

Killara HIGH SCHOOL

MATHEMATICS DEPARTMENT



Student Name

Course: Advanced Mathematics

Year: 11

Task Number: 3

Weighting: 40%

Task Type: Assignment

Date issued: Monday 23/08/21 (Week 7)

Date Due: Monday 06/09/21 (Week 9)

Task Description	There are two sections to this assignment: - Section 1 a set of multiple choice questions. Q 1-5 - Section 2 is a set of short answer response questions. Q 6-23 - Both sections are to be completed at home. Both sections are compulsory. Failure to complete either section of this assignment will be recorded as non-completion of the assignment with subsequent penalties. If you find that you have an incorrect answer, do not use white-out, but cross out your working with one line and begin your solution again. This demonstrates reflection on your working. The mark allocation is provided for each question below.	
Submission Instructions	- You must submit a PDF on Compass. You may wish to use Adobe Scan on your phone to convert your photos to a PDF - You can choose to print the assignment, complete your working, and then convert to a PDF - or - You can write the solutions on blank paper, clearly labelling each question, and then convert to PDF form - Write using blue or black pen. - Use appropriate mathematical language and notation in each of your responses - Show all relevant mathematical reasoning and/or calculations. - Your written responses must be legible and logically structured - Failure to comply with these instructions may incur a penalty	
Section I	Multiple Choice	/5
Section II	Short Answer Response	/65
		/70

Topics and Outcomes	• Working with Functions, Ch 1, 2, 3, 4, 6 • Trigonometric Functions Ch 5,9 • Introduction to Differentiation Ch 8 • Probability and Discrete Probability Distributions Ch 10, 11 • MA11-2: uses the concepts of functions and relations to model, analyse and solve practical problems • MA11-3:uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes • MA11-4:uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities • MA11-5:interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems • MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions
If you need help...	• Reach out to your teacher on your class online learning platform or via email. • All official advice will be given on the combined Year 11 Advanced Google Classroom ONLY
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Section I

5 marks

Allow about 5 minutes for this section

Use the multiple-choice sheet for Question 1–5

- 1** What is the angle of inclination of the line $3x + 2y = 7$ with the positive direction of the x -axis?

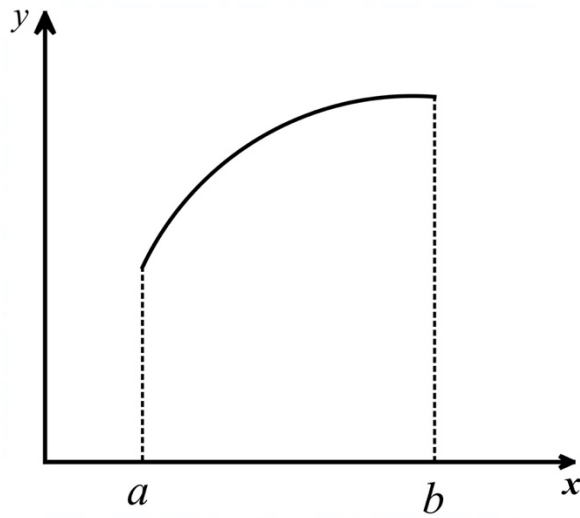
- (A) $33^\circ 41'$
- (B) $56^\circ 19'$
- (C) $123^\circ 41'$
- (D) $146^\circ 19'$

- 2** If $f(x) = \frac{1}{\sqrt{x+1}}$ and $g(x) = 35 - x^2$

What is the domain of $f(g(x))$?

- (A) $x \geq 6, x \leq -6$
- (B) $-6 \leq x \leq 6$
- (C) $-6 < x < 6$
- (D) $x > 6, x < -6$

- 3 For the function $y = f(x)$, $a < x < b$ graphed below:

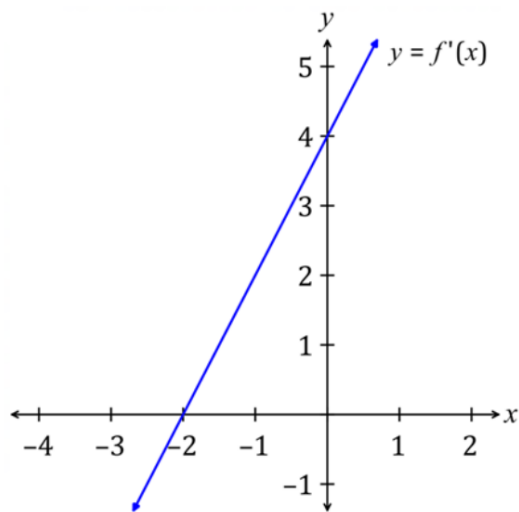


Which of the following is true?

- (A) $f'(x) > 0$ and $f''(x) > 0$
- (B) $f'(x) > 0$ and $f''(x) < 0$
- (C) $f'(x) < 0$ and $f''(x) > 0$
- (D) $f'(x) < 0$ and $f''(x) < 0$
- 4 Sixty tickets are sold in a raffle. There are two prizes. Rachel buys five tickets. Which expression gives the probability that Rachel wins both prizes?

- (A) $\frac{5}{60} + \frac{4}{59}$
- (B) $\frac{5}{60} + \frac{4}{60}$
- (C) $\frac{5}{60} \times \frac{4}{59}$
- (D) $\frac{5}{60} \times \frac{4}{60}$

- 5 The graph $y = f'(x)$ is shown below.



