Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2020 Year 11

Mathematics Advanced

Task 3

Date 31/08/2020

Q	Marks
MC	
11	
12	
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31	
Total	

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I - 10 marks

- Allow about 15 minutes for this section

Section II - 90 marks

- Allow about 2 hours and 45 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 40% of the course.

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Section I

10 marks
Allow about 15 minutes for this section

Use the multiple-choice sheet for Question 1–10

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1 What is the value of $f'(1)$ if $f(x) = x^2(x-2)$?

(A) -2

(B) -1

(C) 1

(D) 7

2 Which of the following is *not* a one-to-one function?

(A) $y = 3x^2 - 3x, \ \ x \geq 1$

(B) $y = \sqrt{3 - x^2}$

(C) $y = 3x^2, \ \ x \geq 0$

(D) $y = 3x$

3 If the parabola $y = ax^2 + bx + c$ is positive definite, which one of the following statements is true?

(A) $a > 0$ and $\Delta > 0$

(B) $a > 0$ and $\Delta < 0$

(C) $a < 0$ and $\Delta > 0$

(D) $a < 0$ and $\Delta < 0$

Section I continues on next page

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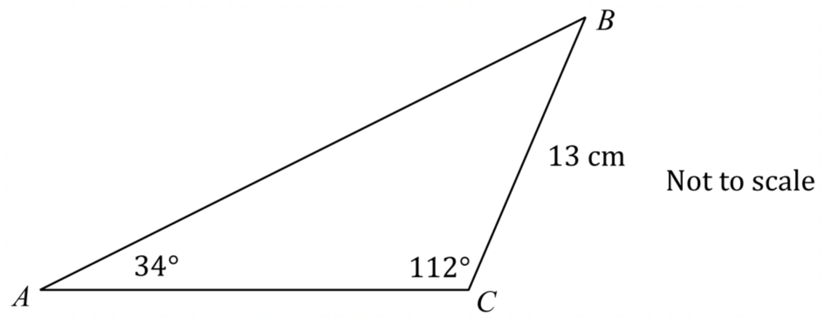


What is the value of AB, correct to one decimal place?

(A) 13.0 cm

(B) 20.9 cm

(C) 21.6 cm

(D) 40.6 cm

5 The line $5x - y - 4 = 0$ passes through the point (k, 26). What is the value of k?

(A) 3

(B) 4

(C) 5

(D) 6

6 If $\dfrac{4}{x-3} + \dfrac{2}{x} = 1, \quad x > 0$ then x is equal to:

(A) $\dfrac{9 \pm \sqrt{57}}{2}$

(B) $\dfrac{9 + \sqrt{57}}{2}$

(C) $\dfrac{9 + \sqrt{105}}{2}$

(D) $\dfrac{9 \pm \sqrt{105}}{2}$

Section I continues on next page

7 What is the derivative of

$$x^{-2} - \frac{1}{x}?$$

(A) $\frac{-2}{x} - \frac{1}{x^2}$

(B) $\frac{-2}{x} + \frac{1}{x^2}$

(C) $\frac{-2}{x^3} - \frac{1}{x^2}$

(D) $\frac{-2}{x^3} + \frac{1}{x^2}$

8 If $\cos\beta = -\frac{2}{5}$ and $\sin\beta < 0$, find the exact value of $\tan\beta$

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{21}}{2}$

(C) $-\frac{\sqrt{21}}{5}$

(D) $\frac{\sqrt{29}}{2}$

9 If f and g are two functions such that $f(x) = 2x$ and $g(x) = 3x + 1$.
The function $f\big(g(x)\big)$ is:

(A) $6x + 1$

(B) $6x + 2$

(C) $6x - 10$

(D) $6x^2 + 1$

10 The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points.
What is the value of k ?

(A) 5

(B) 6

(C) 25

(D) 36

End of Section I

Section II

Allow about 2 hours and 45 minutes for this section

In this section, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (4 marks)

(a) Rationalise the denominator $\dfrac{\sqrt{5}}{4\sqrt{3}+2}$ **2**

(b) Express $(2\sqrt{3}+1)(2-\sqrt{3})$ in the form $a+b\sqrt{3}$, where a and b are integers. **2**

Question 12 (4 marks)

Fully factorise the following expressions.

(a) m^4-1 **2**

(b) $3^x + 3^{x+2}$ **2**

Question 13 (3 marks)

Solve by completing the square. (Leave your answer in surd form) **3**

$$2x^2 - 14x - 1 = 0$$

Question 14 (5 marks)

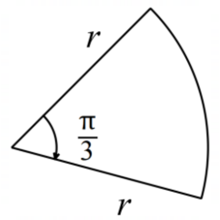
(a) Convert $130°$ into radians (Leave your answer in exact form). **1**

(b) Solve the equation $2\cos\theta = \sqrt{3}$ in the interval $0 \leq \theta \leq 360°$. **2**

(c) Solve the equation $\sin\theta = -\frac{1}{\sqrt{2}}$ in the interval $x \in [0, 2\pi]$. (Leave your answer in terms of π) **2**

Question 15 (2 marks)

The sector below has an area of 3π square metres. **2**



Find the length of the radius in metres. Leave your answer in exact form.

Question 16 (2 marks)

Find the exact value of **2**

$$\sec^2(330°)$$

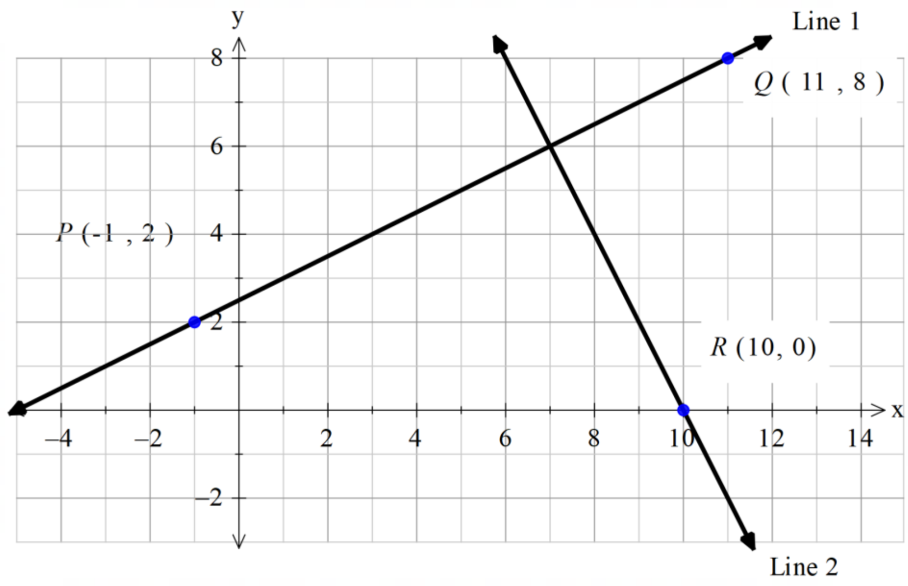
Question 17 (4 marks)

