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Student Number

Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

**2019**

# Year 11
# Mathematics Advanced

## Task 3

Year 11 Exam Block week 3 to 5 (Term 3)

**General Instructions**

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- NESA approved calculators may be used
- A formulae and data sheet is provided at the back of this paper
- Show relevant mathematical reasoning and/or calculations

**Total Marks: 100**

- Section I – Multiple Choice – 10 marks
- Section II – Written response 90 marks
- Attempt Questions 1 – ?

***This question paper must not be removed from the examination room.***
*This assessment task constitutes 40% of the course.*

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## Section I - Multiple Choice

**Please answer on the Multiple Choice answer sheet.**

1. To convert $40^o$ to radians you calculate:

   (A) $40\times\dfrac{180}{\pi}$

   (B) $40\times\dfrac{\pi}{180}$

   (C) $180\times\dfrac{\pi}{40}$

   (D) $40\times\dfrac{\pi}{360}$

2. The range $y \geq 0$ is written in interval notation as:

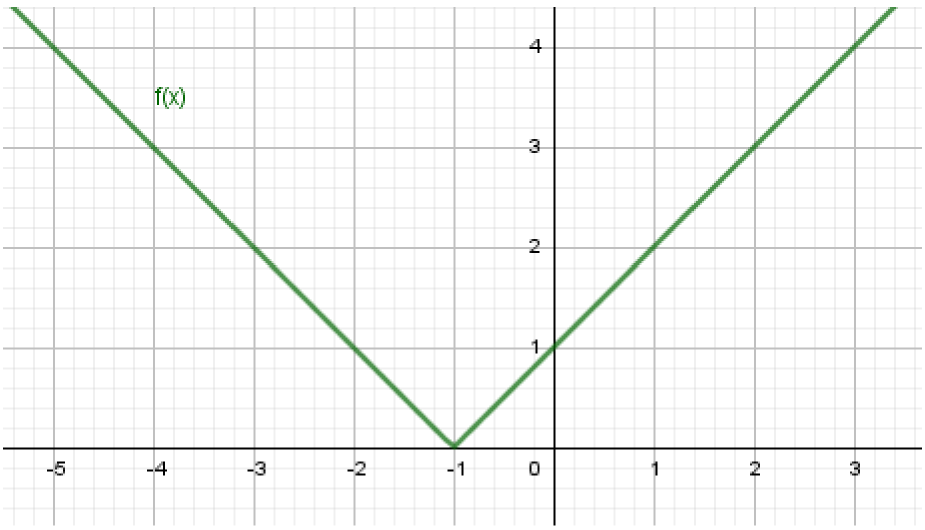
   (A) $y \in (-\infty, \infty)$

   (B) $y \in (0, \infty]$

   (C) $y \in [0, \infty)$

   (D) $y \in (0, \infty)$

3. The graph of $f(x)$ shown below represents:




   (A) $y = |x| + 1$

   (B) $y = |x| - 1$

   (C) $y = |x - 1|$

   (D) $y = |x + 1|$

4. Given $\sin\alpha = -\frac{k}{2}$ , $\sec\alpha$ is equal to what, given $(\pi \leq \alpha \leq \frac{3\pi}{2})$.

(A) $-\frac{\sqrt{4-k^2}}{2}$

(B) $\frac{\sqrt{4-k^2}}{2}$

(C) $\frac{2}{\sqrt{4-k^2}}$

(D) $-\frac{2}{\sqrt{4-k^2}}$

5. Using the function definition

$$f(x) = 2 - x, \quad x \geq 3$$
$$= x^2 + 1, \quad x<3$$

Calculate $f(3) + f(-3)$.

(A) 9

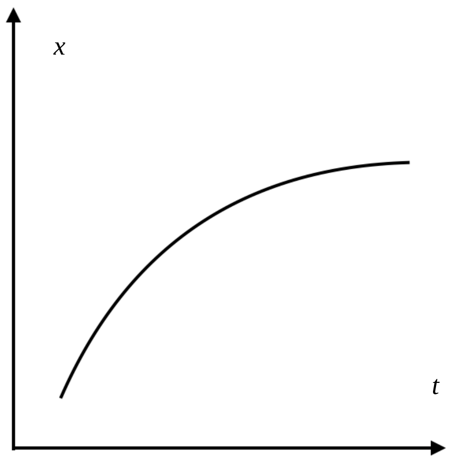
(B) $-9$

(C) 20

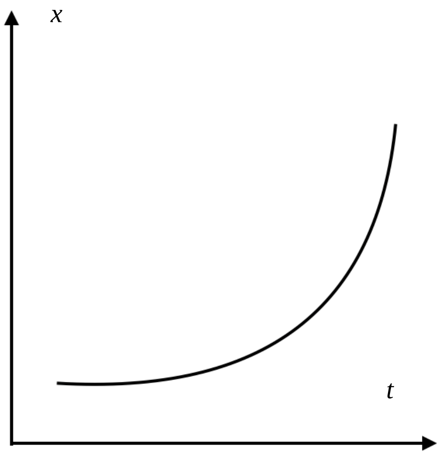
(D) 15

6. A particle is moving so that, for $0 < t < 1$, its velocity is positive, and its acceleration is negative. Which graph could represent the displacement of this particle?

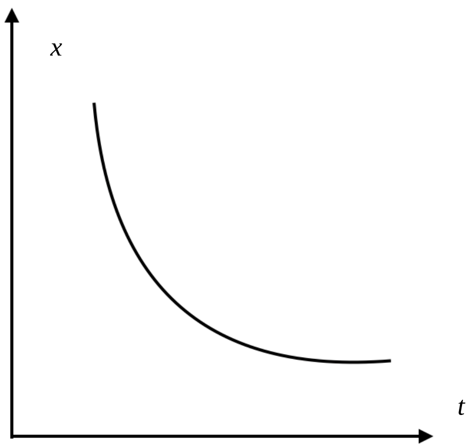
(A)



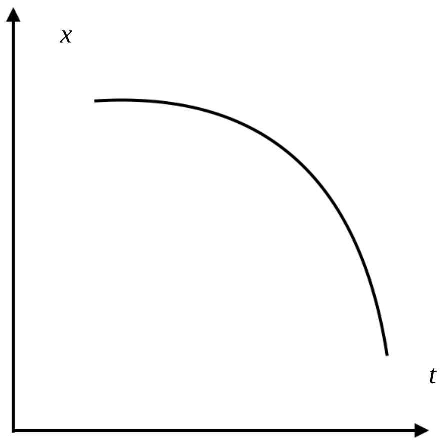

(B)




(C)




(D)




7. Which expression is equivalent to $3x^2 - x - 2$?

(A) $(3x - 1)(x + 2)$

(B) $(3x + 1)(x - 2)$

(C) $(3x - 2)(x + 1)$

(D) $(3x + 2)(x - 1)$

8. Which of the following is equal to the given fraction?

$$\frac{1}{2\sqrt{5}+\sqrt{3}}$$

(A) $\frac{2\sqrt{5}-\sqrt{3}}{7}$

(B) $\frac{2\sqrt{5}+\sqrt{3}}{7}$

(C) $\frac{2\sqrt{5}-\sqrt{3}}{17}$

(D) $\frac{2\sqrt{5}+\sqrt{3}}{17}$

9. Simplify the following:

$$\frac{x}{y}+\frac{y}{x}$$

(A) $x^2+y^2$

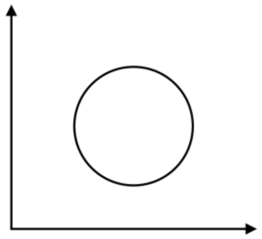
(B) 1

(C) $\frac{x+y}{x-y}$

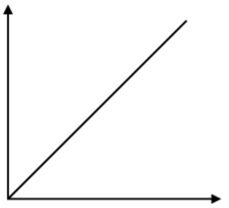
(D) $\frac{x^2+y^2}{xy}$

10. Select the graph that represents a one-to-many relationship.

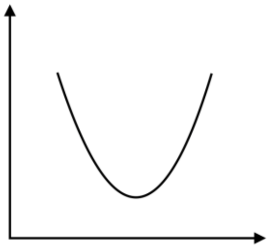
(A)



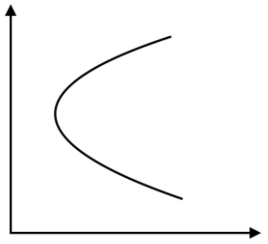
(B)



(C)



(D)



## Section II – Written response

**Attempt All Questions**

Show all necessary working.

**Question 11** (7 marks)

(a) Show algebraically whether $f(x) = x^2 + 2$ is a function that is odd, even or neither. **2**

(b) If $f(x) = x + 1$ and $g(x) = x^2 + x$. What is $g\left[f(c)\right]$ equal to? **2**

(c) For what values of $k$ is the parabola $x^2 - (k+3)x + 4k = 0$ positive definite? **3**

**Question 12** (4 marks)

Given the points $A(1,-2),\ and\ B(-5,6)$

(a) Determine the equation of the line through $AB$ in general form. **2**

(b) What is the angle made by the line AB with the $x-axis$ in the positive direction (to the nearest minute)? **1**

(c) $M\left(\frac{7}{2},-3\right)$ is the midpoint of $A$ and $C$. What are the coordinates of $C$? **1**

**Question 13** (4 marks)

An old fridge costs 90c/day to run. A new fridge is for sale for $800, and only costs 10c/day to run.

