

2022 Year 11 Preliminary Examination

Mathematics Advanced

22/08/2022

General

Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks:
100

Section I - 10 marks (pages 3–8)

- Allow about 15 minutes for this section

Section II - 90 marks (pages 9–41)

- Allow about 2 hours and 45 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 40% of the course.

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Section I

10 marks

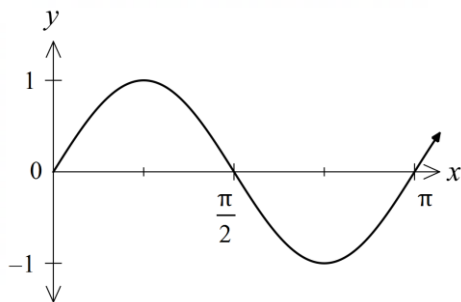
Attempt Questions 1–10

Allow about 15 minutes for this section

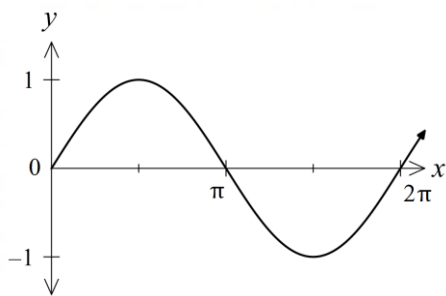
Use the multiple-choice sheet for Question 1–10.

1 Which graph represents the curve $y = \sin x$?

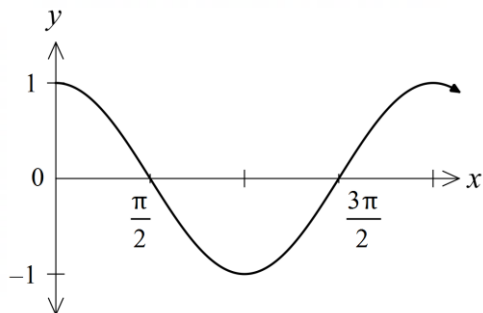
(A)



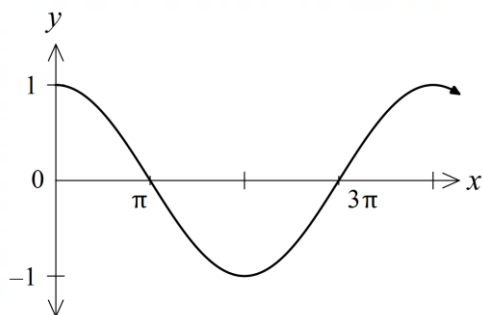
(B)



(C)



(D)



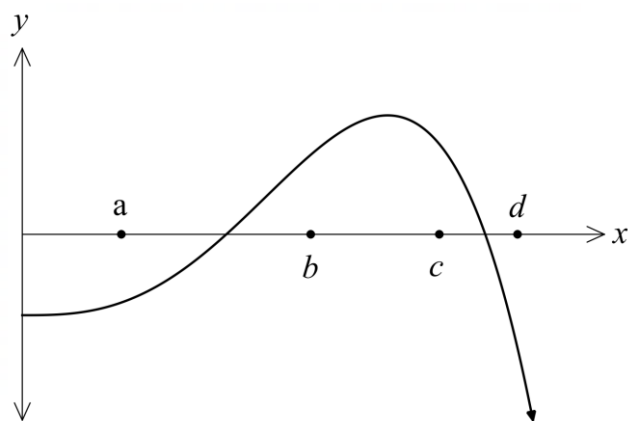
- 2 A spinner can land on the colour red, blue, yellow or green.

After the spinner is spun 50 times, the results are recorded in the table below.

Colour	Red	Blue	Yellow	Green
Frequency	13	12	10	15

What is the relative frequency of spinning a red or green?

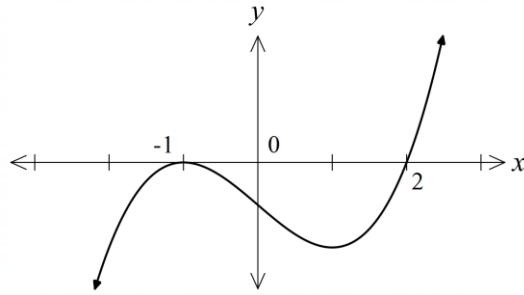
- (A) $\frac{14}{25}$
- (B) $\frac{27}{50}$
- (C) $\frac{1}{2}$
- (D) $\frac{23}{50}$
- 3 The following graph shows a function $y = f(x)$.



At which x value is $f(x) < 0$ and $f'(x) > 0$?

- (A) $x = a$
- (B) $x = b$
- (C) $x = c$
- (D) $x = d$

- 4 Which equation represents the cubic curve shown?



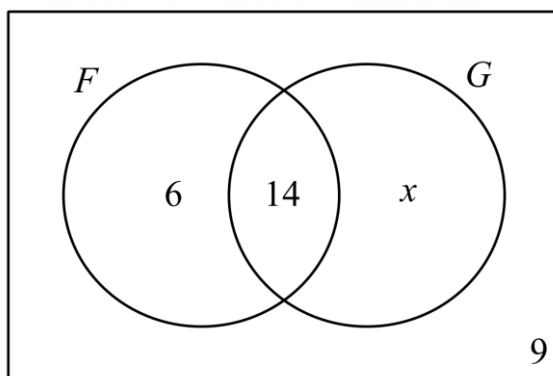
- (A) $y = (x + 1)(x - 2)^2$
- (B) $y = (x + 1)^2(x - 2)$
- (C) $y = (x - 1)(x + 2)^2$
- (D) $y = (x - 1)^2(x + 2)$
- 5 What is the exact value of $\cos 30^\circ + \sin 135^\circ$?

- (A) $\frac{\sqrt{3} - 1}{2}$
- (B) $\frac{\sqrt{3} + 1}{2}$
- (C) $\frac{\sqrt{3} - \sqrt{2}}{2}$
- (D) $\frac{\sqrt{3} + \sqrt{2}}{2}$

6 The graph $y = -\sqrt{2 - x^2}$ can be described as

- (A) One-to-one
- (B) One-to-many
- (C) Many-to-one
- (D) Many-to-many

7 The Venn diagram shows the number of scores in two sets F and G .



If $P(\bar{G}) = \frac{1}{2}$, what is the value of x .

- (A) $x = 1$
- (B) $x = 8$
- (C) $x = 11$
- (D) $x = 29$

- 8 Let f be the function defined by the following:

$$f(x) \begin{cases} \tan x, & x < 0 \\ -x^2, & 0 \leq x \leq 2 \\ x + 2, & 2 \leq x \end{cases}$$

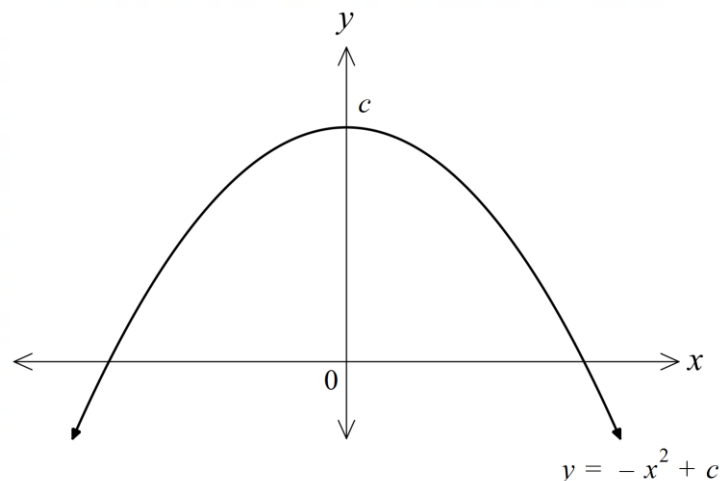
For which x value(s) is f not continuous?

- (A) 0 only
 - (B) 2 only
 - (C) Both 0 and 2
 - (D) Neither 0 nor 2
- 9 The true bearing of Point A to Point B is X° , while the true bearing of Point B to Point A is Y° , where $0^\circ \leq X < 360^\circ$ and $0^\circ \leq Y < 360^\circ$.

Which relationship must be true for X and Y ?

- (A) $|X^\circ - Y^\circ| = 90^\circ$
- (B) $|X^\circ - Y^\circ| = 180^\circ$
- (C) $X^\circ + Y^\circ = 180^\circ$
- (D) $X^\circ + Y^\circ = 360^\circ$

- 10 The following curve shows a parabola of the form $y = -x^2 + c$.



The line $y = \frac{1}{c}x + c$ is graphed on the same Cartesian plane as $y = -x^2 + c$, for $c \neq 0$.

Which statement is correct about the number of points of intersection between the two graphs?

- (A) The two graphs will never intersect for any $c > 0$ values.
- (B) The graphs will always intersect exactly once.
- (C) The graph will intersect once for some c values, and twice for other c values.
- (D) The graphs will always intersect exactly twice.

End of Section I



Killara
HIGH SCHOOL

MATHEMATICS DEPARTMENT

Student Number

2022 Year 11 Preliminary Examination

Mathematics Advanced

Section II Answer Booklet 1

22/08/2022

Section II

90 Marks

Attempt Questions 11–34

Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–26 (59 marks)

Booklet 2 – Attempt Questions 27–34 (31 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 27 & 28. If you use this space, clearly indicate which question you are answering.

