Student Name: _____

2024 **YEAR 11 MATHEMATICS ADVANCED PRELIMINARY**

Mathematics

General Instructions

- Reading time – 10 minutes
- Working time – 120 minutes
- Write using black or blue pen
- Diagrams are not to scale
- NESA Approved calculators may be used
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (page 1–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6–34)

- Attempt Questions 11–30
- Allow about 1 hours and 45 minutes for this section

Section I

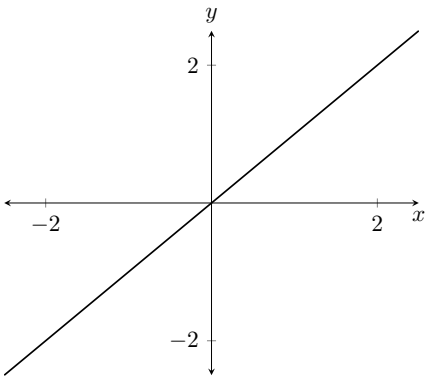
10 marks
Attempt Questions 1–10
Each question in this section is worth 1 mark
Allow about 15 minutes for this section

Circle the correct answer.

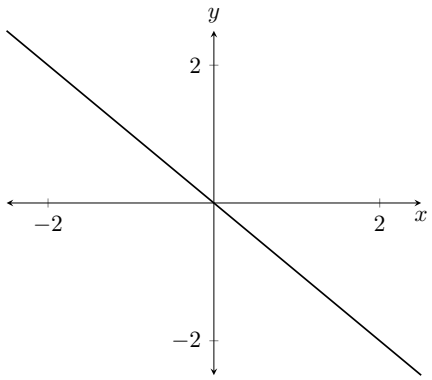
Question 1

Which of the following could be the graph of $2x - 3y = 4$

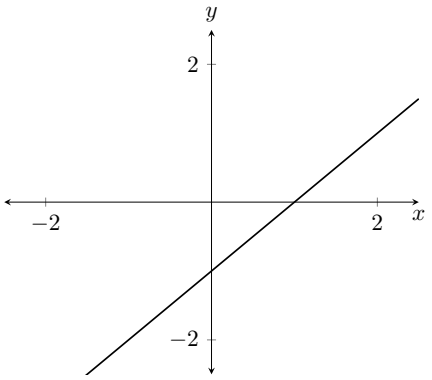
A.



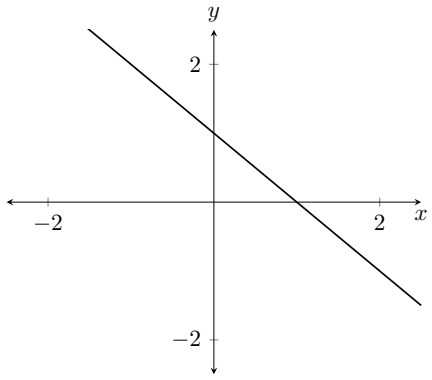
C.



B.



D.



Question 2

Which of the following angles is between $30°$ and $60°$ in size?

A. 0.3 radians

B. 0.5 radians

C. 0.8 radians

D. 1.1 radians

Question 3

Which interval notation gives the domain of $f(x) = \dfrac{1}{x-2}$?

A. $(-\infty, 2) \cap (2, \infty)$

B. $(-\infty, 2) \cup (2, \infty)$

C. $(-\infty, 2] \cap [2, \infty)$

D. $(-\infty, 2] \cup [2, \infty)$

Question 4

Which of the following fractions is equivalent to $\dfrac{3x-1}{2} - \dfrac{x-3}{4}$?

A. $\dfrac{5x-5}{4}$

B. $\dfrac{5x+1}{4}$

C. $\dfrac{10x-10}{4}$

D. $\dfrac{10x+2}{4}$

Question 5

Which of the following expressions is equivalent to $\log_a 12 - \log_a 6$ where $a > 1$?

A. $\log_e(2a)$

B. $\log_e(\frac{2}{a})$

C. $\log_e(2) \times \log_e(a)$

D. $\dfrac{\log_e(2)}{\log_e(a)}$

Question 6

Which of the following expressions is equivalent to $\tan^2\theta - \sin^2\theta$?

A. $\sin^2\theta + \tan^2\theta$

B. $\sin^2\theta - \tan^2\theta$

C. $\sin^2\theta \times \tan^2\theta$

D. $\sin^2\theta \div \tan^2\theta$

Question 7

Which of the following equations represents a one-to-one function.

A. $y = |x - 3|$

B. $y = \sqrt{16 - x^2}$

C. $y = (x - 1)(x + 3)$

D. $y = 2x - 5$

Question 8

For the polynomial $p(x) = 3x(x + 4)(x + 5)(x - 1)(x - 7)$, which statement is true?

A. The degree of $p(x)$ is 5.

B. The constant term of $p(x)$ is -140

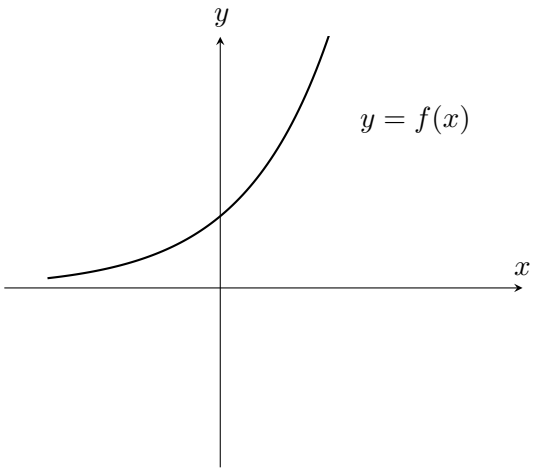
C. The leading coefficient is 10.

D. $p(x)$ has a zero at $x = 5$.

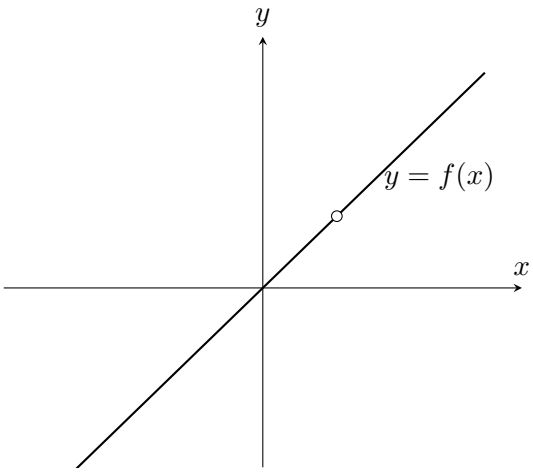
Question 9

Which of the graphs shows the curve $y = f(x)$ which is continuous across the domain shown?

A.



C.



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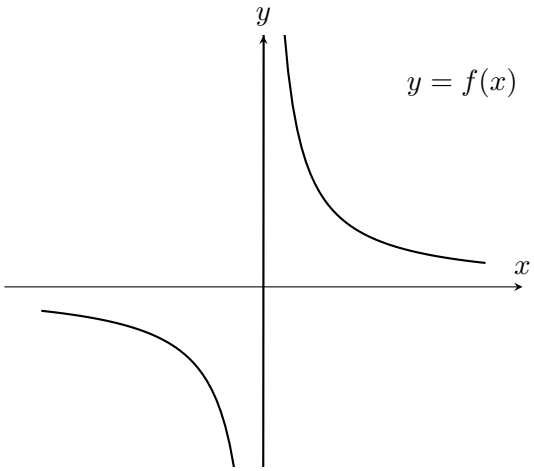
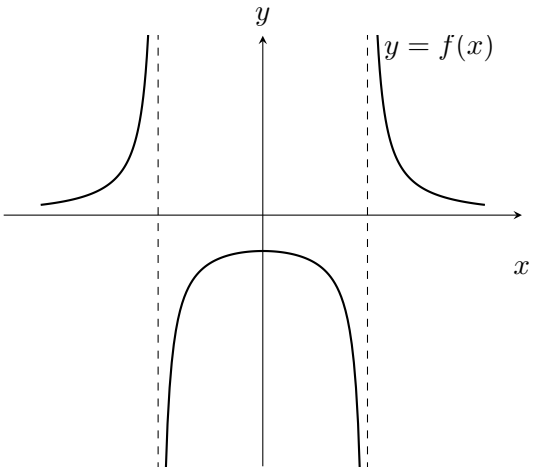
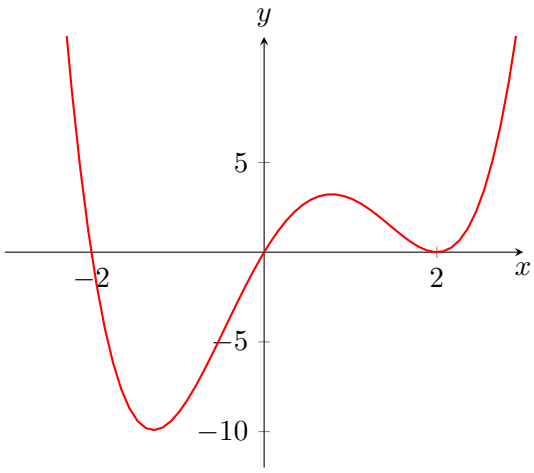


D.