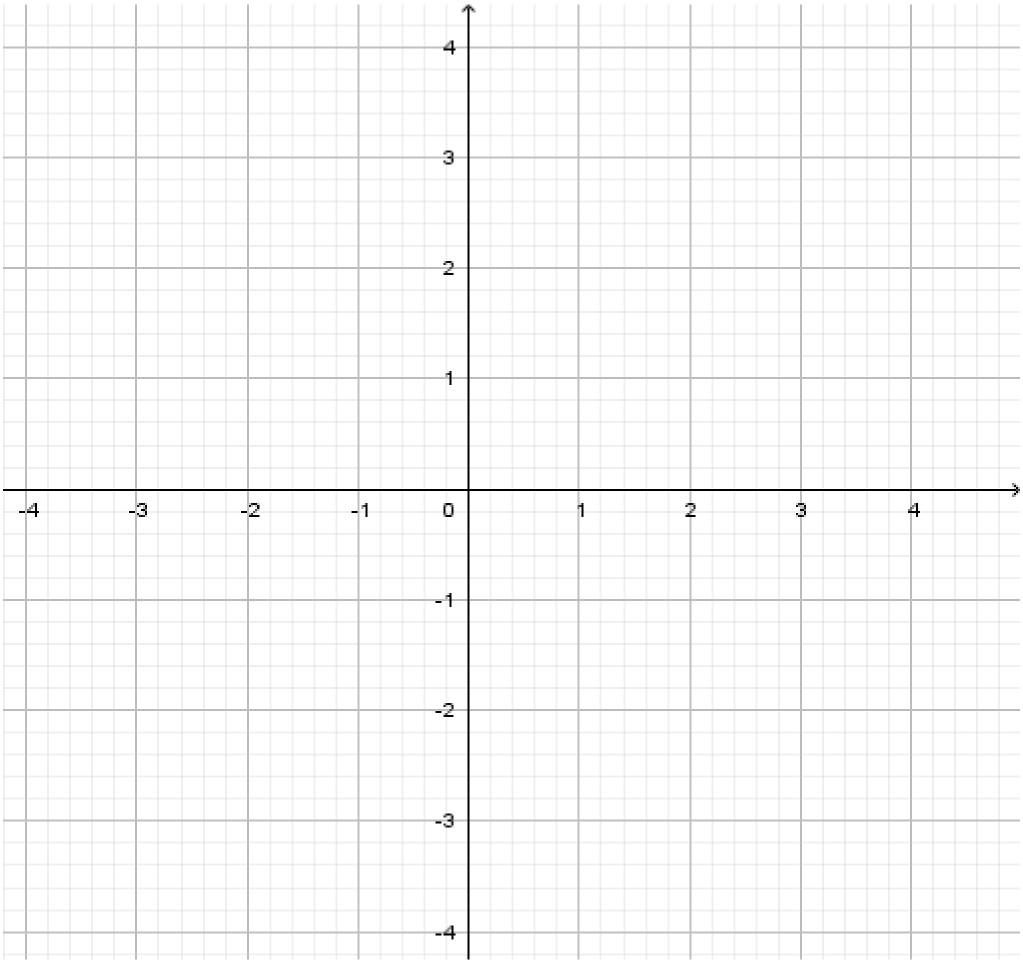
(a) Write an equation for the cost of running the old fridge 'L' in $ per day (*d*). **1**

(b) Write an equation for the cost of running the new fridge 'N' in $ per day. (taking into account the purchase price as part of the cost) **1**

(c) After how many days does the cost of the new fridge 'break-even' with the cost of the old fridge? **2**

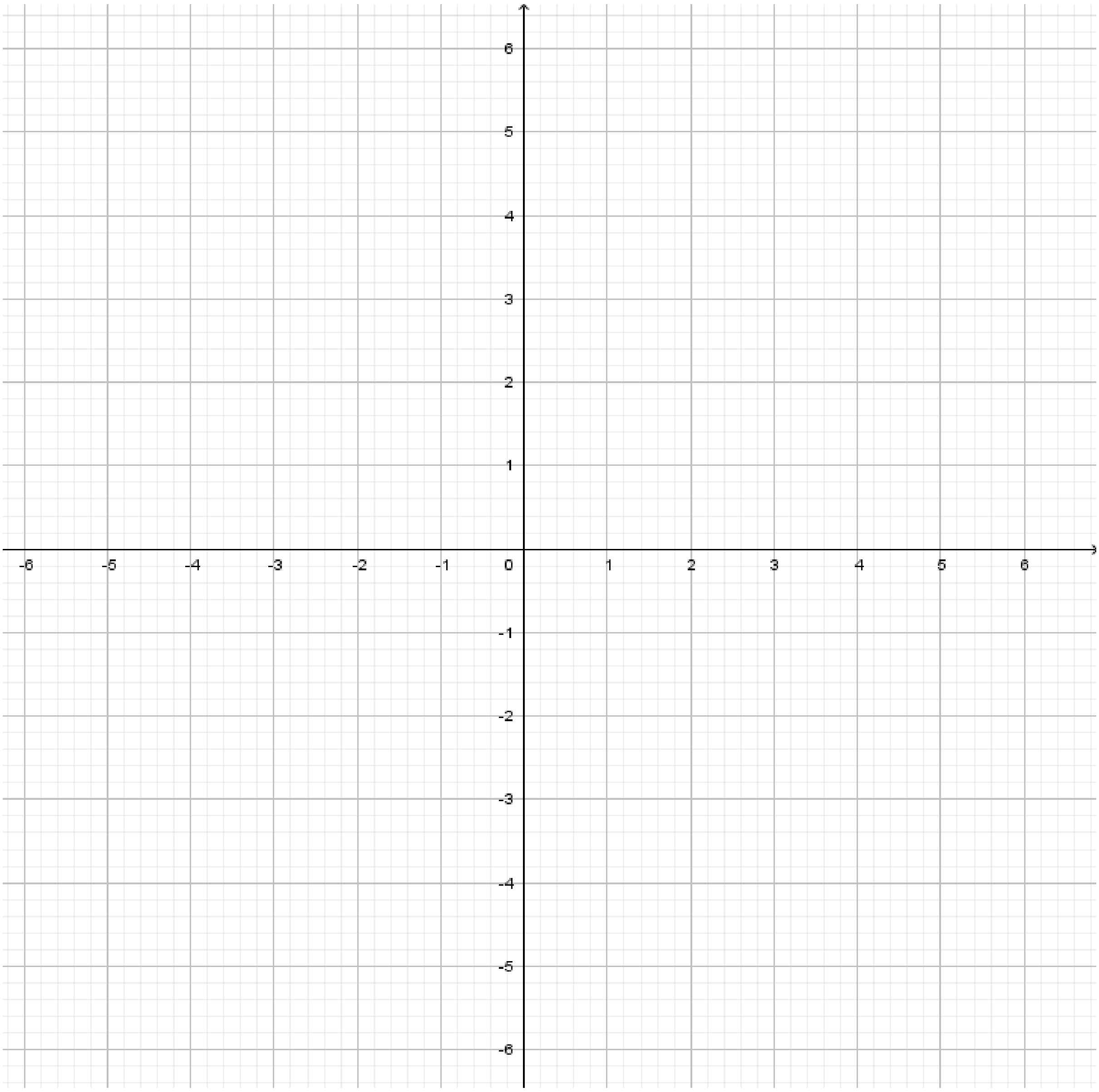
**Question 14** (2 marks)

(a) Sketch the graph of $f(x) = 2(x-1)(x-2)\left(x+\frac{1}{2}\right)$ on the axes below, showing all intercepts. **2**



**Question 15** (3 marks)

(a) Sketch the hyperbola $y = 3 + \dfrac{1}{x-1}$ on the axes below, clearly indicating asymptotes and intercepts. **3**



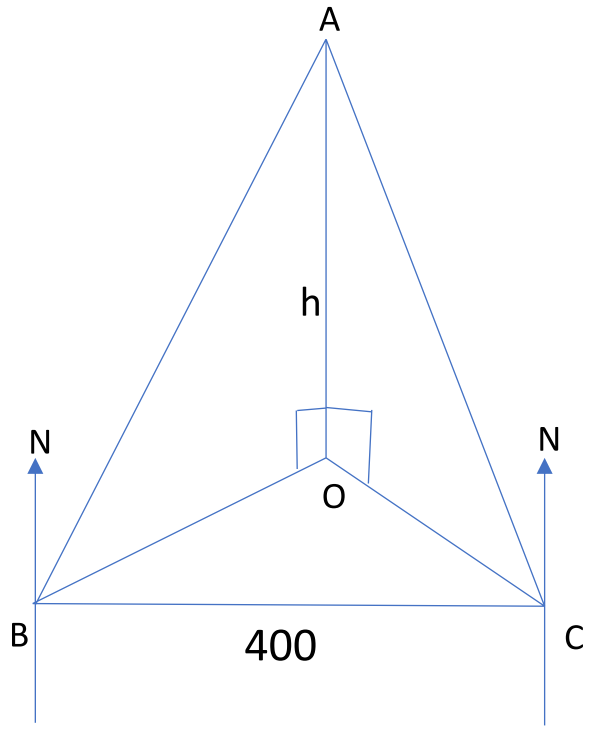
**Question 16** (5 marks)

(a) Solve the equation $|x+4| = 2$. **2**

(b) Find the centre and radius of the circle $x^2 + 4x + y^2 - 2y = 4$. **3**

**Question 17** (5 marks)

Sam walks due East along a straight, level road. At point B he notices a radio antenna (A) on a Bearing of $044^{o}T$, at angle of elevation of $58^{o}$. After walking 400m due East he reaches point C where he notes that the Bearing of the antenna is $322^{o}T$.