Q	Marks
11	/2
12	/5
13	/3
14	/2
15	/4
16	/4
17	/4
18	/3
19	/4
20	/3
21	/4
22	/3
23	/3
24	/4
25	/6
26	/5
Total	/59

Please turn over

Section II

In Questions 11 - 27, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Solve the equation $|3x + 2| = 14$

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Do NOT write in this area.

Question 12 (5 marks)

Differentiate the following functions:

(a)
 $f(x) = x^{15}$
1

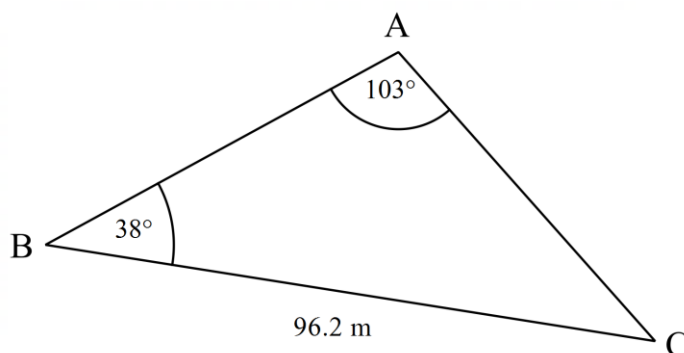
(b)
 $g(x) = 5\sqrt{x} + 4$
2

(c)
 $h(x) = \frac{1}{x^3} - 11x$
2

Question 13 (3 marks)

The diagram shows a triangle ABC .

3



Use the sine rule to find length AC , correct to 3 significant figures.

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Question 14 (2 marks)

For stringed instruments, the frequency of vibrations (in cycles per second) varies inversely with the length of the string (in cm).

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A 64 cm stringed instrument has a frequency of 150 cycles per second. Find the frequency of a 60 cm string.

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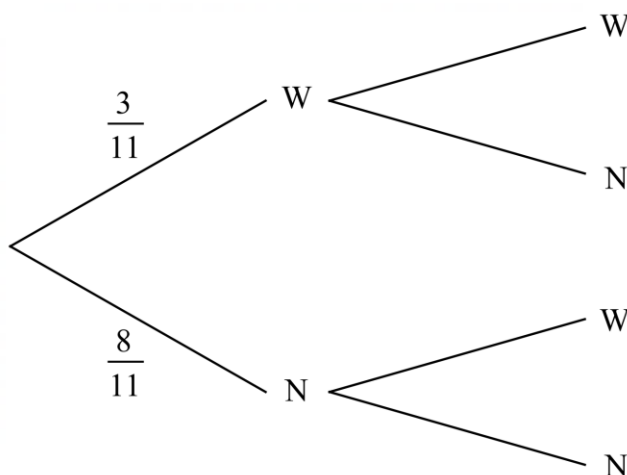
Question 15 (4 marks)

Ms Taylor is competing on a game show. In this show, there are a total of 11 suitcases, 3 of which contain prize money. She chooses 2 suitcases, and wins the prize money contained in those suitcases, if there is any.

Below, W represents Ms Taylor winning prize money, and N represents Ms Taylor not winning money,

- (a) Complete the branches on the diagram below for the second stage.

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- (b) What is the probability that Ms Taylor does not win a prize?

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- (c) What is the probability that Ms Taylor wins at least one prize?

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- (d) Given that the first suitcase Ms Taylor opens has no money, what is the probability that the second suitcase will contain money?

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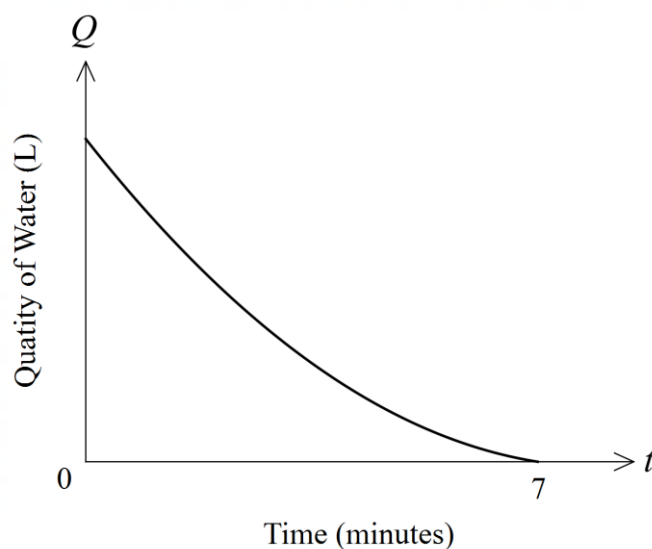
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Question 16 (4 marks)

A tank of water is being drained, and the quantity (Q) of water remaining in the tank after t minutes can be shown by the graph. It takes 7 minutes to completely drain the tank.

The graph has the equation $Q(t) = 20(t - 8)^2 - 20$



- (a) Show that the average rate of change is -180 L/ min.

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Question 16 continues on page 15

Question 16 (continued)

- (b) Find the time at which the instantaneous rate of change is -180 L/ min.

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End of Question 16

Do NOT write in this area.

Question 17 (4 marks)

Solve the following equations for $0 \leq x \leq 2\pi$.

(a)

$$\sin x = \frac{1}{\sqrt{2}}$$

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(b)

$$\sec x = -\frac{2}{\sqrt{3}}$$

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Question 18 (3 marks)

Consider the function

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$$f(x) = \frac{a}{x} + 2x^{\frac{3}{2}}$$

for some constant a .

It is known that $f'(4) = 7$. Find the value of a .

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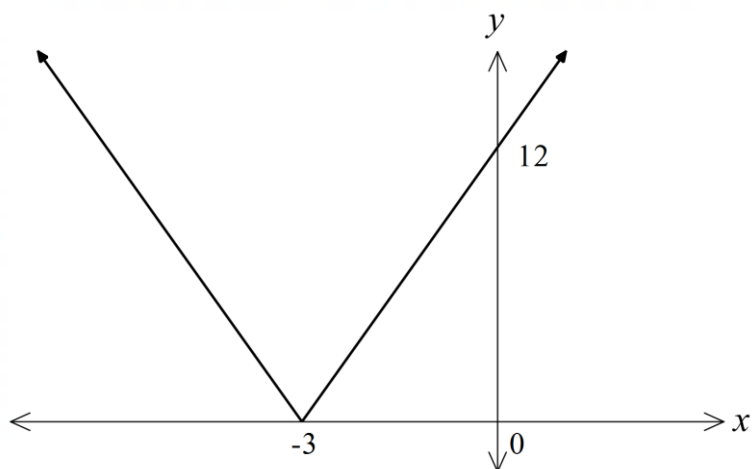
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Question 19 (4 marks)

The following graph shows the relationship $y = f(x)$, where $f(x) = |ax + b|$.



- (a) What is one set of possible values for a and b ?

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- (b) Sketch the graph $y = f(-x)$, showing any intercepts.

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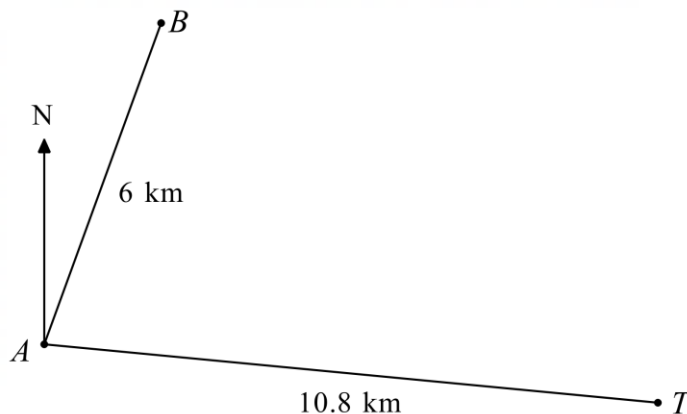
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Question 20 (3 marks)

Ms Taylor lives in Town A , and went on a hiking trip. She hiked 10.8 km, on a bearing of 095°T to point T .

Another town, Town B , is located 6 km away from Town A , on a bearing of 019°T .



- (a) Explain why angle BAT is 76° .

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- (b) Ms Taylor needs to return to one of the towns, Town A or Town B . Which town is closer to her from point T ? Justify your answer with appropriate calculations.

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Question 21 (4 marks)

For the following, find $\frac{dy}{dx}$.

(a)

$$y = (7x + 4)(x + 1)^{\frac{4}{3}}$$

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(b)

$$y = \frac{2x + 5}{x^2 + x}$$

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Question 22 (3 marks)

Determine the number of points of intersection between the following graphs:

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$$y = 3x^2 + 7x + 4$$

$$y = -x^2 + 1$$

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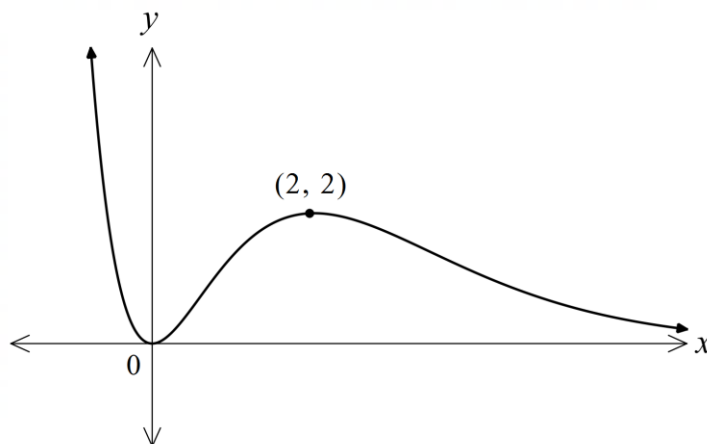
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Question 23 (3 marks)

The following graph shows a function $y = f(x)$.