Find the domain and range of:

(a) $y = |x| + 1$ **2**

(b) $f(x) = \sqrt{4 - x^2}$ **2**

Question **18** (6 marks)



(a) Line 1 (l_1) passes through the points $P(-1, 2)$ and $Q(11, 8)$. Find an equation for l_1 in the form $y = mx + c$, where m and c are constants. **2**

(b) Line 2 (l_2) passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S. Show that the coordinates of S are (7,6). **3**

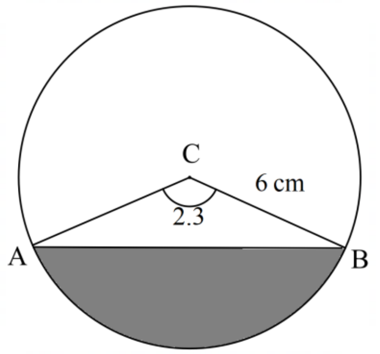
(c) Find the length of RS. (Leave your answer in exact form) **1**

Question 19 (4 marks)

(a) Does $y^2 = x + 1,$ for $x \in [-1, \infty)$ represent a function? Justify your answer. **2**

(b) Test whether $f(x) = \frac{6x^3}{x^2+x^4}$ is odd, even or neither. Justify your answer. **2**

Question 20 (3 marks)



In a circle of radius 6 cm, C is the centre. AC and BC are both radii of the circle.
∠ACB is 2.3 radians.

(a) What is the length of the minor arc AB? **1**

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(b) Calculate the size of the shaded area, correct to two decimal places. **2**

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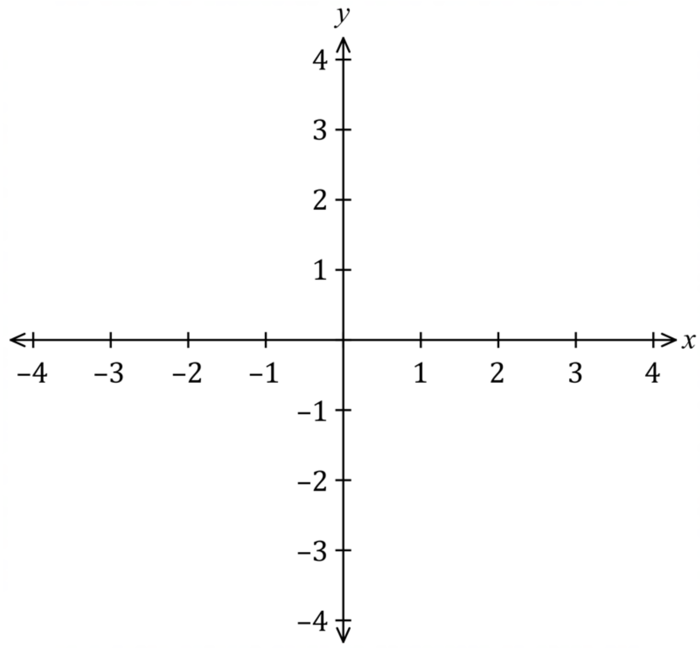
Questions 1-20 are worth 47 marks in total.

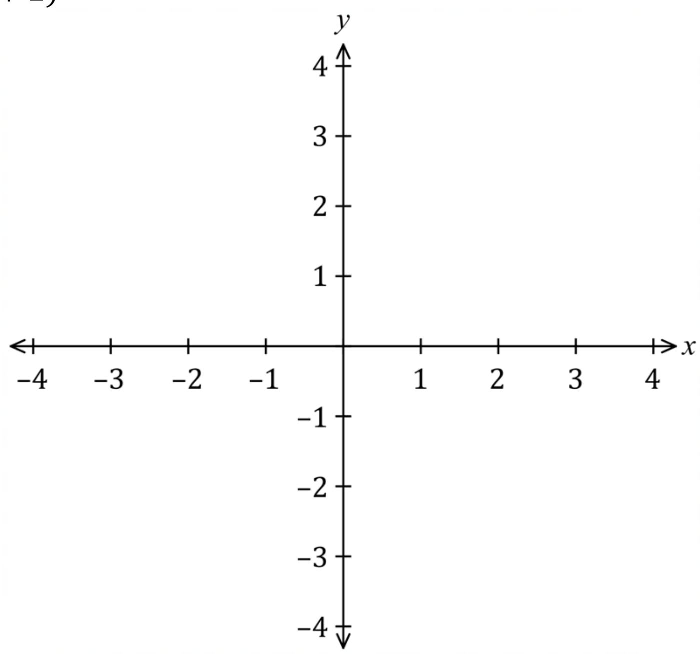
Question 21 (5 marks)

Sketch the graphs of the following functions, showing important features:

(a) $y = \dfrac{3}{2x - 1}$ **2**



(b) $y = (x - 1)^2(x + 2)$ **3**



Question 22 (6 marks)

Differentiate, with respect to x.

(a) $f(x) = x^2(4x + 1)^3$ **2**

(b) $f(x) = (5 + x^2)^{\frac{1}{2}}$ **2**

(c) $f(x) = \dfrac{3x^2 - 5}{2x + 1}$ **2**

Question 23 (2 marks)

Find the values of m for which the equation below has real roots: **2**

$$x^2 + 2mx + (3m - 2) = 0$$

Question 24 (3 marks)

A student was asked to differentiate $f(x) = 3x^2 + 7$ from first principles. **3**

The student began the solution as shown below.

Complete the solution.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Question 25 (2 marks)

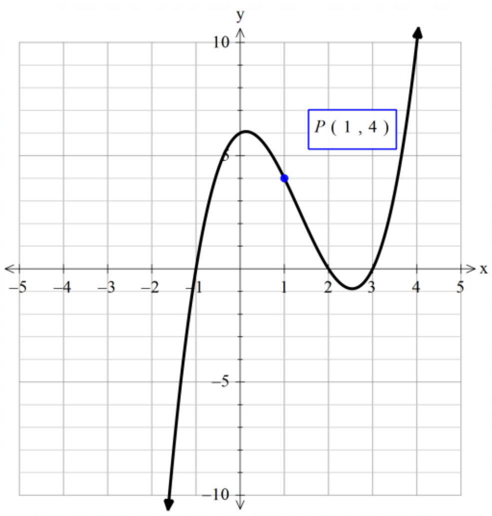
Find the equation of the axis of symmetry of the parabola $y = 5 + 4x - x^2$ and hence find the maximum value of the expression $5 + 4x - x^2$. **2**

Question 26 (2 marks)

Prove **2**

$$\frac{1 - \sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$$

Question 27 (7 marks)



(a) $P(1, 4)$ lies on the curve $y = x^3 - 4x^2 + x + 6$. Find the equation of the tangent to the curve at P. **2**

