

Student Number

Teacher's Name

2023 Year 11 Preliminary Examination

Mathematics Advanced

11/9/2023

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks:
100

Section I - 10 marks (pages 1 – 9)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - 90 marks (pages 10 – 43)

- Attempt Questions 11 – 33
- Allow about 2 hours and 45 minutes for this section

This question paper must not be removed from the examination room

This assessment task constitutes 40% of the course.

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Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1. Factorise the following expression.

$$x^2 - 2x - 3$$

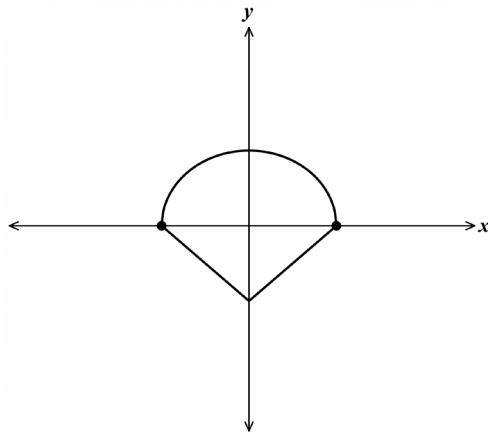
- A. $(x - 3)(x - 1)$
 - B. $(x - 3)(x + 1)$
 - C. $(x + 3)(x + 1)$
 - D. $(x + 3)(x - 1)$
2. What is the coefficient of x^2 in the following algebraic expression?

$$(3x - 4)(x^2 - 2x + 5)$$

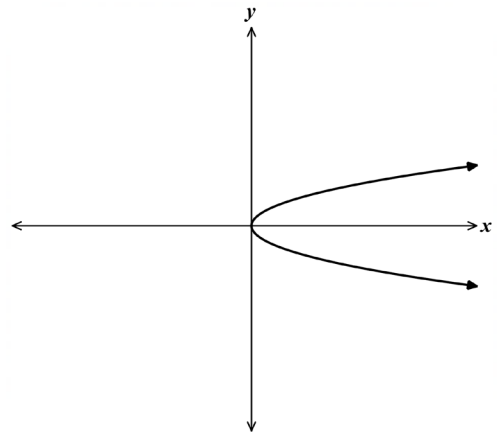
- A. 3
- B. 23
- C. -10
- D. -20

3. Which of the following graph classifies as a function?

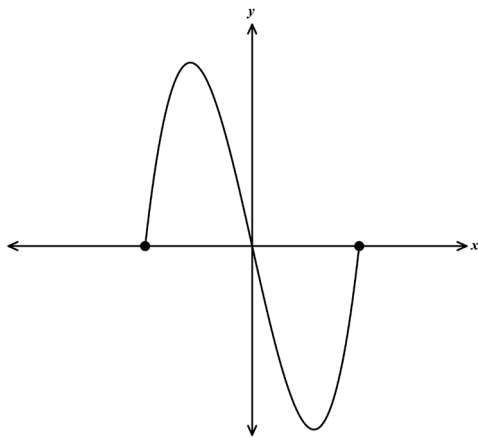
A.



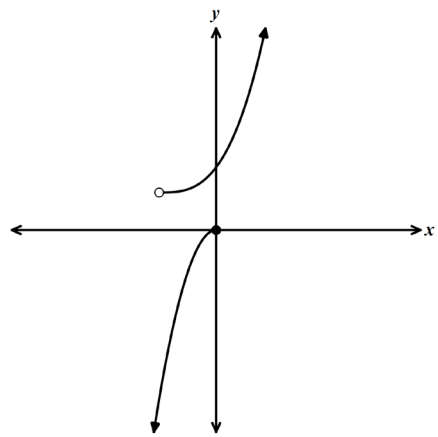
B.



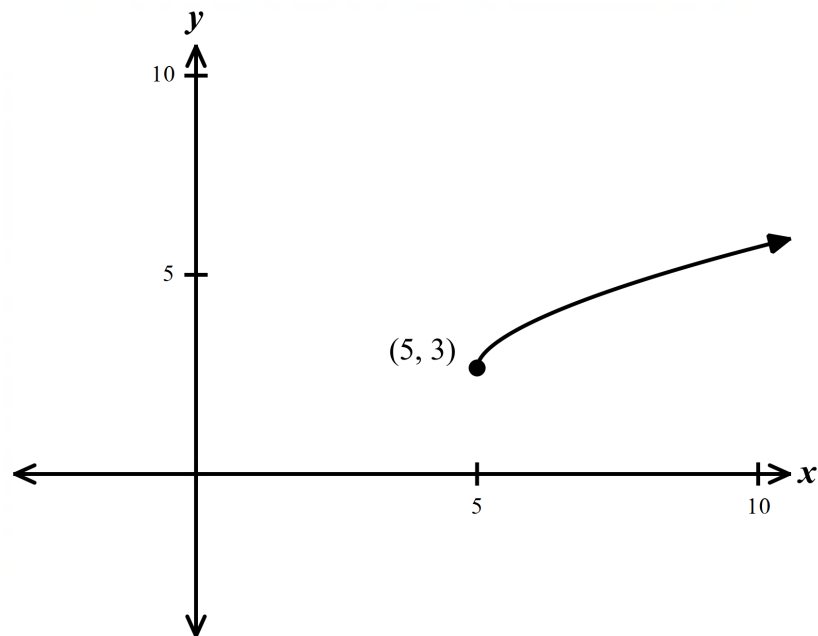
C.



D.

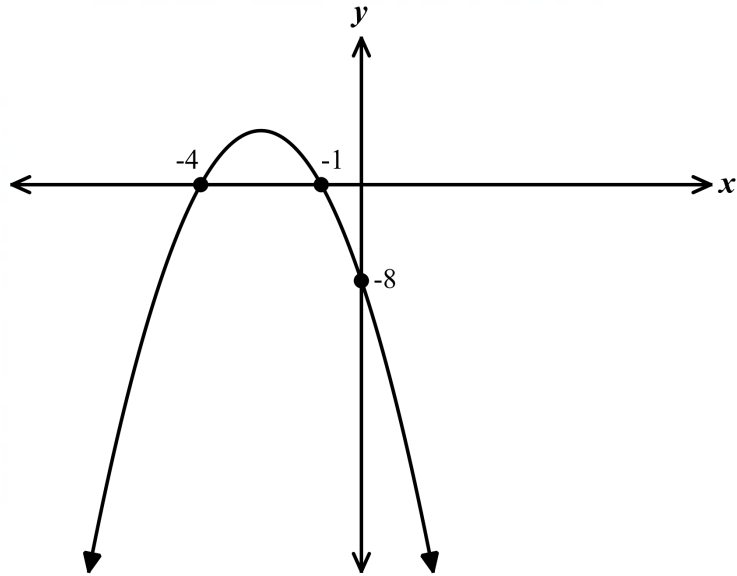


4. What is the domain and range of the following function?



- A. **Domain:** $x \in [5, \infty)$
 Range: $y \in [3, \infty)$
- B. **Domain:** $x \in [3, \infty)$
 Range: $y \in [5, \infty)$
- C. **Domain:** $x \in [5, \infty)$
 Range: $y \in [0, \infty)$
- D. **Domain:** $x \in [0, \infty]$
 Range: $y \in [5, \infty]$

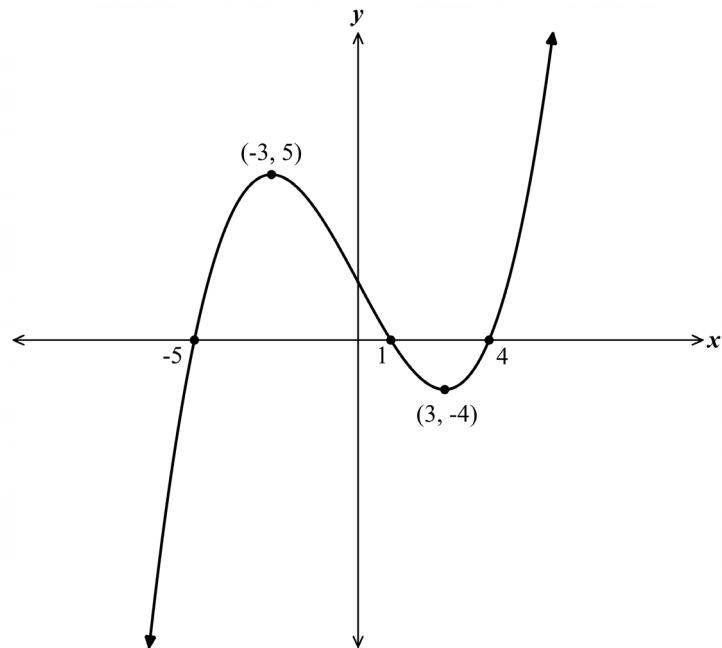
5. The equation of the quadratic function shown in the diagram is



- A. $y = (x - 1)(x - 4)$
- B. $y = -(x + 1)(x + 4)$
- C. $y = -2(x + 1)(x + 4)$
- D. $y = -2(x - 1)(x - 4)$
6. If $\cos \theta = x$ and $0^\circ < \theta < 90^\circ$, then $\tan \theta$ is equal to:

- A. $\frac{1}{\sqrt{1 - x^2}}$
- B. $\sqrt{1 - x^2}$
- C. $\frac{\sqrt{1 - x^2}}{x}$
- D. $\frac{x}{\sqrt{1 - x^2}}$

7. For the graph of $y = f(x)$ shown below, $f'(x)$ is negative when

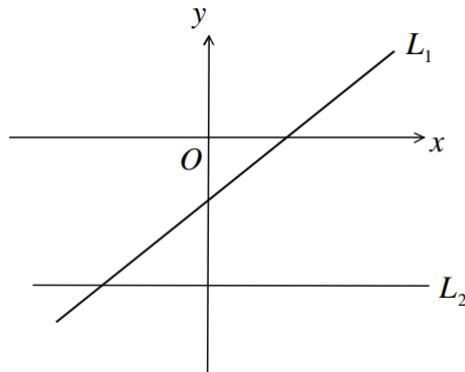


- A. $-3 < x < 3$
- B. $-3 \leq x \leq 3$
- C. $x < -3$ or $x > 3$
- D. $x \leq -3$ or $x \geq 3$
8. If $A + B = 180^\circ$, which of the following statement is true?

- I. $\sin A = \sin B$
- II. $\cos A = \cos B$
- III. $\tan A = \tan B$

- A. I only
- B. II only
- C. III only
- D. None of them

9. In the figure shown below, the equations of the straight lines L_1 and L_2 are $bx + ay = ab$ and $y = \frac{1}{c}$ respectively. Where a, b and c are constants. Which of the following statements are true?



- I. $c < 0$
 - II. The gradient of $L_1 = -\frac{b}{a}$
 - III. $0 < bc < 1$
- A. I and II only
- B. I and III only
- C. II and III only
- D. I, II and III

10. A game show host puts two balls randomly into three different boxes (each box may contain 0 balls, 1 ball or 2 balls).

Participants of the game can choose to open any two of the three boxes. If two balls are drawn, they can get a prize.

Find the probability that the participant can get a prize.

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{4}{9}$

D. $\frac{5}{9}$

End of Section I

Student Number

Teacher Name

2023 Year 11 Preliminary Examination

Mathematics Advanced

Section II Answer Booklet 1

11/09/2023

Section II

90 Marks

Attempt Questions 11 - 33

Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 27 (58 marks)

Booklet 2 – Attempt Questions 28 – 33 (32 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 29 - 30
- If you use this space, clearly indicate which question you are answering.

Q	Marks
11	/2
12	/3
13	/4
14	/3
15	/2
16	/3
17	/2
18	/2
19	/2
20	/6
21	/6
22	/6
23	/3
24	/3
25	/3
26	/2
27	/6
Total	/58

Section II

In Questions 11 - 27, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Simplify the following indices using index laws.

2

$$(q^2)^3 \times \left(\frac{q^5}{q}\right)^2$$

Question 12 (3 marks)

Rationalise the following surds. Leave your answer in the simplest form.