The curve $y = f(x)$ has a minimum value of 6. What is the equation of the curve?

- (A) $y = x^2 - 4x + 2$
- (B) $y = x^2 - 4x + 10$
- (C) $y = x^2 + 4x + 2$
- (D) $y = x^2 + 4x + 10$

Section II

In Questions 6 – 23, your response should include relevant mathematical reasoning and/or calculations.

Question 6 (4 marks)

(a) Rationalise

2

$$\frac{3}{\sqrt{7} + 2}$$

(b) Simplify the following:

2

$$\frac{5}{x^2 - 4} - \frac{3}{x^2 - 3x + 2}$$

Question 7 (3 marks)

If $f(x) = x^2 - 4x + 8$, show that $f'(x) = 2x - 4$ using first principles.

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Question 8 (2 marks)

Fully simplify

$$\frac{\sin 33^\circ + \cos 57^\circ}{\cos 57^\circ}$$

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Question 9 (4 marks)

Given the piecewise function below,

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$$f(x) = \begin{cases} 2x - 1 & x < -5 \\ 3 & -5 \leq x < 0 \\ (x - 1)^2 + 2 & x \geq 0 \end{cases}$$

Solve the following:

(a) $f(-7) + f(-2)$

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(b) $f(a^2)$ in simplest form.

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Question 10 (3 marks)

A factory produces strawberry milkshake and vanilla milkshake both in regular and low fat options. Each day, $\frac{3}{5}$ of the milkshakes produced are strawberry flavoured and the rest are vanilla flavoured. In terms of demand, $\frac{5}{7}$ of the strawberry milkshakes are regular and the rest of the strawberry milkshakes are low fat in option. $\frac{2}{3}$ of the vanilla milkshakes are low fat and the rest of the vanilla milkshakes are regular.

- (a) Draw a probability tree diagram below, labelling all the branches with the correct probabilities. **1**

Use the symbols given below in your tree diagram.

- S represents the strawberry milkshake
- V represents the vanilla milkshake
- L represents a low fat milkshake
- R represents a regular milkshake

- (b) A milkshake is selected at random. Given that a low-fat milkshake is selected, find the probability that the milkshake was vanilla flavoured. **2**

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Question 11 (6 marks)

Differentiate the following functions.

(a) $y = \frac{x^3 - 3x}{x^2}$ **2**

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(b) $y = (4x^3 - x)^7$ **2**

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(c) $y = \frac{x}{x^2 + 1}$ **2**

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Question 12 (3 marks)

Prove the following identity:

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$$\frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2 \sin^2 x$$

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Question 13 (3 marks)

Find the equation of the tangent to $y = x\sqrt{7 - 2x}$ at $x = 1$.

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Question 14 (3 marks)

A company tracked the number of sales made by each sales staff in a day. Using the data from a large number of days, the company estimated the probability of the number of sales made by each sales staff in a day. 3

Let the random variable S be the number of sales made by Khaled per day. The following table shows the probability distribution for Khaled's daily sales.

s	2	3	4	5	6	7
$P(S = s)$	0.05	0.1	0.15	0.35	0.15	0.2

Khaled is expected to make 5.05 sales per day.

Find Khaled's variance and standard deviation to 2 decimal places.

Hint: Add any required rows in the table using space above.

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Question 15 (2 marks)

If $\sin\theta = \frac{4}{7}$, find the exact value of $\sec\theta$

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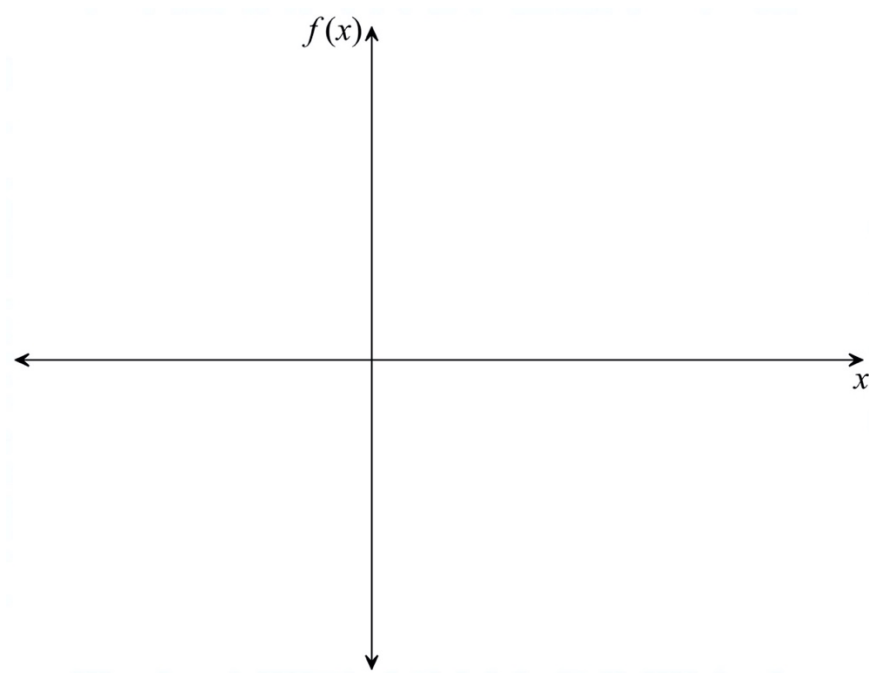
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Question 16 (3 marks)