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Sketch the graph of $y = f'(x)$.

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Question 24 (4 marks)

The equation $y = 2x^2 - 5x + 1$ has a tangent at a point (x_1, y_1) with a gradient of 3.

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Find the equation of the normal to the curve at the point (x_1, y_1) .

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Question 25 (6 marks)

The displacement (in metres) of a particle after t seconds is given by the relationship

$$x(t) = \frac{1}{t^2 + 1}$$

where $t \geq 0$.

- (a) State the initial position of the particle.

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- (b) Show that the velocity can be given by $\frac{dx}{dt} = -\frac{2t}{(t^2+1)^2}$

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Question 25 continues on page 25

Question 25 (continued)

- (c) Find the time(s) when the acceleration is 0 m/s^2 .

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End of Question 25

Question 26 (5 marks)

- (a) Show that 2

$$2 \cos^2 x - 3 \sin^2 x + 1 = 5 \cos^2 x - 2$$

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- (b) Hence, solve the following equation for $0^\circ \leq x \leq 360^\circ$, giving answers to the nearest degree. 3

$$2 \cos^2 x - 3 \sin^2 x + 1 = 0$$

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Proceed to Booklet 2 for Questions 27 – 34

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Section II extra writing space

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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Killara
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MATHEMATICS DEPARTMENT

Student Number

2022 Year 11 Preliminary Examination

Mathematics Advanced

Section II Answer Booklet 2

22/08/2022

Q	Marks
27	/5
28	/4
29	/5
30	/3
31	/3
32	/5
33	/4
34	/2
Total	/31

Booklet 2 – Attempt Questions 27–34 (31 marks)

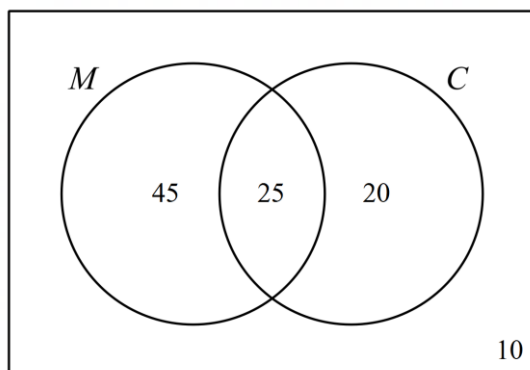
Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 41 & 42. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 27 (5 marks)

In a group of 100 students, some study Mathematics (M), some study Chemistry (C). Some study both Mathematics and Chemistry, while some study neither.



- (a) How many students are in the set $\bar{M} \cup C$?

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- (b) Explain why the events M and C are not mutually exclusive.

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- (c) If a student is chosen at random from this group, what is $P(C|M)$?

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Question 27 continues on page 31

Question 27 (continued)

- (d) Determine whether the events M and C are independent.

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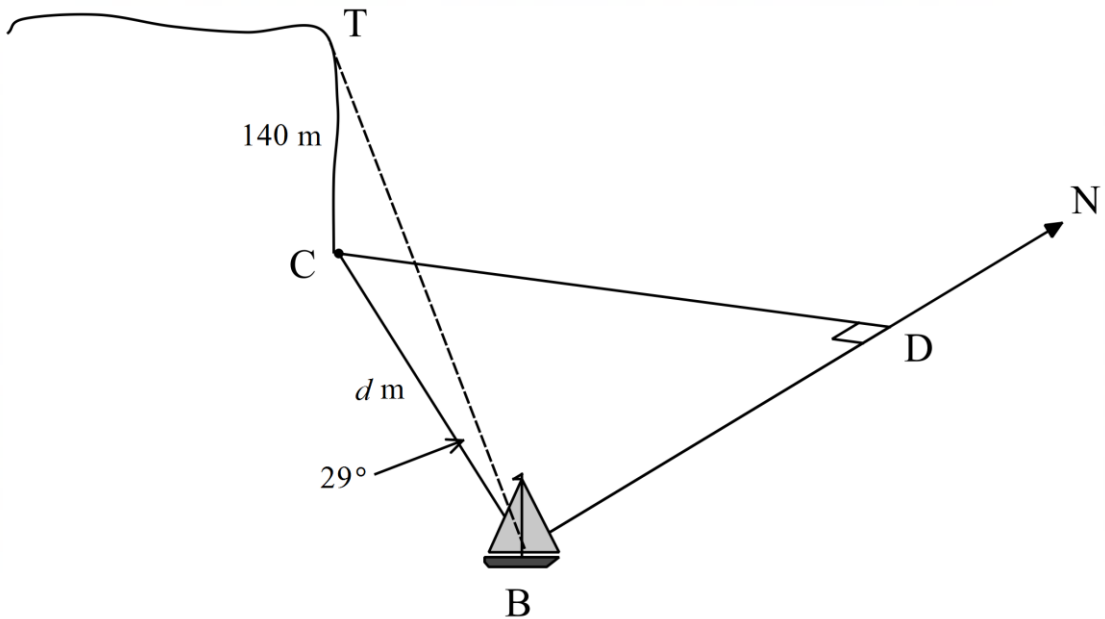
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Question 28 (4 marks)

A travelling boat spots a 140 metres vertical cliff on a true bearing of 312° . The angle of elevation from the boat to the top of the cliff is 29° .



- (a) Show that the distance d metres from the boat to the base of the cliff can be given by

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$$d = \frac{140}{\tan 29^\circ}$$

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Question 28 continues on page 33

- 3

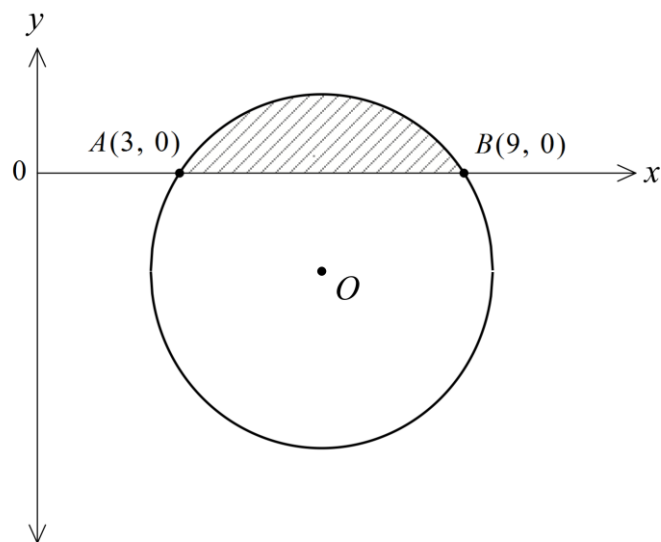
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– 33 –

Question 29 (5 marks)

The following equation represents the graph of the circle shown, centred at O .

$$x^2 - 12x + y^2 + 4y + 27 = 0$$



- (a) Show that the radius of the circle is $\sqrt{13}$ units.

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Question 29 continues on page 35

Question 29 (continued)

(b) Find the angle AOB , correct to the nearest minute 2

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(c) Hence, find the shaded area, correct to 3 significant figures. 2

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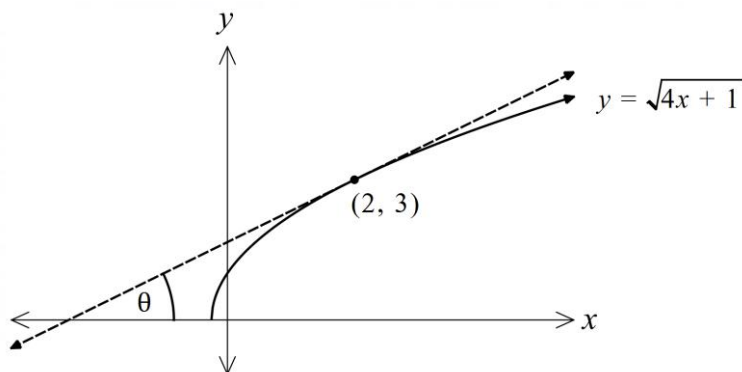
End of Question 29

Question 30 (3 marks)

The function $y = \sqrt{4x + 1}$ has been graphed below. The dotted line shows the tangent to the curve at the point $(2, 3)$.

3

Find the angle the tangent line makes to the x -axis from the positive side (θ), correct to the nearest degree.



Do NOT write in this area.

Question 33 (3 marks)

Let $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x^2-3}$.

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By finding an expression for $g(f(x))$, sketch the function $y = g(f(x))$ over its natural domain.

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Question 34 (5 marks)

Use the following information for both parts (a) and (b).

For an angle θ , where $\pi \leq \theta \leq 2\pi$, $\cos \theta = \frac{2}{3}$.

- (a) Without finding θ , explain why $\sin \theta = -\frac{\sqrt{5}}{3}$

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- (b) Simplify the following expression, leaving your answer in the form $a\sqrt{5} + b$, where a and b are real numbers.