(a)

$$\frac{6}{\sqrt{3}}$$

1

(b)

$$\frac{3}{\sqrt{2} + 1}$$

2

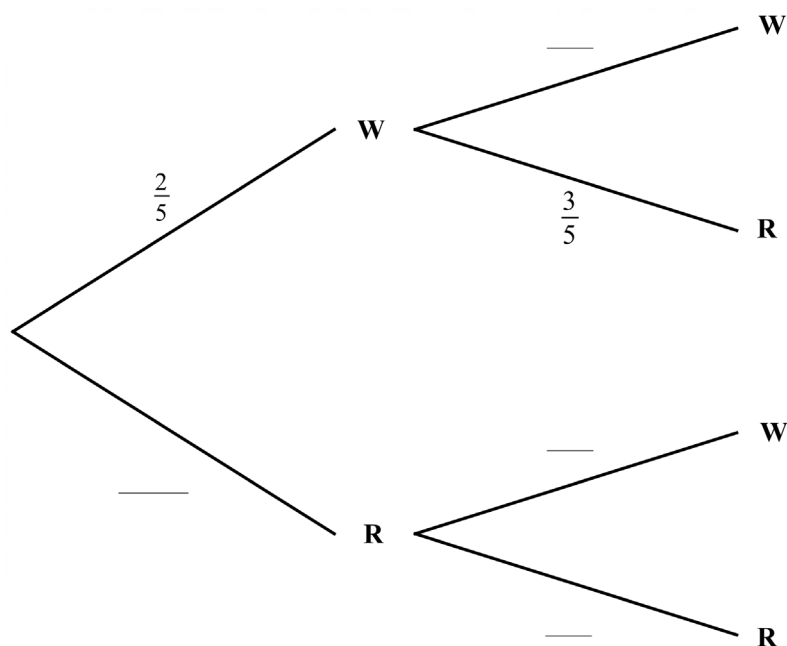
Question 13 (4 marks)

Ms Taylor is picking two marbles from a box. The box contains 2 white and 3 red marbles. A marble is drawn at random from the box and returned to the box after its colour has been noted, then the second marble is drawn at random from the box.

In the probability tree diagram below, W represents white marbles, and R represents red marbles.

- (a) Complete the following tree diagram.

1



- (b) What is the probability that Ms Taylor picks two marbles of the same colour?

1

.....

- (c) What is the probability that Ms Taylor picks at least one white marble?

1

.....

- (d) Given that the first marble that Ms Taylor picked is red, what is the probability that the second marble is white?

1

.....

.....

Question 14 (3 marks)

Given that

$$f(x) = x - \frac{1}{x}$$

(a) Evaluate $f(4)$.

1

(b) Show that f is an odd function.

2

Question 15 (2 marks)

Solve the following equation.

2

$$|9 - 2y| = 21$$

[illegible]

Question 16 (3 marks)

For the following discrete probability distribution:

x	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

- (a) Calculate the expected value.

1

[illegible]

- (b)** Hence calculate the variance.

2

[illegible]

Question 17 (2 marks)

Differentiate $f(x) = x^2 + 3$ using the first principle.

2

The first principle is given below,

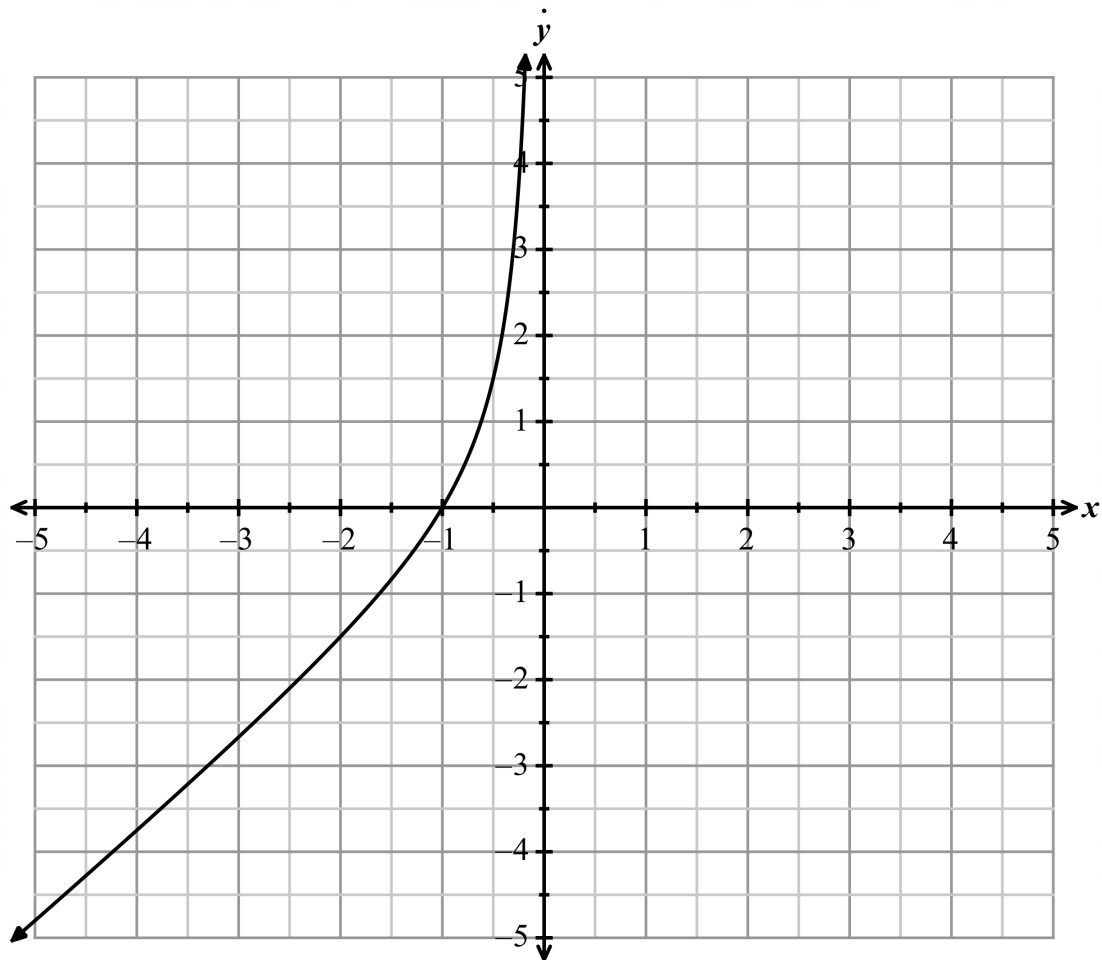
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

[illegible]

Question 18 (2 marks)

A part of an odd function $g(x)$ is shown on the diagram below. Complete the following diagram for $g(x)$ over the domain $(0, \infty)$.

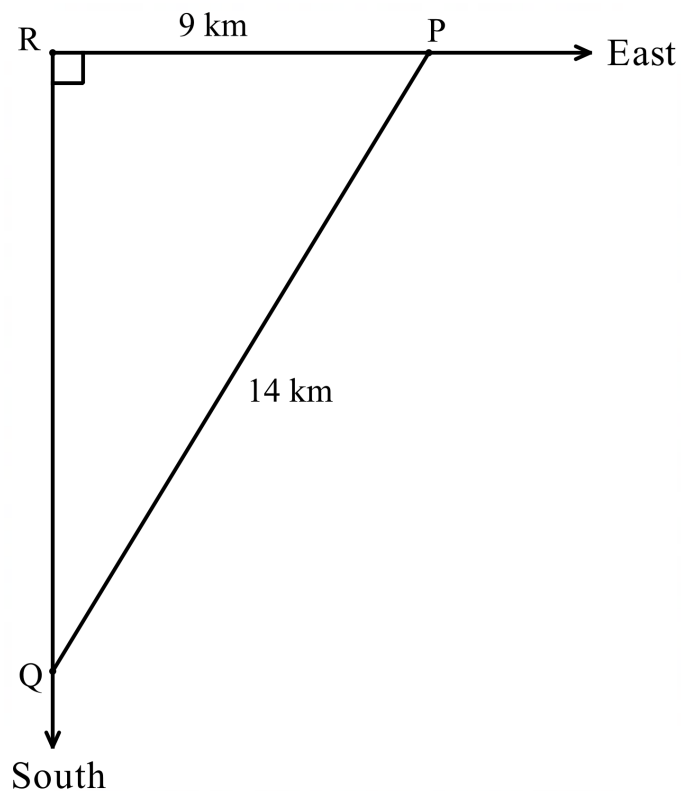
2



Question 19 (2 marks)

In the diagram shown below, P, Q and R are three campsites. P is 9 km due east of R and Q is due south of R. The distance between P and Q is 14 km. Find the bearing of Q from P to the nearest degree.

2



Question 20 (6 marks)

Solve the following trigonometric equations for $0 \leq \theta \leq 2\pi$.

(a) $\cos \theta = \frac{\sqrt{2}}{2}$ 2

(b) $\operatorname{cosec} \theta = -2$ 2

(c) $\sin \theta - \cos \theta = 0$ 2

Question 21 (6 marks)

Differentiate the following functions.

(a)

$$f(x) = x^2 - 5x^{-1}$$

2

(b)

$$f(x) = (4x - 3)^4$$

2

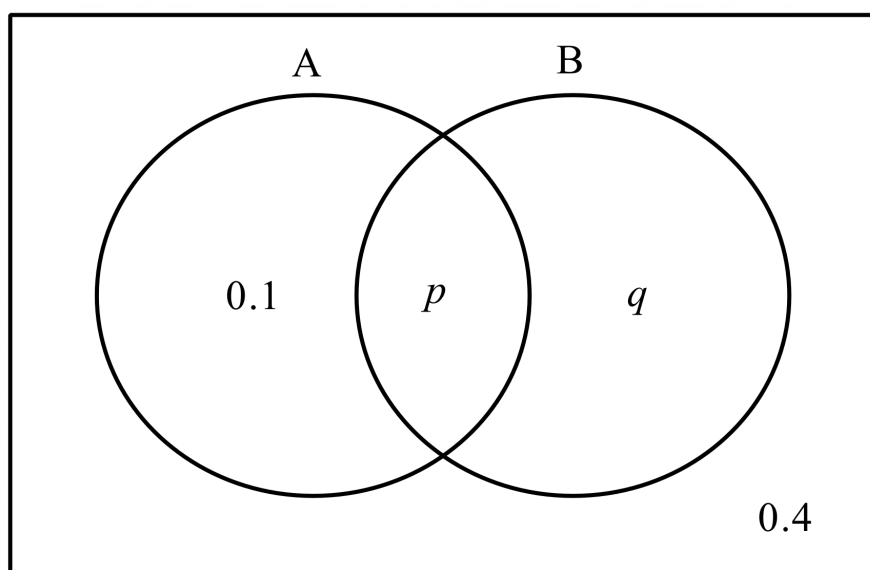
(c)

$$f(x) = \frac{x^2}{2x + 1}$$

2

Question 22 (6 marks)

The following Venn diagram shows events A and B, where $P(A) = 0.3$.



- (a) Find the values of p and q .

2

- (b) Hence, find $P(A' \cup B)$.

1

- (c) Calculate the probability of $P(B|A)$.

1

Question 22 continues on page 22

Question 22 (continued)

- (d) Determine whether events A and B are independent. Justify your answer with the appropriate mathematical calculations. **2**

End of Question 22

Question 23 (3 marks)

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of $g(x)$.

(a) Show that b is 4.

1

(b) The graph $f(x) = x^2$ is transformed to obtain the graph of $g(x)$. Describe this transformation.