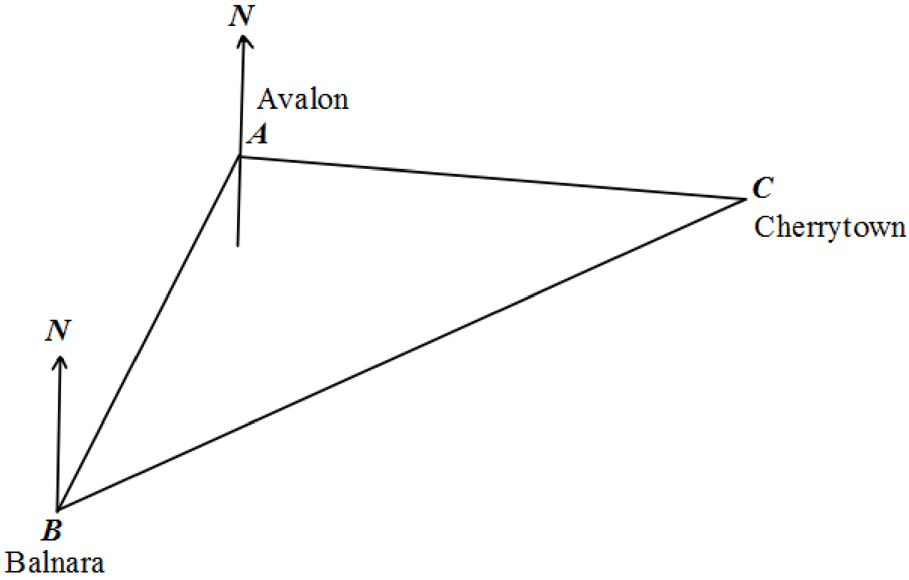
(b) Find the equation of the normal to the curve at P, leave your answer in general form. **2**

(c) The tangent and normal at P intersect the x-axis at T and N respectively. **2**
What are the coordinates of T and N?

(d) Find the area of triangle PTN. **1**

Question **28** (6 marks)

The diagram shows the path of a plane flying from Avalon to Balnara, a distance of 340 kilometres, on a bearing of $220°T$. At Balnara, the plane changed course and flew to Cherrytown, a distance of 620 kilometres on a bearing of $080°T$.



(a) On the diagram above, show the bearings and distances. **1**

(b) Why is $\angle ABC = 40°$? Show clearly on the diagram and give reasons. **1**

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(c) Calculate to the nearest kilometre, the distance the plane has to travel to fly directly from Cherrytown back to Avalon. **2**

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(d) Calculate the bearing from Cherrytown to Avalon. **2**

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Question 29 (6 marks)

A particle moves along a straight line so that its displacement x metres at time t seconds is given by

$$x = t^2 - 7t + 3$$

(a) Find the initial position of the particle. **1**

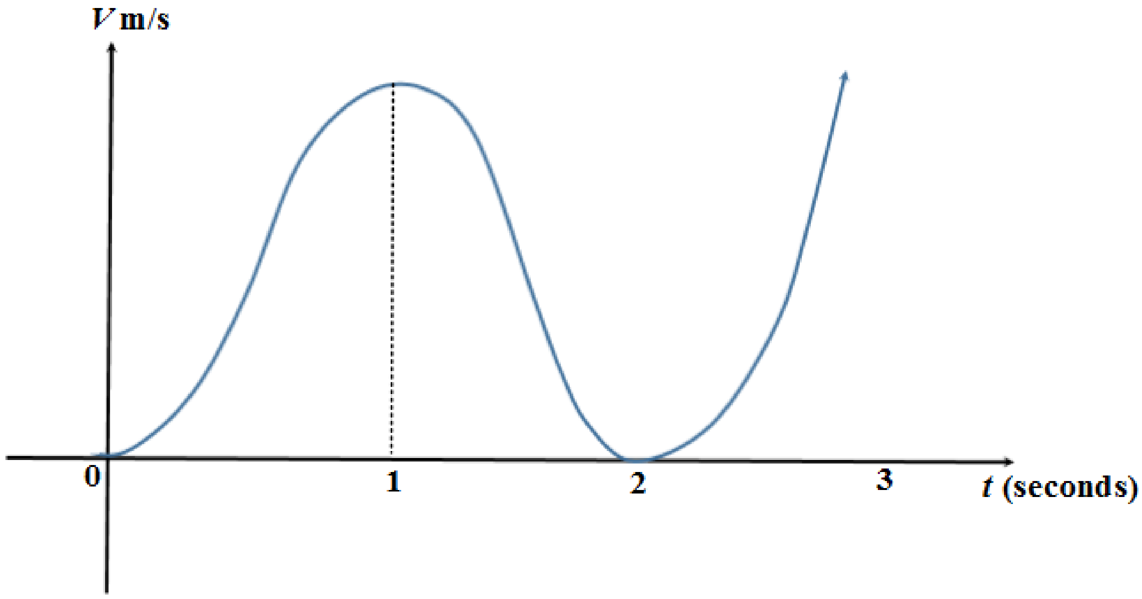
(b) When is the particle at rest? **1**

(c) When is the particle furthest from the origin in the first 8 seconds? **2**

(d) What is the total distance travelled by the particle in the first 8 seconds. **2**

Question 30 (8 marks)

The velocity–time graph of a particle starting from rest from the origin is shown in the diagram.



(a) When is the particle at rest again? **1**

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(b) Does the particle change its direction during the motion? Give reasons. **2**

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(c) Give the first time that the acceleration of the particle is zero. **1**

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(d) Give a physical description of the movement of the particle between $t = 0$ and $t = 1$. **2**

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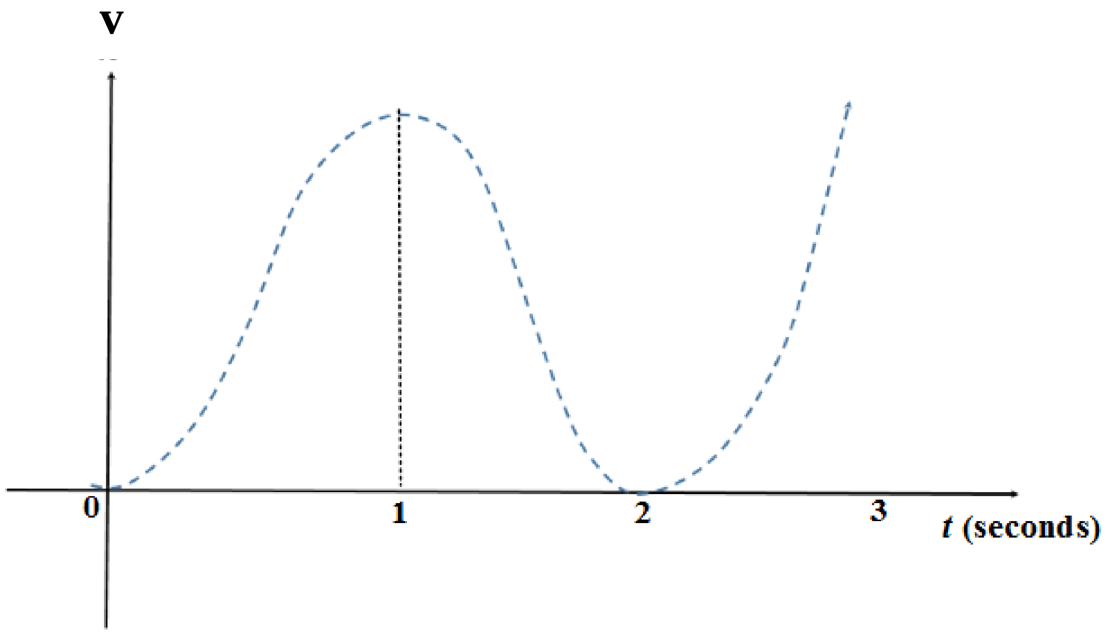
(e) On the $v - t$ graph given below, sketch the Displacement-time graph. **2**



Question 31 (6 marks)

Concrete is pumped from a truck into a building foundation. The volume V m^3 in the foundation is given by the expression V$= 3t^3 - \frac{1}{5}t^5$ for $0 \le t \le 3$, where t is the time measured in hours after the concrete begins to flow.