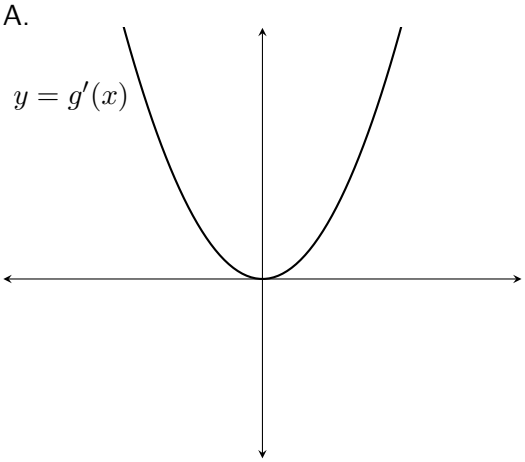
Question 10

The graph below shows the curve $y = g(x)$

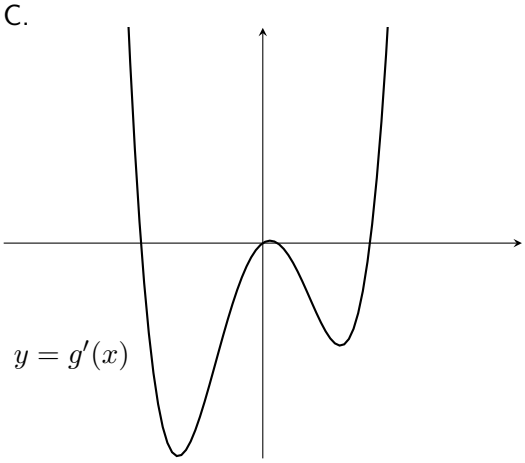


Which of the graphs below shows the curve $y = g'(x)$?

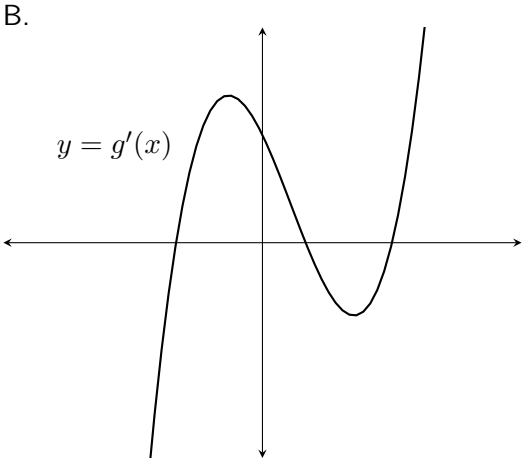
A.



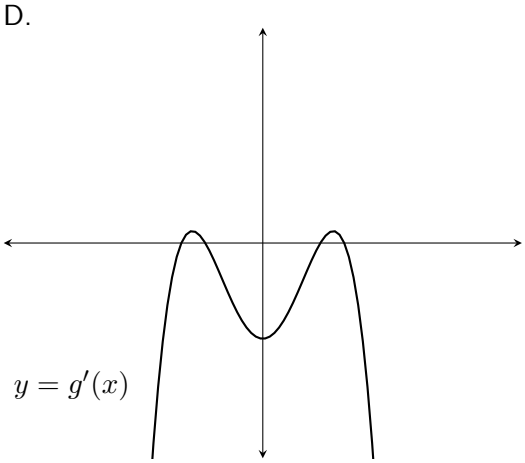
C.



B.



D.



Section II

60 marks
Attempt Questions 11 –30
Allow about 1hrs and 45 minutes for this section

Answer the questions in the spaces provided.
These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks) **Marks**

(a) Solve the equation $\left|\frac{x}{2} - 3\right| = 1$. **2**

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Question 12 (2 marks) **Marks**

(a) Find the centre and radius of the circle represented by the equation **2**

$$x^2 - 6x + y^2 + 10y + 18 = 0.$$

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Question 13 (3 marks) **Marks**

Find the derivative of the following expressions:

(a) $4x^7 + 3x^3$. **1**

(b) $4\sqrt{x^5}$. **1**

(c) $\dfrac{5}{6x^3}$. **1**

Question 14 (3 marks) **Marks**

Draw the graph of $y = |2 - x|$ on the set of axes provided below.

(a) **2**

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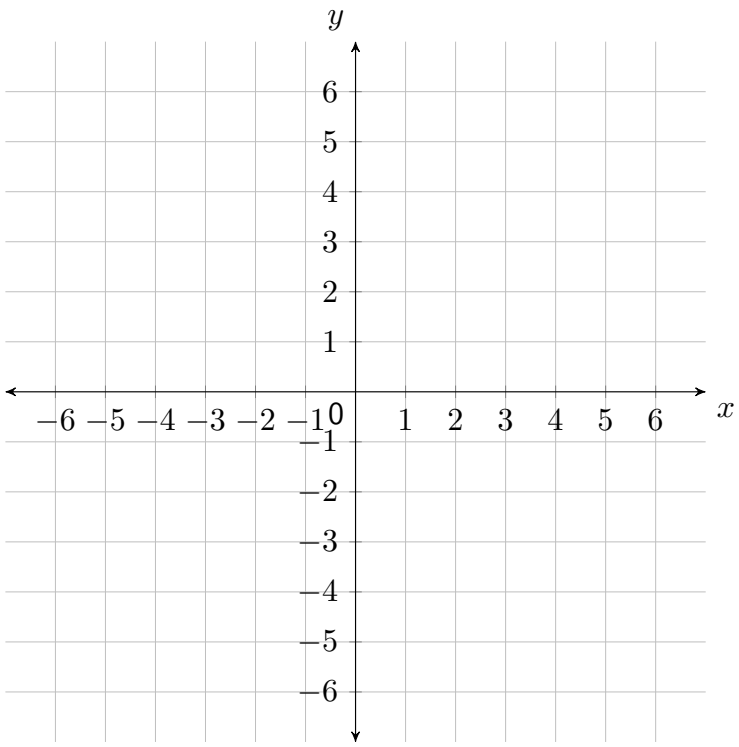

(b) Hence find the solutions for $y = 4$. **1**

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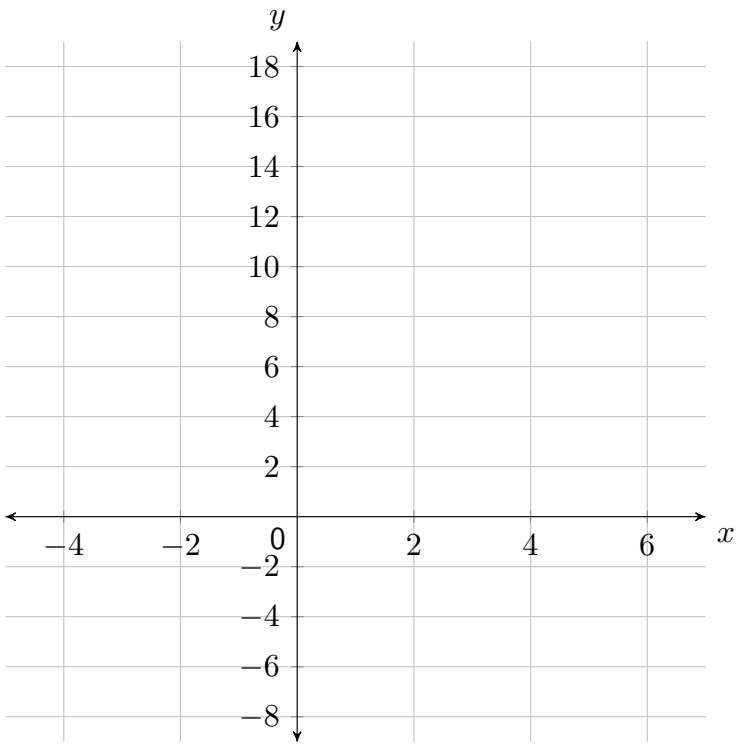
Question 15 (4 marks) **Marks**

(a) Draw the graph of $y = 9 + 7x - 2x^2$ on the set of axes provided, on the following page, showing all intercepts on the axes and turning point(s). **3**



(b) Hence, or otherwise find the range of $y = 9 + 7x - 2x^2$. **1**

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Question 16 (4 marks) **Marks**

(a) Solve the equation, $\log_2(x+20)=5$. **2**

(b) Use the properties of logarithms to show that $\log_3\left(\frac{5}{6}\right)-\log_3\left(22\frac{1}{2}\right)=-3$. **2**

Question 17 (3 marks) **Marks**

For the function $f(x) = 4e^{3x}$.