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$$\frac{\cos \theta + 2 \sin \theta}{\tan \theta - 3 \sin \theta}$$

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Question 35 (4 marks)

A bag contains n identically shaped balls, k of which are black and remaining are white. **4**

- When one ball is chosen at random, the probability of choosing a white ball is $\frac{13}{15}$.
- When two balls are chosen at random without replacement, the probability of choosing two black balls is $\frac{1}{66}$.

Determine the number of balls n in the bag, and the number of black balls k , assuming $k \neq 0$.

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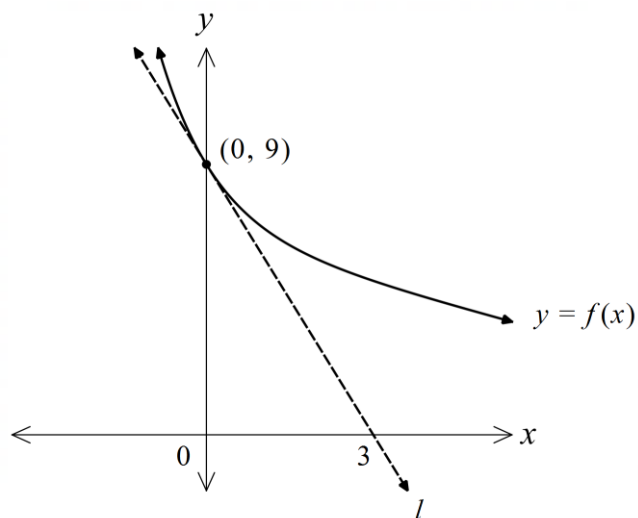
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Question 36 (2 marks)

The graph below shows a function $y = f(x)$, and its tangent at the point $(0, 9)$ represented by line l . 2



Given a new function $g(x) = \sqrt{f(x)}$, find the value of $g'(0)$.

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End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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Section II extra writing space

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Do NOT write in this area.



Mathematics Advanced

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your A ☒ B ☒ C ☐ D ☐ mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐
 correct

Start Here →

1. A ☐ B ☒ C ☐ D ☐
2. A ☒ B ☐ C ☐ D ☐
3. A ☒ B ☐ C ☐ D ☐
4. A ☐ B ☒ C ☐ D ☐
5. A ☐ B ☐ C ☐ D ☒
6. A ☐ B ☐ C ☒ D ☐
7. A ☒ B ☐ C ☐ D ☐
8. A ☐ B ☒ C ☐ D ☐
9. A ☐ B ☒ C ☒ D ☐
10. A ☐ B ☐ C ☐ D ☒



2022 Year 11 Preliminary Examination

Mathematics Advanced
Section II Answer Booklet 1

22/08/2022

Section II

90 Marks

Attempt Questions 11–34

Allow about 2 hours and 45 minutes for this section

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Booklet 2 – Attempt Questions 27–34 (31 marks)

Instructions

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13	/3
14	/2
15	/4
16	/4
17	/4
18	/3
19	/4
20	/3
21	/4
22	/3
23	/3
24	/4
25	/6
26	/5
Total	/59

Please turn over

Section II

In Questions 11 - 27, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Solve the equation $|3x + 2| = 14$

2

$$\begin{cases} 3x + 2 = 14 & (1) \\ 3x + 2 = -14 & (2) \end{cases}$$

$$3x = 12 \quad \text{or} \quad 3x = -16$$

$$\therefore x = 4 \quad \text{or} \quad x = -\frac{16}{3}$$

2 marks.

2 correct answers
with correct working

1 mark, off;

is correct working.

Question 12 (5 marks)

Differentiate the following functions:

(a) $f(x) = x^{15}$ 1

$$f'(x) = 15x^{14}$$

(b) $g(x) = 5\sqrt{x} + 4$ 1 mark for 2

$$g'(x) = 5 \times \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{5}{2\sqrt{x}}$$

either correct
coefficient of $x^{-\frac{1}{2}}$
or correct power.

(c) 1 mark $h(x) = \frac{1}{x^3} - 11x = x^{-3} - 11x$ 2

$$h'(x) = -3x^{-4} - 11$$

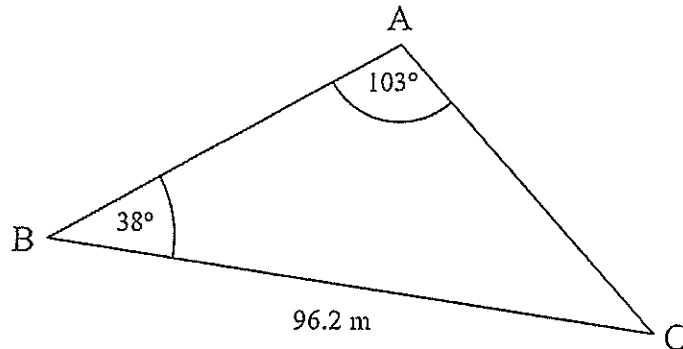
$$= -\frac{3}{x^4} - 11$$

1 mark:
either coefficient
of x^{-4} is correct
or the power
is correct.

Question 13 (3 marks)

The diagram shows a triangle ABC .

3



Use the sine rule to find length AC , correct to 3 significant figures.

$$\frac{AC}{\sin 38^\circ} = \frac{BC}{\sin 103^\circ} \quad / \quad 1 \text{ mark}$$

$$\rightarrow AC = \frac{96.2 \sin 38^\circ}{\sin 103^\circ} \approx 60.8 \text{ m} \quad / \quad 1 \text{ mark}$$

^
2 1 mark

Question 14 (2 marks)

For stringed instruments, the frequency of vibrations (in cycles per second) varies inversely with the length of the string (in cm).

2

A 64 cm stringed instrument has a frequency of 150 cycles per second. Find the frequency of a 60 cm string.

Let f = frequency of vibrations
 l = length of the string

$$f = \frac{k}{l} \quad / \quad 1 \text{ mark;}$$

$$150 = \frac{k}{64} \quad / \quad \text{finding } k \text{ value}$$

$$k = 9600$$

$$f = \frac{9600}{l} \quad / \quad 2 \text{ marks;}$$

$$= \frac{9600}{60} = 160 \text{ cycles per second} \quad / \quad \text{correct answer with working}$$

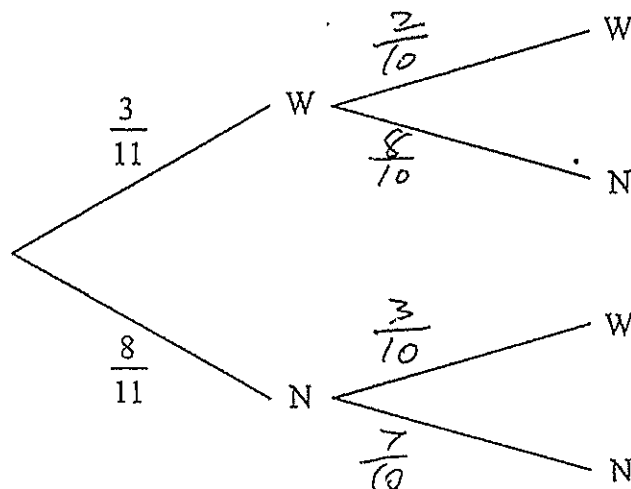
Question 15 (4 marks)

Ms Taylor is competing on a game show. In this show, there are a total of 11 suitcases, 3 of which contain prize money. She chooses 2 suitcases, and wins the prize money contained in those suitcases, if there is any.

Below, W represents Ms Taylor winning prize money, and N represents Ms Taylor not winning money,

- (a) Complete the branches on the diagram below for the second stage.

1



- (b) What is the probability that Ms Taylor does not win a prize?

1

$$\frac{8}{10} \times \frac{7}{10} = \frac{56}{100} = \frac{14}{25}$$

- (c) What is the probability that Ms Taylor wins at least one prize?

1

$$1 - P(\text{no prize}) \\ = 1 - \frac{14}{25} = \frac{11}{25}$$

- (d) Given that the first suitcase Ms Taylor opens has no money, what is the probability that the second suitcase will contain money?

1

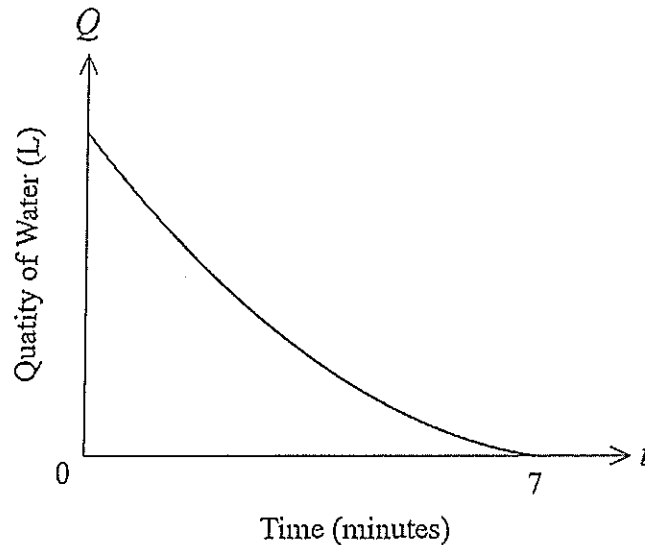
$$P(M / \text{no } M.) = \frac{3}{10} = \frac{3}{10}$$

Since it's independent event.

Question 16 (4 marks)

A tank of water is being drained, and the quantity (Q) of water remaining in the tank after t minutes can be shown by the graph. It takes 7 minutes to completely drain the tank.