(a) Find the average rate of flow between $t = 1$ and $t = 2$. **2**

(b) Find the instantaneous rate of flow at $t = 2$. **2**

(c) Explain why t is restricted to $[0, 3]$. **2**

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End of paper

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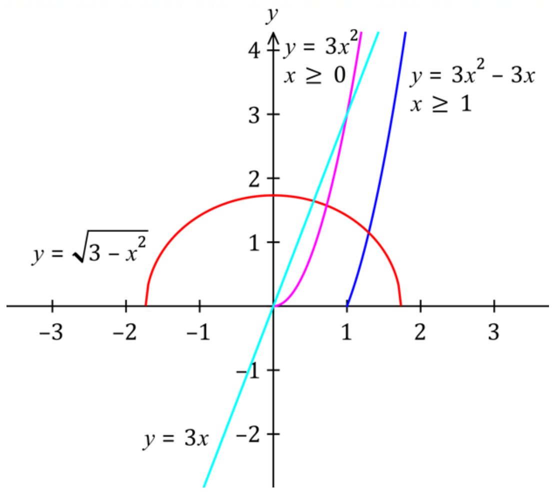
2020 Year 11 Mathematics Advanced Task 3 Worked solutions and marking guidelines

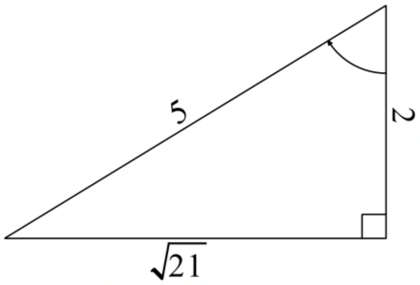
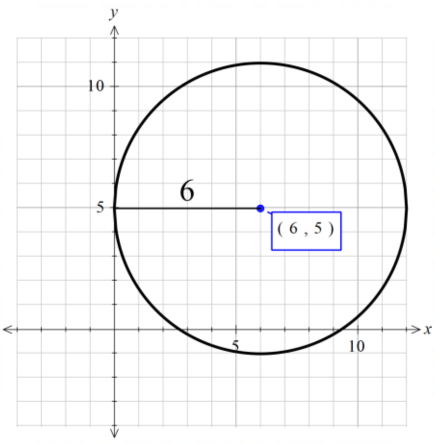
Killara HIGH SCHOOL

CONSERVA PROGREDERE

MATHEMATICS DEPARTMENT

Section I

	Solution	**Criteria**
1.	$f(x) = x^2(x-2)$ $f'(x) = x^2 \times 1 + (x-2) \times 2x$ $= x^2 + 2x^2 - 4x$ $= 3x^2 - 4x$ $f'(1) = 3 \times 1^2 - 4 \times 1 = -1$	1 Mark: B
2.	Function $y = \sqrt{3-x^2}$ $0 = \sqrt{3-x^2}$ $x^2 = 3$ $x = \pm\sqrt{3}$ $(+\sqrt{3}, 0)$ and $(-\sqrt{3}, 0)$ $\therefore \sqrt{3-x^2}$ is not a one-to-one function.  	1 Mark: B
3.	Positive definite, parabola concave up and no real solution $a > 0$ and $\Delta < 0$	1 Mark: B
4.	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\dfrac{AB}{\sin 112°} = \dfrac{13}{\sin 34°}$ $AB = \dfrac{13 \times \sin 112°}{\sin 34°}$ $= 21.5549\ldots \approx 21.6$ cm	1 Mark: C
5.	The point $(k, 26)$ satisfies the equation of the line. $5x - y - 4 = 0$ $5k - 26 - 4 = 0$ $5k = 30$ $k = 6$	1 Mark: D

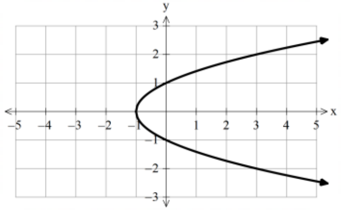
6.	$\frac{4}{x-3}+\frac{2}{x}=1$ $\frac{4x+2(x-3)}{x(x-3)}=1$ $4x+2x-6=x^2-3x$ $x^2-9x+6=0$ $x=\frac{-(-9)\pm\sqrt{(-9)^2-4\times1\times6}}{2}$ $x=\frac{9\pm\sqrt{57}}{2}$ Both $\frac{9+\sqrt{57}}{2}$ and $\frac{9-\sqrt{57}}{2}>0$ $\therefore\ x=\frac{9\pm\sqrt{57}}{2}$	1 Mark: A
7.	$\frac{d}{dx}\left(x^{-2}-\frac{1}{x}\right)=-2x^{-3}-(-1)x^{-2}$ $=\frac{-2}{x^3}+\frac{1}{x^2}$	1 Mark: D
8.	Using Pythagoras's Theorem $\sqrt{5^2-2^2}=\sqrt{21}$  Since $cos\beta<0$ and $sin\beta<0$, β is in $3rd\ quadrant$ $\therefore\ tan\beta>0$ $\therefore\ tan\beta=\frac{\sqrt{21}}{2}$	1 Mark: B
9.	$f\big(g(x)\big)=2(3x+1)=6x+2$	1 Mark: B
10.	$x^2+y^2-12x-10y+k=0$ $x^2-12x+\left(\frac{12}{2}\right)^2-\left(\frac{12}{2}\right)^2+y^2-10y+\left(\frac{10}{2}\right)^2-\left(\frac{10}{2}\right)^2+k$ $=0$ $(x-6)^2+(y-5)^2-36$ $-25+k=0$ To have exactly three intersections with axes radius must equal to 6. $36+25-k=6^2$ $k=36$  	1 Mark: D

