Knox Grammar School

Name: ____________________

2019 Year 11 Yearly Examination **Teacher:** ____________________

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen only.
- Black pen is preferred.
- NESA-approved calculators may be used.
- In Questions 11-20, show relevant mathematical reasoning and/or calculations.

Section I: Pages 3-6

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section
- Use the Multiple Choice Answer Sheet

Section II: Pages 7 - 16

90 marks

- Attempt Questions 11-20
- Allow about 2 hours 45 minutes for this section
- Answer each question in a separate writing booklet

Examiner: Mr Willcocks

Teachers:

Mr Meli
Mr Vuletich
Mr Willcocks
Mr Cheah
Mr Lemaire
Mr Menzies
Mr Naidoo

Write your Name, your Board of Studies Student Number and your Teacher's Name on the front cover of each writing booklet.

This paper MUST NOT be removed from the examination room.

Number of Students in Course: 202

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for questions 1 – 10.

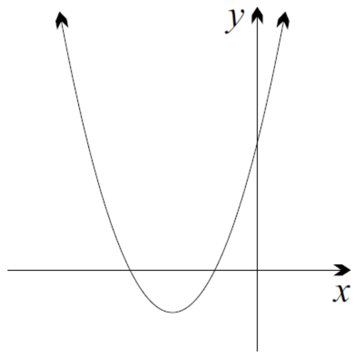
Marks

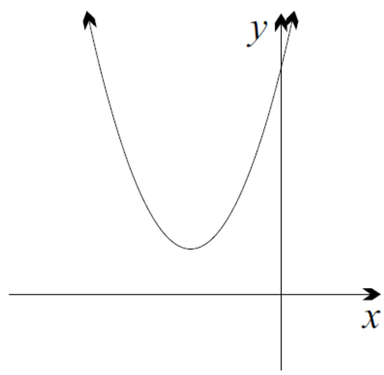
1 Which of the following is equal to $\dfrac{1}{\sqrt{5}+2\sqrt{3}}$? **1**

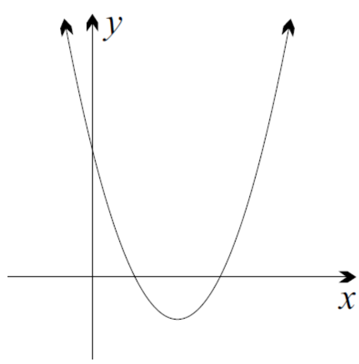
(A) $\dfrac{\sqrt{5}-2\sqrt{3}}{7}$ (B) $\dfrac{2\sqrt{3}+\sqrt{5}}{7}$

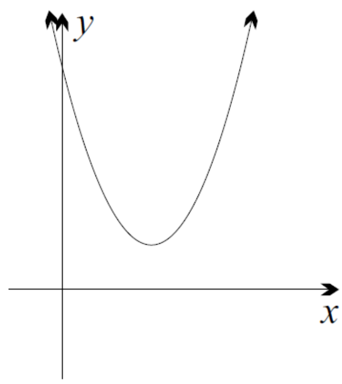
(C) $\dfrac{2\sqrt{3}-\sqrt{5}}{7}$ (D) $\dfrac{\sqrt{5}-2\sqrt{3}}{-7}$.

2 Which of the following graphs best represents $y=(x+2)^2-1$? **1**

(A)


(B)


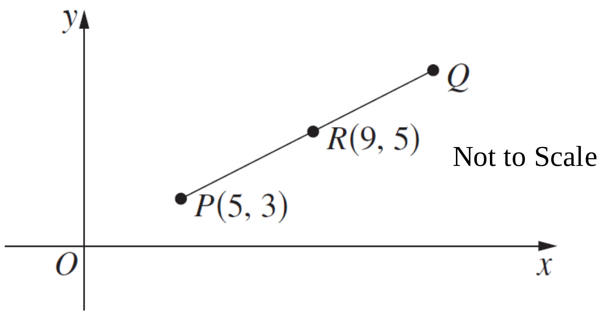
(C)


(D)


Section I continues over the page

Section I continued **Marks**

3 The point $R(9,5)$ is the midpoint of the interval PQ, where P has coordinates $(5,3)$. What are the coordinates of Q? **1**



(A) $(4,7)$ (B) $(7,4)$

(C) $(13,7)$ (D) $(14,8)$.

4 $\cos\left(\frac{-\pi}{6}\right)$ is the same as which of the following: **1**

(A) $-\cos\left(\frac{\pi}{3}\right)$ (B) $-\sin\left(\frac{\pi}{6}\right)$

(C) $\sin\left(\frac{\pi}{3}\right)$ (D) $\cos\left(\frac{2\pi}{3}\right)$.

5 Which of the following is the derivative of x^{n+1} with respect to x ? **1**

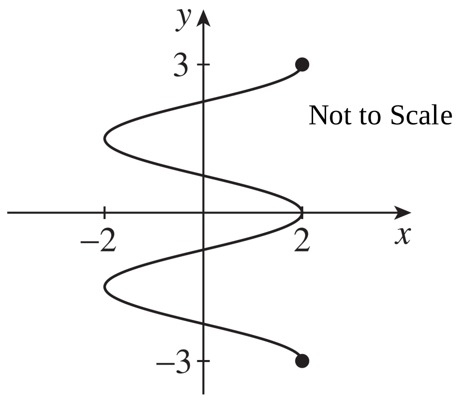
(A) nx^{n+1} (B) $\frac{x^{n+1}}{n+1}$

(C) $(n+1)x^{n-1}$ (D) $(n+1)x^{n}$.