2

Question 24 (3 marks)

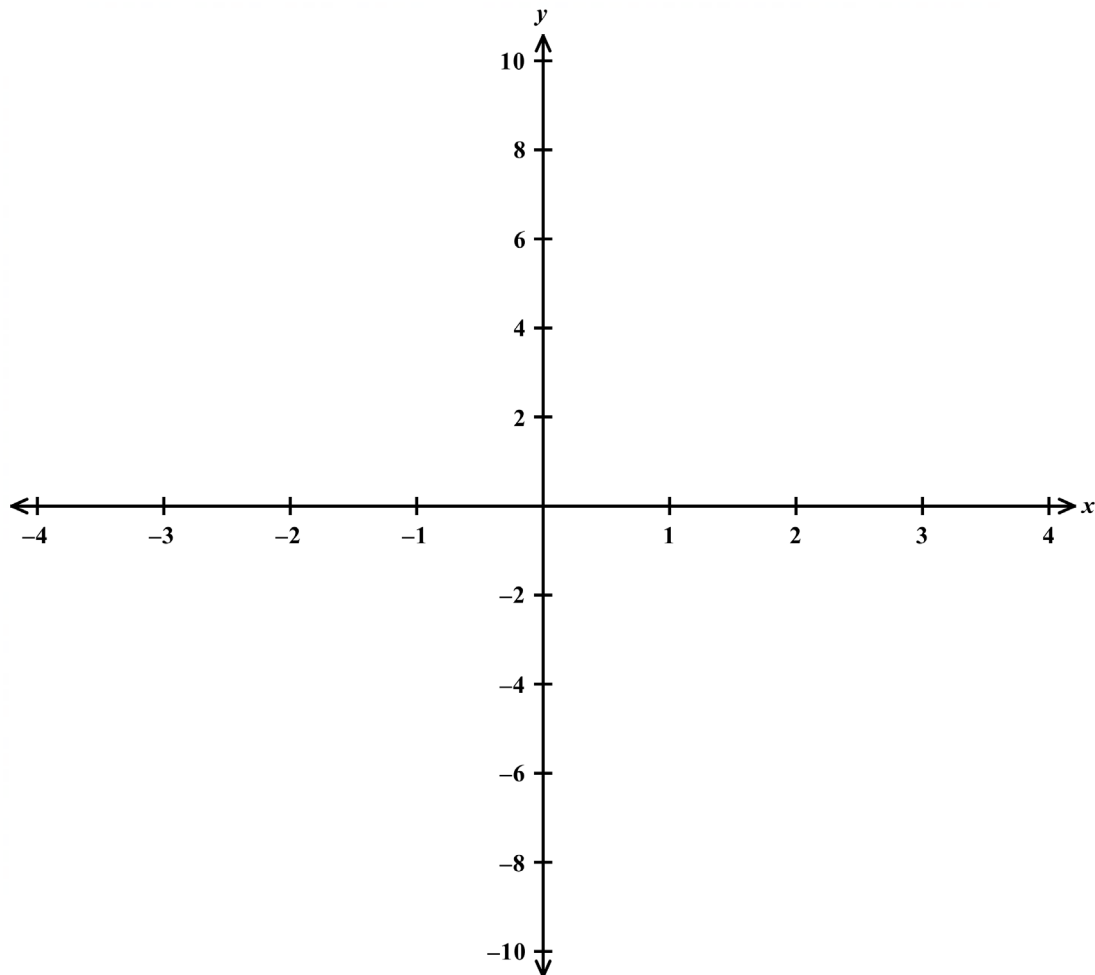
Fully simplify the following expression.

3

$$\frac{\sin \theta}{\cos 60^\circ} + \frac{\cos(270^\circ - \theta)}{\tan 45^\circ}$$

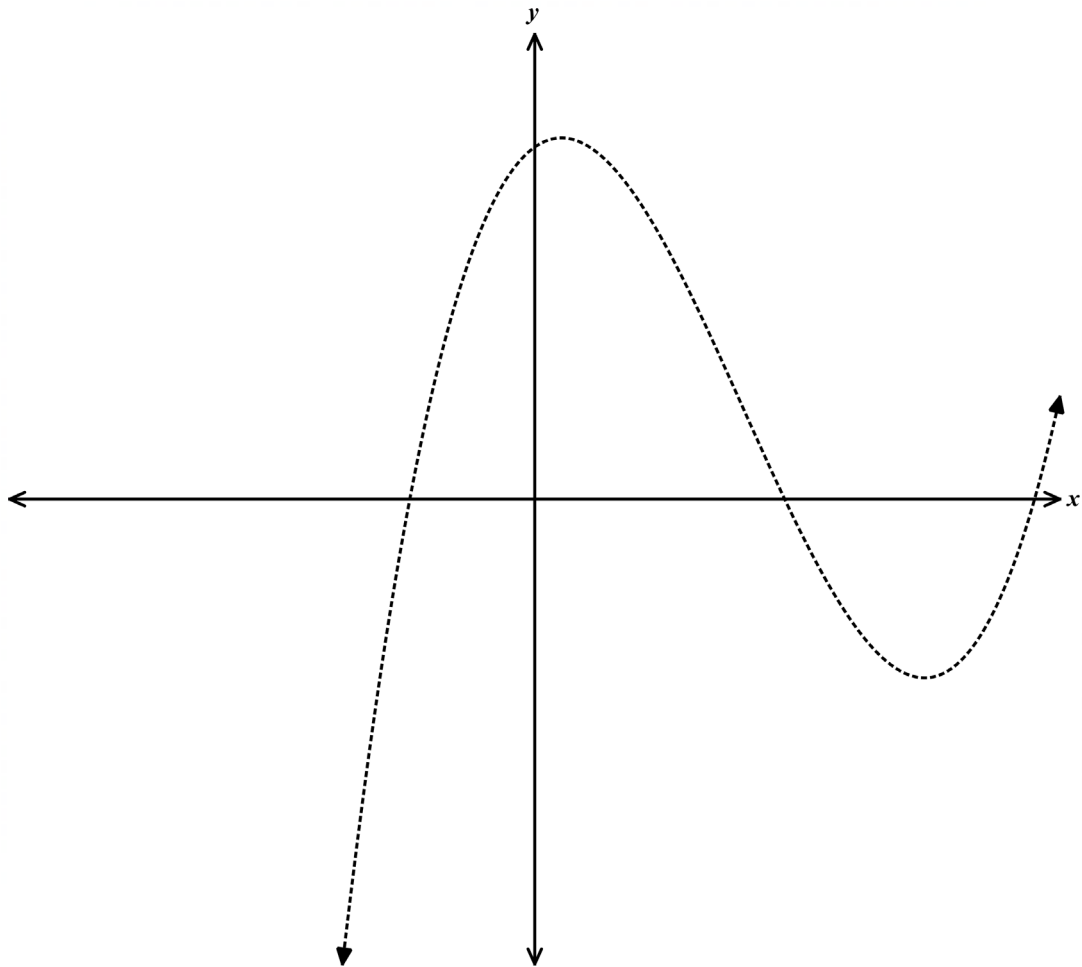
Question 25 (3 marks)

Sketch $f(x) = (x - 1)(x - 3)^2(x + 1)$ on the axis provided below. Labelling the x and y intercepts. **3**



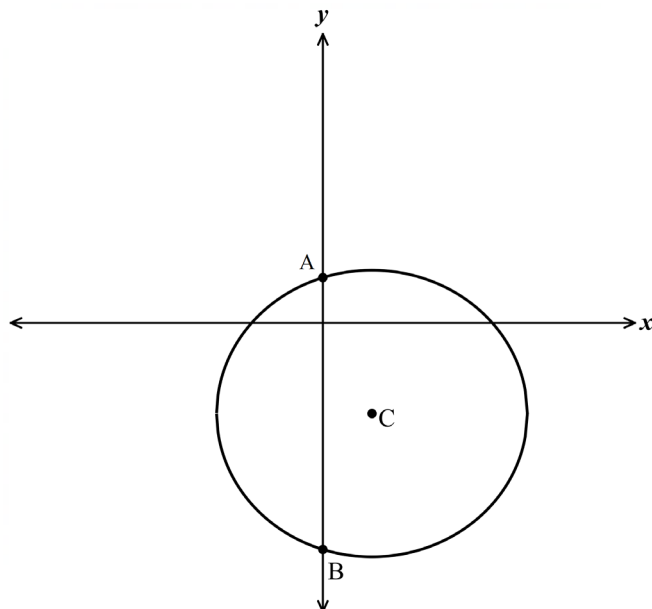
Question 26 (2 marks)

The function $f(x)$ is given on the axis below. Hence sketch the gradient function on the same axis. **2**



Question 27 (6 marks)

The equation of a circle is given by $4x^2 + 4y^2 - 4x + 8y - 5 = 0$. It is given that the circle cuts the y - *axis* at two points A and B respectively as shown on the diagram below.



- (a) Show that the centre C has the coordinate of $\left(\frac{1}{2}, -1\right)$.

2

Question 27 continues on page 27

Question 27 (continued)

(b) Find the radius of this circle.

1

(c) Hence, or otherwise, show that $\angle ACB$ is obtuse.

3

End of Question 27

Proceed to Booklet 2 for Questions 28 - 33

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

Student Number

Teacher name

2023 Year 11 Preliminary Examination

Mathematics Advanced

Section II Answer Booklet 2

11/09/2023

Q	Marks
28	/3
29	/5
30	/6
31	/5
32	/7
33	/6
Total	/32

Booklet 2 – Attempt Questions 28 - 33 (32 marks)

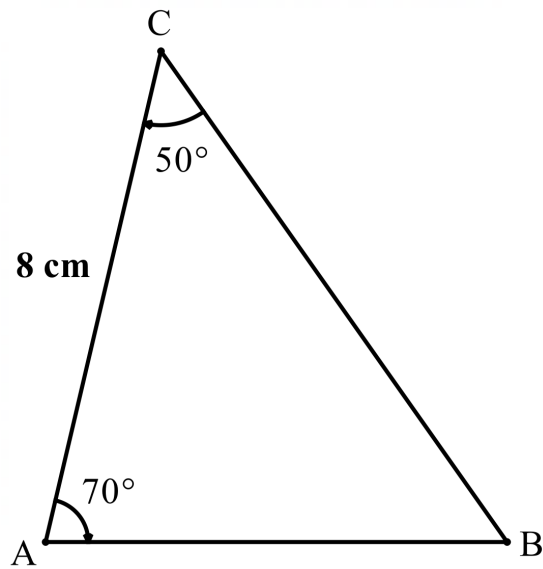
Instructions

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- Extra writing space is provided on page 42 – 43.
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Question 28 (3 marks)

Calculate the area of $\triangle ABC$ to nearest 2 decimal places.

3

This image shows a full page of blank primary-ruled paper. It features ten sets of horizontal lines across the page. Each set consists of a solid top blue line, a dashed middle blue line, and a solid bottom blue line, providing a guide for letter height and placement in handwriting practice. The background is white, and there are no margins or other markings on the page.