(a) Find the exact value of $f(2)$. **1**

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(b) Find the exact value of $f'(2)$. **2**

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Question 18 (2 marks) **Marks**

(a) If $y = \frac{4}{\sqrt{x^2+3x}}$ find $\frac{dy}{dx}$. **2**

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Question 19 (2 marks) **Marks**

(a) If $f(x) = \frac{(3x^2-2)^4}{2x^3}$ find $f'(x)$. **2**

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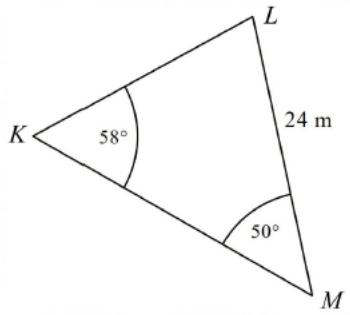
Question 20 (4 marks) **Marks**

The diagram below shows ΔKLM in which $\angle K = 58°$, $\angle M = 50°$ and $LM = 24\,\text{m}$.



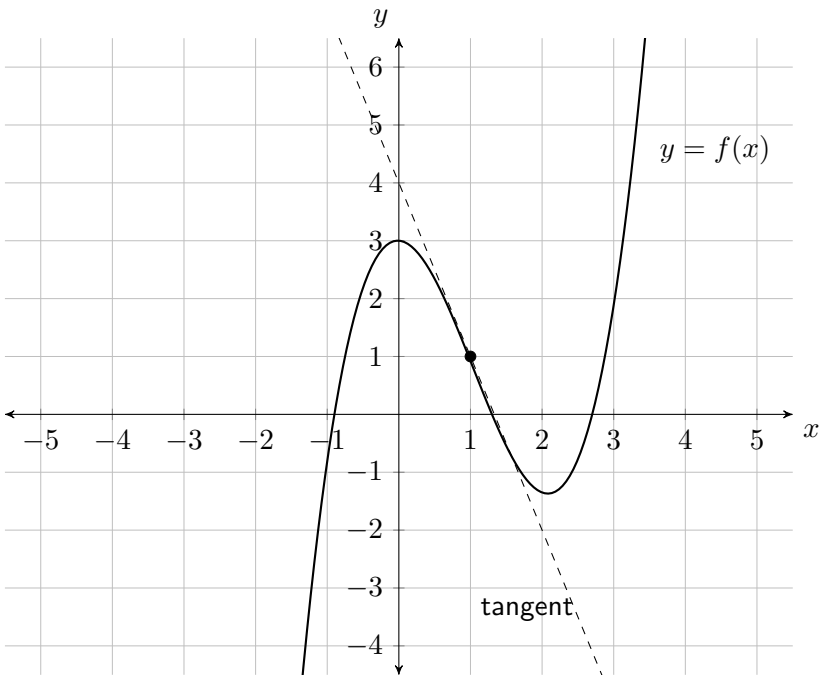
NOT TO SCALE

(a) Find the length of KM, to the nearest metre. **2**

(b) Hence, or otherwise find the area of ΔKLM, to the nearest m^2. **2**

Question 21 (3 marks) **Marks**

The diagram shows the graph of the cubic function, $y = f(x)$, and the tangent to $y = f(x)$ at the point (1,1).



(a) What is the gradient of the tangent at (1,1)? **1**

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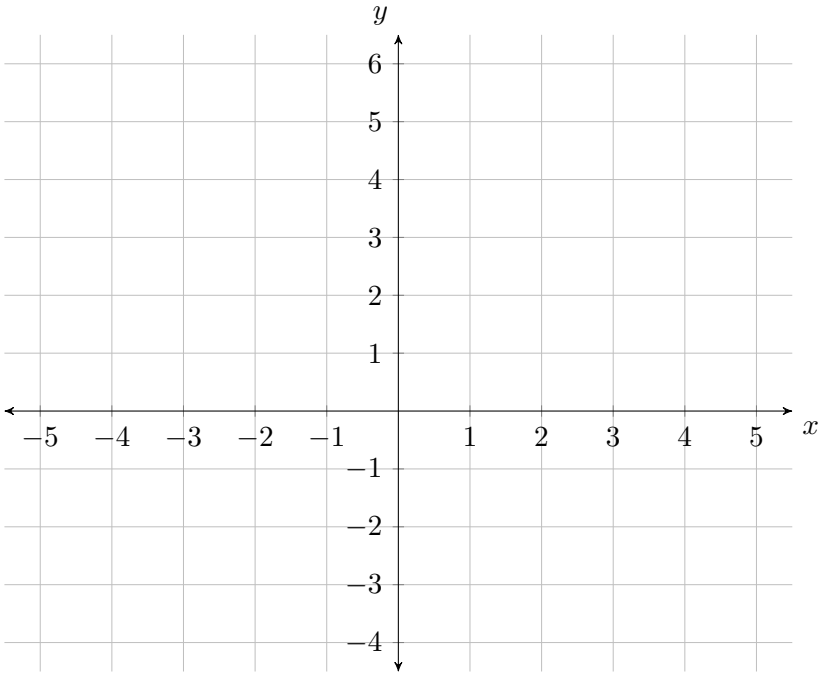
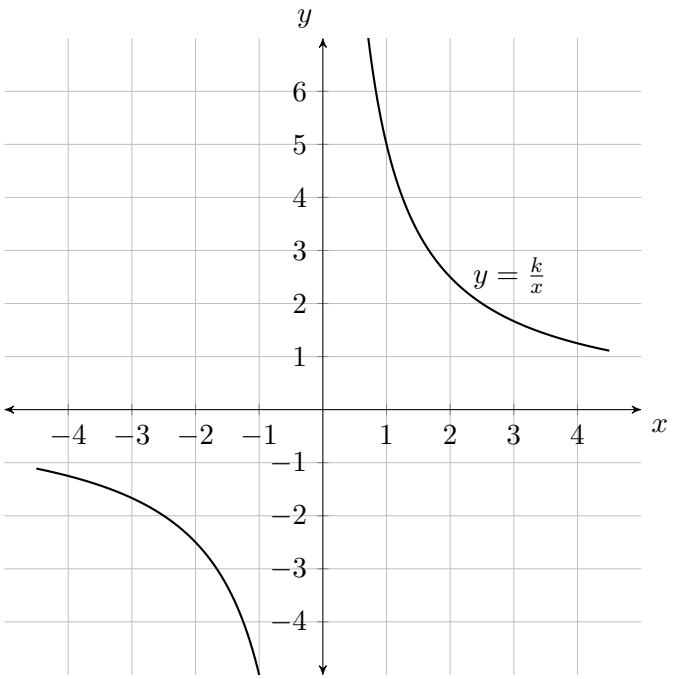
(b) Hence sketch the derivative function, $y = f'(x)$, on the axes below. **2**



Question 22 (2 marks) **Marks**

The graph of $y = \dfrac{k}{x}$ is shown on the axes below.



(a) Find the value of k and solve the equation $\dfrac{k}{x} = 4$ **2**

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Question 23 (3 marks) **Marks**

(a) Find the equation of the normal to the curve $y = x^4 - 2x^3 - x^2 + x$ at the point where $x = 1$. **3**

Question 24 (3 marks) **Marks**

(a) Solve $2\cos x - \sqrt{3} = 0$ in the domain $0 \leq x \leq 2\pi$. **3**

Question 25 (3 marks) **Marks**

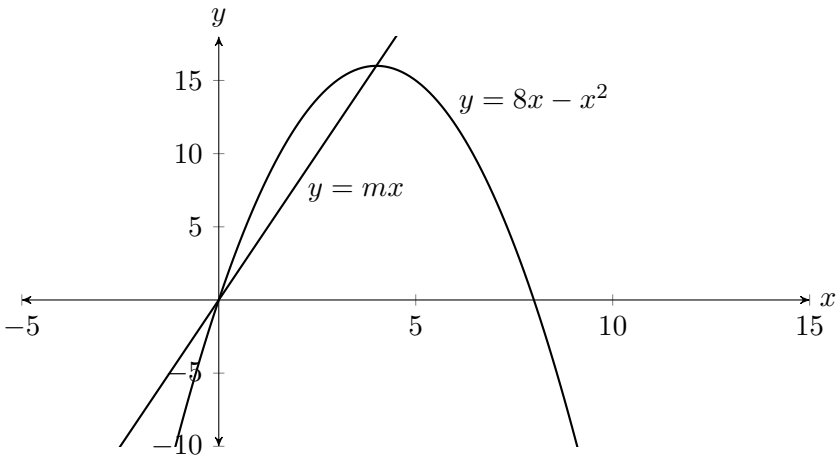
(a) Differentiation from first principles is defined by **3**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Use differentiation from first principles to find the derivative of $x^2 + 3x - 4$.

Question 26 (4 marks) **Marks**

The diagram shows the parabola $y = 8x - x^2$ and the straight line $y = mx$. The parabola and the line intersect at the origin and at the vertex of the parabola.



(a) Show that $m = 4$. **2**

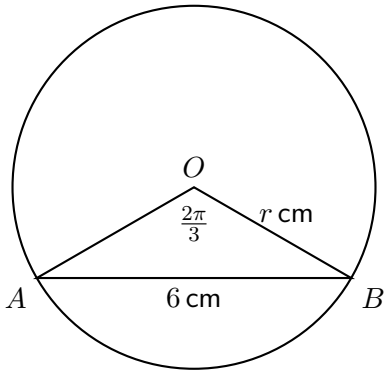
(b) The parabola $y = 8x - x^2$ is translated vertically to give the parabola $y = 8x - x^2 + c$. **2**

The new parabola has only one point of intersection with the line $y = mx$.