The graph has the equation $Q(t) = 20(t - 8)^2 - 20$



- (a) Show that the average rate of change is -180 L/min.

2

1 mark { when $t=0$, $Q = 20 \times (-8)^2 - 20$
 $= 1260$ L

when $Q=0$, $20(t-8)^2 - 20 = 0$
 $(t-8)^2 = 1$, $\therefore t = \pm 1 + 8$
 $t = 7$ or $t = 9$

1 mark { $\therefore$ average rate of change = $\frac{0 - 1260}{7 - 0}$
 $= -180$ L/min

Question 16 continues on page 15

Question 16 (continued)

- (b) Find the time at which the instantaneous rate of change is -180 L/min.

2

1 mark

$$Q'(t) = 20 \times 2(t - 8) = -180$$

1 mark;
correct differentiation

$$t - 8 = -\frac{9}{2}$$

$$t = 8 - \frac{9}{2} = \frac{7}{2}$$

$$= 3.5 \text{ min.}$$

2 marks:
correct answer
with correct
working.

End of Question 16

Question 17 (4 marks)

Solve the following equations for $0 \leq x \leq 2\pi$.

(a)

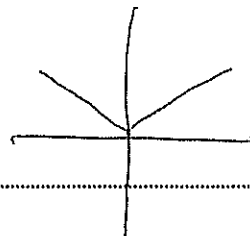
$$\sin x = \frac{1}{\sqrt{2}}$$

2

$$x = \frac{\pi}{4}, \pi - \frac{\pi}{4}$$

$$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}$$

↑ ↑
1 mark 1 mark

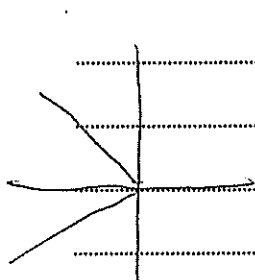


1 mark,
with one correct
angle with

(b)

$$\sec x = -\frac{2}{\sqrt{3}}$$

2



$$\frac{1}{\cos x} = -\frac{2}{\sqrt{3}}$$

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$\text{a coter angle } x = \frac{\pi}{6}$$

$$\therefore x = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$$

$$= \frac{5\pi}{6}, \frac{7\pi}{6}$$

↑ ↑
1 mark 1 mark

Question 18 (3 marks)

Consider the function

3

$$f(x) = \frac{a}{x} + 2x^{\frac{3}{2}}$$

for some constant a .

It is known that $f'(4) = 7$. Find the value of a .

$$f(x) = ax^{-1} + 2x^{\frac{3}{2}}$$

$$f'(x) = -ax^{-2} + 3x^{\frac{1}{2}} \quad \text{--- 1 mark}$$

$$\text{when } x=4, \quad f'(4)=7$$

$$7 = -a \times (4)^{-2} + 3 \times (4)^{\frac{1}{2}}$$

$$7 = \frac{-a}{16} + 6$$

$$1 = -\frac{a}{16}$$

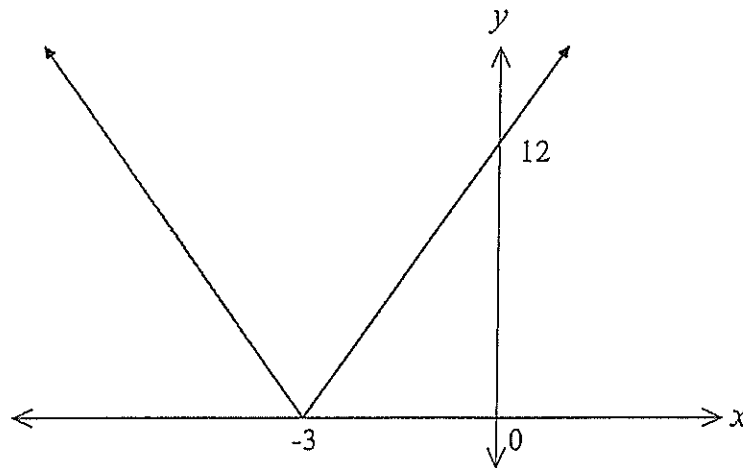
$$\therefore a = -16$$

↑ 1 mark

1 mark.
correct work
working towards
Correct answer.

Question 19 (4 marks)

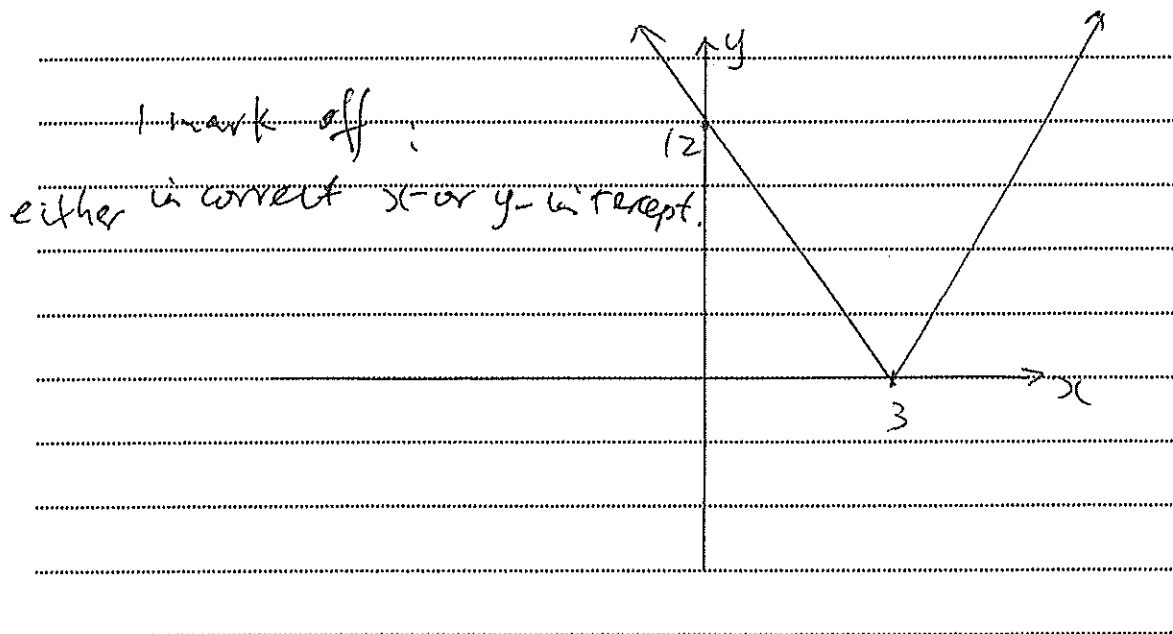
The following graph shows the relationship $y = f(x)$, where $f(x) = |ax + b|$.



- (a) What is one set of possible values for a and b ?

when $x=0$, $f(0)=12$. $|b|=12$, $b=12$. 1 mark
 $x=-3$, $f(-3)=0$, $|-3a+12|=0$ for one correct answer with working.
 $a=4$.

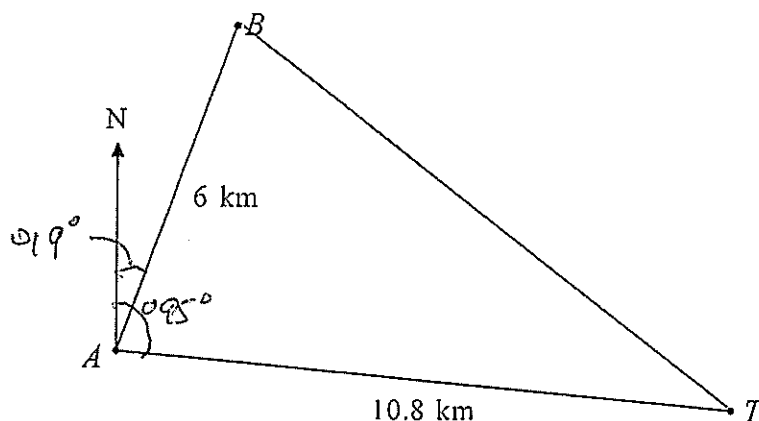
- (b) Sketch the graph $y = f(-x)$, showing any intercepts.



Question 20 (3 marks)

Ms Taylor lives in Town A, and went on a hiking trip. She hiked 10.8 km, on a bearing of 095°T to point T.

Another town, Town B, is located 6 km away from Town A, on a bearing of 019°T .



- (a) Explain why angle BAT is 76° .

1

$$\begin{aligned}\angle BAT &= \angle NAT - \angle NAB \\ &= 095^\circ - 019^\circ \\ &= 076^\circ\end{aligned}$$

- (b) Ms Taylor needs to return to one of the towns, Town A or Town B. Which town is closer to her from point T? Justify your answer with appropriate calculations.

2

$$\begin{aligned}BT^2 &= AB^2 + AT^2 - 2 \times AB \times AT \times \cos 76^\circ \quad \text{1 mark;} \\ BT &= \sqrt{6^2 + 10.8^2 - 2 \times 6 \times 10.8 \times \cos 76^\circ} \quad \text{Correct formula.} \\ &= 11.01 \text{ km}\end{aligned}$$

$$\therefore AT = 10.8 \text{ km}$$

$\therefore$ Town A is closer to point T. 2 marks; correct answer with working

Question 21 (4 marks)

For the following, find $\frac{dy}{dx}$.