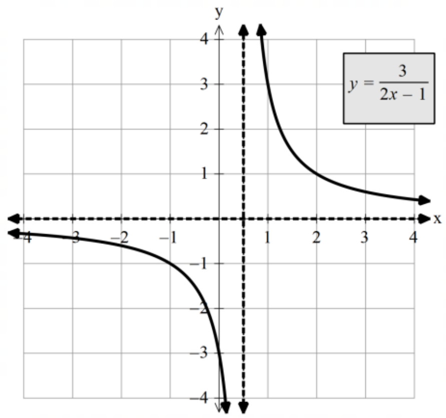
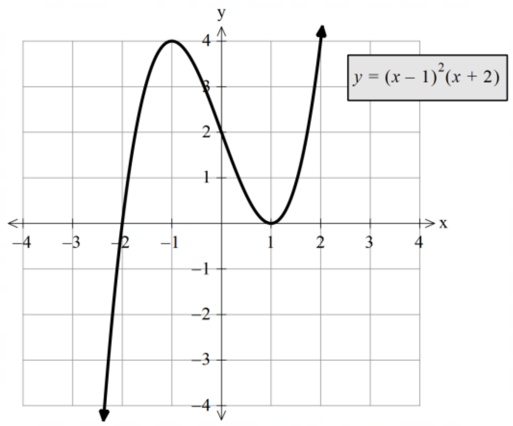
Section II			
	Solution	**Criteria**	**Marking Feedback**
11(a)	$\frac{\sqrt{5}}{4\sqrt{3}+2}=\frac{\sqrt{5}}{4\sqrt{3}+2}\times\frac{4\sqrt{3}-2}{4\sqrt{3}-2}$ $=\frac{\sqrt{5}(4\sqrt{3}-2)}{(4\sqrt{3})^2-(2)^2}$ $=\frac{4\sqrt{15}-2\sqrt{5}}{48-4}$ $=\frac{4\sqrt{15}-2\sqrt{5}}{44}$	2 Marks: Correct answer. 1 Mark:	Most students were able to rationalise the denominator, but a few students made mistakes in expanding the brackets. You can also simplify the answer to: $\frac{2\sqrt{15}-\sqrt{5}}{22}$
11(b)	$(2\sqrt{3}+1)(2-\sqrt{3})=2\times 2\sqrt{3}-2\sqrt{3}\times\sqrt{3}-1\times 2-\sqrt{3}$ $=4\sqrt{3}-6-2-\sqrt{3}$ $=3\sqrt{3}-8$ $a+b\sqrt{3}=3\sqrt{3}-8$ $\therefore a=3, b=-8$	2 Marks: Correct answers for a and b. 1 Mark: Attempt to expand the expression	Most students got this question correct. The most common mistake made was +6 rather that -6.
12(a)	$m^4-1=(m^2-1)(m^2+1)$ $=(m-1)(m+1)(m^2+1)$	2 Marks: Correct answer. 1 Mark: Obtain the first factorisation result.	
12(b)	$3^x+3^{x+2}=3^x+3^x\times 3^2$ $=3^x+3^x\times 9$ $=3^x(1+9)$ $=10\cdot 3^x$	2 Mark: Correct answer. 1 Mark: Attempt to use Index law, or equivalent merit	Many students couldn't do this question. Many students made the error of thinking that $3^x+3^{x+2}=3^{x+x+2}$ which is absolutely incorrect.

13	$2x^2 - 14x - 1 = 0$ $2(x^2 - 7x) - 1 = 0$ $2\left[x^2 - 7x + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2\right] - 1 = 0$ $2\left[\left(x - \frac{7}{2}\right)^2 - \frac{49}{4}\right] - 1 = 0$ $2\left(x - \frac{7}{2}\right)^2 - \frac{49}{2} - 1 = 0$ $2\left(x - \frac{7}{2}\right)^2 - \frac{51}{2} = 0$ $2\left(x - \frac{7}{2}\right)^2 = \frac{51}{2}$ $\left(x - \frac{7}{2}\right)^2 = \frac{51}{4}$ $x - \frac{7}{2} = \pm\frac{\sqrt{51}}{2}$ $x = \pm\frac{\sqrt{51}}{2} + \frac{7}{2}$	3 Mark: Correct answer. 2 Mark: Coefficient is factorised, attempt to complete the square 1 Mark: Attempt to complete the square without factorising	**Students should not use decimals!!** Few students did this well.
14(a)	$130° = 130° \times \frac{\pi}{180°}$ $= \frac{13}{18}\pi$	1 Marks: Correct answer.	Some students used radian and degrees in the same equation, for example $\frac{\frac{\pi}{6}}{360°}$ This should be avoided entirely!

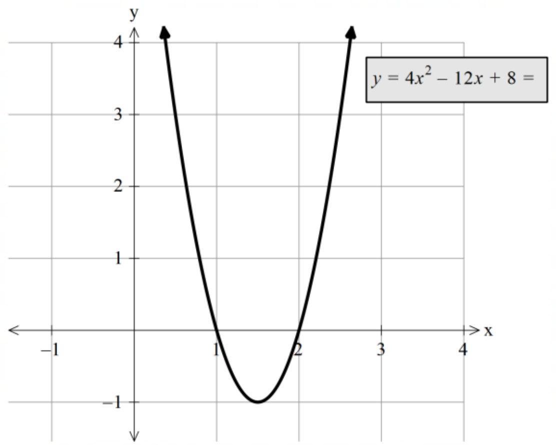
14(b)	$2\cos\theta = \sqrt{3}$ $cos\theta = \frac{\sqrt{3}}{2}$ $related\ angle = 30°$ $\therefore \theta = 30°, 360° - 30°$ $\theta = 30°\ and\ 330°$	2 Marks: Correct answer. 1 Mark: Attempt to find the related angle 30°	Quite a large number of students use the calculator for exact angles 30°, 45° and 60°. We are not interested in whether you can use your calculator for this. We want to know that you can recognise and operate with exact values! Use your reference sheet!
14(c)	$sin\theta = -\frac{1}{\sqrt{2}}$ $related\ angle = \frac{\pi}{4}\ or\ 45°$ $Since\ sin\theta < 0$ $\therefore \theta = \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$ $\theta = \frac{5\pi}{4}\ and\ \frac{7\pi}{4}$	2 Marks: Correct answer. 1 Mark: Finds its related angle	
15	When $\theta\ is\ given\ in\ radian\ measure$ $A_{sec} = \frac{1}{2}\theta r^2$ $3\pi = \frac{1}{2} \times \frac{\pi}{3} \times r^2$ $3\pi = \frac{\pi}{6} \times r^2$ $r^2 = 18$ $r = 3\sqrt{2}$	2 Marks: Correct answer. 1 Mark: Attempts to use the correct formula	
16	$\sec^2(330°) = [\sec(330°)]^2$ $= \left[\frac{1}{\cos 330°}\right]^2$ $= \left[\frac{1}{\quad}\right]^2$	2 Marks: Correct answer. 1 Mark: Attempts to convert sec to cos	Same comment as for 14(b)

	$= \frac{4}{3}$		
17(a)	Domain: $x \in R$ Range: $y \in [1, \infty)$	2 Marks: Correct answer. 1 Mark: Correct domain OR range	
17(b)	Domain: $x \in [-2,2]$ Range: $y \in [0,2]$	2 Mark2: Correct answer. 1 Mark: Correct domain OR range	Few students received full marks
18(a)	Gradient of the l_1 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{11-(-1)} = \frac{1}{2}$ Equation of l_1 $y - y_1 = m(x - x_1)$ $y - 2 = \frac{1}{2}\big(x - (-1)\big)$ $y = \frac{1}{2}x + \frac{5}{2}$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the line or shows some understanding.	Some students lost marks for not reading the question and putting the equation in general form instead of gradient-intercept form.
18(b)	Gradient of the l_2 $m_1 m_2 = -1$ $\frac{1}{2} \times m_2 = -1$ $m_2 = -2$ Equation of l_2 $y - y_1 = m(x - x_1)$ $y - 0 = -2(x - 10)$ $y = -2x + 20$ Point S is the intersection of l_1 and l_2	3 Marks: Correct answer. 2 Marks: Finds the equation of l_2. 1 Mark: Finds the gradient of l_2 or makes some progress finding the equation of l_2.	Very few student showed working to calculate gradient.