(a) Calculate $\angle OBC$ and $\angle OCB$ and label the value of $\angle OBC, \angle OCB$ and $\angle ABO$ on the diagram given above. **2**

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(b) Calculate the length OB to 1 decimal place. **2**

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Question 17 (continued)

(c) Calculate the height ‘h’ of the antenna to 1 decimal place. **1**

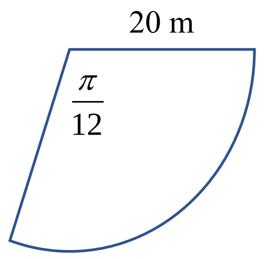
**Question 18** (3 marks)

(a) Solve by completing the square $2x^2 + x - 2 = 0$. **3**

**Question 19** (3 marks)

(a) Convert the formula for the Area of a sector $A = \dfrac{\theta}{360}\pi r^2$ to radians, show all working. **1**

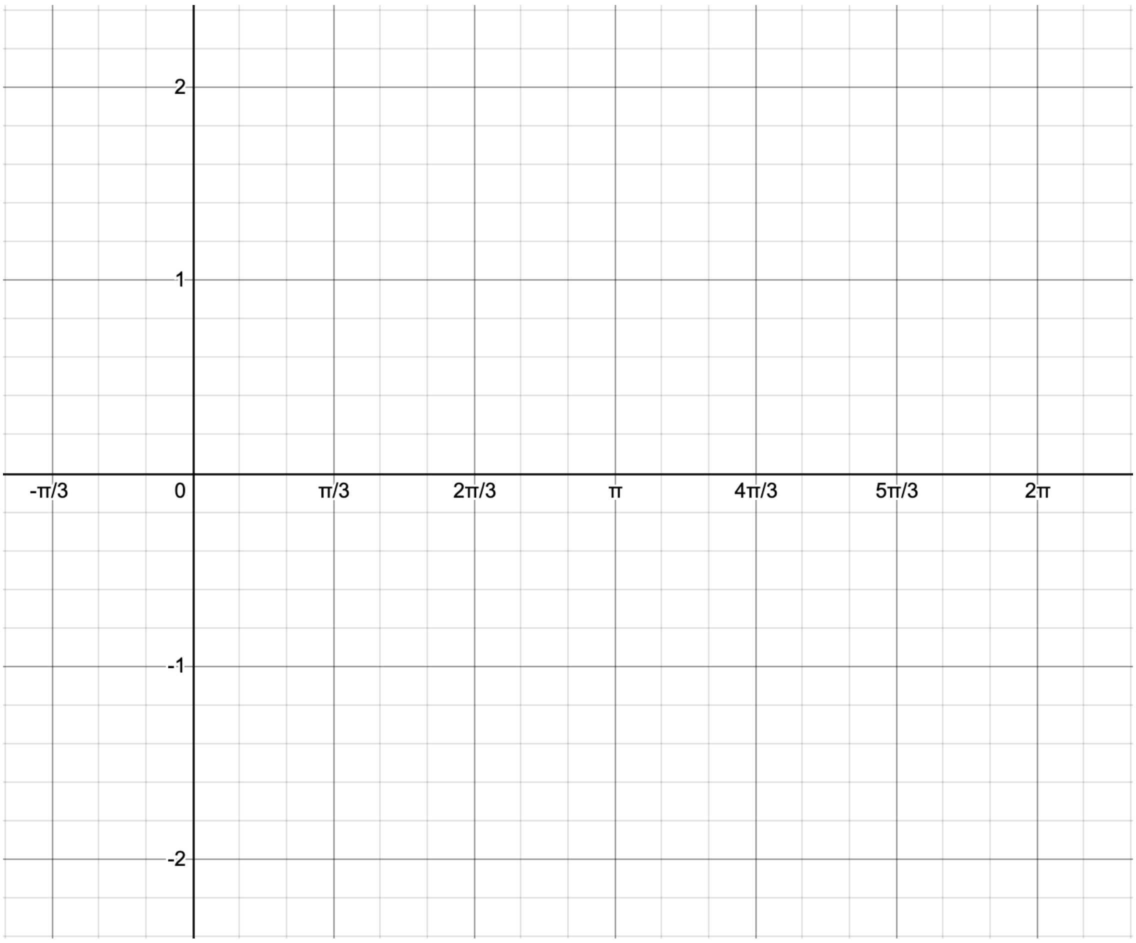
(b) Calculate the Area of the sector shown. **1**




(c) Calculate the arc length of the sector above. **1**

**Question 20** (5 marks)

(a) On the axes below sketch $y = \cos x$ *and* $y = \sec x$ *for* $0 \leq x \leq 2\pi$ . **3**




(b) Solve $\sin 2\theta = \frac{1}{2}$ for $0^o \leq \theta \leq 360^o$. **2**

**Question 21** (3 marks)

(a) Prove: **3**

$$\frac{\sin\theta}{1-\sin\theta}+\frac{\sin\theta}{1+\sin\theta}=2\tan\theta\sec\theta$$

**Question 22** (1 mark)

(a) Differentiate $y = 4x^7$ **1**

**Question 23** (4 marks)

(a) Differentiate the function: **2**

$$f(x) = \sqrt{x}(x - 1)$$

(b) Differentiate the function: **2**

$$f(x) = \frac{x^3 - 2}{x^2}$$

**Question 24** (4 marks)

(a) Differentiate $\sqrt{2x + 1}$ **2**

(b) Hence, show that $y \times y' = 1$. **1**

(c) What does this imply about the behaviour of $y'$ as $y$ gets large? **1**

**Question 25** (3 marks)

(a) Differentiate from first principles, the function: **3**

$$f(x) = \frac{1}{4}x^2$$

**Question 26** (6 marks)

(a) Find the $x$-coordinate of the points on the curve $y = x^3 - 12x + 1$ where the normal has gradient $\frac{1}{9}$. **3**

(b) Show that the gradient of the tangent to the curve $y = \frac{x}{x^2+1}$ is zero at $x = -1$ and $x = 1$. **3**

**Question 27** (2 marks)

For the given function, find the value of *a* so that the function is continuous. **2**

$$f(x) = \begin{cases} 2x + 3 & for & x < 1 \\ -x^2 + a & for & x \geq 1 \end{cases}$$

**Question 28** (7 marks)

A particle's motion can be described by the function $x(t) = 3t^2 - t^3$ where $t$ is measured in seconds.

(a) Find $\dot{x}$ and $\ddot{x}$. **1**

(b) Find times at which the particle is stationary. **2**

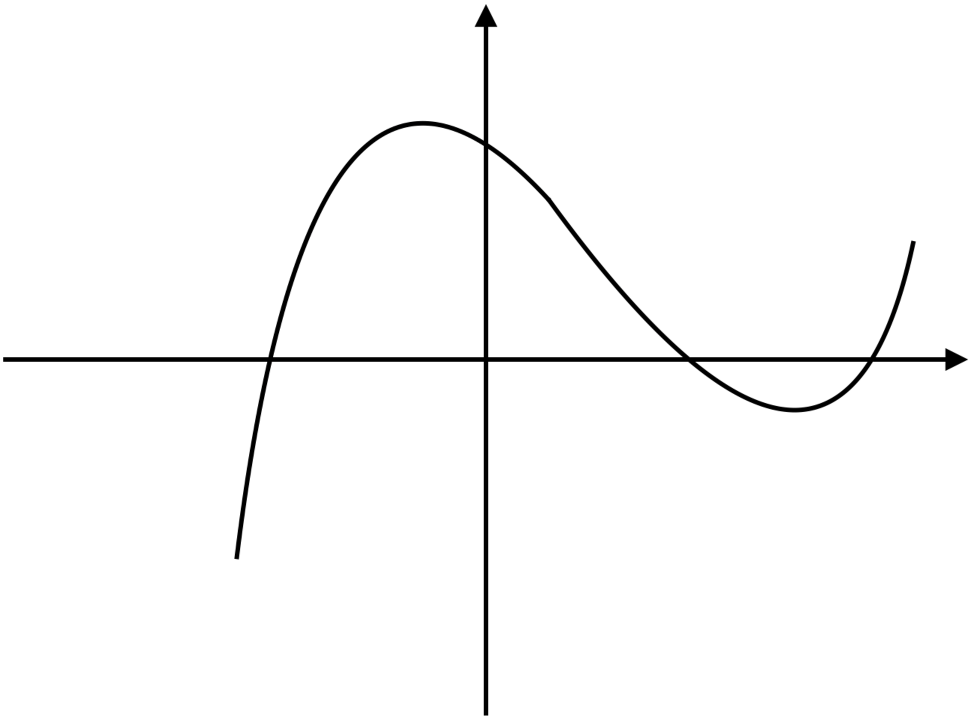
(c) Describe the motion of the particle at time $t = 0$. **2**