(a)

$$y = (7x + 4)(x + 1)^{\frac{4}{3}}$$

2

$$\left. \begin{aligned} \text{Let } u &= 7x + 4, & u' &= 7 \\ v &= (x + 1)^{\frac{4}{3}}, & v' &= \frac{4}{3}(x + 1)^{\frac{1}{3}} \end{aligned} \right\} 1 \text{ mark}$$

$$\frac{dy}{dx} = u'v + uv'$$

$$= 7 \times (x + 1)^{\frac{4}{3}} + (7x + 4) \times \frac{4}{3}(x + 1)^{\frac{1}{3}}$$

$$= 7(x + 1)^{\frac{4}{3}} + \frac{4}{3}(7x + 4)(x + 1)^{\frac{1}{3}}$$

↑ 1 mark

(b)

$$y = \frac{2x + 5}{x^2 + x}$$

2

$$\left. \begin{aligned} \text{Let } u &= 2x + 5, & u' &= 2 \\ v &= x^2 + x, & v' &= 2x + 1 \end{aligned} \right\} 1 \text{ mark}$$

$$y' = \frac{u'v - uv'}{v^2}$$

$$= \frac{2(x^2 + x) - (2x + 5)(2x + 1)}{(x^2 + x)^2}$$

$$= \frac{2x^2 + 2x - 4x^2 - 12x - 5}{(x^2 + x)^2}$$

$$= \frac{-2x^2 - 10x - 5}{(x^2 + x)^2} \quad \uparrow 1 \text{ mark}$$

Question 22 (3 marks)

Determine the number of points of intersection between the following graphs:

3

$$y = 3x^2 + 7x + 4$$

$$y = -x^2 + 1$$

$$-x^2 + 1 = 3x^2 + 7x + 4$$

$$4x^2 + 7x + 3 = 0 \quad - \quad 1 \text{ mark}$$

$$A = 7^2 - 4 \times 4 \times 3$$

$$= 49 - 48$$

$$= 1 > 0$$

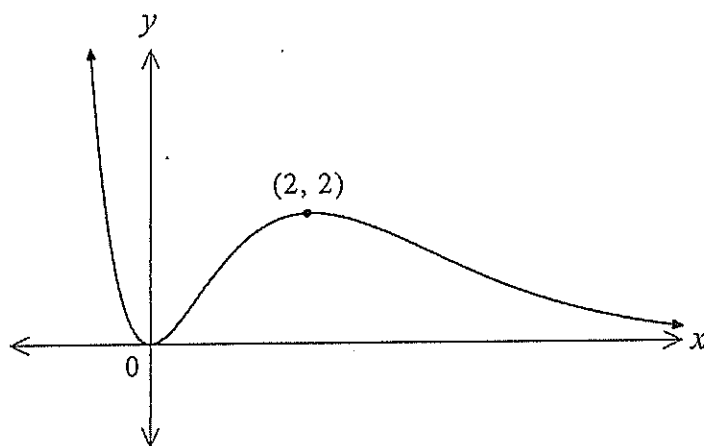
} 1 mark

$\therefore$ There are two intersect
points $\uparrow$ 1 mark

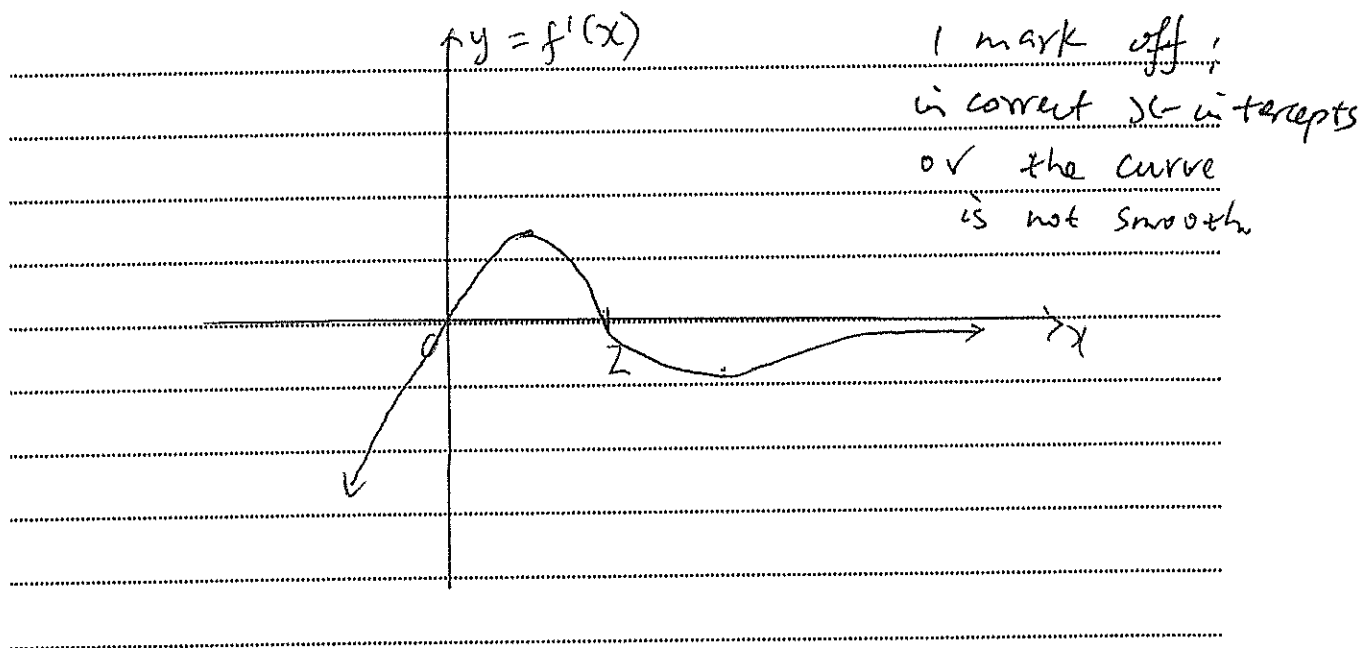
Question 23 (3 marks)

The following graph shows a function $y = f(x)$.

3



Sketch the graph of $y = f'(x)$.



Question 24 (4 marks)

The equation $y = 2x^2 - 5x + 1$ has a tangent at a point (x_1, y_1) with a gradient of 3.

4

Find the equation of the normal to the curve at the point (x_1, y_1) .

$$m_1 = 3.$$

$$m_2 \text{ of normal} = -\frac{1}{3} \quad \text{--- 1 mark}$$

$$\therefore y = -\frac{1}{3}x + b \quad \text{--- 1 mark}$$

$$\text{when } x = x_1, \quad y = y_1$$

$$y_1 = -\frac{1}{3}x_1 + b$$

$$\therefore b = \frac{1}{3}x_1 + y_1 \quad \leftarrow 1 \text{ mark}$$

$\therefore$ The eqn. of the normal:

$$y = -\frac{1}{3}x + \frac{1}{3}x_1 + y_1 \quad \leftarrow 1 \text{ mark}$$

Question 25 (6 marks)

The displacement (in metres) of a particle after t seconds is given by the relationship

$$x(t) = \frac{1}{t^2 + 1}$$

where $t \geq 0$.

- (a) State the initial position of the particle.

1

when $t=0$, $x(0)=1$

- (b) Show that the velocity can be given by $\frac{dx}{dt} = -\frac{2t}{(t^2+1)^2}$

2

$$x(t) = (t^2 + 1)^{-1}$$

1 mark off;

$$\frac{dx}{dt} = -(t^2 + 1)^{-2} \times 2t$$

no negative

sign.

$$= -2t$$

$$(t^2 + 1)^2$$

1 mark;

Correct denominator
with correct power.

Question 25 continues on page 25

Question 25 (continued)

- (c) Find the time(s) when the acceleration is 0 m/s^2 .

3

$$\frac{dx}{dt} = \frac{-2t}{(t^2+1)^2}$$

$$\text{Let } u = -2t, \quad u' = -2$$

$$v = (t^2+1)^2, \quad v' = 2(t^2+1) \times 2t \\ = 4t(t^2+1)$$

} 1 mark

$$\frac{d^2x}{dt^2} = \frac{-2(t^2+1)^2 - (-2t) \times 4t(t^2+1)}{(t^2+1)^4}$$

1 mark

$$= \frac{-2(t^2+1)^2 + 8t^2(t^2+1)}{(t^2+1)^4}$$

working towards
correct answer.

$$= \frac{-2(t^2+1)(t^2+1-4t^2)}{(t^2+1)^4}$$

$$= \frac{-2(-3t^2+1)}{(t^2+1)^3}, \quad \text{when } \frac{d^2x}{dt^2} = 0$$

$$-3t^2+1=0, \quad t^2=\frac{1}{3}, \quad t=\pm\frac{1}{\sqrt{3}}$$

$$\therefore t \approx \frac{1}{\sqrt{3}} \text{ seconds}$$

$$\text{or } \approx 0.6 \text{ seconds}$$

End of Question 25

Question 26 (5 marks)

(a) Show that

$$2 \cos^2 x - 3 \sin^2 x + 1 = 5 \cos^2 x - 2$$

2

$$\begin{aligned} \text{LHS} &= 2 \cos^2 x - 3(1 - \cos^2 x) + 1 \\ &= 2 \cos^2 x - 3 + 3 \cos^2 x + 1 \\ &= 5 \cos^2 x - 2 \\ &= \text{RHS} \end{aligned}$$

1 mark:

Correct working

for towards

correct answer

(b) Hence, solve the following equation for $0^\circ \leq x \leq 360^\circ$, giving answers to the nearest degree.