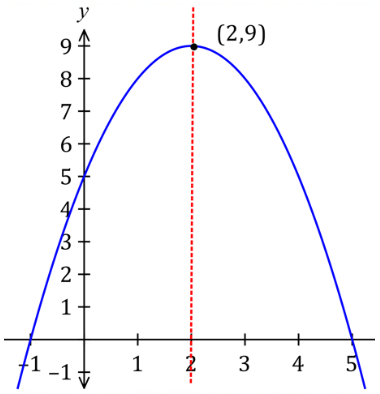
	$\frac{1}{2}x+\frac{5}{2}=-2x+20$ $\frac{5}{2}x=\frac{35}{2}$ $x=7$ and $y=6$ $\therefore S(7,6)$ is shown		
18(c)	To find the distance between $R(10,0)$ and $S(7,6)$ (Pythagoras) $RS^2=(10-7)^2+(0-6)^2$ $RS=\sqrt{45}$ $=3\sqrt{5}$	1 Mark: Correct answer.	Many students forgot to simplify
19(a)	$y^2=x+1$ does not pass vertical line testing. It is a one-to-many relation. Therefore, it is not a function. 	2 Mark: Correct answer with explanation. 1 Mark: Attempt to sketch graph or States the relation is not a function without a sufficient reason.	Very few students achieved full marks as many did not recognise two possible values (positive and negative).
19(b)	$f(-x)=\frac{6(-x)^3}{(-x)^2+(-x)^4}$ $=\frac{-6x^3}{x^2+x^4}$ $=-f(x)$ $\therefore f(x) is\ an\ odd\ function.$	2 Mark: Correct answer. 1 Mark: Attempt to find $f(-x)$.	
20(a)	$l=\theta r$ $l=2.3\times 6$ $=13.8\ cm$	1 Mark: Correct answer.	Many students found the chord and not the arc. Not reading question.

20(b)	$A_{shaded} = \frac{1}{2}\theta r^2 - \frac{1}{2}sin\theta \times AC \times CB$ $= \frac{1}{2} \times 2.3 \times 6^2 - \frac{1}{2} \times \sin\left(2.3 \times \frac{180°}{\pi}\right) \times 6 \times 6$ $= 27.977306 \ldots$ $= 27.98cm^2\ (2d.p.)$	2 Mark: Correct answer. 1 Mark: Correctly finds one of the areas.	Most students found sector area, but did not convert between radians and degree.
21(a)	 	2 Mark: Correct answer 1 Mark: Shows features of hyperbola and one of the asymptotes	$y = \frac{3}{2x-1}$ One mark was given for clearly stating the asymptote: $x = \frac{1}{2}$. Most did this correctly. One mark was given for drawing the hyperbola either side of the asymptote of $x = \frac{1}{2}$.
21(b)	 	3 Mark: Shows correct graph with all intercepts 2 Mark: Correct graph with one incorrect intercept 1 Mark: Show some understanding of cubic graph sketching or find both intercepts correctly.	$y = (x-1)^2(x+2)$ This was clearly an equation for a cubic. One mark was given for drawing a cubic. One mark was given if only 2 out of the 3 main intercepts were shown. If $x = -2, x = 1\ (double\ intercept)\ and\ y = 2$. Was correctly drawn then 3 marks were allocated.
22(a)	$f'(x) = 2x(4x+1)^3 + x^2 \times 3 \times (4x+1)^2 \times 4$ $= 2x(4x+1)^3 + 12x^2(4x+1)^2$ $= (4x+1)^2[2x(4x+1) + 12x^2]$	2 Mark: Correct answer.	The main aspect of this question was using the product rule. The second part was making sure that $(4x+1)$ *was taken common*.

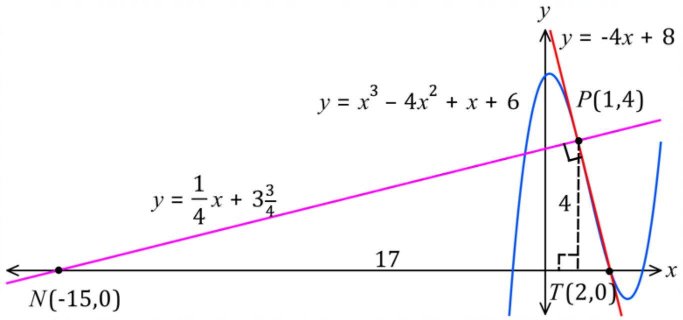
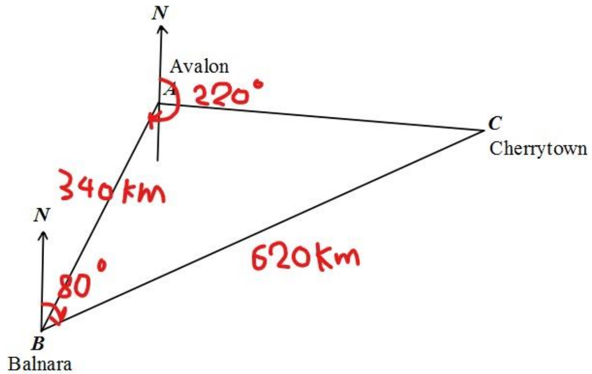
	$= (4x+1)^2(8x^2+2x+12x^2)$ $= (4x+1)^2(20x^2+2x)$ $= (4x+1)^2(2x)(10x+1)$ $= 2x(10x+1)(4x+1)^2$	1 Mark: Attempt to apply product rule and chain rule, answer not factorised.	One mark was allocated for working out with $u, u', v \text{ and } v'$ One mark for writing the complete answer of: $2x(4x+1)^2(10x+1)$
22(b)	$\frac{d}{dx}(5+x^2)^{\frac{1}{2}} = \frac{1}{2}(5+x^2)^{-\frac{1}{2}}2x$ $= \frac{x}{\sqrt{5+x^2}}$	2 Marks: Correct answer. 1 Mark: Applies the chain rule or shows some understanding.	The chain rule should have been applied for this question. Most students reached the point of: $\frac{du}{dx} \times \frac{1}{2}u^{-\frac{1}{2}}$ The second mark was given for leaving the question in the format: $\frac{x}{\sqrt{5+\ x^2}}$
22(c)	$\frac{d}{dx}\left(\frac{3x^2-5}{2x+1}\right) = \frac{(2x+1)6x-(3x^2-5)2}{(2x+1)^2}$ $= \frac{12x^2+6x-6x^2+10}{(2x+1)^2}$ $= \frac{6x^2+6x+10}{(2x+1)^2}$	2 Marks: Correct answer. 1 Mark: Applies the quotient rule.	Most students correctly achieved 2 marks for this question. However a mark was NOT given if working out was not shown to get to the final answer. The denominator was: $v^2\ (2x+1)^2$. Many left the denominator as just 2 or just cancelled it out somehow from the numerator. The numerator was: $6x^2+6x+10$