Question 29 (5 marks)

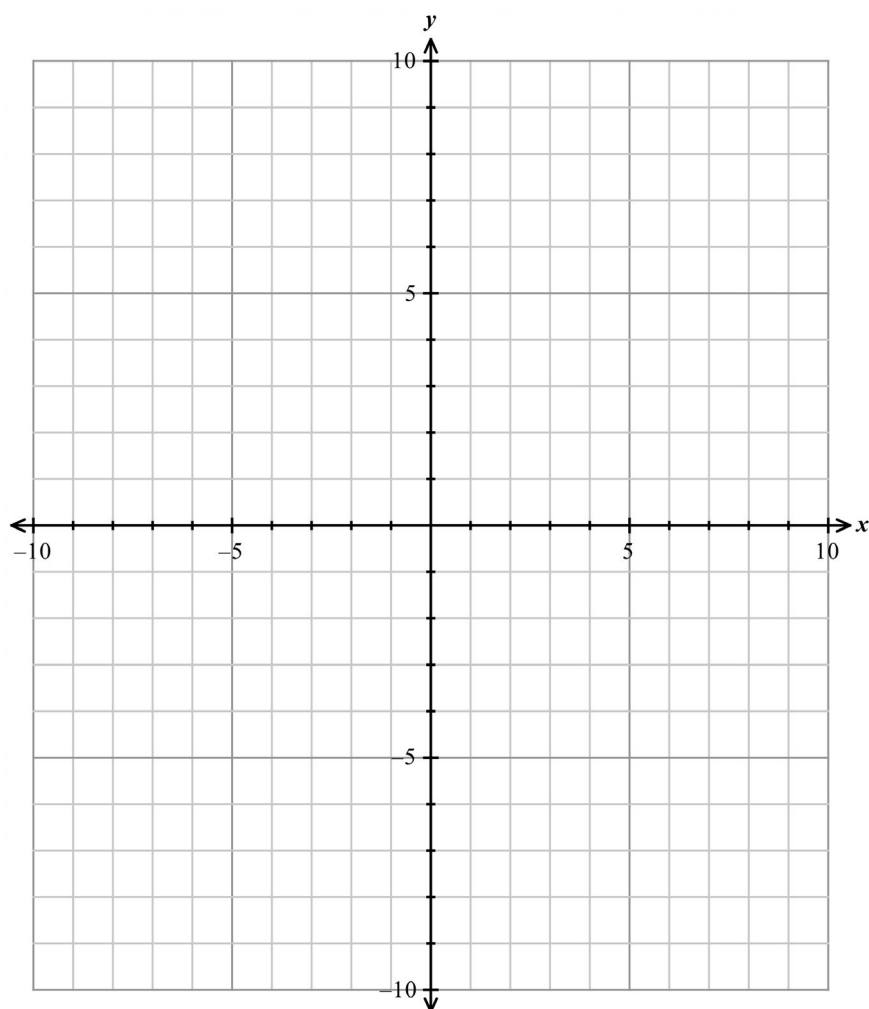
Let $f(x) = \sqrt{x+4}$ and $g(x) = -\frac{1}{x^2-2}$.

- (a) Find an expression for $g(f(x))$ and states its domain.

2

- (b) Hence, sketch $g(f(x))$ over its natural domain.

3



Question 30 (6 marks)

- (a) Prove that the following trigonometric identity is true.

3

$$\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} = \frac{\tan^2 \theta + 1}{\tan^2 \theta - 1}$$

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Question 30 continues on page 35

Question 30 (continued)

- (b)** Hence, find the exact solution for the following equation for $0 \leq \theta \leq \pi$.

3

$$\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} = 2$$

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

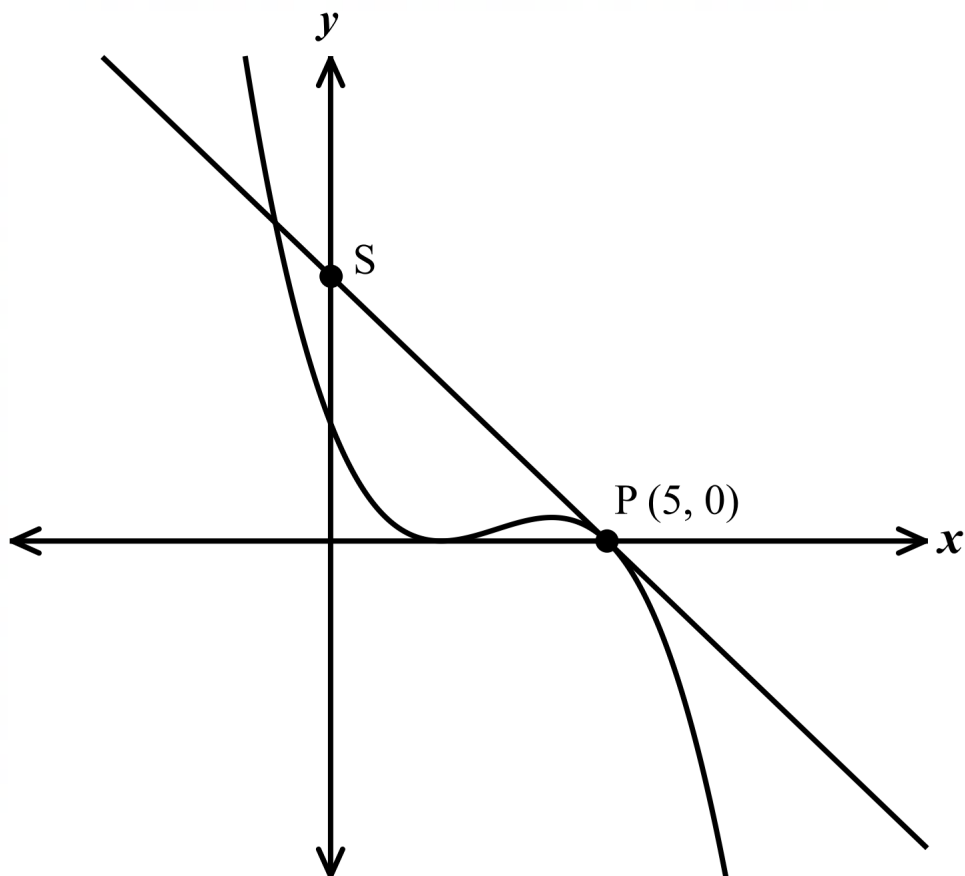
End of Question 30

Question 31 (5 marks)

Consider the following function:

$$f(x) = \frac{1}{5}(x-2)^2(5-x)$$

Point $P(5,0)$ is on a graph of $f(x)$, as shown below. The tangent at P cuts the y -axis at S .



Question 31 continues of page 37

Question 32 (7 marks)

A particle moves in a straight line so that its displacement x cm over time t seconds is given by:

$$x = t\sqrt{25 - t^2}$$

- (a)** Find the initial position of the particle. **1**

- (b)** Show that the velocity as function of t can be given by: **3**

$$\frac{dx}{dt} = \frac{25 - 2t^2}{\sqrt{25 - t^2}}$$

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 32 continues on page 39

Question 32 (continued)

- (c) Hence, find the total distance travelled by the particle in the first 5 seconds.

3

This image shows a full page of a document template designed for handwriting practice. It features a series of evenly spaced, horizontal blue dashed lines extending across the entire width of the page. The background is plain white, providing a clear contrast for the lines. There are no margins, text, or other markings present on the page.

End of Question 32

Question 33 (6 marks)

A biased four-sided die, A, is rolled. Let X be the score obtained when die A is rolled. The probability distribution for X is given in the following table.

x	1	2	3	4
$P(X = x)$	q	q	q	r

(a) Show that

1

$$q = \frac{1}{3}(1 - r)$$

(b) Hence, find the smallest and largest possible values of $E(X)$.

2

Question 33 continues on page 41

Question 33 (continued)

- (c) A second biased four-sided die, B is rolled. Let Y be the score obtained when die B is rolled. The probability distribution for Y is given the following table.

y	1	2	3	4
$P(Y = y)$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{1}{7}$

Agnes and Barbara play a game using the 2 dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Barbara's score is less than Agnes' score is $\frac{1}{2}$.

Hence calculate the value of $E(X)$ for die A.

This image shows a full page of white paper with horizontal blue ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

End of Examination

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The lines are thin and light gray, set against a plain white background. There are no margins, text, or other markings on the page.

Section II extra writing space

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Mathematics Advanced

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Total Marks: 100

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Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1. Factorise the following expression.

$$x^2 - 2x - 3$$

A. $(x - 3)(x - 1)$

☒ B. $(x - 3)(x + 1)$

C. $(x + 3)(x + 1)$

D. $(x + 3)(x - 1)$

2. What is the coefficient of x^2 in the following algebraic expression?

$$(3x - 4)(x^2 - 2x + 5)$$

A. 3

B. 23

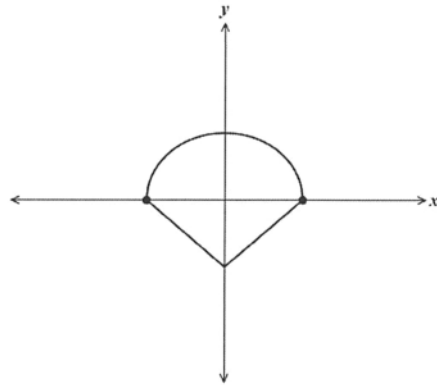
☒ C. -10

D. -20

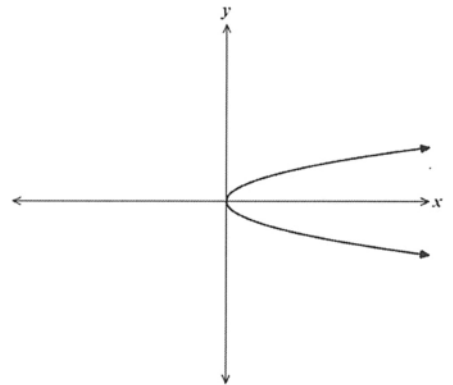
$$\begin{aligned} & 3x^3 - 6x^2 + 15x - 4x^2 + 8x - 20 \\ &= 3x^3 - 10x^2 + 23x - 20 \end{aligned}$$

3. Which of the following graphs classifies as a function?

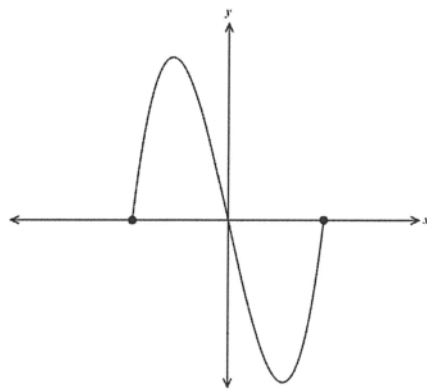
A.



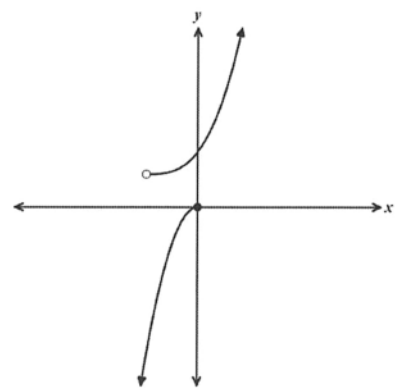
B.



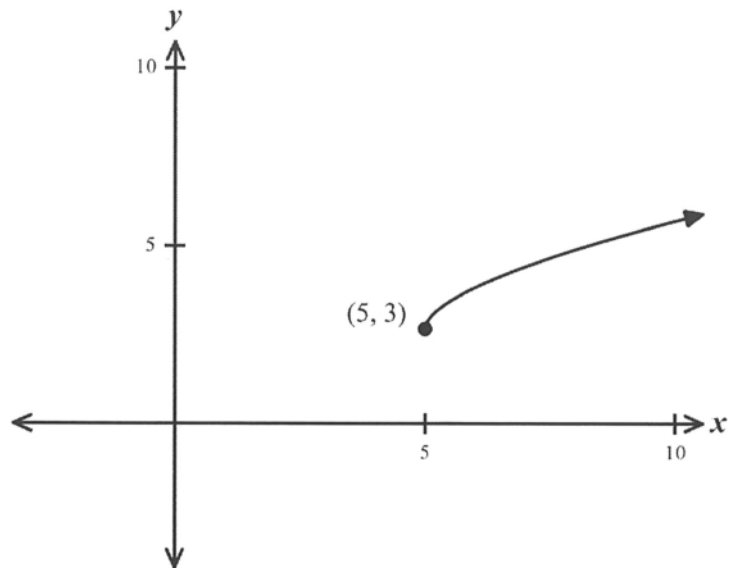
C.



D.