Section I continues over the page

Section I continued **Marks**

6 Classify the graph shown below as one-to-one, one-to-many, many-to-one or many-to-many? **1**



(A) One-to-one (B) One-to-many

(C) Many-to-one (D) Many-to-many.

7 Let $a = e^x$. Which expression is equal to $\log_e\left(a^2\right)$? **1**

(A) e^{2x} (B) e^{x^2}

(C) $2x$ (D) x^2.

8 Solve $\sin 2\theta = \frac{-\sqrt{3}}{2}$ in the domain $0° \le \theta \le 180°$: **1**

(A) $60°$ (B) No solutions

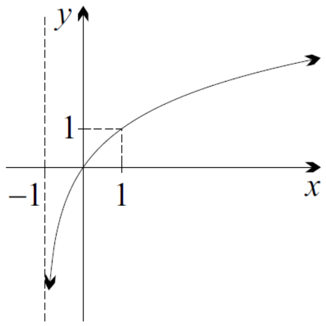
(C) $120°$ (D) $120°, 150°$.

Section I continues over the page

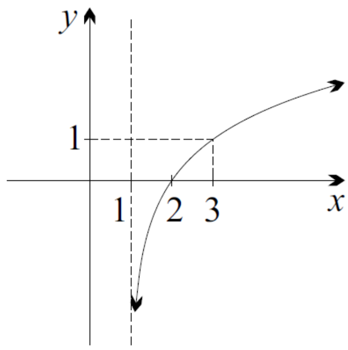
Section I continued **Marks**

9 Which of the following is the graph of $y = -\log_2(x+1)$? **1**

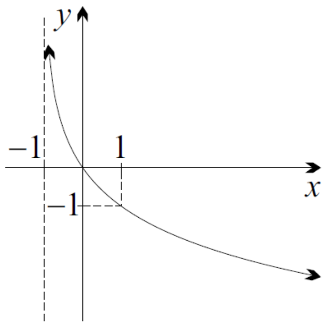
(A)



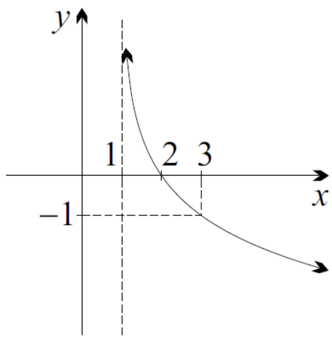
(B)



(C)



(D)



10 Which of the following is an expression for $\dfrac{1-e^{2x}}{1-e^{x}}$? **1**

(A) $1-e^{x}$

(B) $1+\dfrac{e^{2x}}{e^{x}}$

(C) $1+e^{x}$

(D) None of the above.

End of Section I

Section II

90 marks
Attempt Questions 11 – 20
Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (9 marks) **Start a new booklet.** **Marks**

(a) Factorise $2x^2 - 7x - 15$. **1**

(b) Find the centre and radius of the circle given by $x^2 - 6x + y^2 + 8y = 0$. **2**

(c) Determine, showing all working, whether $y = 3^x - 3^{-x}$ is even, odd or neither. **2**

(d) (i) Using a number plane and using half a page, sketch $g(x) = 2 - \dfrac{1}{x+2}$. **2**

(ii) On the same diagram, sketch $y = g(-x)$, showing all asymptotes and intercepts. **1**

(iii) State the range of $y = g(-x)$. **1**

End of Question 11

Question 12 (9 marks) **Start a new booklet.** **Marks**

(a) Let $f(x) = 2x + 3$ and $g(x) = 5x + b$, where b is a constant.

(i) Find expressions for $g\left(f(x)\right)$ and $f\left(g(x)\right)$. **2**

(ii) Hence find a value of b so that $g\left(f(x)\right) \equiv f\left(g(x)\right)$, for all x. **1**

(b) Differentiate $\dfrac{x^3 - 2x^2}{x}$ with respect to x. **1**

(c) Given that $y = 6x\sqrt{x+1}$, find $\dfrac{dy}{dx}$ giving your answer as a single fraction. **2**

(d) Consider the function $f(x) = \dfrac{1}{x}$.

(i) Show that $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-1}{x(x+h)}$. **1**

(ii) Hence show that $f'(x) = \dfrac{-1}{x^2}$. **1**

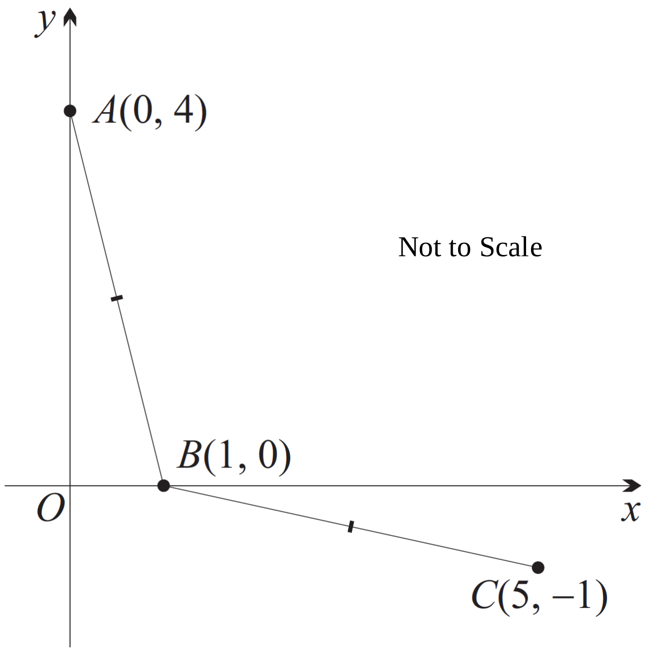
(iii) Describe what happens to the slope of the tangents of $f(x)$ as $x \to 0^+$ and $x \to 0^-$. **1**