Find the value of c.

Question 27 (3 marks) **Marks**

In the diagram, AB is a chord of length 6 cm which subtends an angle of $\frac{2\pi}{3}$ radians at the centre O of a circle with radius r cm.



(a) Use the cosine rule to show that $r = 2\sqrt{3}$. **2**

(b) Find, in simplest exact form, the area of the sector AOB. **1**

Question 28 (2 marks) **Marks**

The number of rabbits in a valley is given by the function $R(t) = 4 \times 2^t + 3$, where t is the time in years.

(a) After how many years will there be 4099 rabbits in the valley? **2**

Question 29 (4 marks) **Marks**

(a) Show that **2**

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta.$$

(b) Hence, or otherwise, find all the values of θ, where $0^\circ \leq \theta \leq 360^\circ$, such that **2**

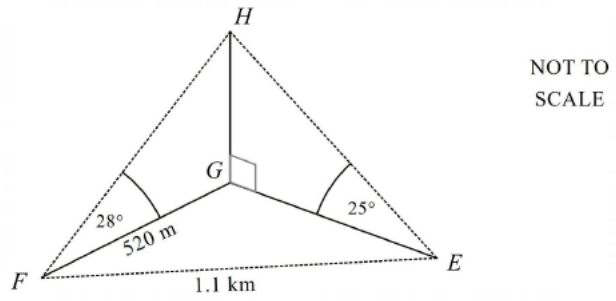
$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2.$$

Question 30 (4 marks) **Marks**

Three points E, F and G lie on a horizontal plain.

The distance $EF = 1.1$km and $FG = 520$m.

A vertical tower HG is constructed so that $\angle HFG = 28°$ and $\angle HEG = 25°$.



NOT TO SCALE

(a) Calculate the height of the tower HG and hence find the distance EG and the size of $\angle EGF$, rounding each appropriately. **4**

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End of Task

Spare Pages

This sheet is to be used if you need more space or to redo a question. Please note on your question sheet to refer to refer to spare page so the marker is aware.

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Question Number: .....

Question Part: .....

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Question Number: .....

Question Part: .....

Student Name: Solutions

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Question 1

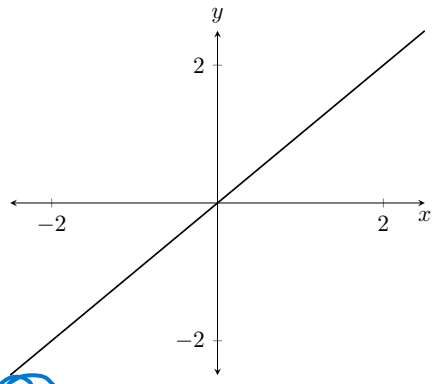
Which of the following could be the graph of $2x - 3y = 4$

$2x - 3y = 4$

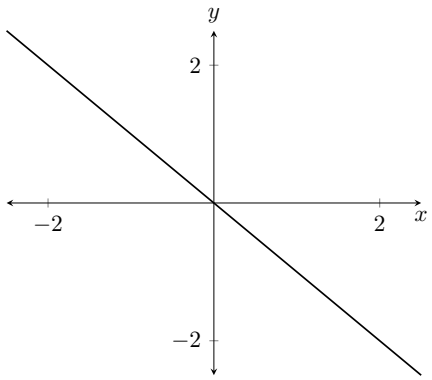
$-3y = -2x + 4$

$y = \frac{2x}{3} - \frac{4}{3}$

A.



C.



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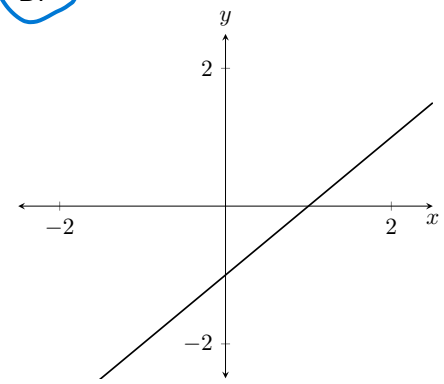
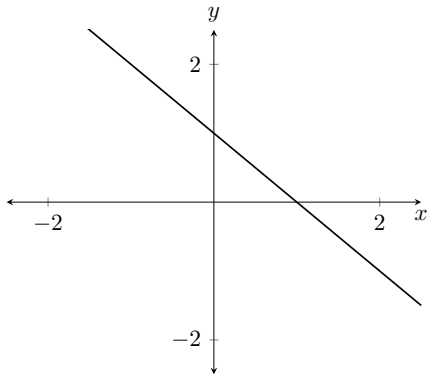


D.



Question 2

Which of the following angles is between $30°$ and $60°$ in size?

A. 0.3 radians

B. 0.5 radians

C. 0.8 radians

D. 1.1 radians

$0.3 \times \frac{180}{\pi} = 17°$

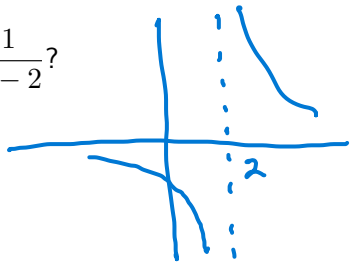
$0.5 \times \frac{180}{\pi} = 29°$

$0.8 \times \frac{180}{\pi} = 46°$

$1.1 \times \frac{180}{\pi} = 63°$

Question 3

Which interval notation gives the domain of $f(x) = \dfrac{1}{x-2}$?



A. $(-\infty, 2) \cap (2, \infty)$

B. $(-\infty, 2) \cup (2, \infty)$

C. $(-\infty, 2] \cap [2, \infty)$

D. $(-\infty, 2] \cup [2, \infty)$

Question 4

Which of the following fractions is equivalent to $\dfrac{3x-1}{2} - \dfrac{x-3}{4}$?

A. $\dfrac{5x-5}{4}$

B. $\dfrac{5x+1}{4}$

C. $\dfrac{10x-10}{4}$

D. $\dfrac{10x+2}{4}$

$$\frac{3x-1}{2} - \frac{x-3}{4}$$
$$= \frac{2(3x-1)-(x-3)}{4}$$
$$= \frac{6x-2-x+3}{4}$$
$$= \frac{5x+1}{4}$$

Question 5

Which of the following expressions is equivalent to $\log_a 12 - \log_a 6$ where $a > 1$?

A. $\log_e(2a)$

B. $\log_e(\frac{2}{a})$

C. $\log_e(2) \times \log_e(a)$

D. $\dfrac{\log_e(2)}{\log_e(a)}$

$$\log_a 12 - \log_a 6$$
$$= \log_a\left(\frac{12}{6}\right)$$
$$= \log_a 2$$
$$= \frac{\log_e 2}{\log_e a}$$

Question 6

Which of the following expressions is equivalent to $\tan^2\theta - \sin^2\theta$?

A. $\sin^2\theta + \tan^2\theta$

B. $\sin^2\theta - \tan^2\theta$

C. $\sin^2\theta \times \tan^2\theta$

D. $\sin^2\theta \div \tan^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$

$= \frac{\sin^2\theta \times \sin^2\theta}{\cos^2\theta}$

$= \sin^2\theta \times \tan^2\theta$

Question 7

Which of the following equations represents a one-to-one function.

A. $y = |x - 3|$

B. $y = \sqrt{16 - x^2}$

C. $y = (x - 1)(x + 3)$

D. $y = 2x - 5$

Question 8

For the polynomial $p(x) = 3x(x + 4)(x + 5)(x - 1)(x - 7)$, which statement is true?

A. The degree of $p(x)$ is 5.

B. The constant term of $p(x)$ is -140

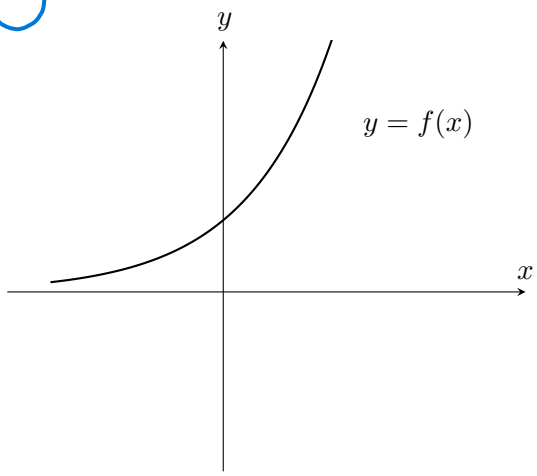
C. The leading coefficient is 10.