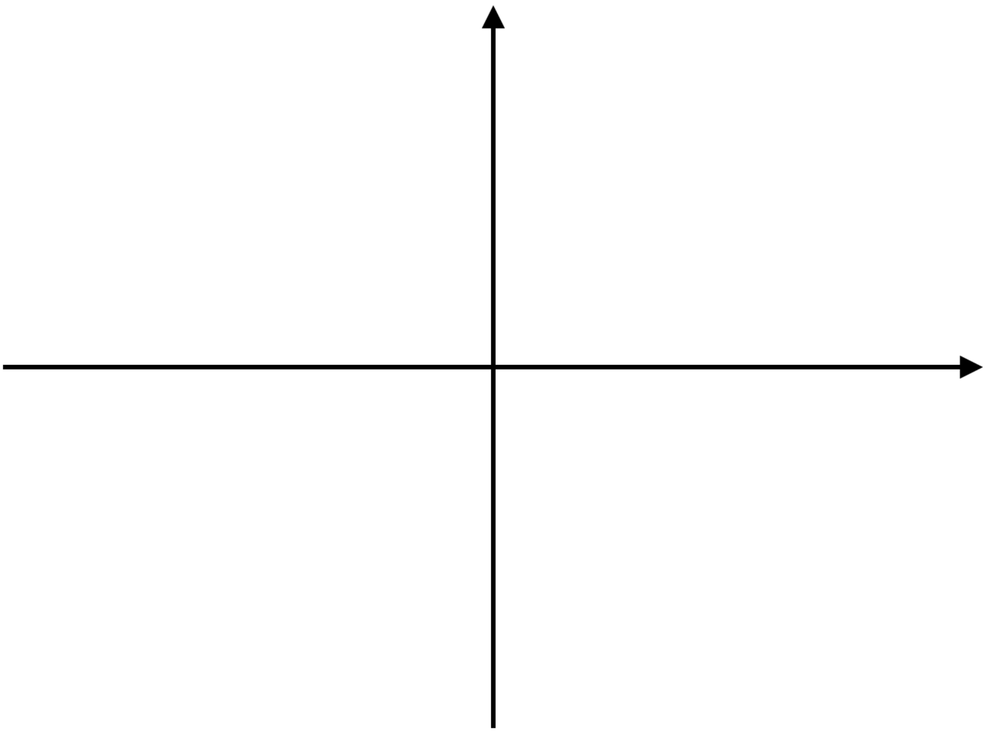
(d) Hence, or otherwise, find the distance travelled by the particle in the first 4 seconds. **2**

**Assessment continues on the next page.**

**Question 29** (9 marks)

(a) Sketch the derivative of the graph given below. **3**





(b) What power would you expect the derivative graph to have? **1**

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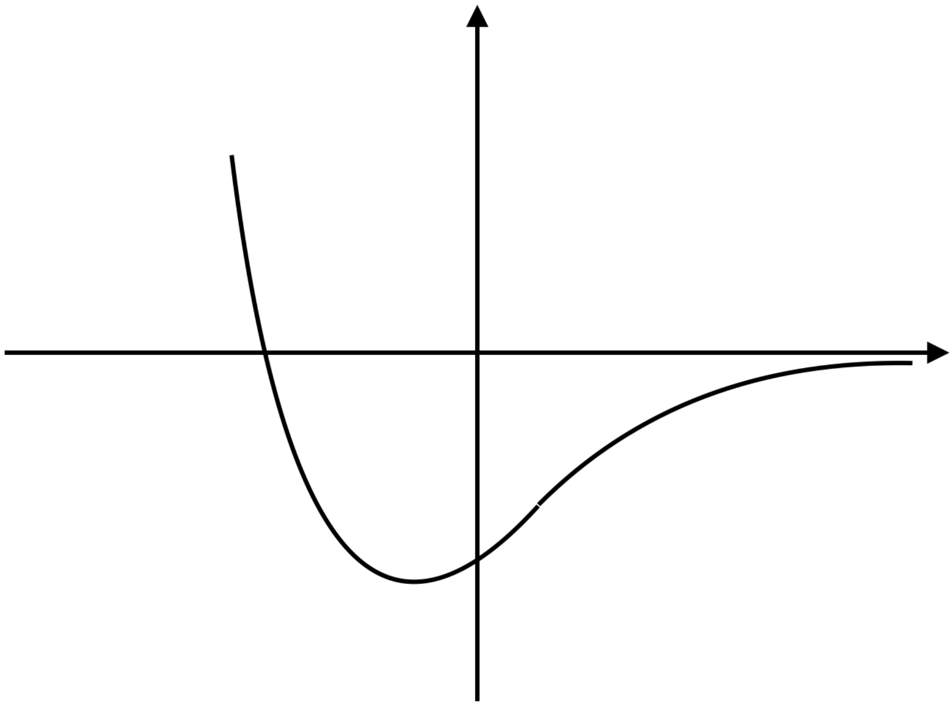
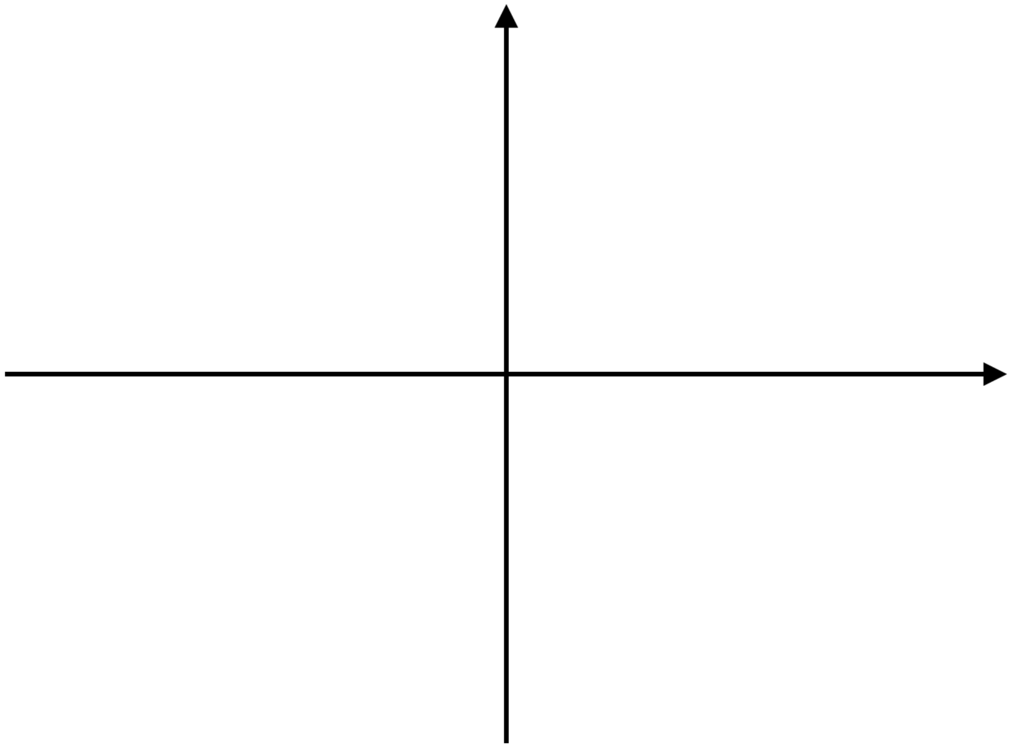
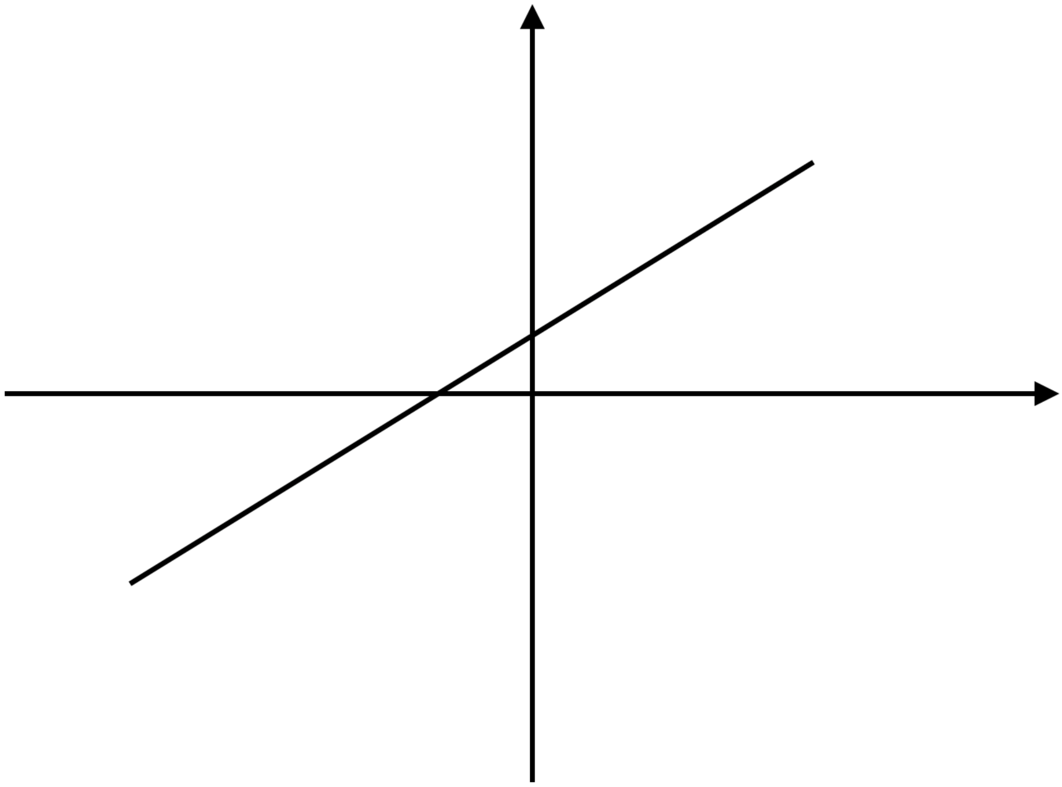
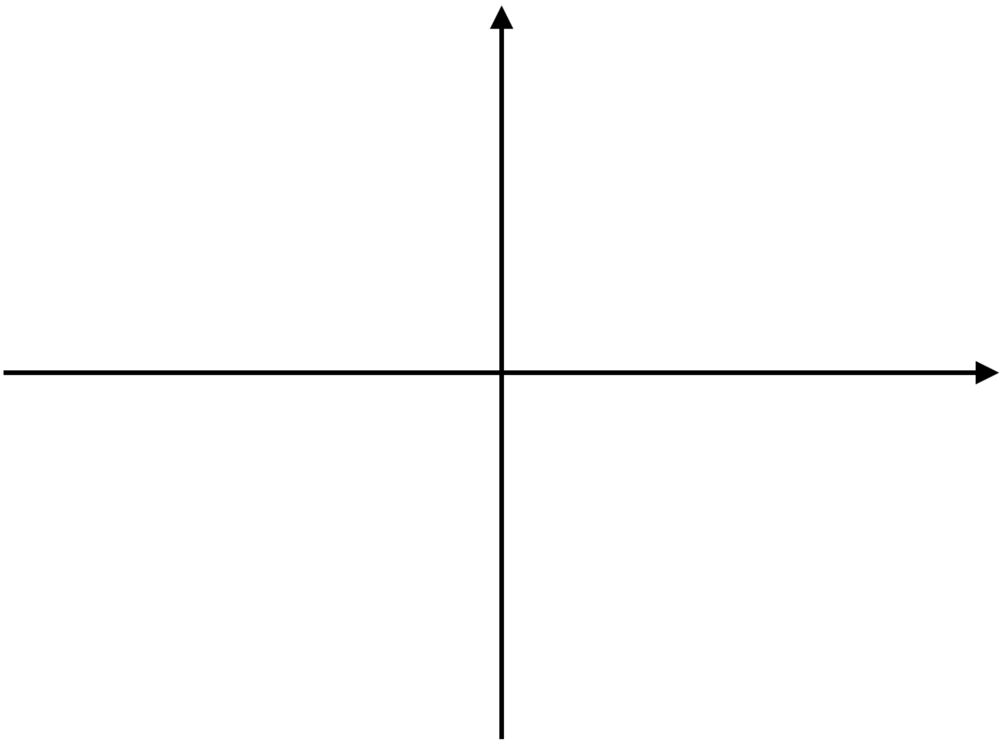
(b) Sketch the derivative of the graph given below. **3**