Sketch the function $y = f(x)$ on the coordinate axes below, given the following information: **3**

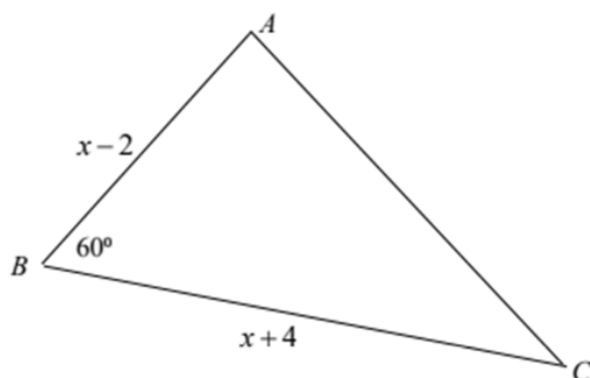
- $f(0) = 3$, $f'(2) = 0$, $f(2) = 6$
- $f'(x) > 0$ for $x < 2$
- $f'(x) < 0$ for $x > 2$
- $f'(x) \rightarrow 0$ as $x \rightarrow \infty$



Question 17 (2 marks)

The area of $\triangle ABC$ is $18\sqrt{3} \text{ cm}^2$. Calculate the value of x .

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NOT TO
SCALE

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Question 18 (3 marks)

Solve $\sqrt{3} \sin^2 \theta = \sin \theta \cos \theta$ for $0 \leq \theta \leq 2\pi$. Give your answers in radians.

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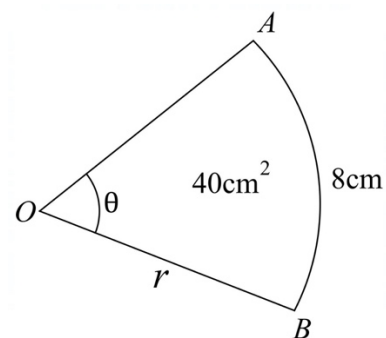
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Question 19 (3 marks)

The diagram shows a sector OAB with measurements as shown.



- (a) Find the length of the radius

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- (b) Hence, or otherwise find the size of angle θ to the nearest degree.

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Question 20 (7 marks)

The displacement in metres of a particle travelling in a straight line is given by $x = 4t^3 - 12t$ where t is the time taken in seconds.

- (a) Find the initial displacement of the particle 1

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- (b) Find the velocity (v) of the particle as a function of time. 1

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- (c) When is the particle stationary? 1

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- (d) When is the particle changing direction? Show full working out. You may draw a diagram to assist you 2

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- (e) What is the total distance travelled by the particle in the first 3 seconds? You may draw a diagram to assist you 2

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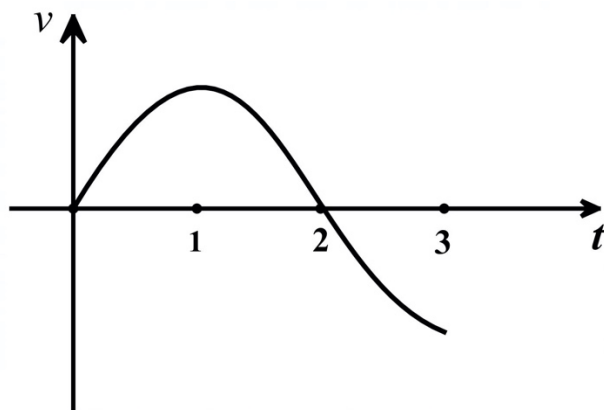
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Question 21 (3 marks)

The graph shows the velocity (v) of a particle moving along a straight line as a function of time (t).

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Describe the motion of the particle between $t = 0$ and $t = 3$ in terms of velocity and acceleration.

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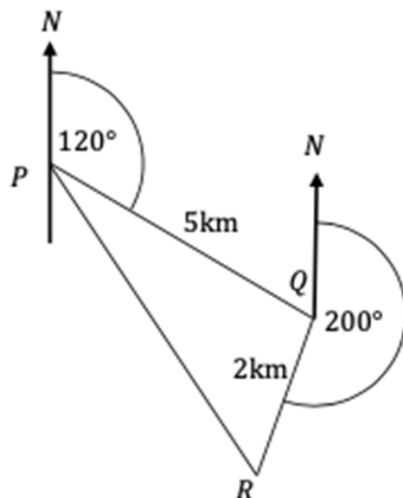
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Question 22 (4 marks)

Chandler starts walking from a camping place P on a bearing of 120° for 5 km to a place Q. He then walks on a bearing of 200° for 2km to a place R.



- (a) What is the size of $\angle PQR$?

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- (b) What is the distance between the camping place P and the place R?
Answer correct to 2 decimal places.