3

$$2 \cos^2 x - 3 \sin^2 x + 1 = 0$$

$$5 \cos^2 x - 2 = 0$$

$$\cos^2 x = \frac{2}{5}$$

$$\cos x = \pm \sqrt{\frac{2}{5}}, \quad - \quad 1 \text{ mark}$$

$$\text{acute angle } x = \cos^{-1}\left(\sqrt{\frac{2}{5}}\right) \approx 51^\circ$$

$$\therefore x = 51^\circ, 180^\circ - 51^\circ, 180^\circ + 51^\circ, 360^\circ - 51^\circ$$

$$= 51^\circ, 129^\circ, 231^\circ, 309^\circ$$

↑

1 mark with two out of four answers

Proceed to Booklet 2 for Questions 27 – 34

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

1



Killara
HIGH SCHOOL

MATHEMATICS DEPARTMENT

Solutions.

Student Number

2022 Year 11 Preliminary Examination

Mathematics Advanced
Section II Answer Booklet 2

22/08/2022

Q	Marks
27	/5
28	/4
29	/5
30	/3
31	/3
32	/5
33	/4
34	/2
Total	/31

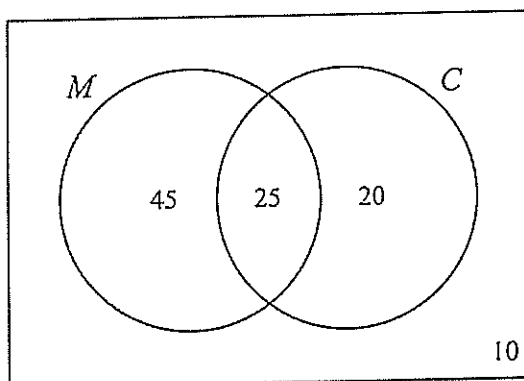
Booklet 2 – Attempt Questions 27–34 (31 marks)

- Instructions**
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided on page 41 & 42. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 27 (5 marks)

In a group of 100 students, some study Mathematics (M), some study Chemistry (C). Some study both Mathematics and Chemistry, while some study neither.



- (a) How many students are in the set $\bar{M} \cup C$? 1

$|\bar{M} \cup C| = 55$ ① for the answer

- (b) Explain why the events M and C are not mutually exclusive. 1

Because the intersection is not empty ① for any correct explanation

- (c) If a student is chosen at random from this group, what is $P(C|M)$? 1

$P(C|M) = \frac{25}{70}$ ① for answer
 $= \frac{5}{14}$ (doesn't need to be simplified)

Question 27 continues on page 31

Question 27 (continued)

- (d) Determine whether the events M and C are independent.

2

$$P(C) = \frac{45}{100}$$

$$= \frac{9}{20}$$

$$\neq \frac{5}{14}$$

$P(C) \neq P(C|M)$, so they are not independent.

- ② Correct calculation + conclusion
* Can also use $P(C)P(M) \neq P(C \cap M)$

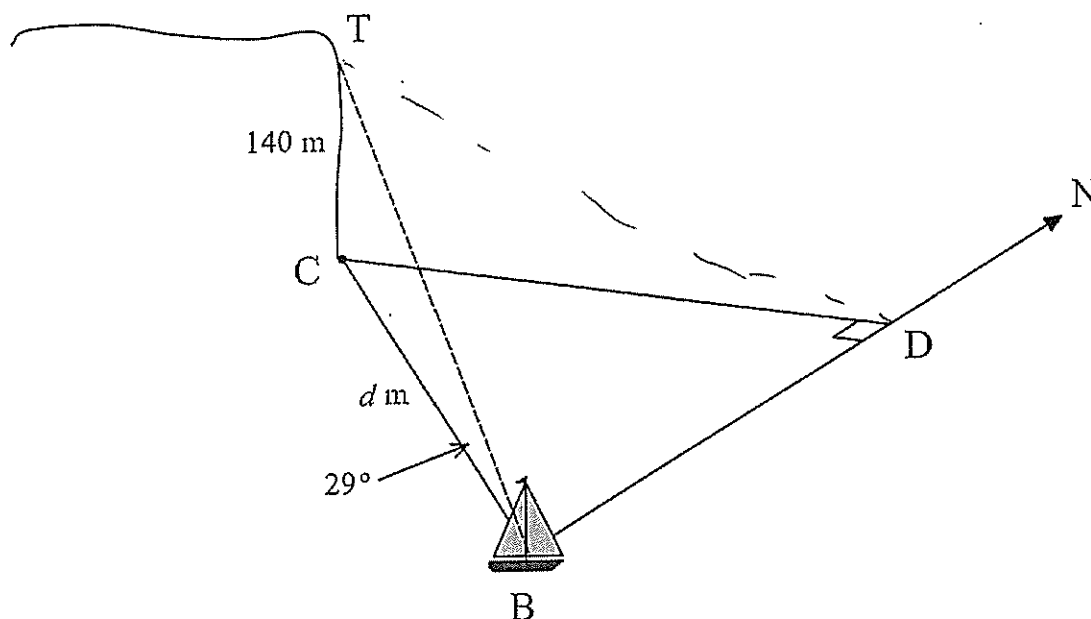
①. calculating $P(C)$

• Attempting to use $P(C) = P(C|M)$

$$P(C)P(M) = P(C \cap M)$$

Question 28 (4 marks)

A travelling boat spots a 140 metres vertical cliff on a true bearing of 312° . The angle of elevation from the boat to the top of the cliff is 29° .



- (a) Show that the distance d metres from the boat to the base of the cliff can be given by

1

$$d = \frac{140}{\tan 29^\circ}$$

$\Rightarrow \tan 29^\circ = \frac{140}{d}$
 So $d = \frac{140}{\tan 29^\circ}$
 ① for the first line followed by the second

Question 28 continues on page 33

Question 28 (continued)

- (b) The boat travels due north, until it is directly east of the base of the cliff, meaning $\angle BDC = 90^\circ$.

3

Find the new angle of elevation of the boat to the top of the cliff, correct to the nearest degree.

$$\sin 48^\circ = \frac{CD}{\left(\frac{140}{\tan 29^\circ}\right)}$$

$$CD = \frac{140 \sin 48^\circ}{\tan 29^\circ}$$

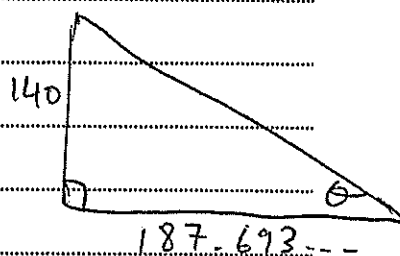
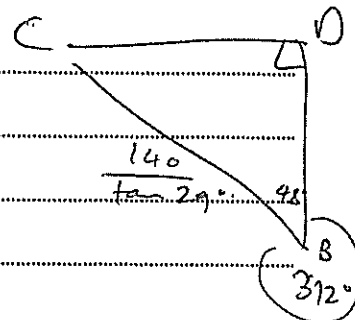
$$= 187.693 \dots$$

$$\tan \theta = \frac{140}{187.693 \dots}$$

$$= 0.745 \dots$$

$$\theta = 36.71 \dots$$

$$= 37^\circ$$



① Finding $\angle DBC = 48^\circ$

② Finding CD

③ Calculating the angle.

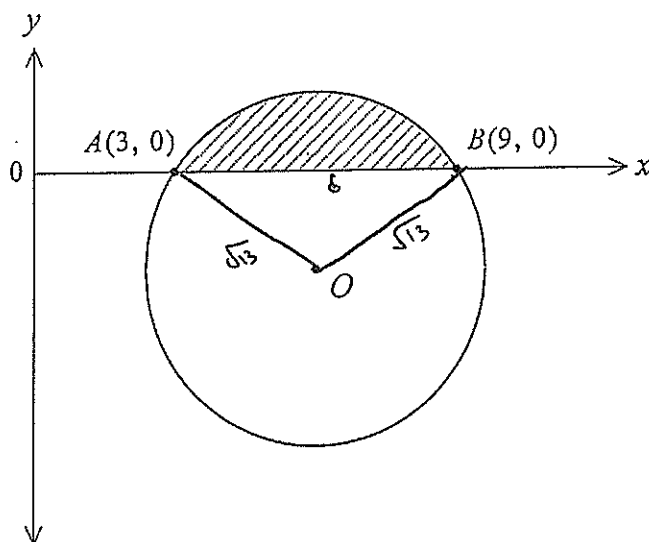
End of Question 28

(rounding not penalised)

Question 29 (5 marks)

The following equation represents the graph of the circle shown, centred at O .

$$x^2 - 12x + y^2 + 4y + 27 = 0$$



- (a) Show that the radius of the circle is $\sqrt{13}$ units.