(c) Sketch the **original** graph, given the **derivative graph** given below. **2**





**Question 30** (3 marks)

(a) Solve $2\sin^2 x + (\sqrt{3} - 2)\cos x + \sqrt{3} - 2 = 0,\ \textit{for}\ 0 \le x \le 2\pi$. **3**

**Question 31** (7 marks)

A pharmaceutical drug is administered and the amount $A$ of the drug in a person's bloodstream is given by the function

$$A = 8t^2 - t^3$$

Where $t$ is the time in hours and $A$ is in milligrams.

(a) Explain why the model is only limited up to a certain value of $t$. **1**

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(b) Find $\frac{dA}{dt}$ and describe its physical meaning. **1**

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(c) When did the body stop absorbing more of the drug? **1**

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(d) During what time interval of $t$ was the drug amount increasing, and during which interval of $t$ was the drug amount decreasing? **2**

(e) What was the highest amount of the drug in the bloodstream? **2**

| A | N | S | W | E | R | | :) | |
|---|---|---|---|---|---|---|---|---|

Student Number

Killara
HIGH SCHOOL
MATHEMATICS DEPARTMENT

| 2019 | Year 11 Mathematics Advanced |
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# Task 3

Year 11 Exam Block week 3 to 5 (Term 3)

**General Instructions**

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- NESA approved calculators may be used
- A formulae and data sheet is provided at the back of this paper
- Show relevant mathematical reasoning and/or calculations

**Total Marks: 100**

- Section I – Multiple Choice – 10 marks
- Section II – Written response 90 marks
- Attempt Questions 1 – 31

***This question paper must not be removed from the examination room.***
*This assessment task constitutes 40% of the course.*

Killara HIGH SCHOOL

Student Number

# Mathematics

## Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 4. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

**Sample:** $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ☒ B ☒ C ○ D ○

Start Here →

| | A | B | C | D | |
|---|---|---|---|---|---|
| 1. | ○ | ● | ○ | ○ | ✓ |
| 2. | ○ | ○ | ● | ○ | ✓ |
| 3. | ○ | ○ | ○ | ● | ✓ |
| 4. | ○ | ○ | ○ | ● | ✓ |
| 5. | ● | ○ | ○ | ○ | ✓ |
| 6. | ● | ○ | ○ | ○ | ✓ |
| 7. | ○ | ○ | ○ | ● | ✓ |
| 8. | ○ | ○ | ● | ○ | ✓ |
| 9. | ○ | ○ | ○ | ● | ✓ |
| 10. | ○ | ○ | ○ | ● | ✓ |