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Question 23 (7 marks)

A petrol tank holds 35 000 litres of petrol. Petrol is draining from the tank and the volume of petrol remaining in the petrol tank after t minutes is given by:

$$V = 35\,000 \left(1 - \frac{t}{30}\right)^2, \text{ where } V \text{ is measured in litres.}$$

- (a) Find the average rate of flow between $t = 10$ and $t = 20$ 2

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- (b) Find the instantaneous rate of flow at $t = 15$ 2

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- (c) Explain why time (t) has to be restricted to $[0, 30]$. 1

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- (d) How much time will it take until only half of the initial volume of petrol remains in the tank? 2

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Section I

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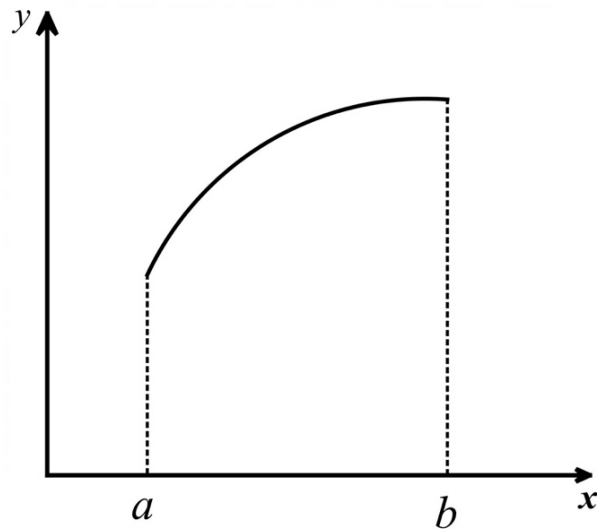
- (A) $33^\circ 41'$
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- 2 If $f(x) = \frac{1}{\sqrt{x+1}}$ and $g(x) = 35 - x^2$

What is the domain of $f(g(x))$?

- (A) $x \geq 6, x \leq -6$
- (B) $-6 \leq x \leq 6$
- (C) $-6 < x < 6$
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- 3 For the function $y = f(x)$, $a < x < b$ graphed below:

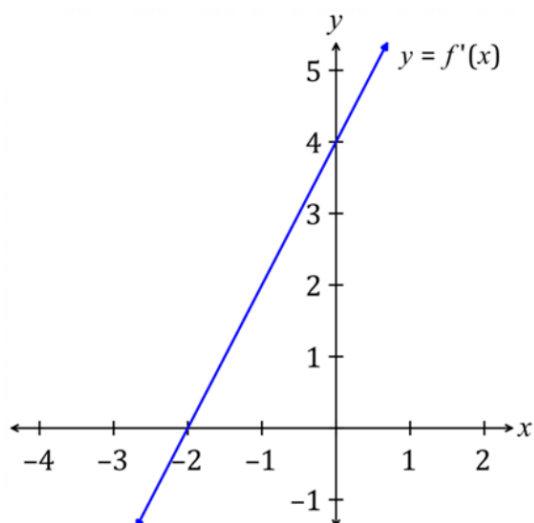


Which of the following is true?

- (A) $f'(x) > 0$ and $f''(x) > 0$
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- (C) $f'(x) < 0$ and $f''(x) > 0$
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- 4 Sixty tickets are sold in a raffle. There are two prizes. Rachel buys five tickets. Which expression gives the probability that Rachel wins both prizes?

- (A) $\frac{5}{60} + \frac{4}{59}$
- (B) $\frac{5}{60} + \frac{4}{60}$
- (C) $\frac{5}{60} \times \frac{4}{59}$
- (D) $\frac{5}{60} \times \frac{4}{60}$

- 5 The graph $y = f'(x)$ is shown below.



Not to scale

The curve $y = f(x)$ has a minimum value of 6. What is the equation of the curve?

- (A) $y = x^2 - 4x + 2$
- (B) $y = x^2 - 4x + 10$
- (C) $y = x^2 + 4x + 2$
- (D) $y = x^2 + 4x + 10$

Section II

In Questions 6 – 23, your response should include relevant mathematical reasoning and/or calculations.

Question 6 (4 marks)

(a) Rationalise 2

$$\frac{3}{\sqrt{7} + 2}$$

$\frac{3(\sqrt{7}-2)}{(\sqrt{7}+2)(\sqrt{7}-2)}$ 1 mark

$= \frac{3(\sqrt{7}-2)}{3}$

$= \sqrt{7}-2$ 1 mark

(b) Simplify the following: 2

$$\frac{5}{x^2 - 4} - \frac{3}{x^2 - 3x + 2}$$

$= \frac{5(x-1)(x-1)}{(x+2)(x-2)(x-1)} - \frac{3(x+2)(x+2)}{(x-1)(x-2)(x+2)}$ 1 mark

$= \frac{5x-5-3x-6}{(x-2)(x+2)(x-1)}$

$= \frac{2x-11}{(x-2)(x+2)(x-1)}$ 1 mark

Question 7 (3 marks)

If $f(x) = x^2 - 4x + 8$, show that $f'(x) = 2x - 4$ using first principles.

3

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - 4(x+h) + 8 - (x^2 - 4x + 8)}{h}$$

1 mark

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 4x - 4h + 8 - x^2 + 4x - 8}{h}$$

1 mark

$$= \lim_{h \rightarrow 0} (h + 2x - 4)$$

1 mark

$$= 2x - 4$$

Question 8 (2 marks)

Fully simplify

2

$$\frac{\sin 33^\circ + \cos 57^\circ}{\cos 57^\circ}$$

$$= \frac{\cos 57^\circ + \cos 57^\circ}{\cos 57^\circ}$$

1 mark

$$= 2$$

1 mark

Question 9 (4 marks)

Given the piecewise function below,

2

$$f(x) = \begin{cases} 2x - 1 & x < -5 \\ 3 & -5 \leq x < 0 \\ (x - 1)^2 + 2 & x \geq 0 \end{cases}$$

Solve the following:

(a) $f(-7) + f(-2)$

$= 2 \times (-7) - 1 + 3$ 1 mark

$= -12$ 1 mark

(b) $f(a^2)$ in simplest form.