End of Question 12

Question 13 (9 marks) **Start a new booklet.** **Marks**

(a) Find the domain of $g\left(h(x)\right)$ if $h(x)=3x-2$ and $g(x)=x^{\frac{1}{2}}$. **1**

(b) In the diagram below, A is $(0,4)$, B is $(1,0)$ and C is $(5,-1)$. The intervals AB and BC are equal in length.



(i) Find the gradient of AC. **1**

(ii) Find the length of AC. **1**

(iii) Show that the equation of AC is $x+y-4=0$. **1**

(iv) Find the coordinates of D so that $ABCD$ is a rhombus. **2**

(v) Find the equation of BD. **1**

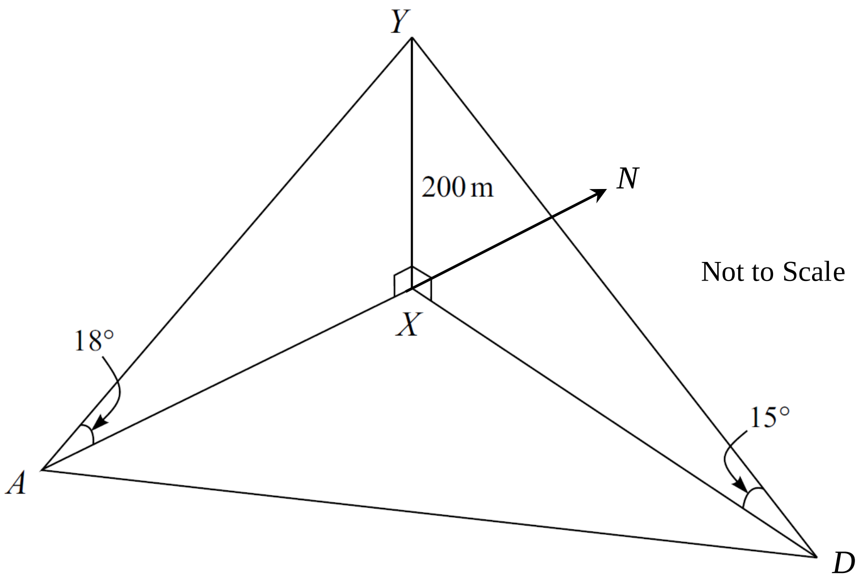
(vi) Find the area of the rhombus $ABCD$. **2**

End of Question 13

Question 14 (9 marks) **Start a new booklet.** **Marks**

(a) Show that $\dfrac{d}{dx}\left[3x^2(4x-1)^3\right] = 6x(10x-1)(4x-1)^2$. **2**

(b) Andrew and Donna are bushwalking on flat ground. They can both see the same mountain, which has a height of 200 m. From Andrew's position A, the mountain is due north and the top of the mountain has an angle of elevation of $18°$. From Donna's position D, the mountain has an angle of elevation of $15°$. Let the base and the top of the mountain be the points X and Y respectively. The bearing of Donna's position from the base of the mountain is $120°\text{T}$ and $069°\text{T}$ from Andrew.



(i) Write down the size of $\angle AXD$. **1**

(ii) Show that $XD = \dfrac{200}{\tan 15°}$. **1**

(i) Using the sine rule, find the distance from Andrew to Donna. Give your answer correct to the nearest metre. **2**

(c) (i) Sketch $h(x) = x^2 - |2x|$, for $-3 \le x \le 3$. **2**

(ii) Indicate where $h(x)$ is not differentiable, giving a reason for your answer. **1**

End of Question 14

Question 15 (9 marks) **Start a new booklet.** **Marks**

(a) Solve $\log_e\left(36-x^2\right)-\log_e x=\log_e 5$ for *x*. **3**

(b) A container of water, originally just below boiling point at $96°\mathrm{C}$, is placed in a fridge whose temperature is $0°\mathrm{C}$. The container is known to cool in such a way that its temperature halves every 20 minutes.

(i) Explain why the temperature $T°\mathrm{C}$ after *n* hours in the fridge is given by the function $T=96\times\left(\frac{1}{2}\right)^{3n}$. **1**

(ii) What is the temperature after 2 hours? **1**

(iii) Show that the time *n* in hours to reach a temperature of $T°\mathrm{C}$ is given by $n=-\frac{1}{3}\log_2\left(\frac{T}{96}\right)$. **2**

(iv) How long does it take, correct to the nearest minute, for the temperature to fall to $1°\mathrm{C}$? **2**

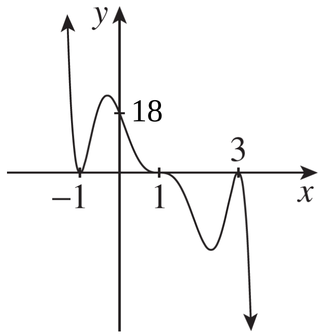
End of Question 15

Question 16 (9 marks) **Start a new booklet.** **Marks**

(a) Solve $\left|2x-1\right|=5$. **2**

(b) Differentiate $\dfrac{5x^2}{7-x}$ giving your answer in simplest form. **2**

(c) The graph shown below is a polynomial which has degree 7.