10/10

## Section I - Multiple Choice

**Please answer on the Multiple Choice answer sheet.**

1. To convert $40^{o}$ to radians you calculate :

   (A) $40\times\frac{180}{\pi}$

   (B) $40\times\frac{\pi}{180}$

   (C) $180\times\frac{\pi}{40}$

   (D) $40\times\frac{\pi}{360}$

2. The range $y\geq 0$ is written in interval notation as :

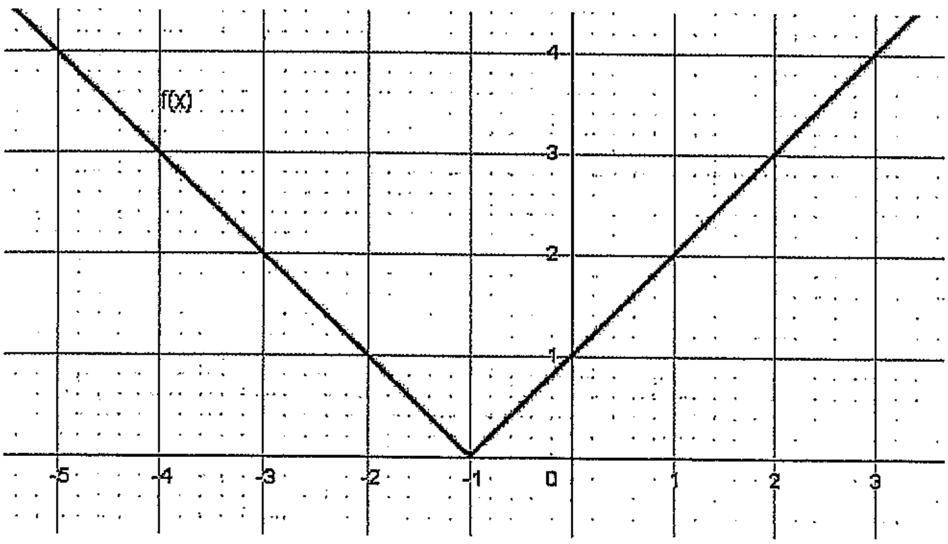
   (A) $y\in(-\infty,\infty)$

   (B) $y\in(0,\infty]$

   (C) $y\in[0,\infty)$

   (D) $y\in(0,\infty)$

3. The graph of $f(x)$ shown below represents :




   (A) $y=|x|+1$

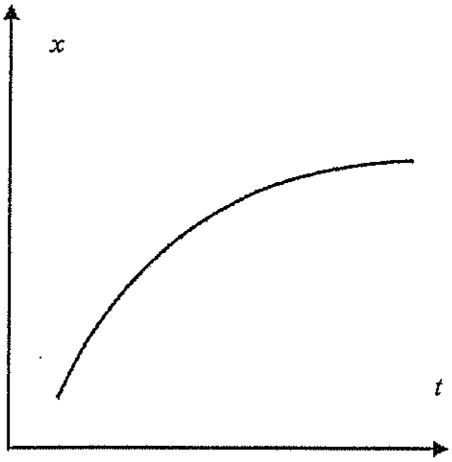
   (B) $y=|x|-1$

   (C) $y=|x-1|$

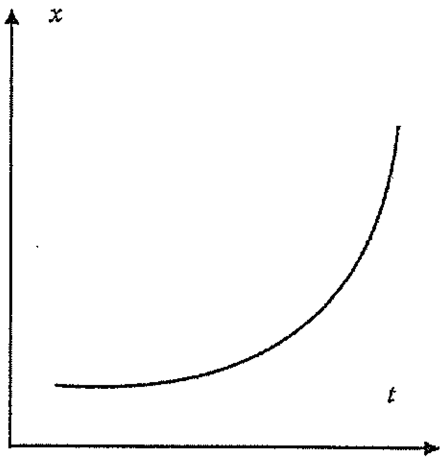
   (D) $y=|x+1|$

6. A particle is moving so that its velocity is positive, and its acceleration is negative. Which graph could represent the displacement of this particle?

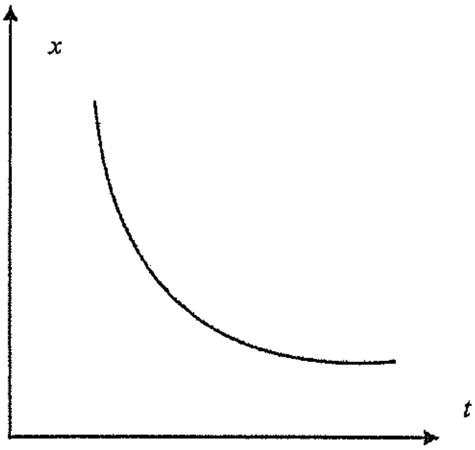
(A)



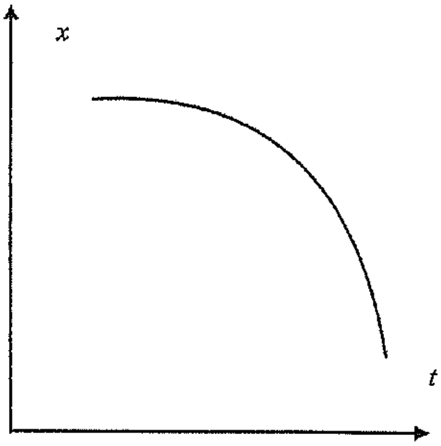

(B)




(C)




(D)




7. Which expression is equivalent to $3x^2 - x - 2$?

(A) $(3x - 1)(x + 2)$ $\quad 3x^2 + \cancel{6x - x} - 2.$

(B) $(3x + 1)(x - 2)$ $\quad 3x^2 - 6x + x.$

(C) $(3x - 2)(x + 1)$ $\quad 3x^2 - 3x + 2x - 2.$

(D) $(3x + 2)(x - 1)$

**Section II – Written response**

**Attempt All Questions**

Show all necessary working.

---

**Question 11** (7 marks)

(a) Show algebraically whether $f(x) = x^2 + 2$ is a function that is odd, even or neither. **2**

$f(-x) = (-x)^2 + 2$

$= x^2 + 2$ [1]

$\therefore$ as $f(x) = f(-x)$

$f(x)$ is EVEN [1]

• If students forgot brackets, 1 mark deducted.
• must state "$f(-x) = f(x)$, $\therefore$ even"
• 0 marks awarded for poor working and correct conclusion.

(b) If $f(x) = x + 1$, *and* $g(x) = x^2 + x$. What is $g[f(c)]$ equal to? **2**

$f(c) = c + 1$

$g(c+1) = (c+1)^2 + c$ [1]

$= c^2 + 2c + 1 + c + 2$

$= c^2 + 3c + 2$ [1]

• 1 mark for $x^2 + 3x + 2$

(c) For what values of $k$ is the parabola $x^2 - (k+3)x + 4k = 0$ positive definite? **3**

Note: $a = 1 \therefore a > 0$

$\Delta = b^2 - 4ac$

$= [-(k+3)]^2 - 4 . 1 . 4(k)$ [1]

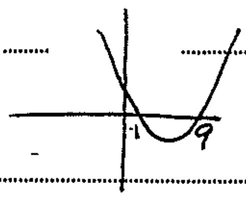
many students forgot the k.

many forgot [ ] resulting in wrong answer.

$= k^2 + 6k + 9 - 16k$

$= k^2 - 10k + 9$

[1] $= (k-1)(k-9)$ $\Rightarrow$



need $\Delta < 0$

[1] $\therefore$ $1 < k < 9$

• many students did not know to find $\Delta < 0$.

**Question 13** (4 marks)

An old fridge costs 90c/day to run.
He is considering buying a new fridge for $800, that only costs 10c/day to run.

(a) Write an equation for the cost of running the old fridge 'L' in $ per day $(d)$ 1

$L = 0\cdot9d$

Some students use $L = 90/d$ instead of $L = 0\cdot90d$

(b) Write an equation for the cost of running the new fridge 'N' in $ per day (taking into account the purchase price as part of the cost) 1

$N = 800 + 0\cdot1d$.

Same mistake $N=800+10d$ instead of $N=800+0\cdot1d$ as part(a)

(c) After how many days does the cost of the new fridge 'break-even' with the cost of the old fridge? 2

$0\cdot9d = 800 + 0\cdot1d$ [1]

$0\cdot8d = 800$

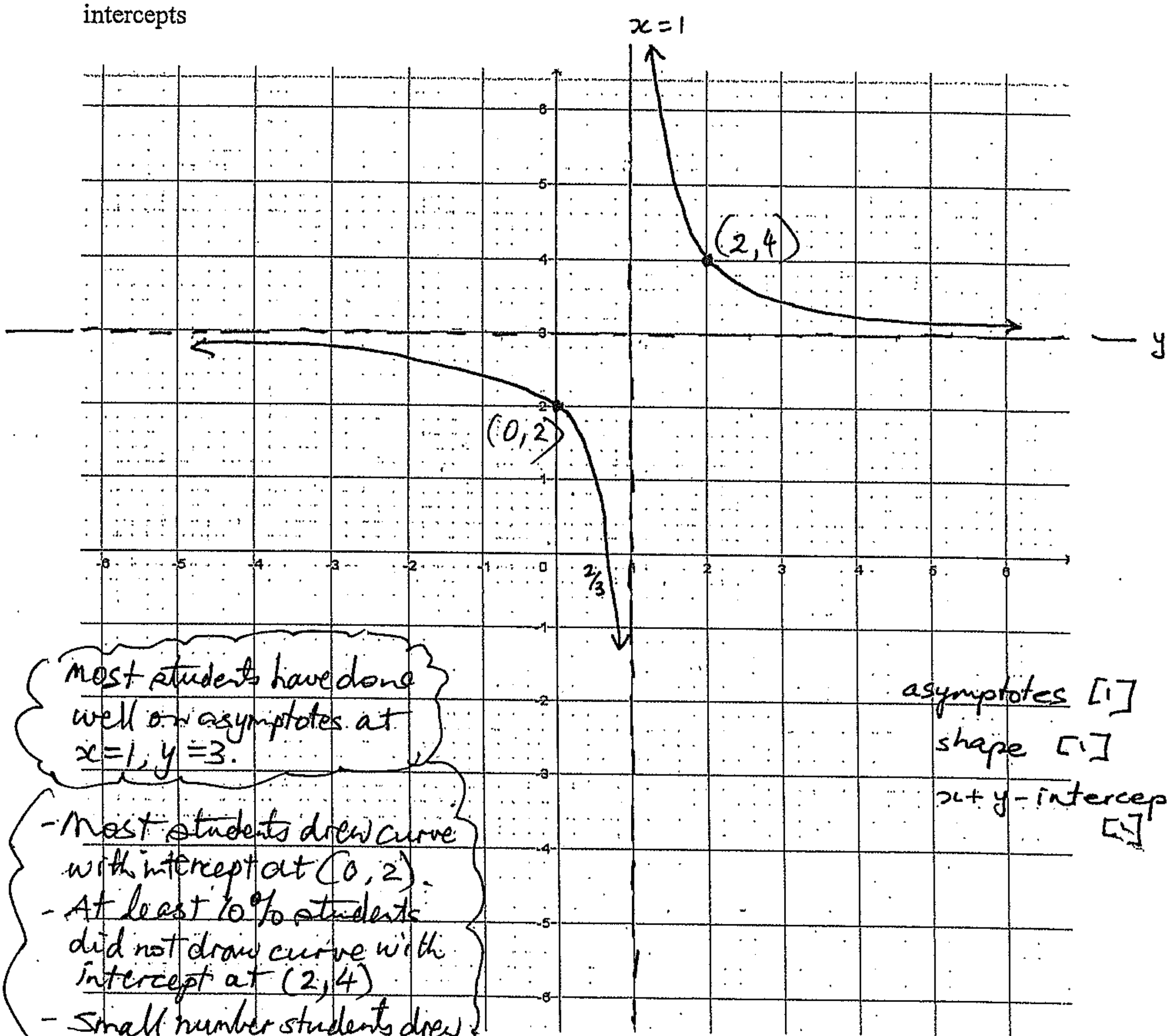
$d = 800 \div 0\cdot8$

$d = 1000$ days [1]

Most of the students did well when the equations for part(a) and (b) were correct

**Question 15** (3 marks)

(a) Sketch the hyperbola $y = 3 + \dfrac{1}{x-1}$ on the axes below, clearly indicating asymptotes and intercepts **3**




most students have done well on asymptotes at $x = 1$, $y = 3$.