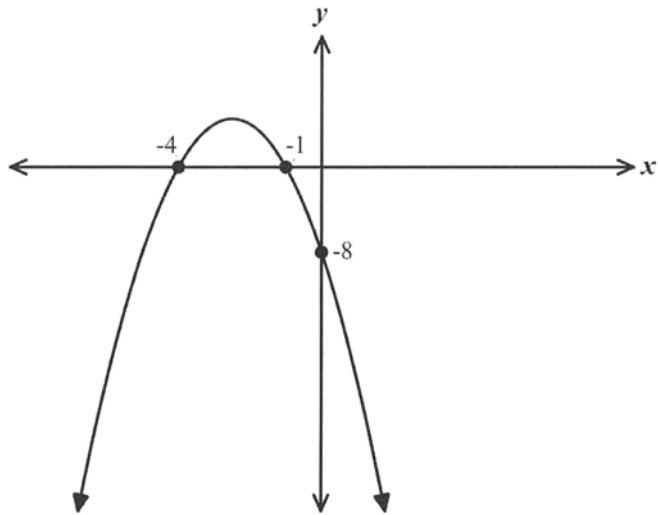


4. What is the domain and range of the following function?



- A. **Domain:** $x \in [5, \infty)$
Range: $y \in [3, \infty)$
- B. **Domain:** $x \in [3, \infty)$
Range: $y \in [5, \infty)$
- C. **Domain:** $x \in [5, \infty)$
Range: $y \in [0, \infty)$
- D. **Domain:** $x \in [0, \infty]$
Range: $y \in [5, \infty]$

5. The equation of the quadratic function shown in the diagram is



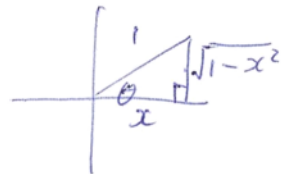
- A. $y = (x - 1)(x - 4)$
 B. $y = -(x + 1)(x + 4)$
 C. $y = -2(x + 1)(x + 4)$
 D. $y = -2(x - 1)(x - 4)$

6. If $\cos \theta = x$ and $0^\circ < \theta < 90^\circ$, then $\tan \theta$ is equal to:

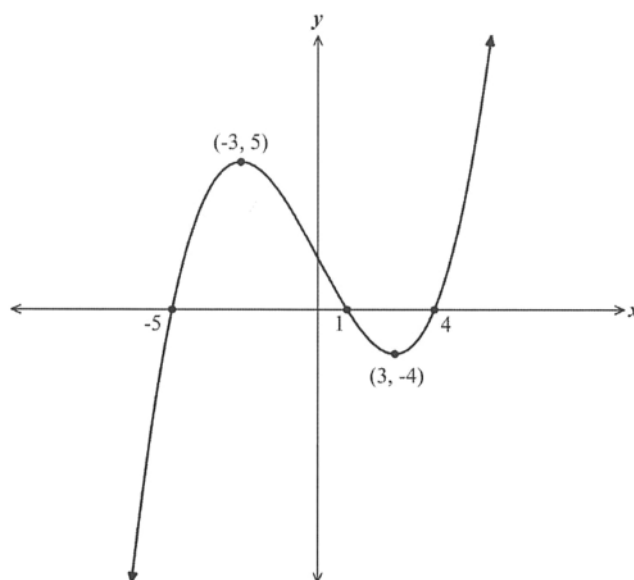
- A. $\frac{1}{\sqrt{1-x^2}}$
 B. $\sqrt{1-x^2}$
 C. $\frac{\sqrt{1-x^2}}{x}$
 D. $\frac{x}{\sqrt{1-x^2}}$

$$\cos \theta = x$$

$$\therefore \tan \theta = \frac{\sqrt{1-x^2}}{x}$$



7. For the graph of $y = f(x)$ shown below, $f'(x)$ is negative when



Correct
↓

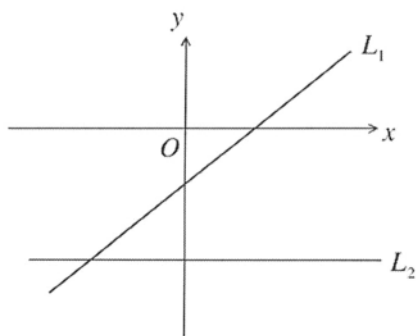
- A. $-3 < x < 3$
B. $-3 \leq x \leq 3$
C. $x < -3$ or $x > 3$
D. $x \leq -3$ or $x \geq 3$

8. If $A + B = 180^\circ$, which of the following statement is true?

- I. $\sin A = \sin B$
II. $\cos A = \cos B$
III. $\tan A = \tan B$

- A. I only
B. II only
C. III only
D. None of them

9. In the figure shown below, the equations of the straight lines L_1 and L_2 are $bx + ay = ab$ and $y = \frac{1}{c}$ respectively. Where a, b and c are constants. Which of the following statements are true?



- I. $c < 0$
- II. The gradient of $L_1 = -\frac{b}{a}$
- III. $0 < bc < 1$

- A. I and II only
- B. I and III only
- C. II and III only
- ☒ D. I, II and III

10. A game show host puts two balls randomly into three different boxes (each box may contain 0 balls, 1 ball or 2 balls).

Participants of the game can choose to open any two of the three boxes. If two balls are drawn, they can get a prize.

Find the probability that the participant can get a prize.

A. $\frac{1}{3}$

B. $\frac{2}{3}$

☒ C. $\frac{4}{9}$

D. $\frac{5}{9}$

End of Section I



Student Number

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Mathematics Advanced

Section II Answer Booklet 1

11/09/2023

Section II

90 Marks

Attempt Questions 11 - 33

Allow about 2 hours and 45 minutes for this section.

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Booklet 2 – Attempt Questions 28 – 33 (32 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 28 - 29
- If you use this space, clearly indicate which question you are answering.

Q	Marks
11	/2
12	/3
13	/4
14	/3
15	/2
16	/3
17	/2
18	/2
19	/2
20	/6
21	/6
22	/6
23	/3
24	/3
25	/3
26	/2
27	/6
Total	/58

Section II

In Questions 11 - 27, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Simplify the following indices using index laws.

2

$$(q^2)^3 \times \left(\frac{q^5}{q}\right)^2$$

$$= q^6 \times q^8$$

$$= q^{14}$$

Criteria	Marks
• Provides correct solution	2
• Correctly simplifies $(q^2)^3$ or $\left(\frac{q^5}{q}\right)^2$	1

Question 12 (3 marks)

Rationalise the following surds. Leave your answer in the simplest form.

(a)

$$\frac{6}{\sqrt{3}}$$

1

$$= \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Criteria	Marks
• Provides correct solution	1

(b)

$$\frac{3}{\sqrt{2}+1}$$

2

$$\frac{3(\sqrt{2}-1)}{2-1} = \frac{3\sqrt{2}-3}{1} = 3\sqrt{2}-3$$

Criteria	Marks
• Provides correct solution	2
• Multiply the denominator by its conjugate	1

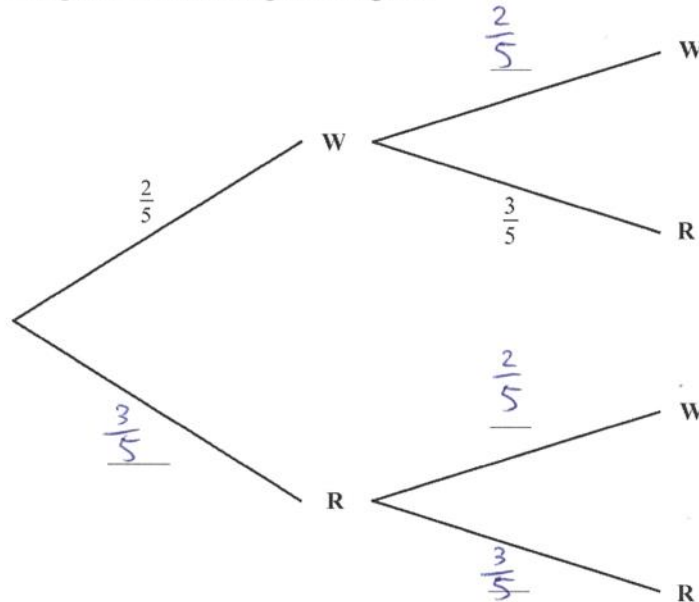
Question 13 (4 marks)

Ms Taylor is picking two marbles from a box. The box contains 2 white and 3 red marbles. A marble is drawn at random from the box and returned to the box after its colour has been noted, then the second marble is drawn at random from the box.

In the probability tree diagram below, W represents white marbles, and R represents red marbles.

- (a) Complete the following tree diagram.

1



- (b) What is the probability that Ms Taylor picks two marbles of the same colour?

1

$$\left(\frac{2}{5} \times \frac{2}{5}\right) + \left(\frac{3}{5} \times \frac{3}{5}\right) = \frac{13}{25}$$

Criteria	Marks
• Provides correct solution	1

- (c) What is the probability that Ms Taylor picks at least one white marble?

1

$$1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25}$$

Criteria	Marks
• Provides correct solution	1

- (d) Given that the first marble that Ms Taylor picked is red, what is the probability that the second marble is white?

1

$$P(W|R) = \frac{2}{5}$$

Criteria	Marks
• Provides correct solution	1

Question 14 (3 marks)

Given that

$$f(x) = x - \frac{1}{x}$$

(a) Evaluate $f(4)$.

1

$$f(4) = 4 - \frac{1}{4}$$

$$= \frac{15}{4}$$

Criteria	Marks
• Provides correct solution	1

(b) Show that f is an odd function.

2

$$-f(-x) = -\left[-x - \frac{1}{-x}\right]$$

$$= -\left[-x + \frac{1}{x}\right]$$

$$= x - \frac{1}{x}$$

$$= f(x)$$

$\therefore f$ is odd.

Criteria	Marks
• Provides correct solution	2
• Attempt to show that $f(x) = -f(-x)$ or $f(-x) = -f(x)$	1

Question 15 (2 marks)

Solve the following equation.

2

$|9 - 2y| = 21$

$9 - 2y = 21$, $9 - 2y = -21$

$2y = -12$, $2y = 30$

$y = -6$, $y = 15$

Criteria	Marks
• Correctly provides both solutions	2
• Attempt to solve $9 - 2y = \pm 21$	1

Question 16 (3 marks)

For the following discrete probability distribution:

x	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

- (a) Calculate the expected value.

1

$$E(X) = 0 + \frac{1}{2} + \frac{2}{4}$$

$$= 1$$

Criteria	Marks
• Provides correct solution	1

- (b) Hence calculate the variance.

2

x	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
x^2	0	1	4
$x^2 P(x)$	0	$\frac{1}{2}$	1

$$\text{Var}(X) = E(X^2) + \mu^2$$

$$= 0 + \frac{1}{2} + 1 + 1$$

$$\text{Var}(X) = \frac{5}{2}$$

Criteria	Marks
• Provides correct solution	2
• Calculate $E(X^2)$ or equivalent merits	1

Question 17 (2 marks)

Differentiate $f(x) = x^2 + 3$ using the first principle.

2

The first principle is given below,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 3 - x^2 - 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h$$

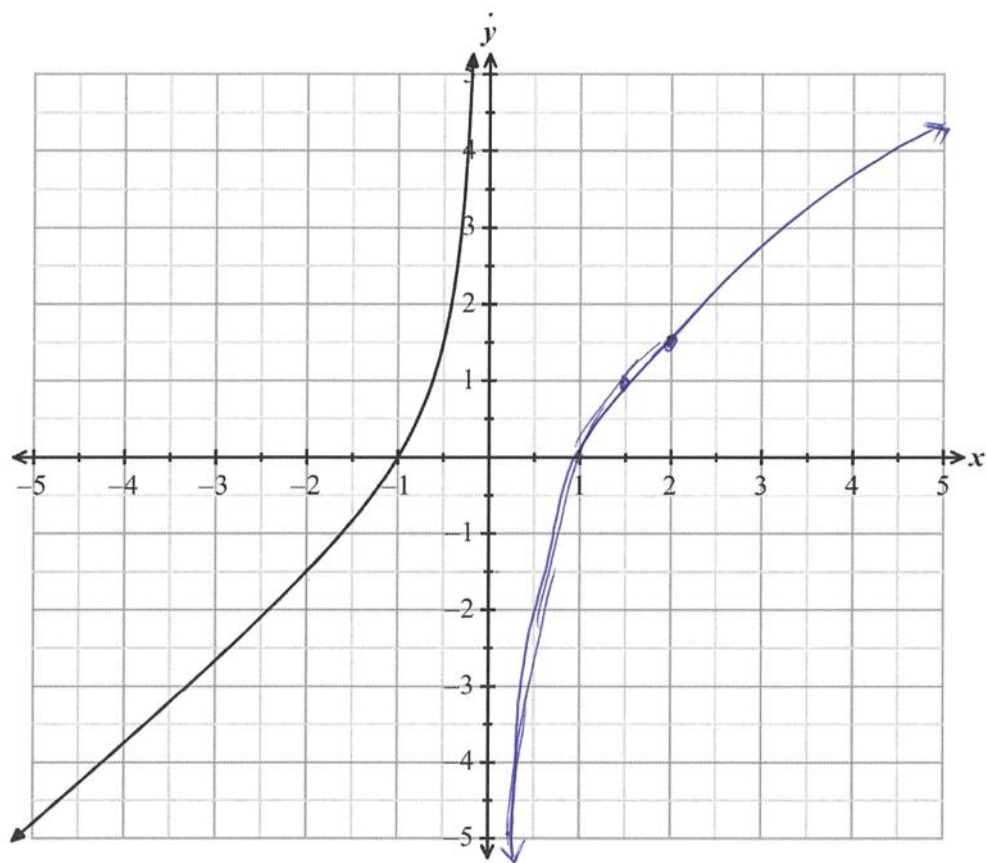
$$\therefore f'(x) = 2x$$

Criteria	Marks
• Provides correct solution	2
• Correctly expand and simplify $\frac{f(x+h) - f(x)}{h}$	1

Question 18 (2 marks)

A part of an odd function $g(x)$ is shown on the diagram below. Complete the following diagram for $g(x)$ over the domain $(0, \infty)$.

