



Knox Grammar School

Name: _____

2020 - Year 11 Yearly Examination

Teacher: _____

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen only.
- Black pen is preferred.
- NESAs-approved calculators may be used.
- In Questions 11-16, show relevant mathematical reasoning and/or calculations.

Examiner: Mr Meli

Teachers:

Mr Charlier
Mr Cheah
Mrs Dempsey
Mr Lemaire
Mr Menzies
Mr Naidoo
Mr Willcocks
Ms Yun
Mrs Zaidi

Write your Name, your Board of Studies Student Number and your Teacher's Name on the front cover of each writing booklet.

This paper MUST NOT be removed from the examination room.

Number of Students in Course: 178

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

1 When fully simplified, $\sqrt{128}$ is written as,

A. $16\sqrt{8}$

B. $8\sqrt{2}$

C. $100\sqrt{28}$

D. $2\sqrt{32}$

2 The gradient of the line $3x - 2y = 0$ is,

A. $\frac{3}{2}$

B. $-\frac{3}{2}$

C. $-\frac{2}{3}$

D. $\frac{2}{3}$

3 What is the value of p so that $\frac{a^2 a^{-3}}{\sqrt{a}} = a^p$?

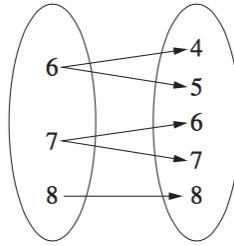
A. -3

B. 12

C. $-\frac{1}{2}$

D. $-\frac{3}{2}$

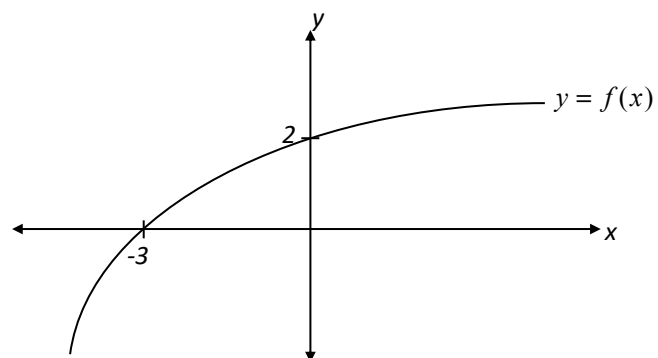
- 4 What type of relation is shown ?



- A. Many to many.
- B. One to many.
- C. One to one.
- D. Many to one.
- 5 Which of the following is equivalent to $(x-3)(x-6) - (x-3)(x+3)$?
- A. $3x+9$
- B. $-9x+9$
- C. $3x+27$
- D. $-9x+27$
- 6 The graph of $y = 2x^2 - x + 1$ is reflected in the y axis. The equation of the new function is,
- A. $y = -2x^2 + x - 1$
- B. $y = -2x^2 - x - 1$
- C. $y = 2x^2 + x + 1$
- D. $y = 2x^2 - x - 1$

- 7 When evaluated, $\log_3 15 + \log_3 18 - \log_3 10 =$
- A. 1
 - B. 2
 - C. 3
 - D. $\log_3 23$
- 8 Given that $g(x)$ is an odd function and that $g(2) = -4$, the value of $g(-2)$ is,
- A. 2
 - B. -2
 - C. -4
 - D. 4
- 9 Given that $x \in [0, 2\pi]$, how many solutions has the equation $e^x(\cos x + 1) = 0$?
- A. 0
 - B. 1
 - C. 2
 - D. 3

- 10 The graph of $y = f(x)$ is shown below with $f(-3) = 0$ and $f(0) = 2$.



Which of the following pairs of graphs will both pass through the origin ?

- A. $y = f(x-3)$ and $y = f(x)-2$
- B. $y = f(x)-3$ and $y = f(x-2)$
- C. $y = f(x+3)$ and $y = f(x)-2$
- D. $y = f(x)-3$ and $y = f(x+2)$

Section II

90 marks

Attempt Questions 11–16

Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new Writing Booklet.

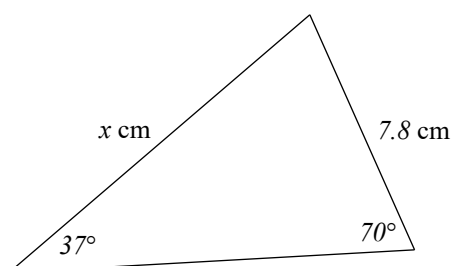
(a) Factorise fully,

(i) $4x^2 + 4x - 15$. **1**

(ii) $9x - xy^2$. **2**

(b) Simplify the expression $\frac{1}{n-1} - \frac{1}{n}$. **2**

(c) Find the value of x in the diagram below, giving your answer correct to one decimal place. **2**



(d) Solve the inequation **3**

$$\frac{1}{2}(y-3) - \frac{1}{3}(3+2y) \geq -2.$$

(e) Find the exact values of a and b if $\frac{2}{\sqrt{5}+2} = \sqrt{a} + b$. **3**

(f) Find the exact value of $\operatorname{cosec} 225^\circ$. **2**

End of Question 11

Question 12 (15 marks) Start a new Writing Booklet.