Write down its equation as a product of its linear factors. **2**

(d) (i) What is the domain of the function $g(x)=\dfrac{x^4-x^2}{x^2-1}$. **2**

(ii) Sketch $y=g(x)$. **1**

End of Question 16

Question 17 (9 marks) **Start a new booklet.** **Marks**

(a) A bush turkey plague hit the Northern Beaches this year but was eventually brought under control. The council estimated that the bush turkey population P in hundreds, t months after 1st January, was given by $P = 15 + 14t - t^2$.

(i) Find the bush turkey population on 1st February. **1**

(ii) At what rate were they increasing at this time? **1**

(iii) Calculate the average rate of increase over the first month and explain why it is different to your answer in part (ii). **2**

(iv) In what months does the bush turkey population decrease? **2**

(b) Solve $7^x = 6 \times 5^{x-3}$ for x, leaving your answer in terms of $\log_{10}$. **3**

End of Question 17

Question 18 (9 marks) **Start a new booklet.** **Marks**

(a) (i) Show that the equation of the tangent to $y = 1 - e^{-x}$ at the origin is $y = x$. **2**

(ii) Deduce the equation of the normal at the origin without further use of calculus. **1**

(iii) Sketch the curve, showing the points T and N where the tangent and normal respectively cut the asymptote. **2**

(b) (i) Show that $\sqrt{\dfrac{\text{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = |\tan x|$. **2**

(ii) Hence, find the value(s) of x if $\sqrt{\dfrac{\text{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = 1$ for $0° \le x \le 180°$. **2**

End of Question 18

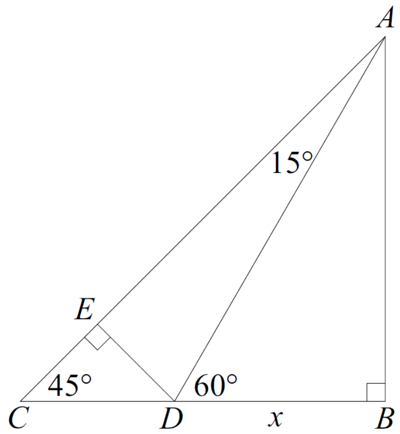
Question 19 (9 marks) **Start a new booklet.** **Marks**

(a) (i) Write down the expansion for $(a+b)^2$. **1**

(ii) Use the expansion in part (i) to show that $\left(\sqrt{6+\sqrt{11}}+\sqrt{6-\sqrt{11}}\right)^2$ is an integer. **2**

(iii) Hence simplify $\sqrt{6+\sqrt{11}}+\sqrt{6-\sqrt{11}}$. **1**

(b) In the diagram below, ΔABC is a right-angled isosceles triangle. The point D is chosen on BC so that $\angle ADB = 60°$, and DE is drawn perpendicular to AC. Let $BD = x$.



Not to Scale

(i) Show that $DE = \dfrac{\left(\sqrt{3}-1\right)x}{\sqrt{2}}$. **3**

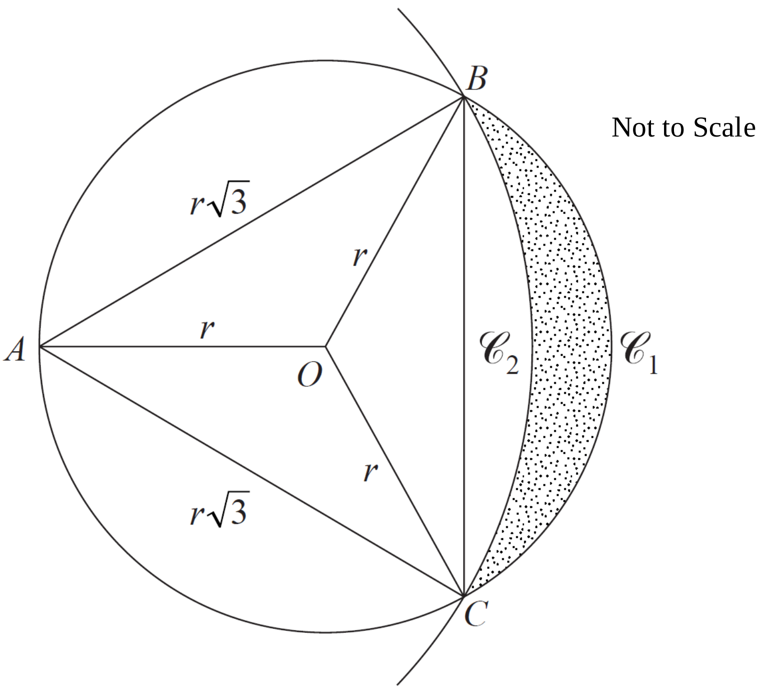
(ii) Hence, or otherwise, find the exact value for $\cos 15°$ with a rational denominator. **2**

End of Question 19

Question 20 (9 marks) **Start a new booklet.** **Marks**

(a) Find the values of k for which $(k-2)x^2-2kx-1=0$ has equal roots. **2**

(b) The point A lies on the circle $\mathscr{C}_1$ with centre O and radius r. The circle $\mathscr{C}_2$ with centre A and radius $r\sqrt{3}$ intersects $\mathscr{C}_1$ at the points B and C, as shown in the diagram.