2

$= (a^2 - 1)^2 + 2$ 1 mark

$= a^4 - 2a^2 + 1 + 2$

$= a^4 - 2a^2 + 3$ 1 mark

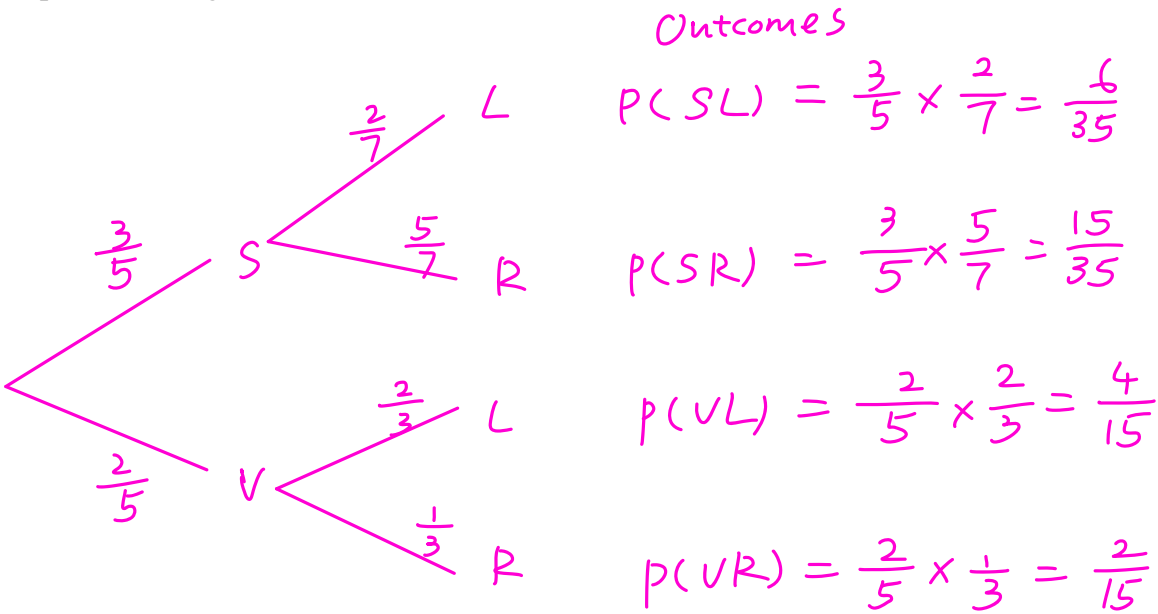
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A factory produces strawberry milkshake and vanilla milkshake both in regular and low fat options. Each day, $\frac{3}{5}$ of the milkshakes produced are strawberry flavoured and the rest are vanilla flavoured. In terms of demand, $\frac{5}{7}$ of the strawberry milkshakes are regular and the rest of the strawberry milkshakes are low fat in option. $\frac{2}{3}$ of the vanilla milkshakes are low fat and the rest of the vanilla milkshakes are regular.

- (a) Draw a probability tree diagram below, labelling all the branches with the correct probabilities. 1

Use the symbols given below in your tree diagram.

- S represents the strawberry milkshake
- V represents the vanilla milkshake
- L represents a low fat milkshake
- R represents a regular milkshake



- (b) A milkshake is selected at random. Given that a low-fat milkshake is selected, find the probability that the milkshake was vanilla flavoured. 2

$$P(L) = P(SL) + P(VL)$$
$$= \frac{6}{35} + \frac{4}{15} \text{ 1mark}$$
$$= \frac{46}{105}$$

$$\frac{P(V \cap L)}{P(L)} = \frac{\frac{4}{15}}{\frac{46}{105}}$$
$$= \frac{14}{23} \text{ 1mark}$$

Question 11 (6 marks)

Differentiate the following functions.

(a) $y = \frac{x^3 - 3x}{x^2}$ 2

$$y = x - 3x^{-1}$$

$$y' = 1 - 3x(-1)x^{-2} \quad \text{1 mark}$$

$$y' = 1 + 3x^{-2} \text{ or } 1 + \frac{3}{x^2} \quad \text{1 mark}$$

(b) $y = (4x^3 - x)^7$ 2

$$y' = 7(4x^3 - x)^6 (12x^2 - 1)$$

$$\downarrow \quad \downarrow$$

$$\text{1 mark} \quad \text{1 mark}$$

(c) $y = \frac{x}{x^2 + 1}$ 2

$$u = x \quad v = x^2 + 1$$

$$u' = 1 \quad v' = 2x$$

$$y' = \frac{x^2 + 1 - x \cdot 2x}{(x^2 + 1)^2} \quad \text{1 mark}$$

$$y' = \frac{1 - x^2}{(x^2 + 1)^2} \quad \text{1 mark}$$

Question 12 (3 marks)

Prove the following identity:

3

$$\frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2 \sin^2 x$$

$$\text{LHS} = \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} \quad 1 \text{ mark}$$

