for correct substitution

(b) Find the dimensions of the rectangle that will give the minimum perimeter.
Give your answer correct to 1 decimal place. **3**

Let $P' = 0$ for st. pt.

① Mark for st. pt.

$P = 2x + 8000x^{-1}$

② Mark for proving minimum

$P' = 2 - 8000x^{-2}$

$P' = 2 - \dfrac{8000}{x^2}$

③ Mark for finding dimensions of rectangle

let $P' = 0$

$\dfrac{8000}{x^2} = 2$

$8000 = 2x^2$

CAUTION: No marks deducted for not explaining why positive v positive and negative. Always communicate mathematically why you chose positive. Otherwise, full marks will not be awarded in the future.

$x^2 = 4000$

$x = \pm\sqrt{4000} = \sqrt{4000} \; (x > 0)$

$P'' = 16000x^{-3}$

$P'' = \frac{16000}{x^3}$

sub $x = \sqrt{4000}$

$= \frac{16000}{(\sqrt{4000})^3}$

$= 0.06\ldots$

$\therefore$ minimum

Max 3 marks awarded if the stationary point was not proven to be a minimum via a table of values or 2nd derivative test.

$\therefore$ at $x = \sqrt{4000}$ it will give minimum

$A = x \times y$

$4000 = \sqrt{4000} \times y$

$y = \frac{4000}{\sqrt{4000}} \times \frac{\sqrt{4000}}{\sqrt{4000}}$

$= \sqrt{4000}$

End of Task