(i) Using the cosine rule, or otherwise, find $\angle BAO$. **2**

(ii) Find the area of the sector BAC of $\mathscr{C}_2$. **1**

(iii) Find the area of the sector BOC of $\mathscr{C}_1$. **2**

(iv) Hence, or otherwise, find the area of the shaded region in terms of r. **2**

End of the Examination

Multiple Choice — Suggested Solution (s)	Comments
see below	

Multiple Choice

1. $\dfrac{1}{\sqrt{5}+2\sqrt{3}} \times \dfrac{\sqrt{5}-2\sqrt{3}}{\sqrt{5}-2\sqrt{3}}$

$= \dfrac{\sqrt{5}-2\sqrt{3}}{5-12}$

$= \dfrac{2\sqrt{3}-\sqrt{5}}{7}$ or $\dfrac{\sqrt{5}-2\sqrt{3}}{-7}$

$\Rightarrow$ C/D

2. $y = (x+2)^2 - 1$

move down 1

shift left 2

positive quadratic $\Rightarrow$ A

3. let $Q(x,y)$

$\dfrac{5+x}{2} = 9 \quad \therefore x = 13$

$\Rightarrow$ C

4. $\cos\left(-\frac{\pi}{6}\right)$

$= \cos\left(\frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{3}\right) \Rightarrow$ C

5. $\dfrac{d}{dx}\left(x^{n+1}\right)$

$= (n+1)x^{n+1-1}$

$= (n+1)x^n \Rightarrow$ D

6. For at least one number in the domain there is more than one number in range, every number in range there is only one number in domain $\Rightarrow$ B

7. $\log_e (e^x)^2$

$= \log_e e^{2x}$

$= 2x \Rightarrow$ C

8. $2\theta = 180 + 60°,$

$360 - 60°$

$\therefore \theta = 120°, 150°$

$\Rightarrow$ D.

9. $y = -\log_2(x+1)$

flip x — shift left

$\Rightarrow$ C

10. $\dfrac{(1+e^x)\cancel{(1-e^x)}}{\cancel{(1-e^x)}}$

$= 1 + e^x \Rightarrow$ C

Summary

1. C/D 2. A 3. C 4. C 5. D
6. B 7. C 8. D 9. C 10. C

Question. 11 Suggested Solution (s)	Comments

(a) $(2x+3)(x-5)$ ✓

(b) $x^2-6x+9+y^2+8y+16=25$

$(x-3)^2+(y+4)^2=5^2$

centre $(3,-4)$ ✓ radius $=5$ ✓

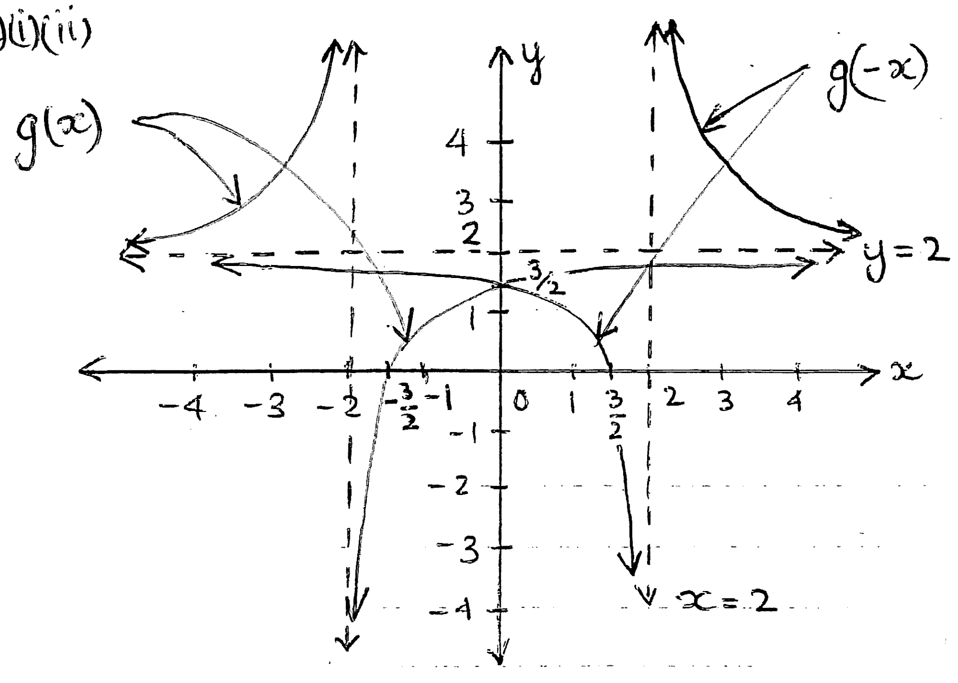
(c) $f(-x) = 3^{-x}-3^{--x}$

$= -(3^x-3^{-x})$ ✓

$= -f(x)$

$\therefore$ odd ✓

(d)(i)(ii)