$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} \quad 1 \text{ mark}$$

$$= 1$$

$$= \cos^2 x - \sin^2 x$$

$$= 1 - \sin^2 x - \sin^2 x \quad 1 \text{ mark}$$

$$= 1 - 2 \sin^2 x = \text{RHS}$$

Question 13 (3 marks)

Find the equation of the tangent to $y = x\sqrt{7-2x}$ at $x = 1$.

$$u = x$$

$$v = (7-2x)^{\frac{1}{2}}$$

$$u' = 1$$

$$v' = \frac{1}{2}(7-2x)^{-\frac{1}{2}} \cdot (-2)$$

$$v' = -\frac{1}{\sqrt{7-2x}}$$

$$y' = uv' + uv''$$

$$y' = \sqrt{7-2x} - \frac{x}{\sqrt{7-2x}}$$

$$\bullet \text{ find gradient at } x=1 \quad m = \sqrt{5} - \frac{1}{\sqrt{5}} = \frac{4}{\sqrt{5}} \quad 1 \text{ mark}$$

$$\bullet \text{ point } (1, \sqrt{5}) \quad 1 \text{ mark}$$

$$\bullet \text{ equation } y - \sqrt{5} = \frac{4}{\sqrt{5}}(x-1)$$

$$y - \sqrt{5} = \frac{4x}{\sqrt{5}} - \frac{4}{\sqrt{5}}$$

$$y = \frac{4x}{\sqrt{5}} + \frac{1}{\sqrt{5}} \quad 1 \text{ mark}$$

$$\text{or } y = \frac{4x+1}{\sqrt{5}}$$

Question 14 (3 marks)

A company tracked the number of sales made by each sales staff in a day. Using the data from a large number of days, the company estimated the probability of the number of sales made by each sales staff in a day.

Let the random variable S be the number of sales made by Khaled per day. The following table shows the probability distribution for Khaled's daily sales.

s^2	4	9	16	25	36	49	
s	2	3	4	5	6	7	
$P(S = s)$	0.05	0.1	0.15	0.35	0.15	0.2	sum
ps/s^2	0.2	0.9	2.4	8.75	5.4	9.8	27.45

Khaled is expected to make 5.05 sales per day.

Find Khaled's variance and standard deviation to 2 decimal places.

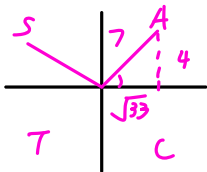
Hint: Add any required rows in the table using space above.

$$E(S^2) = 27.45 \text{ 1mark}$$
$$Var(S) = E(S^2) - E(S)^2$$
$$= 27.45 - 5.05^2$$
$$= 1.95 \text{ 1mark}$$

$$\sigma(S) = \sqrt{Var}$$
$$= 1.40 \text{ 1mark}$$

Question 15 (2 marks)

If $\sin\theta = \frac{4}{7}$, find the exact value of $\sec\theta$



$$\cos\theta = \pm \frac{\sqrt{33}}{7} \text{ 1mark}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$= \pm \frac{7}{\sqrt{33}}$$
$$= \pm \frac{7\sqrt{33}}{33} \text{ 1mark}$$

$$\text{or } \frac{7\sqrt{33}}{33}$$
$$\text{(1mark)}$$

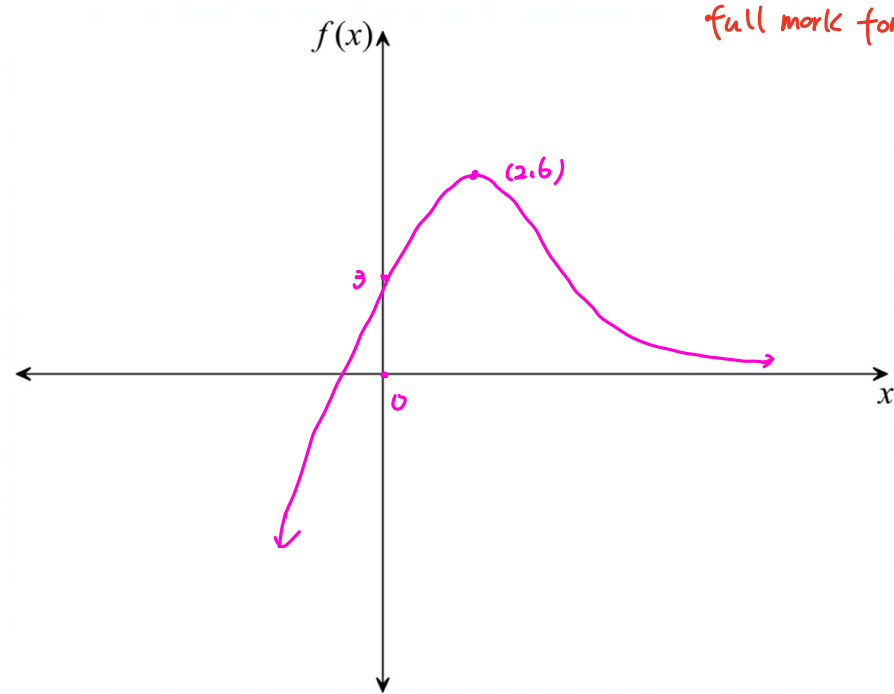
$$-\frac{7\sqrt{33}}{33}$$
$$\text{(1mark)}$$