23	$\Delta = b^2 - 4ac$ $= (2m)^2 - 4 \times (3m - 2)$ $= 4m^2 - 12m + 8$ Real roots means $\Delta \geq 0$. $4m^2 - 12m + 8 \geq 0$ $m^2 - 3m + 2 \geq 0$ $(m-1)(m-2) \geq 0$ $m \leq 1 \; or \; m \geq 2$ 	2 Marks: Correct answer. 1 mark: Attempt to find the discriminant in terms of m and $\Delta \geq 0$	Many students did not get this question correct as they initially did NOT allow delta $\Delta \geq 0$. One mark was allocated for working out if the roots were left as 1 and 2. The answer was $x \geq 1$ and $x \geq 2$

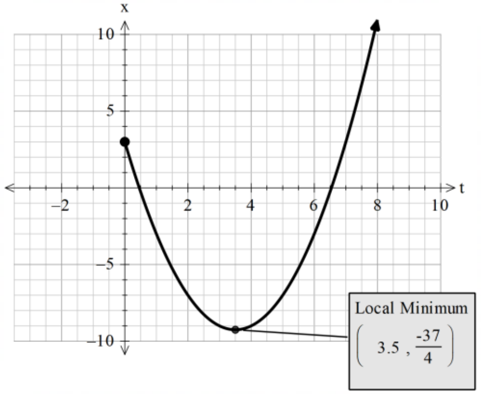
24	$f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $= \lim\limits_{h\to 0}\dfrac{3(x+h)^2+7-(3x^2+7)}{h}$ $= \lim\limits_{h\to 0}\dfrac{3x^2+6xh+h^2+7-3x^2-7}{h}$ $= \lim\limits_{h\to 0}\dfrac{6xh+h^2}{h}$ $= \lim\limits_{h\to 0} 6x+h$ $= 6x$ $f'(x) = 6x$	3 Marks: Correct answer. 2 Marks: 1 Mark: Attempt to use the formula with some substitution errors	One mark for substituting correctly: $f(x+h) = 3(x+h)^2+7$ One mark was given for the correct substitution. $f(x) = 3(x)^2+7$ $\lim\limits_{h-0}\frac{3x^2+6xh+3h^2+7-3x^2-7}{h}$ One mark for simplifying to the correct answer. Lim h →0 should have been incorporated at 6x + 3h to result in an answer of 6x. One mark for substituting 0.
25	$y = 5+4x-x^2$ $= -(x-2)^2+9$ $\therefore$ Axis of symmetry is $x = 2$ Or $Axis\ of\ symmetry = -\dfrac{b}{2a} = -\dfrac{4}{-2} = 2$ $y_{max} = 5+4\times 2-(2)^2$ $= 9$	2 Mark: Correct answer. 1 Mark: finds the axis of symmetry	It is much better to either use the vertex form of a parabola, or to use the axis of symmetry. Very few of those who attempted to average the roots did this effectively. Students must check that they have answered the question before moving on. Many students found $x = 2$ and did not sub this value back into the original equation to find the y-value.

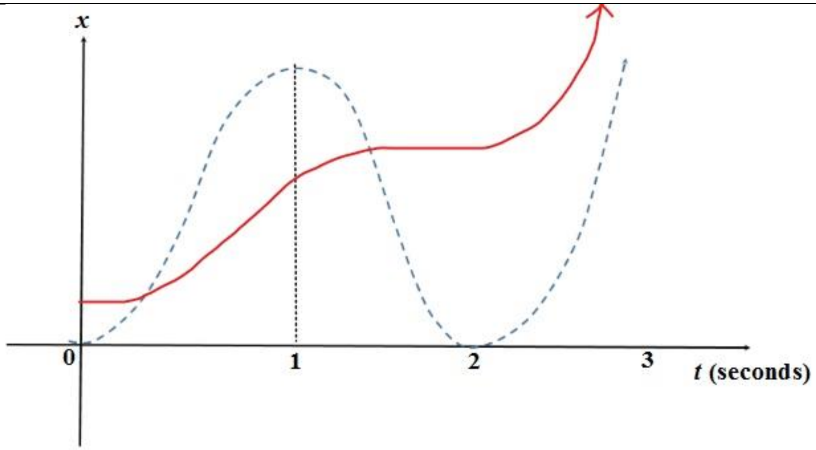
			
26	$\frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$ $LHS = \frac{\sin^2\theta + \cos^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$ $= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$ $= \tan^2\theta + 1 - \sin^2\theta$ $= \tan^2\theta + \cos^2\theta$ $= RHS$	2 Marks: Correct answer. 1 Mark: Attempts to use some correct Pythagorean identity	There were many alternate solutions to this problem. Many students did this well,
27(a)	$y = x^3 - 4x^2 + x + 6$ $\frac{dy}{dx} = 3x^2 - 8x + 1$ Gradient of the tangent at $P(1, 4)$ $m = 3 \times (1)^2 - 8 \times 1 + 1 = -4$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the tangent at P.	Some students attempted to find the gradient WITHOUT differentiating. You can not be sure this will give you an exact value.

	Equation of the tangent at $P(1, 4)$ $y - y_1 = m(x - x_1)$ $y - 4 = -4(x - 1)$ $y = -4x + 8$ $\therefore$ Equation of the tangent is $y = -4x + 8$		
27(b)	Gradient of the normal at $P(1, 4)$ $m_1 m_2 = -1$ $m \times -4 = -1$ $m = \frac{1}{4}$ Equation of the normal at $P(1, 4)$ $y - y_1 = m(x - x_1)$ $y - 4 = \frac{1}{4}(x - 1)$ $y = \frac{1}{4}x + \frac{15}{4}$ $\frac{1}{4}x - y + \frac{15}{4} = 0$ $x - 4y + 15 = 0$ $\therefore$ Equation of the normal in general form is $x - 4y + 15 = 0$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the normal at P or equation not in general form.	Many students found $m_2 = -\frac{1}{4}$ and need to revise operations with negative numbers. Most completed the question very well but lost a mark for not putting the equation in general form. Which means, x comes first and with a positive integer coefficient.
27(c)	Intercept of the tangent (y = 0) $0 = -4x + 8$ $x = 2$ Intercept of the normal (y = 0) $0 = \frac{1}{4x} + 3\frac{3}{4}$ $x = -15$	2 Marks: Correct answer. 1 Mark: Finds either point T or N.	Generally done well. Again, you can not read values off the graph. You must show substitution.