D. $p(x)$ has a zero at $x = 5$.

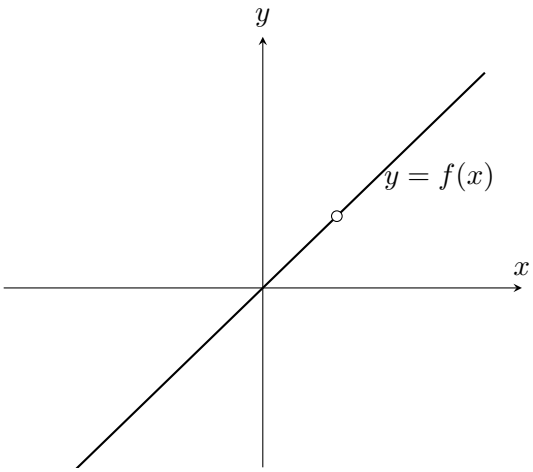
Question 9

Which of the graphs shows the curve $y = f(x)$ which is continuous across the domain shown?

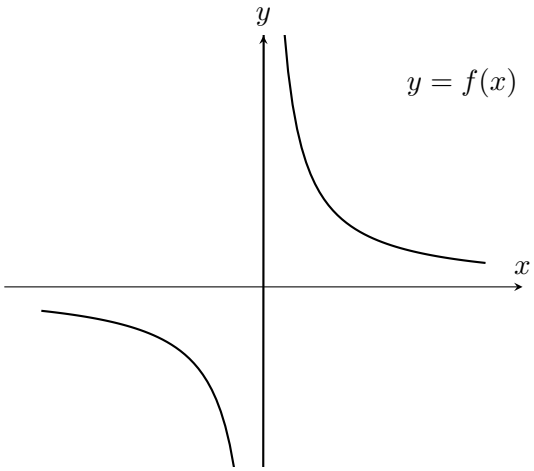
A.



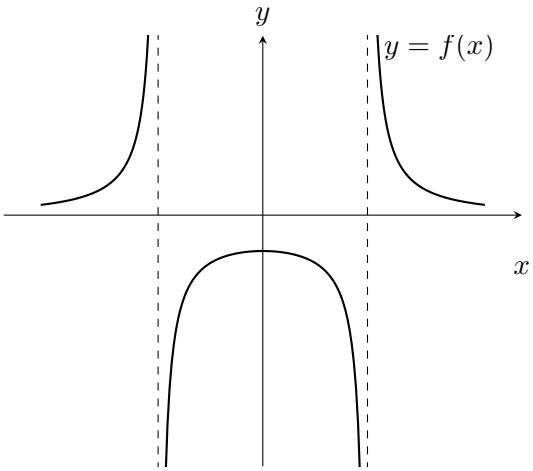
C.



B.

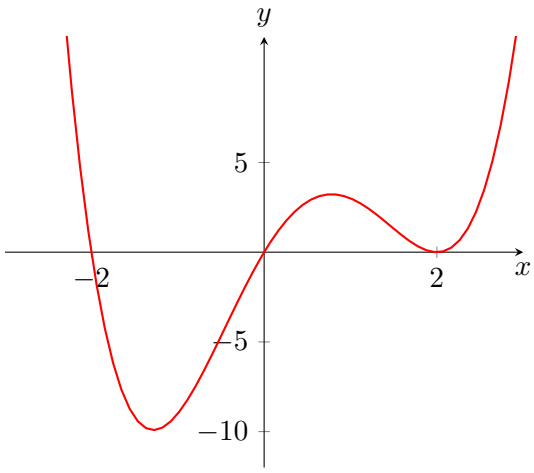


D.



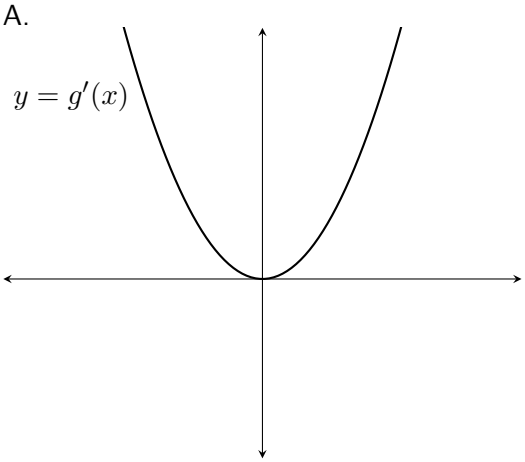
Question 10

The graph below shows the curve $y = g(x)$

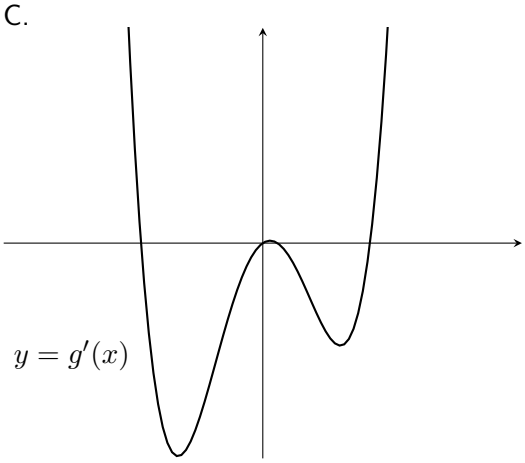


Which of the graphs below shows the curve $y = g'(x)$?

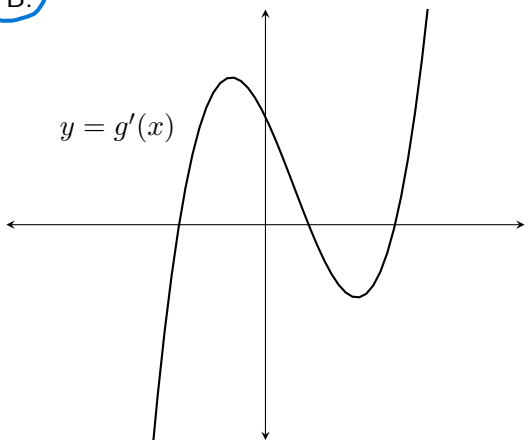
A.



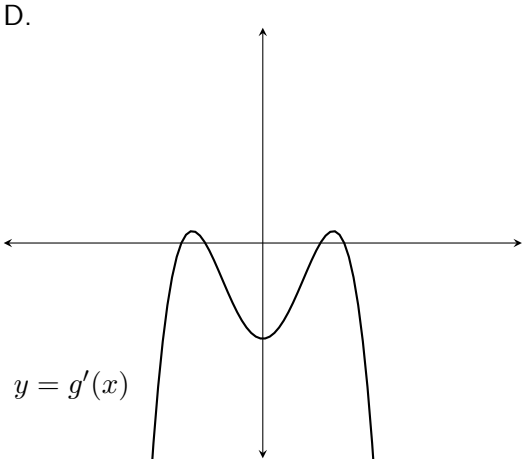
C.



B.



D.



Section II

60 marks
Attempt Questions 11 –30
Allow about 1hrs and 45 minutes for this section

Answer the questions in the spaces provided.
These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks) **Marks**

(a) Solve the equation $\left|\frac{x}{2} - 3\right| = 1$. **2**

$\frac{x}{2} - 3 = 1$ or $-\left(\frac{x}{2} - 3\right) = 1$

$\frac{x}{2} = 4$ $\frac{x}{2} - 3 = -1$

$x = 8$ $\frac{x}{2} = 2$

$x = 4$

$\therefore$ $x = 4$ or 8

1 mark for each correct solution

Question 12 (2 marks) **Marks**

(a) Find the centre and radius of the circle represented by the equation **2**

$$x^2 - 6x + y^2 + 10y + 18 = 0.$$

$(x^2 - 6x + (\frac{6}{2})^2) + (y^2 + 10y + (\frac{10}{2})^2) = -18 + (\frac{6}{2})^2 + (\frac{10}{2})^2$

$(x-3)^2 + (y+5)^2 = -18 + 9 + 25$

$(x-3)^2 + (y+5)^2 = 16$

$\therefore$ the circle has a centre at $(3, -5)$ and a radius of 4.

1 mark for minor error
2 marks for correct radius and centre

Question 13 (3 marks) **Marks**

Find the derivative of the following expressions:

(a) $4x^7 + 3x^3$. **1**

$\frac{d}{dx}(4x^7+3x^3) = 4\times7x^6 + 3\times3x^2$

$= 28x^6 + 9x^2$

1 mark for correct solution

(b) $4\sqrt{x^5}$. **1**

$4\sqrt{x^5} = 4x^{5/2}$

$\frac{d}{dx}(4x^{5/2}) = 4\times\frac{5}{2}x^{3/2}$

$= 10x^{3/2}$

$= 10\sqrt{x^3}$

1 mark for correct solution

(c) $\frac{5}{6x^3}$. **1**

$\frac{5}{6x^3} = \frac{5}{6}x^{-3}$

$\frac{d}{dx}\left(\frac{5}{6x^3}\right) = \frac{5}{6}\times-3\times x^{-4}$

$= -\frac{5}{2x^4}$

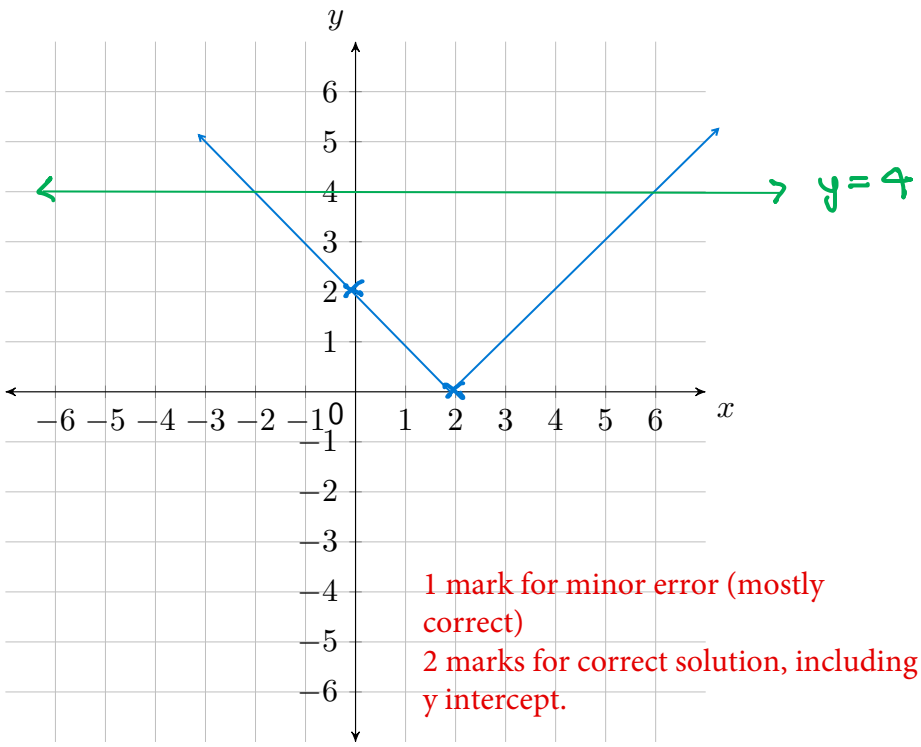
1 mark for correct solution

Question 14 (3 marks) **Marks**

Draw the graph of $y = |2 - x|$ on the set of axes provided below.

(a) $y = 2 - x$ or $y = x - 2$ **2**

* for only positive y values.



1 mark for minor error (mostly correct)
2 marks for correct solution, including y intercept.

(b) Hence find the solutions for $y = 4$. **1**

$\therefore$ when $y = 4$ $x = -2$ or 6

or coordinate $(-2, 4)$ or $(6, 4)$

1 mark for correct solution.

Question 15 (4 marks) **Marks**

(a) Draw the graph of $y = 9 + 7x - 2x^2$ on the set of axes provided, on the following page, showing all intercepts on the axes and turning point(s). **3**

$y = 9 + 7x - 2x^2$

x intercepts when $y = 0$

$0 = -2x^2 + 7x + 9$

PSF

$P = -2 \times 9 = -18$

$S = 7$

$F = +9, -2$

$0 = -2x^2 + 9x - 2x + 9$

$= x(-2x + 9) + (-2x + 9)$

$0 = (x + 1)(9 - 2x)$

$\therefore x = -1$ or $x = \frac{9}{2}$

$= 4.5$

ie $(-1, 0)$ $(4.5, 0)$ 1 mark for correct x intercepts

y intercept when $x = 0$

$y = -2(0)^2 + 7(0) + 9$

$= 9$

$(0, 9)$

1 mark for correct y intercept

Parabola is concave downwards as the leading term is negative.

$\therefore$ maximum turning point

$x = \frac{-b}{2a}$

$= \frac{-7}{2x-2}$

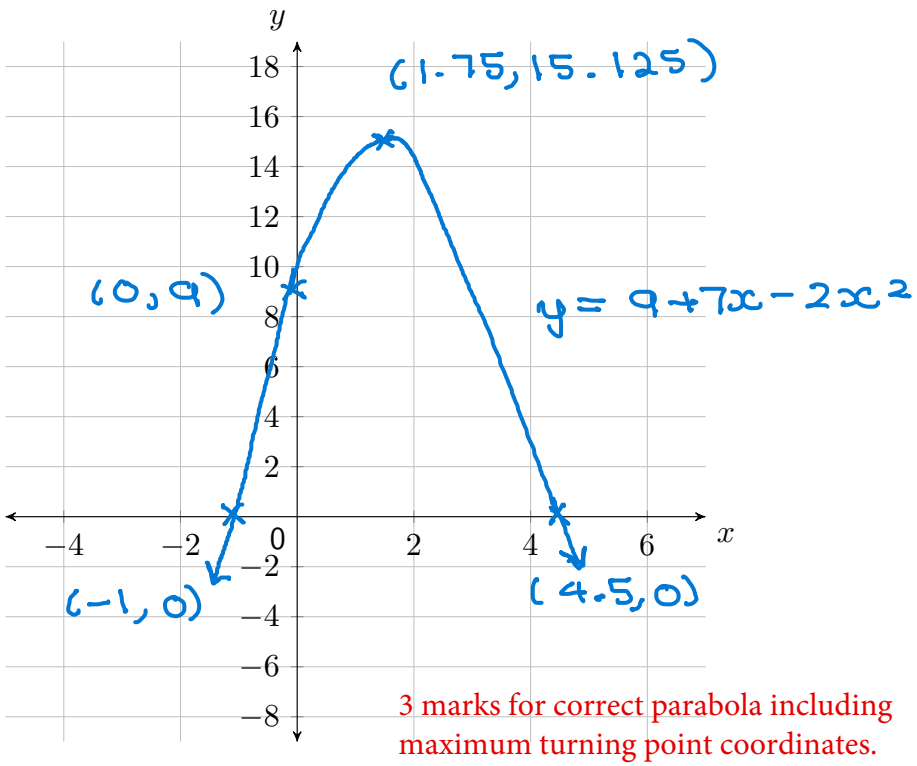
$= \frac{7}{4}$

$= 1.75$

$y = -2(1.75)^2 + 7(1.75) + 9$

$= 15.125$

$\therefore$ the maximum point is $(1.75, 15.125)$



3 marks for correct parabola including maximum turning point coordinates.
2 marks for significant progress towards solution or minor error

(b) Hence, or otherwise find the range of $y = 9 + 7x - 2x^2$. **1**

Range $[15.125, \infty)$

or $-\infty < y \leq 15.125$

1 mark for correct solution

Question 16 (4 marks) **Marks**

(a) Solve the equation, $\log_2(x+20)=5$. **2**

$2^5 = x+20$

$32 = x+20$

$x = 12$

1 mark for minor error
2 marks for correct solution

(b) Use the properties of logarithms to show that $\log_3\left(\frac{5}{6}\right) - \log_3\left(22\frac{1}{2}\right) = -3$. **2**

$LHS = \log_3\left(\frac{5}{6} \div 22\frac{1}{2}\right)$

$= \log_3\left(\frac{5}{6} \div \frac{45}{2}\right)$

$= \log_3\left(\frac{5}{6_3} \times \frac{2}{45_9}\right)$

$= \log_3\left(\frac{1}{27}\right)$

$= \log_3 3^{-3}$

$= -3\log_3 3$

$LHS = -3$

$\therefore$ LHS = RHS as required

1 mark for minor error
2 marks for correct solution

Question 17 (3 marks) **Marks**

For the function $f(x) = 4e^{3x}$.

(a) Find the exact value of $f(2)$. **1**

$f(2) = 4e^{3\times 2}$

$= 4e^{6}$

1 mark for correct solution

(b) Find the exact value of $f'(2)$. **2**

$f(x) = 4e^{3x}$

$f'(x) = 4\times 3e^{3x}$

$f'(x) = 12e^{3x}$

$f'(2) = 12e^{3\times 2}$

$= 12e^{6}$

1 mark for correct differentiation
2 marks for correct solution

Question 18 (2 marks) **Marks**

(a) If $y = \frac{4}{\sqrt{x^2+3x}}$ find $\frac{dy}{dx}$. **2**

a) $y = 4(x^2+3x)^{-\frac{1}{2}}$

$\frac{dy}{dx} = 4 \times -\frac{1}{2} \times (x^2+3x)^{-\frac{3}{2}} \times (2x+3)$

$= -2(2x+3)(x^2+3x)^{-\frac{3}{2}}$

$= -\frac{2(2x+3)}{\sqrt{(x^2+3x)^3}}$

OR

$= -\frac{4x+3}{\sqrt{(x^2+3x)^3}}$

1 mark for minor error
2 marks for correct solution

Question 19 (2 marks) **Marks**

(a) If $f(x) = \frac{(3x^2-2)^4}{2x^3}$ find $f'(x)$. **2**

let $v = 2x^3$ $u = (3x^2-2)^4$

$v' = 6x^2$ $u' = 4(3x^2-2)^3 \times 6x$

$= 24x(3x^2-2)^3$

$f'(x) = \frac{2x^3 \times 24x(3x^2-2)^3 - 6x^2(3x^2-2)^4}{(2x^3)^2}$

$= \frac{48x^4(3x^2-2)^3 - 6x^2(3x^2-2)^4}{4x^4}$

$= \frac{24x^2(3x^2-2)^3 - 3(3x^2-2)^4}{2x^2}$

OR

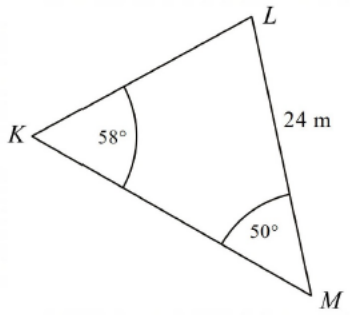
$= \frac{(3x^2-2)^3[24x^2 - 9x^2 + 6]}{2x^2}$

$= \frac{(3x^2-2)^3(15x^2+6)}{2x^2}$

1 mark for minor error
2 marks for correct solution

Question 20 (4 marks) **Marks**

The diagram below shows ΔKLM in which $\angle K = 58°$, $\angle M = 50°$ and $LM = 24\,\text{m}$.