(i) max 2 marks
-1 no intercepts
-1 no asymptotes

(ii) can be on separate graphs - no penalty
must show reflection about y-axis,

(iii) range $y \in \mathbb{R}, y \neq 2$

Question 12 Suggested Solution (s)	Comments
(a)(i) $g(f(x)) = 5(2x+3)+b = 10x+15+b$ ✓ $f(g(x)) = 2(5x+b)+3 = 10x+3+2b$ ✓ (ii) $15+b = 3+2b$ $b = 12$ ✓ (b) $= \frac{d}{dx}(x^2-2x)$ $= 2x-2$ ✓ (c) $u = 6x$ $\quad v = (x+1)^{\frac{1}{2}}$ $u' = 6$ $\quad v' = \frac{1}{2}(x+1)^{-\frac{1}{2}}$ $= 6(x+1)^{\frac{1}{2}} + 3x(x+1)^{-\frac{1}{2}}$ ✓ $= \frac{3(2x+1)}{\sqrt{x+1}}$ or $\frac{3(2x+1)\sqrt{x+1}}{x+1}$ ✓ either (d)(i) $\frac{f(x+h)-f(x)}{h}$ $= \frac{\frac{x\times 1}{x\times(x+h)} - \frac{1\times(x+h)}{x\times(x+h)}}{h}$ $= \frac{x-(x+h)}{xh(x+h)}$ $= \frac{-h}{xh(x+h)}$ $= \frac{-1}{x(x+h)}$ (ii) $f'(x)$ = the limit of $h \to 0$ of $\frac{f(x+h)-f(x)}{h}$ so that the gradient of the secant becomes the gradient of the tangent at point x. (ii) $f'(x) = h \to 0$ of $\frac{f(x+h)-f(x)}{h}$ $= \frac{-1}{x(x+0)}$ $= \frac{-1}{x^2}$ student must convey that $h \to 0$ (iii) $f'(x) \to -\infty$ ✓	1 mark either line

Question . 13 Suggested Solution (s)	Comments
(a) $g(h(x)) = \sqrt{3x-2}$ domain $3x - 2 \geqslant 0$ $x \geqslant 2/3$ ✓ "must have "greater than or equal to." (b) (i) $m_{AC} = \frac{-1-4}{5-0}$ $= -1$ ✓ (ii) $d_{AC} = \sqrt{(-1-4)^2 + (5-0)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$ } either ✓ (iii) $y - 4 = -1(x-0)$ ✓ $y + x - 4 = 0$ (iv) let $D(x, y)$ $\frac{1+x}{2} = \frac{5}{2}$ and $\frac{0+y}{2} = \frac{3}{2}$ $\therefore x = 4$ ✓, $y = 3$ ✓ (v) $y - 0 = 1(x-1)$ $x - y - 1 = 0$ ✓ ...or equivalent. (vi) $d_{BD} = \sqrt{(3-0)^2 + (4-1)^2}$ $= \sqrt{18}$ $= 3\sqrt{2}$ ✓ $A_{ABCD} = \frac{1}{2} \times 3\sqrt{2} \times 5\sqrt{2}$ $= 15u^2$ ✓ or $2\times\left(\frac{1}{2}\times 5 \times 5 - \frac{1}{2}\times 4 \times 1 - \frac{1}{2}(1+5)\right)$ ✓ $= 15u^2$ ✓	

Question . 14 Suggested Solution (s)	Comments

(a) let $u = 3x^2$, $v = (4x-1)^3$

$u' = 6x$ $\quad v' = 12(4x-1)^2$ ✓ both

$$\frac{d}{dx}\left(3x^2(4x-1)^3\right) = 6x(4x-1)^3 + 36x^2(4x-1)^2$$

$$= 6x(4x-1)^2(4x-1+6x)$$

$$= 6x(10x-1)(4x-1)^2$$

must show factoring line for 2 marks

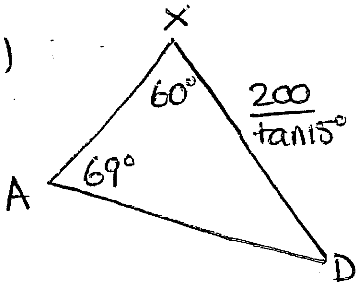
(b)(i) $\angle AXD = 180° - 120°$

$= 60°$ ✓

(ii) $\tan 15° = \frac{200}{XD}$ ✓

$XD = \frac{200}{\tan 15°}$ or $200\cot 15°$

(iii)



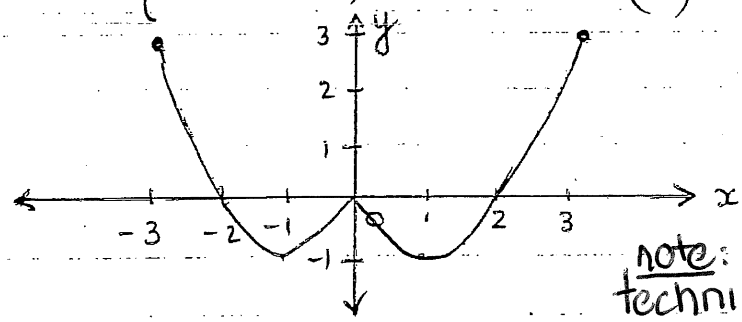
$$\frac{AD}{\sin 60°} = \frac{200}{\tan 15°} \div \sin 69° \checkmark$$

$$AD = \frac{100\sqrt{3}}{\tan 15° \sin 69°}$$

$AD = 692.399\ldots$

$AD = 692$ m (nearest metre) ✓

(c)(i) $h(x) = \begin{cases} x^2 - 2x, & x \geq 0 \\ x^2 + 2x, & x < 0 \end{cases}$ $\qquad h(3) = 3$, $\quad h(-3) = 3$



note: technically not differentiable at end points also ⇒ no marks allocated here