1

$$x^2 - 12x + 36 + y^2 + 4y + 4 = 36 + 4 - 27$$

$$(x-6)^2 + (y+2)^2 = 13 = r^2$$

$$\text{so } r = \sqrt{13}$$

① for completing the square
to show $r^2 = 13$
so $r = \sqrt{13}$

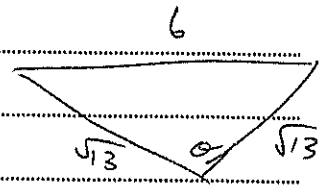
Question 29 continues on page 35

Question 29 (continued)

- (b) Find the angle AOB , correct to the nearest minute

2

$$\cos \theta = \frac{(\sqrt{13})^2 + (\sqrt{13})^2 - 6^2}{2(\sqrt{13})(\sqrt{13})}$$



$$= \frac{-10}{26}$$

① for using cos rule
* if they have incorrect rounding

$$\theta = \cos^{-1}\left(\frac{-10}{26}\right)$$

$$= 112^{\circ}37'$$

② correct working
rounded to the nearest minute.

- (c) Hence, find the shaded area, correct to 3 significant figures.

2

$$A = \frac{112^{\circ}37'}{360} \times \pi(13) - \frac{1}{2}(13) \sin 112^{\circ}37'$$

$$= 6.78 \text{ u}^2$$

① for either correct area of a sector or triangle

② correct area (don't care about rounding)

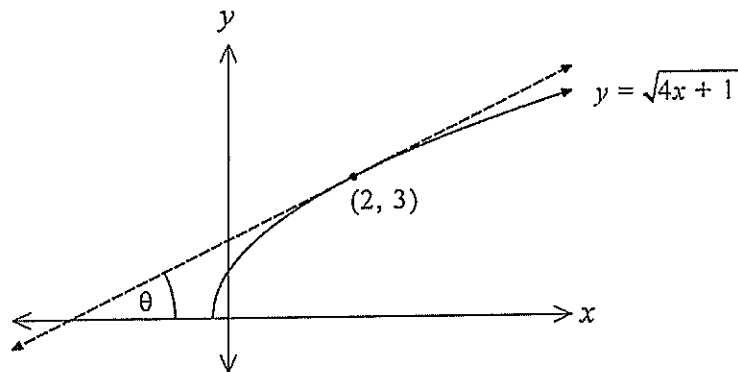
End of Question 29

Question 30 (3 marks)

The function $y = \sqrt{4x + 1}$ has been graphed below. The dotted line shows the tangent to the curve at the point $(2, 3)$.

3

Find the angle the tangent line makes to the x -axis from the positive side (θ), correct to the nearest degree.



$$y = (4x + 1)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(4)(4x + 1)^{-\frac{1}{2}}$$

$$= \frac{2}{\sqrt{4x + 1}}$$

① for the derivative

$$y'(2) = \frac{2}{\sqrt{9}}$$

$$= \frac{2}{3}$$

② for the gradient value

$$\frac{2}{3} = \tan \theta$$

$$\theta = 33.69 \dots$$
$$= 34^\circ$$

③ for the angle

Question 33 (3 marks)

Let $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x^2-3}$.

3

By finding an expression for $g(f(x))$, sketch the function $y = g(f(x))$ over its natural domain.

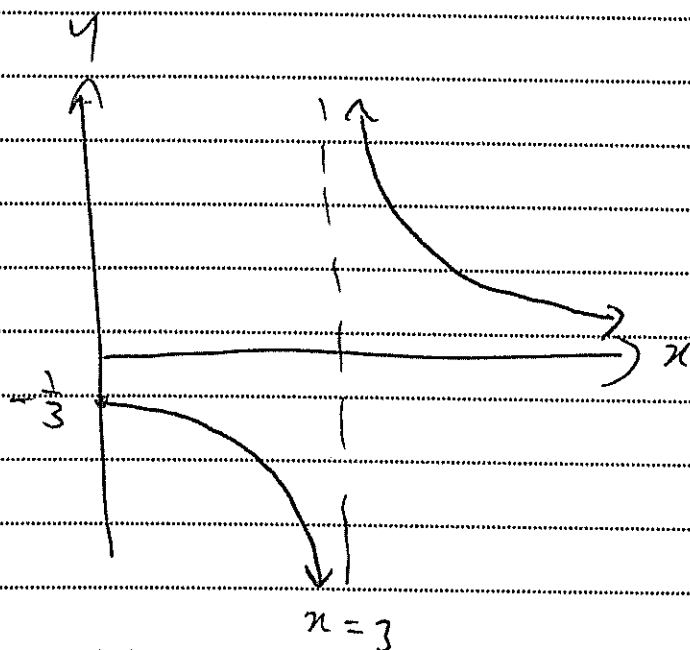
$$g(f(x)) = g(\sqrt{x})$$

$$= \frac{1}{(\sqrt{x})^2 - 3}$$

$$= \frac{1}{x-3}$$

Domain

$$x \geq 0$$



① finding $g(f(x))$

② graphing $y = \frac{1}{x-3}$
w/o restricted domain

③ correct graph

y-intercept: $g(f(0)) = -\frac{1}{3}$

Question 34 (5 marks)

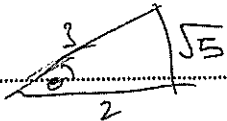
Use the following information for both parts (a) and (b).

For an angle θ , where $\pi \leq \theta \leq 2\pi$, $\cos \theta = \frac{2}{3}$.

$\nearrow$ 3rd, 4th quad

- (a) Without finding θ , explain why $\sin \theta = -\frac{\sqrt{5}}{3}$

2


 $\sin \theta \leftarrow$ in the 3rd/4th quad

(1) $\rightarrow$ So $\sin \theta = -\frac{\sqrt{5}}{3}$ (7)

- (b) Simplify the following expression, leaving your answer in the form $a\sqrt{5} + b$, where a and b are real numbers.

3

$$\frac{\cos \theta + 2 \sin \theta}{\tan \theta - 3 \sin \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= -\frac{\sqrt{5}}{3} \times \frac{3}{2}$$

$$= -\frac{\sqrt{5}}{2}$$

(1) for $\tan \theta$

$$\frac{\cos \theta + 2 \sin \theta}{\tan \theta - 3 \sin \theta} = \frac{\frac{2}{3} + \frac{2\sqrt{5}}{3}}{-\frac{\sqrt{5}}{2} + \sqrt{5}} \times 6$$

$$= \frac{4 - 4\sqrt{5}}{-3\sqrt{5} + 6\sqrt{5}}$$

(2) substituting
in w/o
simplifying

$$= \frac{4 - 4\sqrt{5}}{3\sqrt{5}}$$

$$= \frac{4\sqrt{5} - 20}{15}$$

(3) correct
simplification

$$= \frac{4}{15}\sqrt{5} - \frac{4}{3}$$

Question 35 (4 marks)

A bag contains n identically shaped balls, k of which are black and remaining are white.

4

1. When one ball is chosen at random, the probability of choosing a white ball is $\frac{13}{15}$.
2. When two balls are chosen at random without replacement, the probability of choosing two black balls is $\frac{1}{66}$.

Determine the number of balls n in the bag, and the number of black balls k , assuming $k \neq 0$.

$$1. \quad \frac{n-k}{n} = \frac{13}{15}$$

① for setting up either 1 or 2

$$15n - 15k = 13n$$

$$2n = 15k$$

$$n = \frac{15}{2}k$$

② for setting up both 1 & 2 algebraically

$$2. \quad \frac{k}{n} \times \frac{k-1}{n-1} = \frac{1}{66}$$

$$66k^2 - 66k = n^2 - n$$

$$66k^2 - 66k = \frac{225}{4}k^2 - \frac{15}{2}k$$

$$264k^2 - 225k^2 = 264k - 30k$$

$$39k^2 = 234k$$

$$k = 0, 6 \rightarrow k = 6$$

③ setting up a quadratic to find k or n

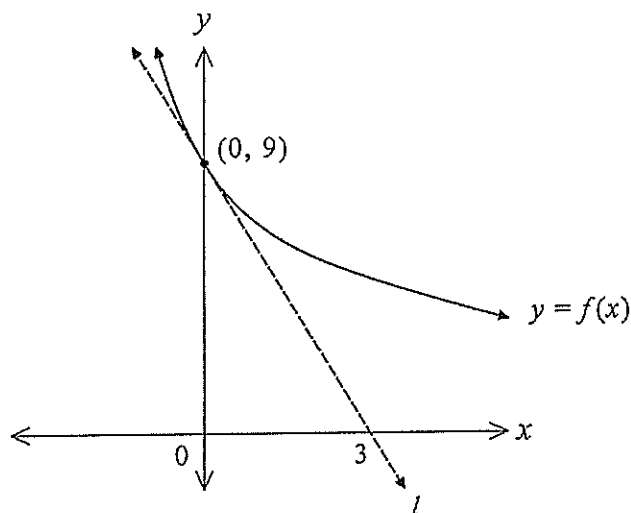
$$n = \frac{15}{2}(6)$$

$$= 45$$

④ correctly solving for both k & n

Question 36 (2 marks)

The graph below shows a function $y = f(x)$, and its tangent at the point $(0, 9)$ represented by line l . 2



Given a new function $g(x) = \sqrt{f(x)}$, find the value of $g'(0)$.

$$g'(x) = \frac{1}{2} (f(x))^{-\frac{1}{2}} \times f'(x) \quad \leftarrow \textcircled{1}$$

$$g'(0) = \frac{f'(0)}{2\sqrt{f(0)}}$$

$$= \frac{-3}{2\sqrt{9}}$$

$$= -\frac{1}{2} \quad \leftarrow \textcircled{2}$$

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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[The page contains dense horizontal noise patterns consisting of various symbols and characters.]