- Most students drew curve with intercept at $(0, 2)$.
- At least 10% students did not draw curve with intercept at $(2, 4)$
- Small number students drew the curves away from the asymptotes

asymptotes [1]
shape [1]
x + y - intercep [1]

like $\dfrac{1}{x-1}$ pushed up 3 units


when $y = 0$ (x-int)

$\dfrac{1}{x-1} = -3$

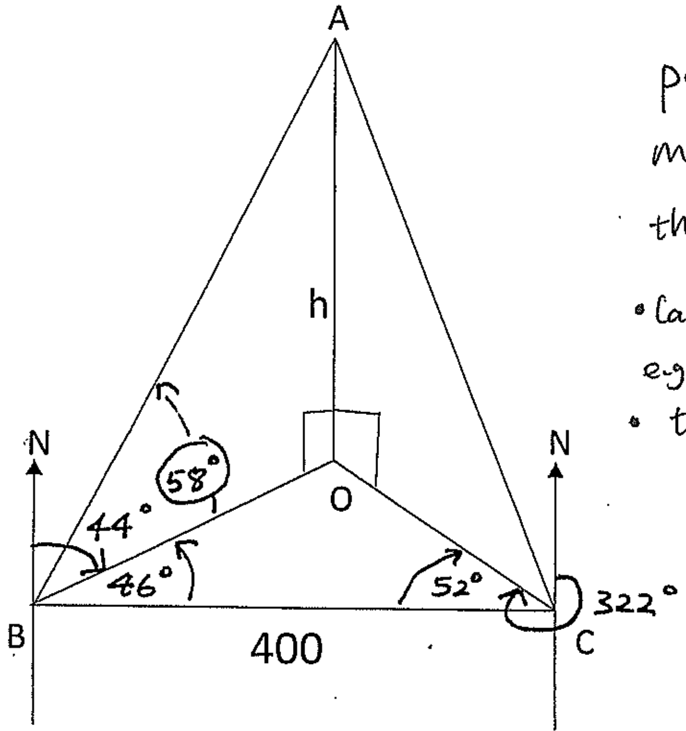
$1 = -3x + 3$

$-3x = -2$

$x = \frac{2}{3}$

**Question 17** (5 marks)

James walks due East along a straight, level road. At point B he notices a radio antenna (A) on a Bearing of $044^{o}T$, at angle of elevation of $58^{o}$. After walking 400m due East he reaches point C where he notes that the Bearing of the antenna is $322^{o}T$.




poorly done.
many students couldnot picture the 3-D graph.

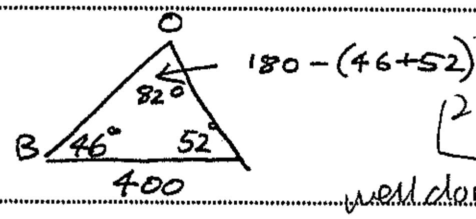
- label bearings & elevation wrong e.g. take $\angle ABC = 44°$
- take $\angle BOC = 90°$

(a) Calculate $\angle OBC$ *and* $\angle OCB$ and label them and $\angle ABO$ on the diagram above. **2**

[1] $\angle OBC = 90 - 44 = 46°$

$\angle OCB = 322 - 270 = 52°$ { all 3 and on diagram } [1]

(b) Calculate the length OB to 1 decimal place. **2**




well done

$\dfrac{OB}{\sin 52°} = \dfrac{400}{\sin 82°}$ [1]

$\therefore OB = \dfrac{400}{\sin 82°} \times \sin 52° = 318.3\text{ m}$ [1]

2 marks awarded for carry forward mistake if Sine rule is used and the calculation is correct

**Question** 19 (3 marks)

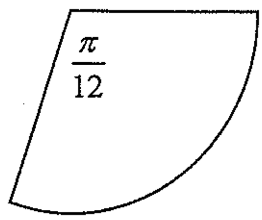
(a) Convert the formula for the Area of a sector $A = \frac{\theta}{360}\pi r^2$ to radians, show all working **1**

lots of students could not see $360° = 2\pi$.

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\theta}{2\cancel{\pi}} \times \cancel{\pi} r^2 = \frac{1}{2} r^2 \theta$ [1]

(b) Calculate the Area of the sector shown below **1**




$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 20^2 \times \frac{\pi}{12}$

$= \frac{50\pi}{3}$ m$^2$ [1]

- majority students ~~assumed the answer~~ in 2 decimal places.

- 1 mark off between b) & c) for answering in decimal places.

(c) Calculate the arc length of the sector above **1**

$l = r\theta$

$= 20 \times \frac{\pi}{12}$

$= \frac{5\pi}{3}$ m [1]

Q21

Generally well done. Quite a few students did not show this step:

$$\frac{2\sin\theta}{\cos^2\theta} = \frac{2\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}$$
$$= 2\tan\theta\sec\theta$$

Q22

Well done.

Q23

Many students struggled with derivative of '$\sqrt{\ }$' or did not remember product rule.

Q24 Generally well done. Quite a few students did not simplify their answer.

**Question 21** (3 marks)

Prove: **3**

$$\frac{\sin\theta}{1-\sin\theta}+\frac{\sin\theta}{1+\sin\theta}=2\tan\theta\sec\theta$$

LHS

$$= \frac{\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}+\frac{\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{\sin^2\theta+\sin\theta}{1-\sin^2\theta}+\frac{\sin\theta-\sin^2\theta}{1-\sin^2\theta} \quad [1]$$

as $1-\sin^2\theta=\cos^2\theta$

$$= \frac{2\sin\theta}{\cos^2\theta} \quad [1]$$

$$= \frac{2\times\sin\theta}{\cos\theta}\times\frac{1}{\cos\theta} = 2\sec\theta\tan\theta \quad [1]$$

**Question 22** (1 mark)

Differentiate $y = 4x^7$ **1**

$y' = 7\times4\times x^6$

$y' = 28x^6$

**Question 24** (4 marks)

(a) Differentiate $\sqrt{2x+1}$ **2**

$y = (2x+1)^{1/2}$

$y' = (2x+1)^{-1/2}$ — ① for use of chain rule

$= \frac{1}{\sqrt{2x+1}}$ — ① for final answer

(b) Hence, show that $y \times y' = 1$. **1**

$y \times y' = \sqrt{2x+1} \times \frac{1}{\sqrt{2x+1}} = 1$

(c) What does this imply about the behaviour of $y'$ as $y$ gets large? **1**

this implies as $y \to \infty$, $y' \to \frac{1}{\infty}$

or $y'$ gets smaller as $y$ gets large.

**Question 25** (3 marks)

Differentiate from first principles, the function: 3

$$f(x) = \frac{1}{4}x^2$$

$f(x) = \frac{1}{4}x^2 \qquad f(x+h) = \frac{1}{4}(x+h)^2$

$= \frac{1}{4}(x^2+2xh+h^2)$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

① for $f(x+h)$

① for correct substitution

$= \lim_{h \to 0} \frac{\cancel{\frac{x^2}{4}} + \frac{xh}{2} + \frac{h^2}{4} - \cancel{\frac{x^2}{4}}}{h}$

① for taking the limit.

$= \lim_{h \to 0} \frac{\cancel{h}\left(\frac{x}{2} + \frac{h}{4}\right)}{\cancel{h}} \quad = \lim_{h \to 0} \left(\frac{x}{2} + \frac{h}{4}\right) = \boxed{\frac{x}{2}}$

[question was poorly completed some students did not understand the meaning of the limit.