2

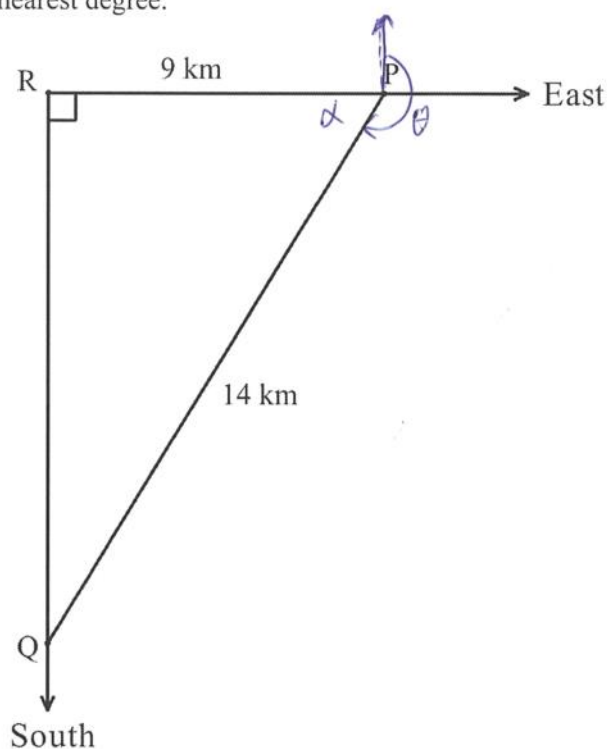


Criteria	Marks
• Provides correct solution	2
• Correct general shape of the curve or reflect one correct point about the origin.	1

Question 19 (2 marks)

In the diagram shown below, P, Q and R are three campsites. P is 9 km due east of R and Q is due south of R. The distance between P and Q is 14 km. Find the bearing of Q from P to the nearest degree.

2



$$\cos \alpha = \frac{9}{14}$$

$$\alpha = \cos^{-1}\left(\frac{9}{14}\right)$$

$$\approx 49.99479\dots^\circ$$

$$\begin{aligned} \therefore \theta &= 360 - 90 - 49.994\dots \\ &= 220.005\dots^\circ \\ &\approx 220^\circ \end{aligned}$$

Criteria	Marks
• Provides correct solution	2
• Calculate $\angle RPQ$	1

Question 20 (6 marks)

Solve the following trigonometric equations for $0 \leq \theta \leq 2\pi$.

(a)

$$\cos \theta = \frac{\sqrt{2}}{2}$$

2

$$\begin{aligned}\theta &= \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) \\ &= \pi/4, 7\pi/4\end{aligned}$$

Criteria	Marks
• Provides both correct solutions	2
• Correctly provide one correct solution	1

(b)

$$\operatorname{cosec} \theta = -2$$

2

$$\begin{aligned}\frac{1}{\sin \theta} &= -2 \\ \sin \theta &= -\frac{1}{2} \\ \theta &= 7\pi/6, 11\pi/6\end{aligned}$$

Criteria	Marks
• Provides both correct solutions	2
• Show that $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$	1

(c)

$$\sin \theta - \cos \theta = 0$$

2

$$\begin{aligned}\sin \theta &= \cos \theta \\ \tan \theta &= 1 \\ \theta &= \frac{\pi}{4}, \frac{5\pi}{4}\end{aligned}$$

Criteria	Marks
• Provides both correct solutions	2
• Show that $\sin \theta - \cos \theta = 0$ is equal to $\tan \theta = 1$	1

Question 21 (6 marks)

Differentiate the following functions.

(a) $f(x) = x^2 - 5x^{-1}$ 2

$$f'(x) = 2x + 5x^{-2}$$

Criteria	Marks
• Provides correct solution	2
• Correctly differentiate x^2 or $-5x^{-1}$	1

(b) $f(x) = (4x - 3)^4$ 2

$$f'(x) = 4(4)(4x-3)^3$$

$$= 16(4x-3)^3$$

Criteria	Marks
• Provides correct solution	2
• Applying the chain rule correctly	1

(c) $f(x) = \frac{x^2}{2x+1}$ 2

$$u = x^2$$

$$v = 2x+1$$

$$u' = 2x$$

$$v' = 2$$

$$f'(x) = \frac{2x(2x+1) - 2x^2}{(2x+1)^2}$$

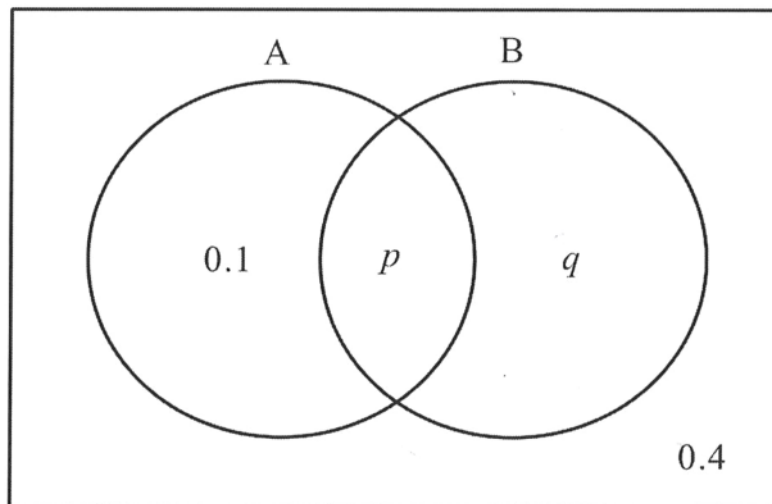
$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2}$$

$$f'(x) = \frac{2x^2 + 2x}{(2x+1)^2}$$

Criteria	Marks
• Provides correct solution	2
• Applying the quotient rule correctly	1

Question 22 (6 marks)

The following Venn diagram shows events A and B, where $P(A) = 0.3$.



- (a) Find the values of p and q .

2

$$P = 0.2, \quad q = 1 - 0.3 - 0.4 = 0.3$$

Criteria	Marks
• Provides both correct solutions	2
• Calculate p or q	1

- (b) Hence, find $P(A' \cup B)$

1

$$0.3 + 0.4 = 0.7$$

Criteria	Marks
• Provides correct solution	1

- (c) Calculate the probability of $P(B|A)$.

1

$$P(B|A) = 0.2$$

Criteria	Marks
• Provides correct solution	1

Question 22 continues on page 22

Question 22 (continued)

- (d) Determine whether events A and B are independent. Justify your answer with the appropriate mathematical calculations. 2

$$\begin{aligned} P(B) &= 0.2 + 0.3 \\ &= 0.5 \\ &\neq P(B|A) \end{aligned}$$

Since $P(B) \neq P(B|A)$, so they are not independent.

End of Question 22

Criteria	Marks
• Provides correct solution with relevant justifications	2
• Calculate the value of $P(B)$	1

Question 23 (3 marks)

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of $g(x)$.

- (a) Show that b is 4.

1

$$\begin{aligned} g(-1) &= (-1)^2 - b + 11 = 8 \\ 1 - b + 11 &= 8 \\ b &= 4 \end{aligned}$$

Criteria	Marks
• Provides correct solution	1

- (b) The graph $f(x) = x^2$ is transformed to obtain the graph of $g(x)$. Describe this transformation.

2

$$\begin{aligned} g(x) &= x^2 + 4x + 11 \\ &= x^2 + 4x + 4 - 4 + 11 \\ &= (x+2)^2 + 7 \end{aligned}$$

$f(x)$ is shifted left by 2 and up by 7.

Criteria	Marks
• Provides correct solution	2
• Complete the square on $g(x)$ or equivalent merits	1

Question 24 (3 marks)

Fully simplify the following expression.

3

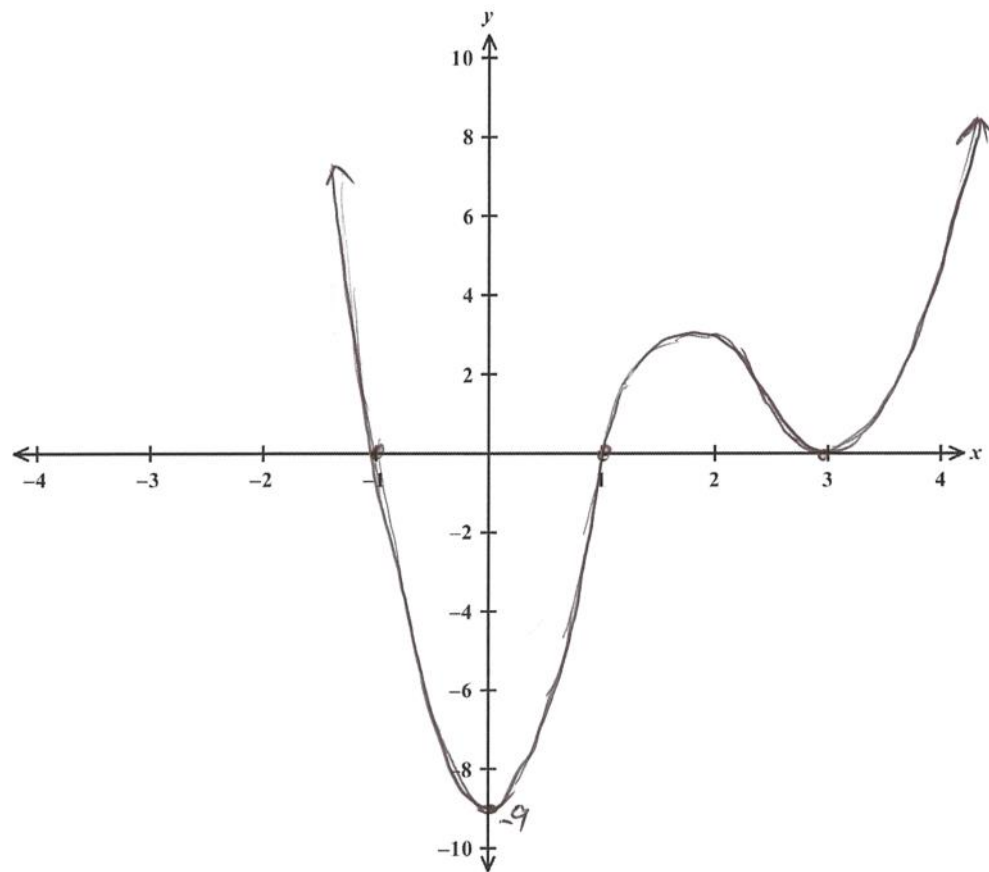
$$\frac{\sin \theta}{\cos 60^\circ} + \frac{\cos(270^\circ - \theta)}{\tan 45^\circ}$$

$$\begin{aligned} &= \frac{\sin \theta}{\frac{1}{2}} + \frac{-\sin \theta}{1} \\ &= 2\sin \theta - \sin \theta \\ &= \sin \theta \end{aligned}$$