Question 16 (3 marks)

Sketch the function $y = f(x)$ on the coordinate axes below, given the following information: 3

- $f(0) = 3$, $f'(2) = 0$, $f(2) = 6$
- $f'(x) > 0$ for $x < 2$
- $f'(x) < 0$ for $x > 2$
- $f'(x) \rightarrow 0$ as $x \rightarrow \infty$

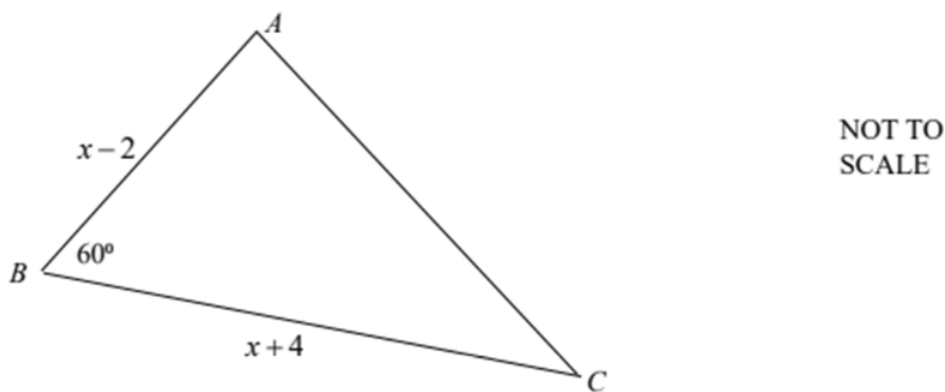
- 1 mark for key points (0,3) (2,6)
- 1 mark for gradients for $x < 2$ and $x > 2$
correct
- 1 mark for gradient approaching 0 when $x \rightarrow \infty$
- full mark for all details correct



Question 17 (2 marks)

The area of $\triangle ABC$ is $18\sqrt{3} \text{ cm}^2$. Calculate the value of x .

2



$$A = \frac{1}{2}(x-2)(x+4)\sin 60^\circ = 18\sqrt{3}$$

$$\frac{1}{2}(x-2)(x+4)\frac{\sqrt{3}}{2} = 18\sqrt{3} \quad 1 \text{ mark}$$

$$x^2 - 2x - 8 = 72$$

$$(x-8)(x+10) = 0$$

$$x = 8 \quad 1 \text{ mark}$$

Question 18 (3 marks)

Solve $\sqrt{3} \sin^2 \theta = \sin \theta \cos \theta$ for $0 \leq \theta \leq 2\pi$. Give your answers in radians.

3

$$\sqrt{3} \sin^2 \theta - \sin \theta \cos \theta = 0$$

$$\sin \theta (\sqrt{3} \sin \theta - \cos \theta) = 0$$

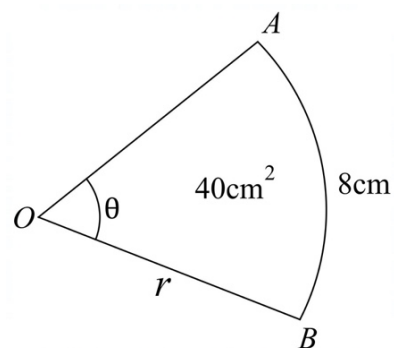
$$\sin \theta = 0 \text{ or } \sqrt{3} \sin \theta - \cos \theta = 0 \quad 1 \text{ mark}$$

$$\sin \theta = 0 \text{ or } \tan \theta = \frac{1}{\sqrt{3}} \quad 1 \text{ mark}$$

$$\theta = 0, \pi, 2\pi \text{ or } \frac{\pi}{6}, \frac{7\pi}{6} \quad 1 \text{ mark}$$

Question 19 (3 marks)

The diagram shows a sector OAB with measurements as shown.



- (a) Find the length of the radius

2

$$\theta r = 8$$

$$\frac{1}{2}\theta r^2 = 40 \quad 1 \text{ mark}$$

$$\theta r \cdot \frac{1}{2}r = 40$$

$$8 \times \frac{1}{2}r = 40$$

$$r = 10 \text{ cm} \quad 1 \text{ mark}$$

- (b) Hence, or otherwise find the size of angle θ to the nearest degree.

1

$$\theta r = 8$$

$$\theta = \frac{8}{10}$$

$$\theta = 0.8 \times \frac{180}{\pi}$$

$$\approx 46^\circ$$

Question 20 (7 marks)

The displacement in metres of a particle travelling in a straight line is given by $x = 4t^3 - 12t$ where t is the time taken in seconds.

- (a) Find the initial displacement of the particle

1

when, $t=0$ $x=0$ at the origin

- (b) Find the velocity (v) of the particle as a function of time.

1

$$v = \dot{x}$$

$$v = 12t^2 - 12$$

- (c) When is the particle stationary?