NOT TO SCALE

(a) Find the length of KM, to the nearest metre. **2**

$\angle KLM = 180 - 58 - 50$ (angle sum of a triangle)

$= 72°$

$\frac{KM}{\sin 72} = \frac{24}{\sin 58}$

$KM = \frac{24}{\sin 58} \times \sin 72$

$= 26.915...$

$\doteq 27$ m (nearest metre)

1 mark for correct angle KLM or minor error
1 mark for correct solution

(b) Hence, or otherwise find the area of ΔKLM, to the nearest m^2. **2**

$A = \frac{1}{2} \times 27 \times 24 \times \sin 50$

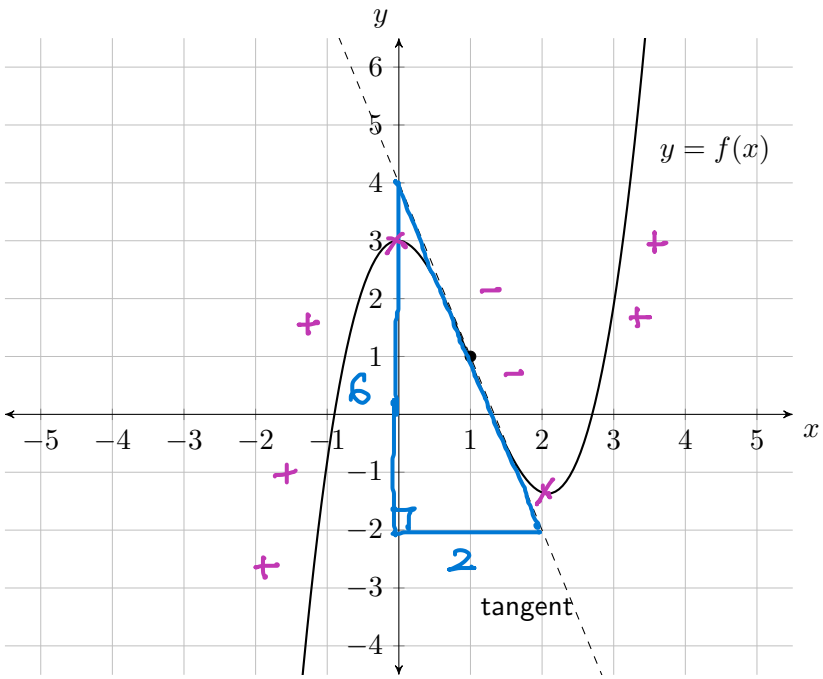
$= 247.279...$

$\doteq 247\ \text{m}^2$ (nearest m^2)

1 mark for correct formula substitution or minor error
1 mark for correct solution to 'nearest m^2'

Question 21 (3 marks) **Marks**

The diagram shows the graph of the cubic function, $y = f(x)$, and the tangent to $y = f(x)$ at the point (1,1).



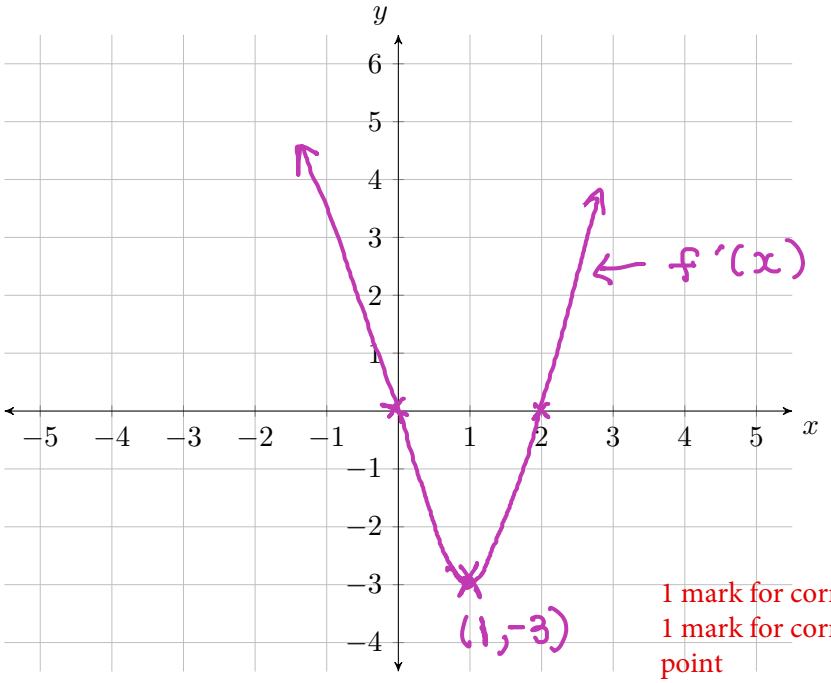
(a) What is the gradient of the tangent at (1,1)? **1**

gradient = $\frac{VR}{HR} = \frac{-6}{2} = -3$

$\therefore$ the gradient at (1,1) is -3.

1 mark for correct gradient

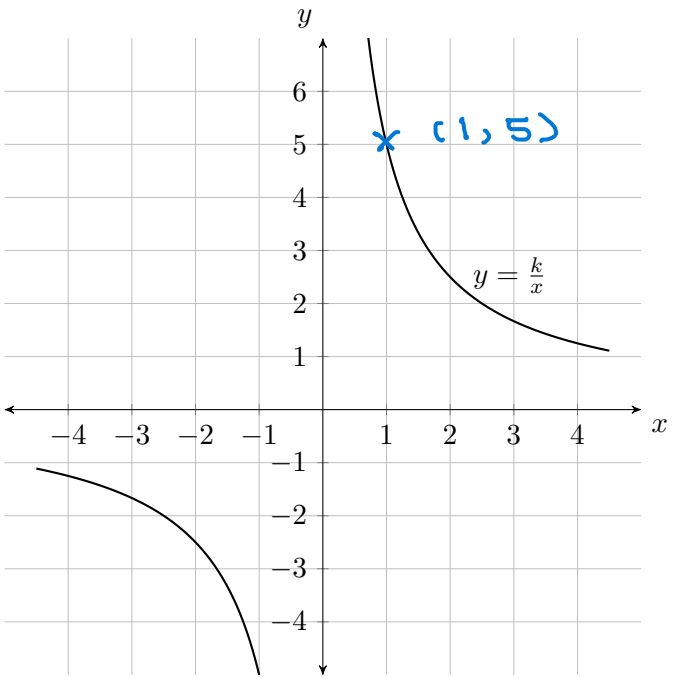
(b) Hence sketch the derivative function, $y = f'(x)$, on the axes below. **2**



1 mark for correct curve,
1 mark for correct minimum point

Question 22 (2 marks) **Marks**

The graph of $y = \dfrac{k}{x}$ is shown on the axes below.



(a) Find the value of k and solve the equation $\dfrac{k}{x} = 4$ **2**

when $x = 1$

$y = 5$

$y = \frac{k}{x}$ hyperbola.

$5 = \frac{k}{1}$

$\therefore k = 5$ 1 mark for correct k

solve $\frac{k}{x} = 4$

$\frac{k}{4} = x$

$\therefore x = \frac{5}{4}$

OR

$x = 1.25$ 1 mark for correct solution

Question 23 (3 marks) **Marks**

(a) Find the equation of the normal to the curve $y = x^4 - 2x^3 - x^2 + x$ at the point where $x = 1$. **3**

$y = x^4 - 2x^3 - x^2 + x$

$\frac{dy}{dx} = 4x^3 - 6x^2 - 2x + 1$ 1 mark for correct differentiation

when $x = 1$ $\frac{dy}{dx} = 4 - 6 - 2 + 1 = -3$

ie when $x = 1$ the gradient is -3

the gradient of the normal is $\frac{1}{3}$

when $x = 1$ $y = 1 - 2 - 1 + 1 = -1$

$\therefore$ the coordinate is $(1, -1)$ 1 mark for correct gradient for the equation of the normal

$y - y_1 = m(x - x_1)$

$y + 1 = \frac{1}{3}(x - 1)$

$y = \frac{1}{3}x - \frac{1}{3} - 1$

$y = \frac{1}{3}x - \frac{4}{3}$

OR 1 mark for correct solution

$3y = x - 4$

$x - 3y - 4 = 0$

Question 24 (3 marks) **Marks**

(a) Solve $2\cos x - \sqrt{3} = 0$ in the domain $0 \leq x \leq 2\pi$. **3**

$\cos x = \frac{\sqrt{3}}{2}$

1 mark for correct rearranging of equation.