Criteria	Marks
• Provides correct solution	3
• Simplify $\cos(270 - \theta)$ to $-\sin \theta$	2
• Simplify $\cos 60^\circ$ or $\tan 45^\circ$	1

Question 25 (3 marks)

Sketch $f(x) = (x - 1)(x - 3)^2(x + 1)$ on the axis provided below. Labelling the x and y intercepts. 3

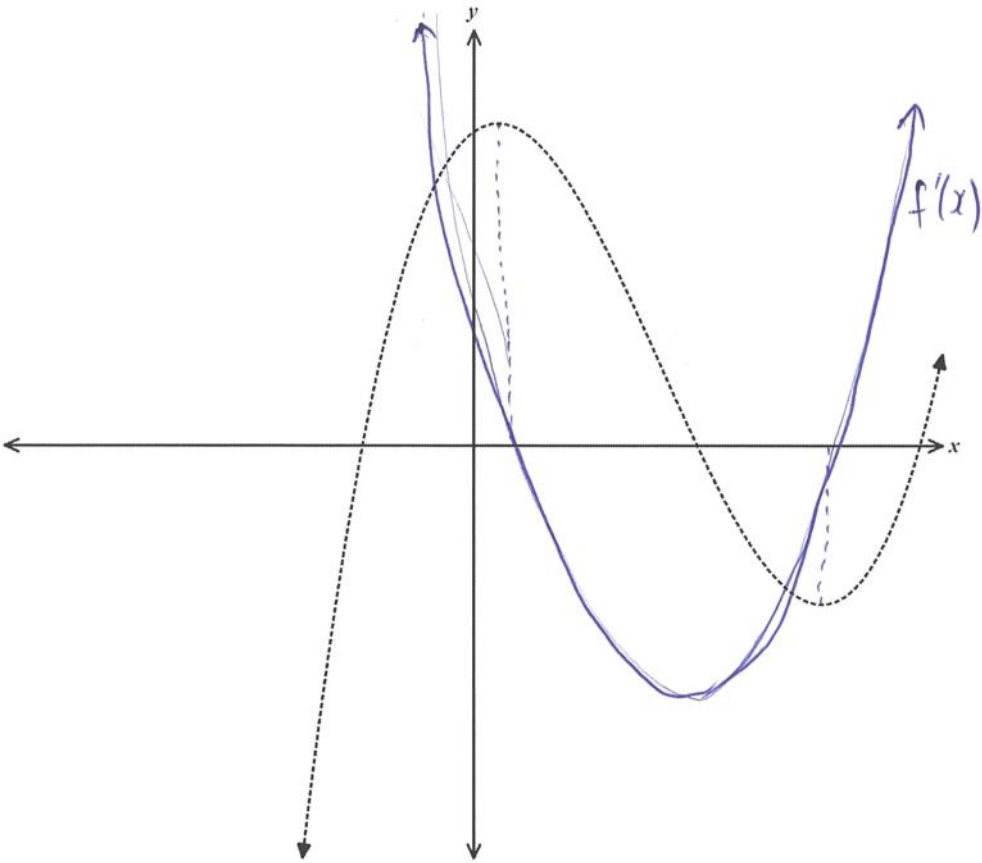


$$\begin{aligned} \text{Let } x=0, \quad f(0) &= (0-1)(0-3)^2(0+1) \\ &= -1 \times (9) \times 1 \\ &= -9 \end{aligned}$$

	Criteria	Marks
	• Provides correct solution	3
	• Correct shape of the curve along with the correct x intercepts	2
	• Calculate the y intercepts	1

Question 26 (2 marks)

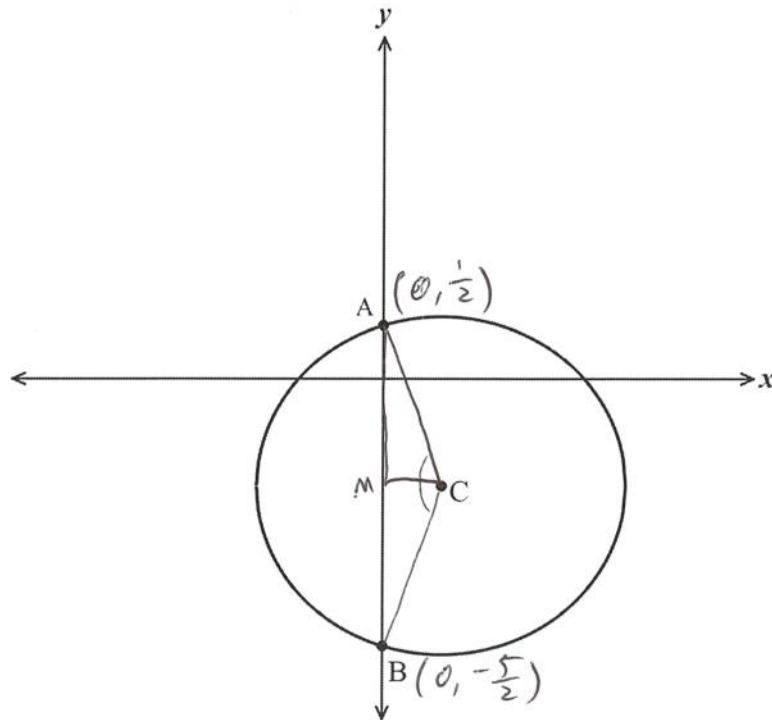
The function $f(x)$ is given on the axis below. Hence sketch the gradient function on the same axis. 2



Criteria	Marks
• Provides correct solution showing explicitly the stationary points	2
• Correct shape of the gradient function	1

Question 27 (6 marks)

The equation of a circle is given by $4x^2 + 4y^2 - 4x + 8y - 5 = 0$. It is given that the circle cuts the y -axis at two points A and B respectively as shown on the diagram below.



- (a) Show that the centre C has the coordinate of $(\frac{1}{2}, -1)$.

2

$$\begin{aligned}
 4(x^2 - x) + 4(y^2 + 2y) &= 5 \\
 4(x^2 - x + \frac{1}{4} - \frac{1}{4}) + 4(y^2 + 2y + 1 - 1) &= 5 \\
 4(x - \frac{1}{2})^2 - 1 + 4(y + 1)^2 - 5 &= 5 \\
 4(x - \frac{1}{2})^2 + 4(y + 1)^2 &= 10
 \end{aligned}$$

Criteria	Marks
• Provides correct solution	2
• Complete the square on the circle function	1

Question 27 continues on page 27

Question 27 (continued)

- (b) Find the radius of this circle.

1

$$\therefore r = \frac{\sqrt{10}}{2}$$

Criteria	Marks
• Provides correct solution	1

- (c) Hence, or otherwise, show that $\angle ACB$ is obtuse.

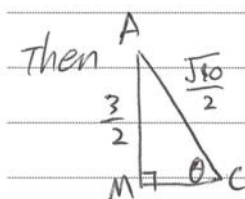
3

$$\text{If } x=0, \quad 4(0)^2 + 4y^2 - 4(0) + 8y - 5 = 0$$

$$4y^2 + 8y - 5 = 0$$

$$(2y-1)(2y+5) = 0$$

$$\therefore y = \frac{1}{2}, -\frac{5}{2}$$



$$\theta = \sin^{-1}\left(\frac{\frac{3}{2}}{\frac{\sqrt{10}}{2}}\right) \approx 71.565 \dots$$

$$\therefore \angle ACB = 2 \times \theta$$

$$\approx 143.13^\circ \quad \therefore \angle ACB \text{ is obtuse.}$$

End of Question 27

Proceed to Booklet 2 for Questions 28 - 33

Criteria	Marks
• Provides correct solution	3
• Calculate $\angle ACM$ or $\angle ACB$	2
• Calculate the value of A and B	1



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Student Number

Teacher name

2023 Year 11 Preliminary Examination

Mathematics Advanced

Section II Answer Booklet 2

11/09/2023

Q	Marks
28	/3
29	/5
30	/6
31	/5
32	/7
33	/6
Total	/32

Booklet 2 – Attempt Questions 28 - 33 (32 marks)

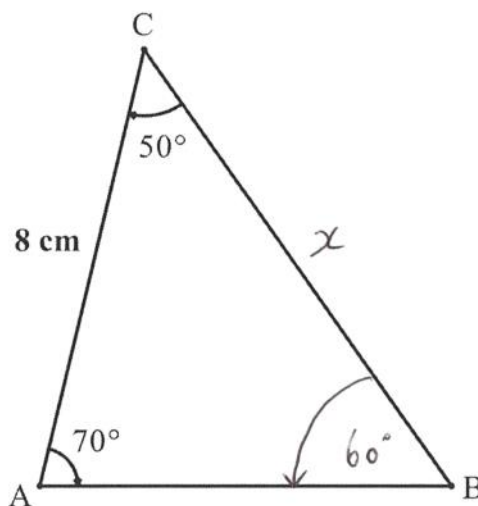
Instructions

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- Extra writing space is provided on page 42 – 43.
- If you use this space, clearly indicate which question you are answering.

Question 28 (3 marks)

Calculate the area of $\triangle ABC$ to nearest 2 decimal places.

3



$$\frac{x}{\sin 70^\circ} = \frac{8}{\sin 60^\circ}$$

$$x = \frac{8 \sin 70^\circ}{\sin 60^\circ}$$

$$\approx 8.8107\dots$$

$$A = \frac{1}{2} \times 8 \times 8.8107\dots \times \sin 50^\circ$$

$$= 24.9205\dots$$

$$= 24.92 \text{ cm}^2$$

Criteria	Marks
• Provides correct solution	3
• Calculate AB or BC	2
• Calculate $\angle ABC$	1

Question 29 (5 marks)

Let $f(x) = \sqrt{x+4}$ and $g(x) = -\frac{1}{x^2-2}$.

- (a) Find an expression for $g(f(x))$ and states its domain

2

$$g(f(x)) = -\frac{1}{(\sqrt{x+4})^2 - 2}$$

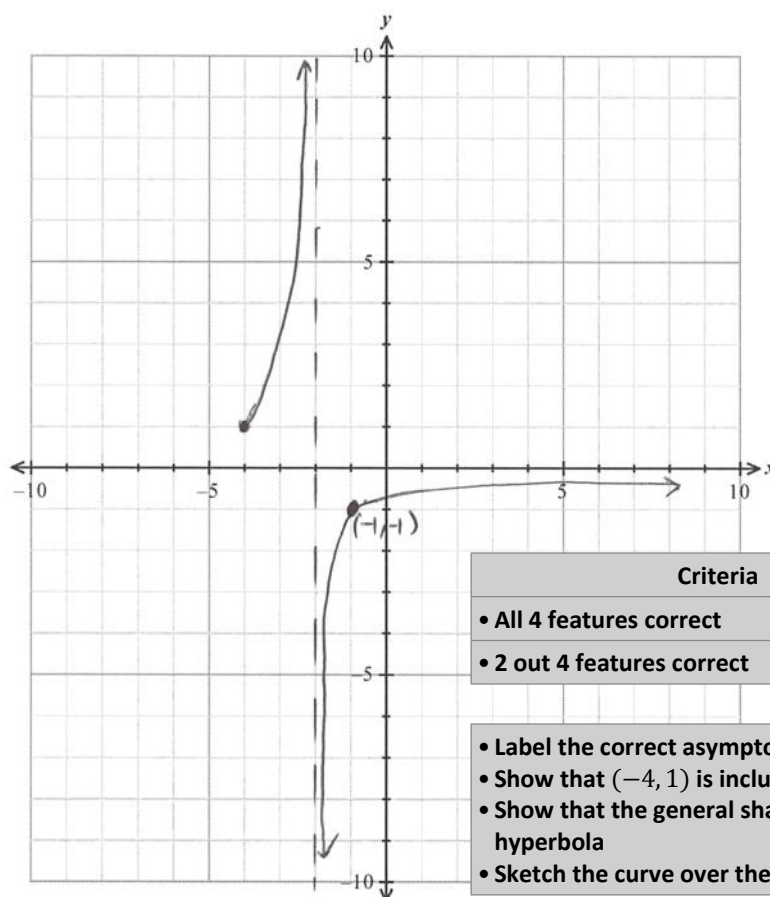
$$= -\frac{1}{x+4-2}$$

$$= -\frac{1}{x+2}, \quad \text{Domain: } x \geq -4, x \neq -2$$

Criteria	Marks
• Provide both correct solutions	2
• Find $g(f(x))$ or state the correct domain	1

- (b) Hence, sketch $g(f(x))$ over its natural domain.