1

$$v = 12t^2 - 12 = 0$$

$$t = 1$$

- (d) When is the particle changing direction? Show full working out. You may draw a diagram to assist you

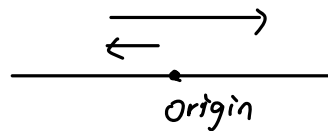
2

$$v=0 \text{ and } a \neq 0$$

From part (c) when $t=1$ $v=0$ 1 mark

$$a = \ddot{x} = 24 \neq 0 \quad \text{1 mark}$$

$\therefore$ when $t=1$ the particle is changing direction



- (e) What is the total distance travelled by the particle in the first 3 seconds? You may draw a diagram to assist you

2

- distance travelled between $t=0$ and $t=1$ is 8m

$$x(0) = 0 \quad x(1) = -8 \quad \text{1 mark}$$

- distance travelled between $t=1$ and $t=3$ is 80

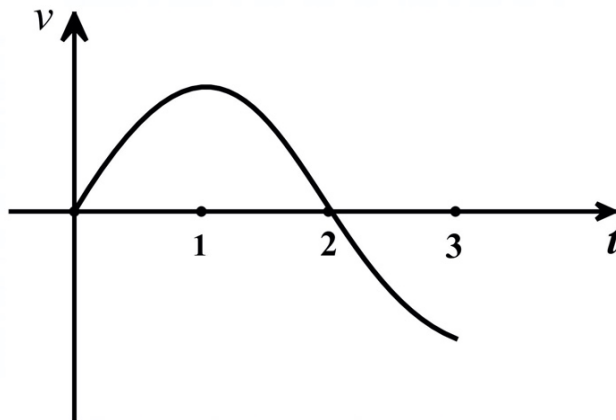
$$x(1) = -8 \quad x(3) = 72$$

$\therefore$ distance travelled between $t=0$ and $t=3$ is 88 1 mark

Question 21 (2 marks)

The graph shows the velocity (v) of a particle moving along a straight line as a function of time (t).

3

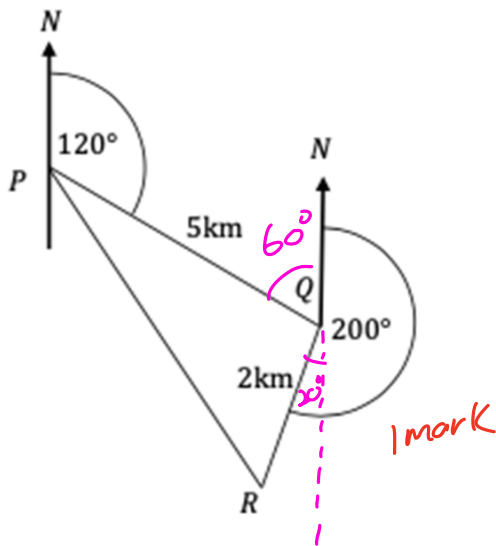


Describe the motion of the particle between $t = 0$ and $t = 3$ in terms of velocity and acceleration.

- At $t=0$ the particle is at rest, about to move to the right and speed up, and when $t=1$ it reaches its maximum speed. (1 mark)
- between $1 < t < 2$, the particle slows down with positive velocity and negative acceleration. It comes to rest at $t=2$, and it is furthest from the origin at this point (1 mark)
- when $t > 2$, the particle begins to move to the left ($v < 0$ and $a < 0$) until it reaches $t=3$ (1 mark)

Question 22 (4 marks)

Chandler starts walking from a camping place P on a bearing of 120° for 5 km to a place Q. He then walks on a bearing of 200° for 2km to a place R.



- (a) What is the size of $\angle PQR$?2

$\angle PQR = 100^\circ$ 1 mark

- (b) What is the distance between the camping place P and the place R?2
Answer correct to 2 decimal places.

$PR^2 = 5^2 + 2^2 - 2 \times 5 \times 2 \times \cos 100^\circ$
 $PR \approx 5.70 \text{ km}$

Question 23 (8 marks)

A petrol tank holds 35 000 litres of petrol. Petrol is draining from the tank and the volume of petrol remaining in the petrol tank after t minutes is given by:

$$V = 35\,000 \left(1 - \frac{t}{30}\right)^2, \text{ where } V \text{ is measured in litres.}$$

- (a) Find the average rate of flow between $t = 10$ and $t = 20$

2

$$r = \frac{V(20) - V(10)}{20 - 10} \quad \text{1 mark}$$

$$= \frac{-3500}{3} \text{ L/m} \quad \text{1 mark}$$

$$(-1166.67 \text{ L/m})$$

- (b) Find the instantaneous rate of flow at $t = 15$

2

$$V' = 35000 \times 2 \left(1 - \frac{t}{30}\right) \times \left(-\frac{1}{30}\right)$$

$$V' = -\frac{7000}{3} \left(1 - \frac{t}{30}\right) \quad \text{1 mark}$$

$$V'(15) = -\frac{3500}{3} \text{ L/m} \quad \text{1 mark}$$

$$(-1166.67 \text{ L/m})$$

- (c) Explain why time (t) has to be restricted to $[0, 30]$.

1

$$V' = -\frac{7000}{3} \left(1 - \frac{t}{30}\right)$$

petrol is draining $V' < 0$

$$1 - \frac{t}{30} > 0$$

$$\text{when } t > 30 \quad V' > 0$$

- (d) How much time will it take until only half of the initial volume of petrol remains in the tank?

2

$$17500 = 35000 \left(1 - \frac{t}{30}\right)^2$$

$$\frac{1}{2} = \left(1 - \frac{t}{30}\right)^2$$

$$1 - \frac{t}{30} = \frac{\sqrt{2}}{2}$$

$$30 - t = 15\sqrt{2}$$

$$t = 30 - 15\sqrt{2}$$

$$t \approx 8.79 \text{ m}$$