(ii) as $x \to 0^-$, $m > 0$

$x \to 0^+$, $m < 0$

not smooth at $x = 0$ (ie cusp) ✓

Question 15 Suggested Solution (s)	Comments
(a) $\log_e\left(\frac{36-x^2}{x}\right) = \log_e 5$ $\therefore\ 36 - x^2 = 5x$ $x^2 + 5x - 36 = 0$ $(x+9)(x-4) = 0$ ✓ $\therefore\ x = 4$ ✓ $\because\ 0 < x < 6$ or demonstrated that for $\log x$, $x > 0$ (b) (i) starts at 96°C and halves every 20 mins. Therefore $3n$ is the number of 20min periods in n hours. ✓ (ii) $T = 96 \times \left(\frac{1}{2}\right)^6$ $= 1.5°C$ ✓ (iii) $\frac{T}{96} = \left(\frac{1}{2}\right)^{3n}$ $\log_2 \frac{T}{96} = 3n \log_2\left(\frac{1}{2}\right)$ ✓ $n = \frac{1}{3} \times \frac{\log_2 \frac{T}{96}}{-1}$ ✓ either $\log_2 2^{-1}$ or -1 this needs to be clear for second mark. $n = -\frac{1}{3}\log_2\left(\frac{T}{96}\right)$ (iv) $n = \frac{\log 96 - \log 1}{3 \log 2}$ $= 2.194\ldots$ hours ✓ change of base $= 2°11'41.96''$ $= 2$ hours 12 mins ✓	

Question. 16 Suggested Solution (s)	Comments

(a) $2x-1=5$ and $2x-1=-5$

$2x=6$ $\quad$ $2x=-4$

$x=3$ ✓ $\quad$ $x=-2$ ✓

(b) $u=5x^2$ $\quad v=7-x$ ✓

$u'=10x$ $\quad v'=-1$

$$\frac{10x(7-x)+5x^2}{(7-x)^2} \checkmark$$

$$=\frac{70x-5x^2}{(7-x)^2}$$

$$=\frac{5x(14-x)}{(7-x)^2}$$

(c) $y=a(x+1)^2(x-1)^3(x-3)^2$ ✓

sub in $(0,18)$ to find a

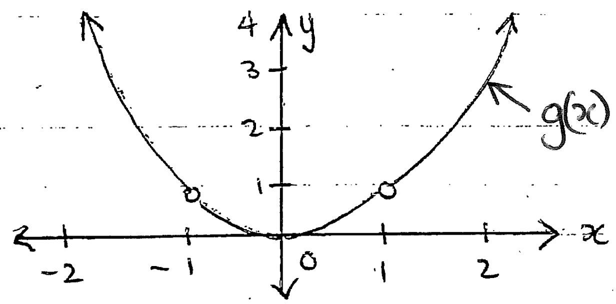
$18=-9a$

$\therefore a=-2$ and $y=-2(x+1)^2(x-1)^3(x-3)^2$ ✓

(d)(i) $g(x)=\dfrac{x^2\cancel{(x^2-1)}}{\cancel{x^2-1}}$ $\quad$ domain: $x\in\mathbb{R},\ x\neq\pm1$ ✓

x^2 ✓, $x\neq\pm1$

(ii)



✓ ○ no holes

Question. 17 Suggested Solution (s)	Comments
(a)(i) $P = 15 + 14(1) - 1^2$ $P = 28$ $\therefore$ population 1st Feb $= 2800$ ✓ (ii) $\frac{dP}{dt} = 14 - 2t$ at $t = 1$ $\frac{dP}{dt} = 12$ $\therefore$ increasing at 1200/month. ✓ (iii) increase is $2800 - 1500 = 1300$ average rate is 1300/month ✓ the rate in part (ii) is smaller because it is instantaneous and slowing down over time. ✓ (iv) $\frac{dP}{dt} = 0$ $2t = 14$ $(15-t)(1+t) = 0$ $t = 7$ 1 mark for 7 & 15 $7 < t \leq 15$ or Aug – March (following year) ✓ (b) $\log_{10} 7^x = \log_{10} 6 + \log_{10} 5^{x-3}$ ✓ $x\log_{10} 7 = \log_{10} 6 + (x-3)\log_{10} 5$ $x(\log_{10} 7 - \log_{10} 5) = \log_{10} 6 - \log_{10} 5^3$ ✓ $x = \dfrac{\log_{10}\frac{6}{125}}{\log_{10}\frac{7}{5}}$ or $\dfrac{\log_{10} 6 - \log_{10} 125}{\log_{10} 7 - \log_{10} 5}$ ✓	collecting and factoring x