(a) Solve $|3x - 1| = 7$. 2

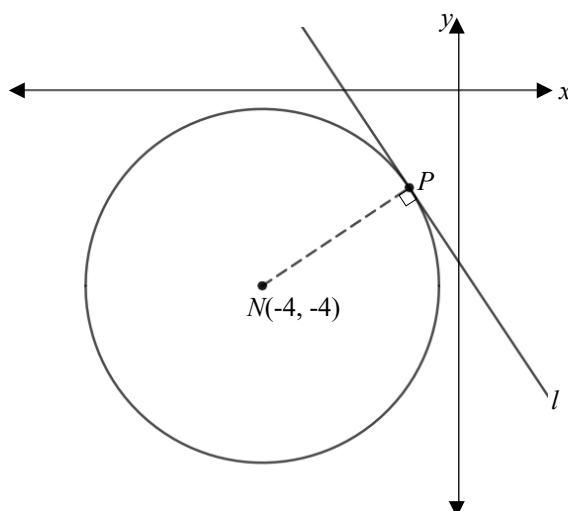
(b) A function is defined by the rule $f(x) = \frac{x^4 - 1}{x}$.

(i) Evaluate $f(-2)$. 1

(ii) Solve $f(x) = 0$. 2

(iii) Show that $f(x)$ is an odd function. 1

- (c) The circle in the diagram below has centre $N(-4, -4)$.
The line l is a tangent to the circle at P and has equation $3x + 2y + 7 = 0$.



(i) Show that the equation of NP is $2x - 3y - 4 = 0$. 2

(ii) Find the co-ordinates of the point P . 2

(iii) Find the exact radius of the circle. 2

(iv) Give the equation of the circle. 1

(d) A student was asked to differentiate $f(x) = 3x - x^2$ from first principles. 2

The student began the solution with the formula, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

Complete the solution.

End of Question 12

Question 13 (15 marks) Start a new Writing Booklet.

(a) Differentiate the following.

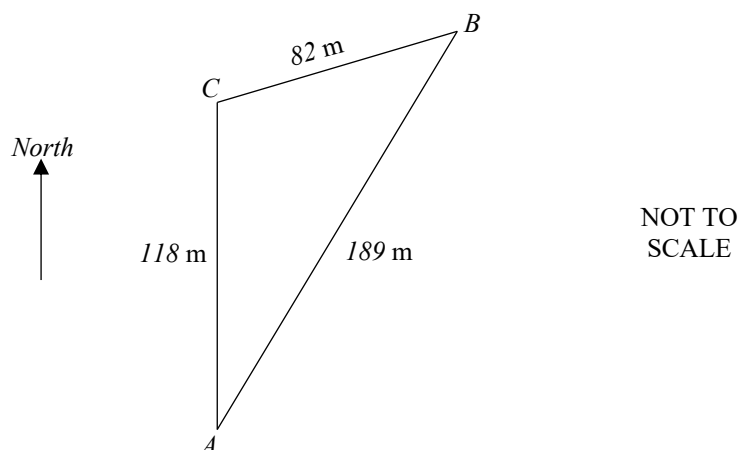
(i) $y = 3x^2 - \frac{1}{x}$ 1

(ii) $y = (2 - x^2)^5$ 1

(iii) $y = \frac{x^2}{x-1}$ 2

(iv) $y = x^3 e^{2x}$ 2

(b) A park is bordered by three straight roads, AB , BC and AC . The road AC runs due North, as shown in the diagram below.



(i) Find the size of $\angle ACB$ to the nearest degree. 2

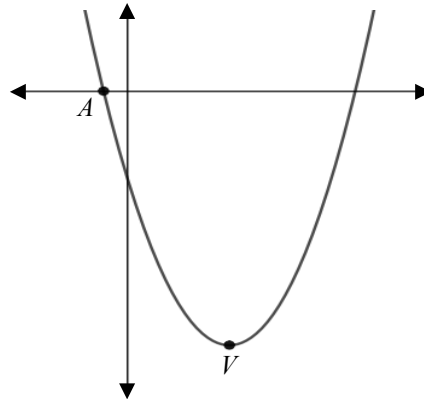
(ii) What is the bearing of B from C ? 1

(c) Determine the domain of the function $f(x) = \frac{\sqrt{x+1}}{x}$. 2

Question 13 continues on page 9

Question 13 (continued)

- (d) The diagram shows the graph of $y = x^2 - 4x - 2$.



- | | | |
|------|--|---|
| (i) | Find the co-ordinates of the vertex, V | 2 |
| (ii) | Find the x co-ordinate of the intercept marked A . | 2 |

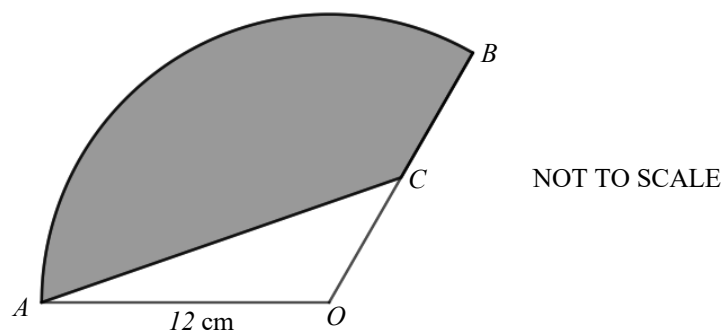
End of Question 13

Question 14 (15 marks) Start a new Writing Booklet.