3/

**Question 26** (6 marks)

(a) Find the $x$-coordinate of the points on the curve $y = x^3 - 12x + 1$ where the normal has gradient $\frac{1}{9}$. 3

$M_N = \frac{1}{9} \qquad y = x^3 - 12x + 1$

$M_T = -9 \qquad y' = 3x^2 - 12$

① for $M_T = -9$

$y' = -9 = 3x^2 - 12$

① for derivation

~~$3x^2 = 4$~~

~~$x^2 = \frac{4}{3}$~~

~~$x = \pm\frac{2}{\sqrt{3}}$~~

$3 = 3x^2$

$x^2 = 1$

$x = \pm 1$

① for 2 answers.

with $\frac{1}{9}$.

$\pm\sqrt{\frac{109}{27}}$

$\pm 2.01$

3/

[Some students forgot $\pm$
Many forgot to find the perpendicular gradient.

**Question 27 (2 marks)**

For the given function, find the value of a so that the function is continuous.

$$f(x) = \begin{cases} 2x+3 \text{ for } x<1 \\ -x^2+a \text{ for } x \geq 1 \end{cases}$$

**Solution:**

$$2x+3 = -x^2+a \quad \text{(1 mark recognising stating/condition of continuity)}$$
$$2(1)+3 = -(1)^2+a$$
$$5 = -1+a$$
$$a = 6 \quad \text{(1 mark for answer)}$$

**Marker feedback:** Student must state condition of continuity by somehow indicating equality, then correctly solved for $a$. A concerning number of students found $a = 4$.

**Question 28 (7 marks)**

A particle's motion can be described by the function $x(t) = 3t^2 - t^3$ where $t$ is measured in seconds.

(a) Find $\dot{x}$ and $\ddot{x}$.

**Solution:**

$$x(t) = 3t^2 - t^3$$
$$\dot{x} = 6t - 3t^2$$
$$\ddot{x} = 6 - 6t \quad \text{(1 mark for both correct)}$$

**Marker feedback:** Mostly well done.

(b) Find times at which the particle is stationary.

**Solution:**

$$\dot{x} = 0 \text{ for stationary}$$
$$6t - 3t^2 = 0 \quad (1 \text{ for statement } \dot{x} = 0)$$
$$3t(2-t) = 0$$
$$t = 0\,,\ t=2 \quad \text{(1 for both answers)}$$

**Marker feedback:** A concerning number of students did not factorise correctly to find two solutions.

(c) Describe the motion of the particle at time $t = 0$.

**Solution:** at $t = 0$

$x = 0$

$v = \dot{x} = 0$

$a = \ddot{x} = 6$

The particle is stationary at the **origin** ($x = 0$ **must be mentioned**), but experiencing acceleration in the positive direction, so it is about to **start moving** ($v = 0$ **must be mentioned**) in the positive direction.

- 1 mark awarded without calculations as long as both **origin** and **start moving** mentioned
- 0 marks "Changing direction"
- 0 marks "not moving but at increasing rate"

(b) Show that the gradient of the tangent to the curve $y = \frac{x}{x^2+1}$ is zero at $x = -1$ and $x = 1$. **3**

$y = \frac{x}{x^2+1}$ $\frac{u}{v}$ $u = x$ $v = x^2+1$

$u' = 1$ $v' = 2x$.

$y' = \frac{(x^2+1)\cancel{4} - 2x^2}{(x^2+1)^2}$.

$= \frac{-\cancel{2}x^2+1}{(x^2+1)^2}$ —① for quotient

$y' = 0 = \frac{-\cancel{3}x^2+1}{(x^2+1)^2}$ —① for equating to zero Not assuming

when $x^2 = 1$ —① for final answer

$x = \pm 1$ as required with full working

**Question 27** (2 marks)

For the given function, find the value of $a$ so that the function is continuous. **2**

$$f(x) = \begin{cases} 2x+3 & for \quad x < 1 \\ -x^2+a & for \quad x \geq 1 \end{cases}$$

at 1 $2x+3 = -x^2+a$ —① for

$2.1+3 = -(1)^2+a$ recognising/stating

$5 = -1+a$. condition for continuity

$a = 6$

—① for answer

(d) Hence, or otherwise, find the distance travelled by the particle in the first 4 seconds. **2**

$x(0) = 0$

$x(2) = 3 \times 2^2 - 2^3$ — ① for

$= 4$ recognising the

$x(4) = 3 \times 4^2 - 4^3.$ particle changes direction

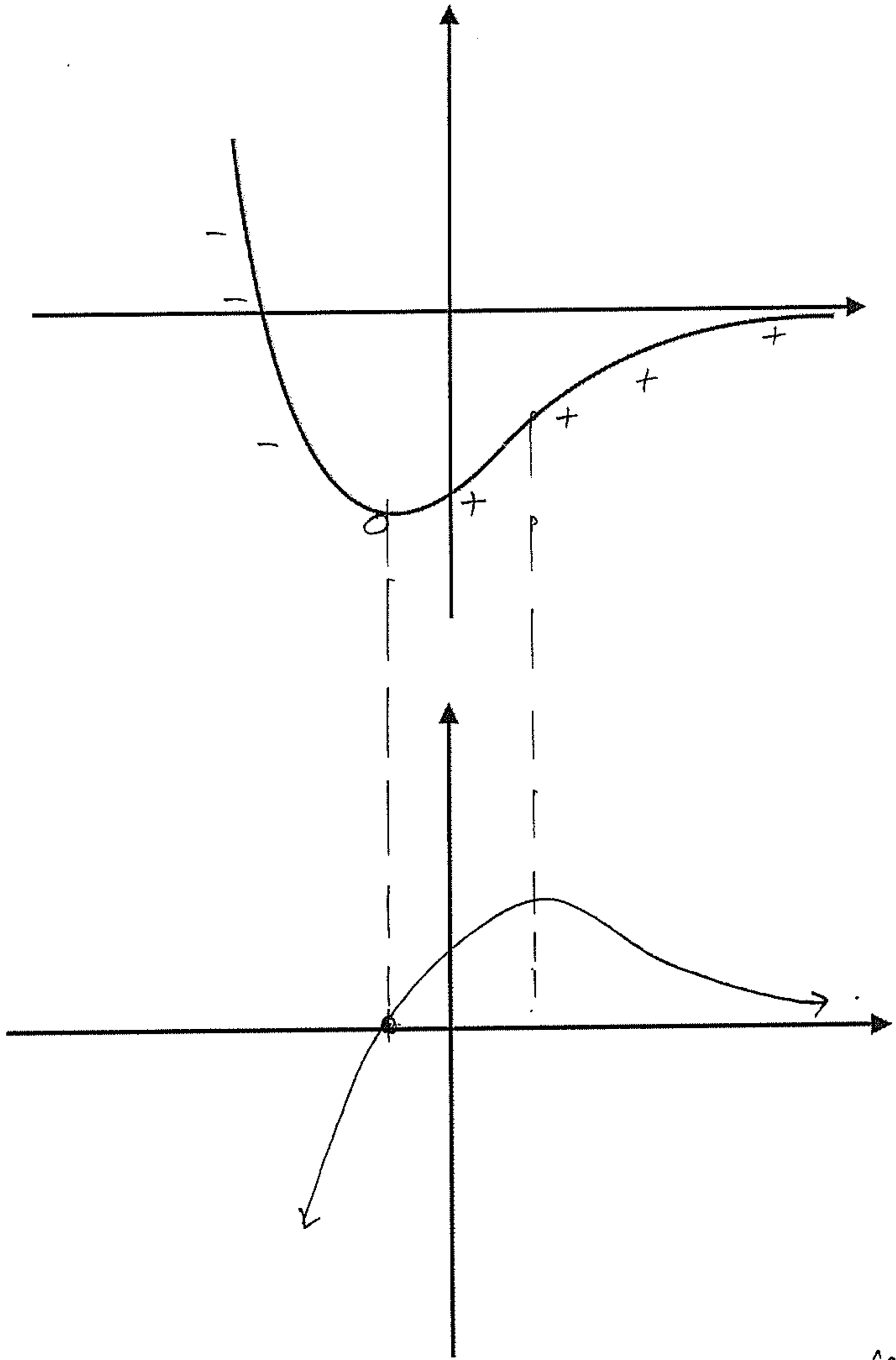
$= -16.$

particle has travelled ~~20~~ 24 units

— ① for correct answer with correct working

**Assessment continues on the next page.**

(b) Sketch the derivative of the graph given below. **3**



1 mark for shape.

1 mark intercepts

1 mark: asymptotes.

Match up corresponding features.

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Assessment continues on the next page

(d) During what time interval of $t$ was the drug amount in the bloodstream increasing, and during which interval of $t$ was the drug amount decreasing? 2

~~$A = 0$~~ Some students set $\frac{d^2A}{dt^2} = 0$ to get the answer?

$0 < t < \frac{16}{3} \rightarrow$ increasing

$16/3 < t < 8 \rightarrow$ decreasing.

(e) What was the highest amount of the drug in the bloodstream? 2

when $t = \frac{16}{3}$

$A = 8 \times \left(\frac{16}{3}\right)^2 - \left(\frac{16}{3}\right)^3$

$= \frac{2048}{27}$ ✓

awarded full marks for ~~and~~ the incorrect value of t carried from part (d)

most of the students got this correct