	$\therefore\ T(2,0)$ and $N(-15,0)$		
27(d)	 $A=\frac{1}{2}bh$ $=\frac{1}{2}\times 17\times 4$ $=34$ square units	1 Mark: Correct answer.	Generally either done well or not attempted.
28(a)		1 Mark: Label all bearing and distances	
28(b)	$\angle BAO=\angle DBA=40°(alternate\ angles\ on\ parallel\ lines)$ $\therefore\ \angle ABC=80°-40°=40°$	1 Mark: correct answer	Formal reasoning was not penalised in this question, so many students who got the mark and are looking to do a higher level of Mathematics will need to revise how to set out

			reasoning.
28(c)	Cosine rule $c^2 = a^2 + b^2 - 2ab\cos C$ $AC^2 = 340^2 + 620^2 - 2 \times 340 \times 620 \times \cos 40°$ $AB = \sqrt{340^2 + 620^2 - 2 \times 340 \times 620 \times \cos 40°}$ $= 420.76\ldots$ ≈ 421 km $\therefore$ The distance AC is 421 km.	2 Marks: Correct answer. 1 Mark: Uses the cosine with at least one correct value.	Generally done well. Some students still had their calculator in radians, which changed their answer.
28(d)	$\frac{340}{\sin \angle ACB} = \frac{421}{\sin 40°}$ $\sin \angle ACB = \frac{340 \times sin40°}{421}$ $\sin \angle ACB = 0.5194 \ldots$ $\angle ACB = 31.293 \ldots$ $\approx 31°$ $\therefore Bearing = 180 + 80 + 31 = 291°T$	2 Marks: Correct answer. 1 Mark: Attempt to use sine rule	Many students did not notice the ambiguous case of the sine rule and so provided an incorrect answer from all bar completely correct working. Many students who calculated 31° did not know how to use it in order to find the bearing/thought that 31° was the bearing.
29(a)	Initial position: t = 0 X = 3 3 metres right of the origin	1 mark: correctly substitutes and explains 3 m right	Most students identified that by substituting $t = 0$ they would be able to find the initial position but most students did not state that the particle is 3 m **to the right** of the origin. Students **must show the substitution**.
29(b)	At rest, $\dot{x} = 0$ $2t - 7 = 0$ $t = 3.5\ sec$	1 mark: sets $\dot{x} = 0$ and gives the correct value of t	Some students did not understand that for a particle to be stationary or at rest, they needed to differentiate and solve for $\dot{x} = 0$

29(c)	$t = 3.5,\ \ x = (3.5)^2 - 7(3.5) + 3 = -9.25\ m$ At t = 8, $x = (8)^2 - 7(8) + 3 = 11\ m$ $11 > \lvert -9.25 \rvert$ The particle is furthest from the origin at t = 8 sec 	2 marks: evaluates x at t = 8, compares with the displacement at the turning point and gives the correct answer. 1 mark: finds the displacement at t = 3.5 sec	This question was done particularly poorly. Many students used a guess and check method to find that when $t = 8$, the particle is furthest from the origin. **This method is incorrect and should not be used.** Students were expected to demonstrate their understanding of the quadratic functions and show that the displacement could be furthest at either the turning point or end points of the domain.
29(d)	Distance travelled = $3 + 2 \times \lvert -9.25 \rvert + 11 = 32.5\ m$	2 mark: Gives the correct answer with working 1 mark: adds at least two of the displacements	Again, this question was done poorly due to a lack of understanding of quadratic functions.
30(a)	$t = 2$ sec	1 Mark: Correct answer	This question was done fairly well.
30(b)	No. the velocity must be negative to indicate a direction change and this is not the case.	2 Mark: Correct answer with explanation 1 Mark: Answer without correct explanation.	This question was mostly done well. If "no" was given but poor reasoning, or reasoning revealed a lack of understanding, a zero was awarded.
30(c)	$t = 1$	1 Mark: Correct answer	Marks were awarded to those who stated $t = 0$ or $t = 1$.

30(d)	The particle starts from a stationary position, moving towards a positive direction between t=0 to t=1.	2 Marks: Correctly stating the stationary position and positive direction. 1 Mark: Only mention one area.	1 mark was awarded for any correct description of the movement, for example, "increasing at an increasing rate before changing to increasing at a decreasing rate" Any incorrect statements were ignored. Students must have stated that the particle was starting from stationary and moving in the positive direction to be awarded the full two marks. Students used the word "deceleration" incorrectly. Both "accelerating" and "decelerating" should be avoided in these descriptions as students use these incorrectly. Better to use "positive acceleration" or "negative acceleration".
30(e)		2 Marks: showing the change of concavity at t=1 and t=2 1 Mark: attempts to show a continuous increasing graph	This question was done poorly. Most students need to review this concept/skill quite rigorously.
31(a)	$Average\ rate\ of\ flow = \frac{V(2)-V(1)}{2-1}$	2 Marks: Correct answer	If students correctly found $V(2)$ and $V(1)$ and used the correct units in their answer, even if

	$= \dfrac{\left[3(2)^3 - \frac{1}{5}\times 2^5\right] - \left[3(1)^3 - \frac{1}{5}\times(1)^5\right]}{2-1}$ $= \dfrac{\frac{88}{5} - \frac{14}{5}}{1}$ $= \dfrac{74}{5} m^3/h$	1 Mark: attempt to substitute to the correct formula	their answer was wrong, they were awarded 1 mark. A large number of students interpreted the "average rate" to mean $\frac{V(1)+V(2)}{2}$. **This is incorrect**. 1 mark was deducted if units were left off or if they were incorrect. Many students interpreted this question to mean to differentiate V.
31(b)	$V' = 9t^2 - t^4$ $= 9 \times (2)^2 - (2)^4$ $= 20m^3/h$	2 Mark: correct answer 1 Mark: attempts to differentiate	Many of those students who differentiated V in part (a), differentiated again to obtain V''. This is incorrect and reveals very little understanding of the difference between **average rate of change** and **instantaneous rate of change** and what the **first and second derivative** indicate. Again, students were penalised for incorrect or leaving off the units.
31(c)	When $t = 3$, $V' = 9 \times (3)^2 - 3^4$ $= 0$ This is when the flow rate stops. $\therefore t \leq 3$ Also $t \geq 0$	2Mark: correct explanation 1Mark: attempt to explain why $t \leq 3$ without calculation.	This question was done very poorly. Only a handful of students provided adequate reasoning here. Many students used $V(4)$ to show that $V < 0$ but forgot that there exists **non-integer** values between 3 and 4 for which $V > 0$.

	$\therefore 0 \leq t \leq 3$		Correct reasoning required the mention of negative rate of flow. Those that mentioned the volume of concrete in the building foundation **decreased** when $t > 3$ were also awarded a mark for sound reasoning.