Domain: $0 \leq x \leq 2\pi$

Related angle: $\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$

1 mark for correct related angle

Quadrants: cos is positive in the 1st and 4th quadrants

Solve: $x = \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$\therefore x = \frac{\pi}{6}, \frac{11\pi}{6}$

1 mark for correct solution

Question 25 (3 marks) **Marks**

(a) Differentiation from first principles is defined by **3**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Use differentiation from first principles to find the derivative of $x^2 + 3x - 4$.

$f'(x) = \lim_{h \to 0} \dfrac{[(x+h)^2 + 3(x+h) - 4] - [x^2 + 3x - 4]}{h}$

1 mark for correct substitution

$= \lim_{h \to 0} \dfrac{[x^2 + 2xh + h^2 + 3x + 3h - 4 - x^2 - 3x + 4]}{h}$

$= \lim_{h \to 0} \dfrac{2xh + h^2 + 3h}{h}$

$= \lim_{h \to 0} \dfrac{h(2x + h + 3)}{h}$

$= \lim_{h \to 0} 2x + h + 3$

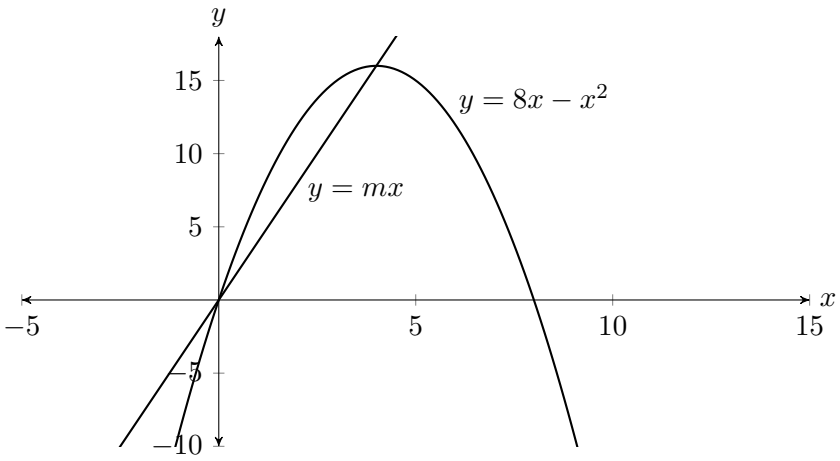
1 mark for significant progress towards solution

$\therefore f'(x) = 2x + 3$

1 mark for correct solution and correct use of the limit sign

Question 26 (4 marks) **Marks**

The diagram shows the parabola $y = 8x - x^2$ and the straight line $y = mx$. The parabola and the line intersect at the origin and at the vertex of the parabola.



(a) Show that $m = 4$. **2**

$y = 8x - x^2$

vertex of parabola

$x = \frac{-b}{2a}$

$= -\frac{8}{2\times -1}$

$= 4$

$y = 8\times 4 - 4^2$

$= 16$

$\therefore$ intersection points are $(0,0)$ and $(4,16)$

$y = mx$

$16 = m\times 4$

$\therefore m = 4$ as required.

1 mark for significant progress towards solution
1 mark for correct solution and clear working out

(b) The parabola $y = 8x - x^2$ is translated vertically to give the parabola $y = 8x - x^2 + c$. **2**

The new parabola has only one point of intersection with the line $y = mx$.

$\therefore \Delta = 0$.

Find the value of c.

The two functions intersect at one point.

$8x - x^2 + c = 4x$

$0 = x^2 - 4x - c$

If one point of intersection

$\Delta = 0$

$\therefore 0 = b^2 - 4ac$

$0 = (-4)^2 - 4 \times 1 \times -c$

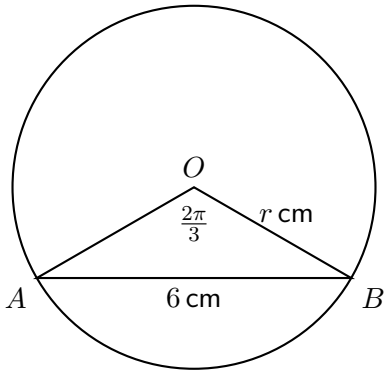
$0 = 16 + 4c$

$\therefore c = -4$

1 mark for correct equating of equations
1 mark for correct solution

Question 27 (3 marks) **Marks**

In the diagram, AB is a chord of length 6 cm which subtends an angle of $\frac{2\pi}{3}$ radians at the centre O of a circle with radius r cm.



(a) Use the cosine rule to show that $r = 2\sqrt{3}$. **2**

$6^2 = r^2 + r^2 - 2r^2\cos\left(\frac{2\pi}{3}\right)$

$36 = 2r^2 - 2r^2\left(-\frac{1}{2}\right)$

$36 = 2r^2 + r^2$

$36 = 3r^2$

$r = +\sqrt{\frac{36}{3}}$ as $r > 0$

$= \sqrt{12}$

$= \sqrt{4 \times 3}$

$\therefore r = 2\sqrt{3}$ as required

1 mark for correct substitution into the cosine ruel
1 mark for correct solution and clear working out

(b) Find, in simplest exact form, the area of the sector AOB. **1**

$\text{Area} = \frac{1}{2} r^2 \theta$ (in radians)

$= \frac{1}{2} \times (\sqrt{12})^2 \times \frac{2\pi}{3}$

$\therefore$ Area $= 4\pi$ cm^2 (exact)

1 mark for correct solution.

Question 28 (2 marks) **Marks**

The number of rabbits in a valley is given by the function $R(t) = 4 \times 2^t + 3$, where t is the time in years.

(a) After how many years will there be 4099 rabbits in the valley? **2**

$R(t) = 4 \times 2^t + 3$

$4099 = 4 \times 2^t + 3$

$\frac{4099 - 3}{4} = 2^t$

$\log_e(1024) = \log_e 2^t$

$\log_e(1024) = t \log_e 2$

$t = \frac{\log_e(1024)}{\log_e 2}$

$\therefore t = 10$ years.

1 mark for significant progress towards solution
1 mark for correct solution

Question 29 (4 marks) **Marks**

(a) Show that **2**

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta.$$

$$\text{LHS} = \frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$$

$$= \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

1 mark for correct expansion of fraction with common denominator

$$= \frac{2}{1-\sin^2\theta}$$

pythagorian identity

$\sin^2\theta+\cos^2\theta=1$

$\cos^2\theta = 1-\sin^2\theta$

$$= \frac{2}{\cos^2\theta}$$

$$\text{LHS} = 2\sec^2\theta$$

$\therefore$ LHS = RHS as required

1 mark for simplifying and showing correct solution

(b) Hence, or otherwise, find all the values of θ, where $0^\circ \le \theta \le 360^\circ$, such that **2**

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2.$$

$2\sec^2\theta = 2$ (from part a)

$\sec^2\theta = 1$

1 mark for correctly using part a and simplifying.

$\therefore \frac{1}{\cos^2\theta} = \frac{1}{1}$

$\cos^2\theta = 1$

$\cos\theta = \pm 1$

$\therefore \theta = 0^\circ, 180^\circ, 360^\circ$

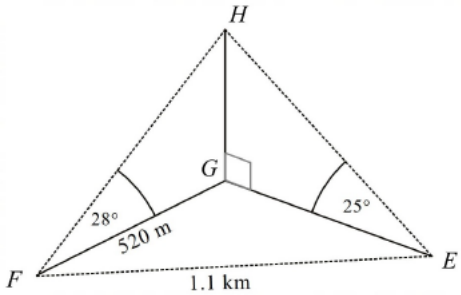
1 mark for correct solution

Question 30 (4 marks) **Marks**

Three points E, F and G lie on a horizontal plain.

The distance $EF = 1.1$km and $FG = 520$m.

A vertical tower HG is constructed so that $\angle HFG = 28°$ and $\angle HEG = 25°$.



NOT TO SCALE

* 1100m

(a) Calculate the height of the tower HG and hence find the distance EG and the size of $\angle EGF$, rounding each appropriately. **4**

$\frac{HG}{520} = \tan 28°$

$HG = \tan 28° \times 520$

$= 276.49...$

$\doteq 276$ m 1 mark for correct HG

$\frac{HG}{EG} = \tan 25°$

$\frac{276}{EG} = \tan 25°$

$\frac{276}{\tan 25} = EG$

$EG = 592.932....$

$\doteq 593$ m 1 mark for correct EG

(592 m if rounding HG)

$\angle EGF = ?$

$\cos \angle EGF = \frac{520^2 + 593^2 - 1100^2}{2 \times 520 \times 593}$

$= -0.95359....$

$\angle EGF = 162.4759...$

$\doteq 162°$

1 mark for correct solution

if rounding and used 592°

$\cos \angle EGF = -0.956886...$

$\angle EGF = 163.1194...$

$\doteq 163°$

End of Task

Spare Pages

This sheet is to be used if you need more space or to redo a question. Please note on your question sheet to refer to refer to spare page so the marker is aware.

Student Name: .....

Question Number: .....

Question Part: .....

Spare Pages

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