3



Criteria	Marks
• All 4 features correct	2
• 2 out of 4 features correct	1

- Label the correct asymptote
- Show that $(-4, 1)$ is included in the function
- Show that the general shape of the curve is a hyperbola
- Sketch the curve over the correct domain

Question 30 (6 marks)

(a) Show that

3

$$\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} = \frac{\tan^2 \theta + 1}{\tan^2 \theta - 1}$$

$$\text{LHS} = \frac{\sin \theta (\sin \theta - \cos \theta) + \cos \theta (\sin \theta + \cos \theta)}{\sin^2 \theta - \cos^2 \theta}$$

$$= \frac{\sin^2 \theta - \sin \theta \cos \theta + \sin \theta \cos \theta + \cos^2 \theta}{\sin^2 \theta - \cos^2 \theta}$$

$$= \frac{1}{\sin^2 \theta - \cos^2 \theta}$$

$$= \frac{\sec^2 \theta}{\tan^2 \theta - 1} \quad \left(\begin{array}{l} \text{divide top \& bottom} \\ \text{by } \cos \theta \end{array} \right)$$

$$= \frac{\tan^2 \theta + 1}{\tan^2 \theta - 1}$$

$$= \text{RHS}$$

Criteria	Marks
• Provides correct solution	3
• Correct working towards the RHS	2
• Express LHS as a single fraction <u>or</u> divide LHS by $\cos \theta$	1

Question 30 continues on page 34

Question 30 (continued)

(b) Hence, find the exact solution for the following equation for $0 \leq \theta \leq \pi$.

3

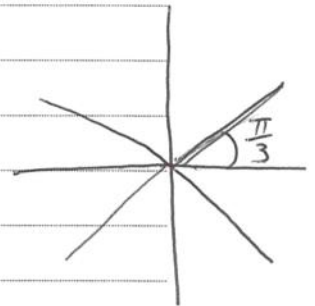
$$\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} = 2$$

$$\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} = 2$$

$$\tan^2 \theta + 1 = 2\tan^2 \theta - 2$$

$$\tan^2 \theta = 3$$

$$\tan \theta = \pm \sqrt{3}$$



$$\theta = \frac{\pi}{3}, \pi - \frac{\pi}{3}$$

$$= \frac{\pi}{3}, \frac{2\pi}{3}$$

End of Question 30

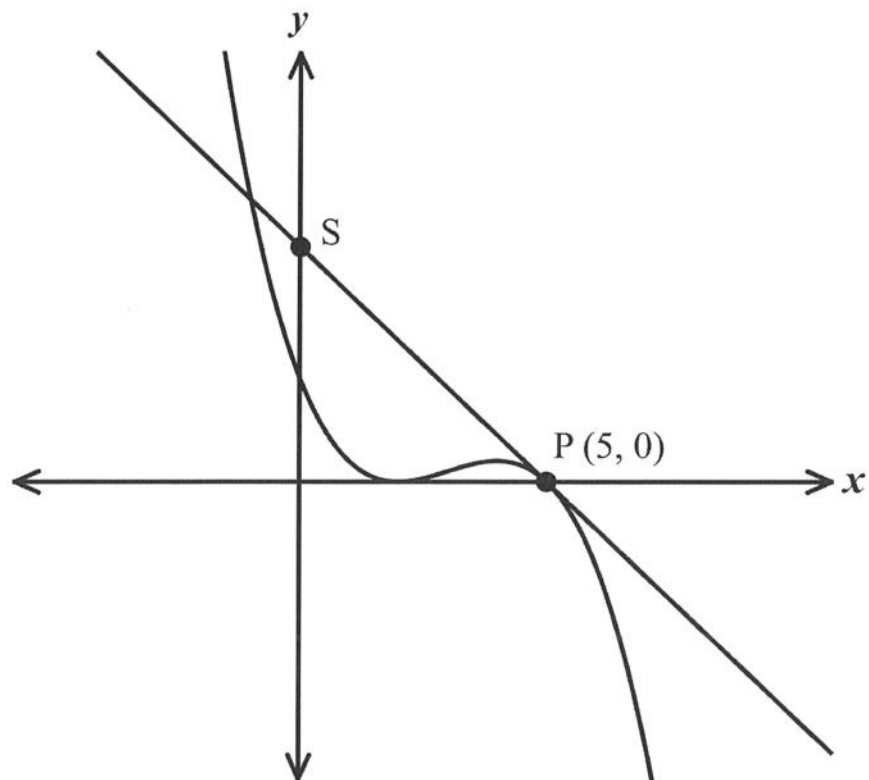
Criteria	Marks
• Provides both correct solutions between $0 \leq \theta \leq \pi$	3
• Correct working towards finding both solutions	2
• Use part (a) correctly to show $\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} = 2$	1

Question 31 (5 marks)

Consider the following function:

$$f(x) = \frac{1}{5}(x-2)^2(5-x)$$

Point $P(5,0)$ is on a graph of $f(x)$, as shown below. The tangent at P cuts the y -axis at S .



Question 31 continues of page 36

- (a) Find the equation of the tangent to the graph $f(x)$ at the $P(5,0)$.

3

$$f(x) = \frac{1}{5}(x-2)^2(5-x) \quad u = \frac{1}{5}(x-2)^2 \quad v = 5-x$$

$$f'(x) = \frac{2}{5}(x-2)(5-x) - \frac{1}{5}(x-2)^2 \quad u' = \frac{2}{5}(x-2) \quad v' = -1$$

$$f'(5) = -\frac{9}{5}$$

$$\therefore y - 0 = -\frac{9}{5}(x - 5)$$

$$5y = -9(x - 5)$$

$$5y = -9x + 45$$

$$9x + 5y - 45 = 0$$

Criteria	Marks
• Provide the correct equation of the tangent	3
• Find the gradient at $P(5,0)$	2
• Differentiate $f(x)$	1

- (b) Hence, or otherwise, find the distance of PS and express it in the form of $\sqrt{b}$, where b is a positive integer.

2

$$\text{Let } x=0, \quad 9(0) + 5y - 45 = 0$$

$$5y = 45$$

$$y = 9$$

$$\therefore S(0,9)$$

$$\therefore PS = \sqrt{9^2 + 5^2}$$

$$= \sqrt{106}$$

Criteria	Marks
• Provide the correct solution	2
• Find the coordinate of S	1

Question 32 (7 marks)

A particle moves in a straight line so that its displacement x cm over time t seconds is given by:

$$x = t\sqrt{25 - t^2}$$

- (a) Find the initial position of the particle.

1

Let $t=0$, $x = 0\sqrt{25-0^2}$
 $x = 0 //$

Criteria	Marks
• Provides correct solution	1

- (b) Show that the velocity as function of t can be given by:

3

$$\frac{dx}{dt} = \frac{25 - 2t^2}{\sqrt{25 - t^2}}$$

$u = t$ $v = (25 - t^2)^{\frac{1}{2}}$
 $u' = 1$ $v' = \frac{1}{2}(-2t)(25 - t^2)^{-\frac{1}{2}}$
 $= -\frac{2t}{2\sqrt{25 - t^2}} = -\frac{t}{\sqrt{25 - t^2}}$
 $\frac{dx}{dt} = (25 - t^2)^{\frac{1}{2}} \cdot -\frac{t}{\sqrt{25 - t^2}}$
 $= \frac{25 - t^2 - t^2}{\sqrt{25 - t^2}} = \frac{25 - 2t^2}{\sqrt{25 - t^2}}$

Question

Criteria	Marks
• Provide the correct solution	3
• Express $x'(t)$ in terms of surds	2
• Differentiate $x(t)$	1

Question 32 (continued)

(c) Hence, find the total distance travelled by the particle in the first 5 seconds.

3

$$\frac{dx}{dt} = \frac{25 - 2t^2}{\sqrt{25 - t^2}} = 0$$

$$25 - 2t^2 = 0$$

$$2t^2 = 25$$

$$t = \pm \sqrt{\frac{25}{2}} \text{ seconds.}$$

$\therefore$ the particle is at rest at $t = \sqrt{12.5}$ seconds.

From $0s - \sqrt{12.5}s$,

$$x = \sqrt{12.5} \times \sqrt{25 - (\sqrt{12.5})^2} = 12.5 \text{ cm}$$

From $\sqrt{12.5}s - 5s$,

$$x = 5 \sqrt{25 - 5^2} = 0$$

$\therefore$ the particle travelled a total of 25 cm in the first 5 seconds.

Criteria	Marks
• Provide the correct solution	3
• Calculate the distance travelled between $t = 0$ to $\sqrt{12.5}$ or $t = \sqrt{12.5}$ to 5	2
• Find t when $\frac{dx}{dt} = 0$	1

End of Question 32

Question 33 (6 marks)

A biased four-sided die, A, is rolled. Let X be the score obtained when die A is rolled. The probability distribution for X is given in the following table.

x	1	2	3	4
$P(X = x)$	q	q	q	r

(a) Show that

1

$$q = \frac{1}{3}(1 - r)$$

Criteria	Marks
• Provides correct solution	1

$$3q + r = 1$$

$$q = \frac{1}{3}(1 - r)$$

(b) Hence, find the smallest and largest possible values of $E(X)$

2

$$E(X) = q + 2q + 3q + 4r$$

$$= 6q + 4r$$

$$= 6\left[\frac{1}{3}(1 - r)\right] + 4r$$

$$= 2 - 2r + 4r$$

$$E(X) = 2 + 2r$$

Since $0 \leq r \leq 1$

then $2 \leq E(X) \leq 4$

Criteria	Marks
• Provide both correct solutions	2
• Show that $E(X) = 2 + 2r$	1

Question 33 continues on page 41

Question 33 (continued)

- (c) A second biased four-sided die, B is rolled. Let Y be the score obtained when die B is rolled. The probability distribution for Y is given the following table. 3

y	1	2	3	4
$P(Y = y)$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{1}{7}$

Agnes and Barbara play a game using the 2 dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Barbara's score is less than Agnes' score is $\frac{1}{2}$.

Hence calculate the value of $E(X)$ for die A.

Outcomes of Barbara's score is less than Agnes,

$(2,1), (3,1), (4,1), (3,2), (4,2), (4,3)$

then, $\frac{2}{7}q + \frac{2}{7}q + \frac{2}{7}r + \frac{2}{7}q + \frac{2}{7}r + \frac{2}{7}r = \frac{1}{2}$

$$\frac{6}{7}q + \frac{6}{7}r = \frac{1}{2}$$

$$\frac{6}{7} \left[\frac{1}{3}(1-r) \right] + \frac{6}{7}r = \frac{1}{2}$$

$$\frac{2}{7} - \frac{2}{7}r + \frac{6}{7}r = \frac{1}{2}$$

$$\frac{4}{7}r = \frac{3}{14}$$

$$r = \frac{3}{8}$$

$$\therefore E(X) = 2 + 2r = 2 + 2\left(\frac{3}{8}\right) = \frac{11}{4}$$

End of Examination

Criteria	Marks
• Provide the correct solution	3
• Calculate the value of $r = \frac{3}{8}$	2
• Show that $\frac{6}{7}q + \frac{6}{7}r = \frac{1}{2}$	1