Question. 18 Suggested Solution (s)	Comments

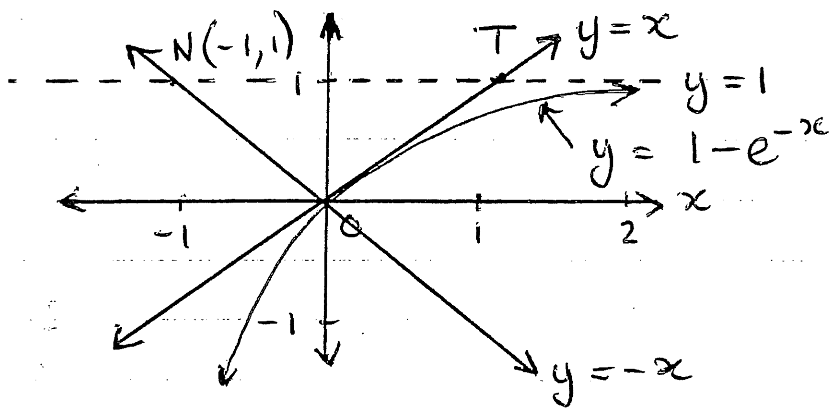
(a) (i) $\frac{dy}{dx} = e^{-x}$ ✓

at $x=0$ $\frac{dy}{dx} = 1$ ✓, $y = 0$

$\therefore y = x$

(ii) normal : $y = -x$ ✓

(iii)



(b)(i) LHS $= \sqrt{\dfrac{1+\cancel{\cot^2 x}-\cancel{\cot^2 x}-\cos^2 x}{\cos^2 x}}$ ✓

$= \sqrt{\dfrac{1-\cos^2 x}{\cos^2 x}}$

$= \sqrt{\dfrac{\sin^2 x}{\cos^2 x}}$ ✓

$= \sqrt{\tan^2 x}$

$= |\tan x|$

(ii) $|\tan x| = 1$

$\tan x = \pm 1$

$\therefore x = 45°, 135°$ 1 each.

Question 19 Suggested Solution (s)	Comments
(a)(i) $a^2 + 2ab + b^2$ ✓ (ii) let $a = \sqrt{6+\sqrt{11}}$, $b = \sqrt{6-\sqrt{11}}$ $(a+b)^2 = 6 + \sqrt{11} + 2\sqrt{36-11} + 6 - \sqrt{11}$ ✓ $= 12 + 10$ $= 22 \in \mathbb{Z}$ ✓ (iii) $= a+b$ $= \sqrt{22}$ ✓ (b)(i) $\tan 60 = \frac{AB}{x}$ $AB = \sqrt{3}x$ $AB = BC$ (isosceles triangle) $\therefore CD = \sqrt{3}x - x$ ✓ $= (\sqrt{3}-1)x$ $CE = DE$ (isosceles triangle) $\therefore 2DE^2 = (\sqrt{3}-1)^2x^2$ ✓ $DE = \frac{(\sqrt{3}-1)x}{\sqrt{2}}$ (ii) $AD^2 = x^2 + (\sqrt{3}x)^2$ $AD = 2x$ $AE^2 = (2x)^2 - \frac{(\sqrt{3}-1)^2x^2}{2}$ ✓ $AE^2 = \frac{8x^2}{2} - \frac{(3-2\sqrt{3}+1)x^2}{2}$ $AE = \frac{\sqrt{8-4+2\sqrt{3}}}{\sqrt{2}}x$ $AE = \frac{\sqrt{2}\sqrt{4+2\sqrt{3}}}{2}x$ $\cos 15° = \frac{AE}{AD} = \frac{\sqrt{8+4\sqrt{3}}}{4}$ ✓✓ note: $\frac{\sqrt{4+2\sqrt{3}}}{2\sqrt{2}}$ 2 marks only.	

Question. 20 Suggested Solution (s)	Comments
(a) $\Delta = b^2 - 4ac$ $0 = 4k^2 - 4(-1)(k-2)$ $\therefore 4k^2 + 4k - 8 = 0$ ✓ $k^2 + k - 2 = 0$ $(k+2)(k-1) = 0$ $\therefore k = -2$ or $k = 1$ ✓ (b) (i) $\cos\angle BAO = \dfrac{r^2 + (r\sqrt{3})^2 - r^2}{2(r)(r\sqrt{3})}$ $= \dfrac{3\cancel{r^2}}{2\sqrt{3}\cancel{r^2}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$ ✓ $= \dfrac{\sqrt{3}}{2}$ $\therefore \angle BAO = \cos^{-1}\dfrac{\sqrt{3}}{2}$ ✓ $= \dfrac{\pi}{6}$ (ii) $A_1 = \frac{1}{2}(r\sqrt{3})^2 \frac{\pi}{3}$ $A_1 = \dfrac{\pi r^2}{2}$ ✓ $\angle BAO = \angle CAO$ (isosceles triangles) $\therefore \angle CAO = \frac{\pi}{3}$ (iii) $\angle BOA = \pi - \frac{\pi}{3}$ ($\angle$ sum Δ) $= \angle AOC$ $\angle BOC = 2\pi - \frac{4\pi}{3}$ $= \dfrac{2\pi}{3}$ ✓ $A_2 = \frac{1}{2}r^2\left(\frac{2\pi}{3}\right)$ $= \dfrac{\pi r^2}{3}$ ✓ (iv) $A_{\Delta AOB} = A_{\Delta AOC} = A_{\Delta BOC} = \frac{1}{2}r^2\sin\left(\frac{2\pi}{3}\right)$ $= \frac{1}{2}r^2\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}r^2$ ✓ $A_{\text{shaded region}} = \dfrac{2\sqrt{3}r^2}{4} - \dfrac{\pi r^2}{2} + \dfrac{\pi r^2}{3}$ $= \left(\frac{\sqrt{3}}{2} - \frac{\pi}{2} + \frac{\pi}{3}\right)r^2 = \left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)r^2$ ✓ or $0.34r^2$	