- (a) Find the equation of the normal to the graph of $y = \sqrt{3x-2}$ at the point where $x = 2$. **3**

- (b) The diagram below shows sector OAB with centre O and a radius of 12 cm.

The point C is the midpoint of OB . The area of $\triangle OAC$ is $18\sqrt{3} \text{ cm}^2$.



- (i) Find the size of the obtuse angle AOC in radians. **2**
- (ii) Find the exact length of the arc AB . **1**
- (iii) Find the shaded area. **2**

Question 14 continues on page 11

Question 14 (continued)

- (c) The population of native parrots on a tropical island has been studied since 1950 for conservation purposes.

The number of parrots, P , can be modelled by the equation $P = 2500e^{-0.036t}$, where t is the number of years after 1950.

- (i) What was the population of native parrots in 1970 ? **1**
- (ii) Show that the parrot population was declining at a rate of approximately 44 parrots per year in 1970. **2**
- (iii) When the parrot population reaches 250, they are listed as being in danger of extinction. **2**

After how many years will this occur ?

- (d) Given that $f(x) = x^2 + 1$ and $g(x) = \sqrt{x - 3}$, sketch $y = f(g(x))$ over its natural domain. **2**

End of Question 14

Question 15 (15 marks) Start a new Writing Booklet.

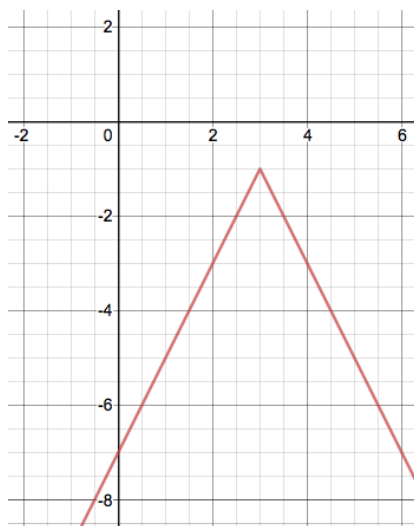
(a) (i) Show that, $\cos^2 \theta (\sec^2 \theta - 1) = \sin^2 \theta$. **2**

(ii) Hence, or otherwise solve,

$$4\cos^2 \theta (\sec^2 \theta - 1) = 1, \quad 0^\circ \leq \theta \leq 2\pi. \quad \mathbf{3}$$

(b) The function $f(x) = |x|$ is transformed and the equation of the new
3
function is in the form $y = kf(x + b) + c$, where k , b , and c are constants.

The graph of the new function is shown.



What are the values of k , b , and c ?

Question 15 continues on page 13

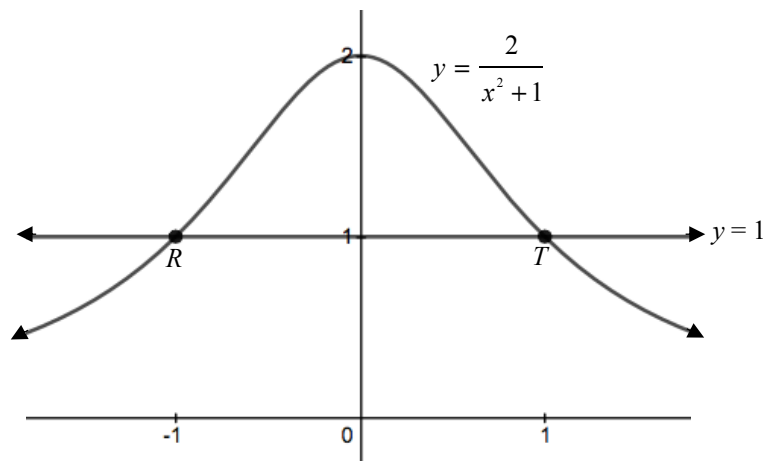
Question 15 (continued)

- (c) (i) Show that $\frac{x}{x+1} = 1 - \frac{1}{x+1}$. **1**
- (ii) Hence, or otherwise, sketch the graph of $y = \frac{x}{x+1}$ showing any asymptotes and intercepts with the axes. **2**
- (d) The line $y = mx$ is a tangent to the curve $y = e^{0.5x}$ at a point A .
- (i) Sketch the line and the curve on a set of axes. **1**
- (ii) Find the coordinates of A . **2**
- (iii) Find the value of m . **1**

End of Question 15

Question 16 (15 marks) Start a new Writing Booklet.

- (a) The diagram below shows the curve $y = \frac{2}{x^2 + 1}$ and the line $y = 1$ which intersects the curve at the points $R(-1, 1)$ and $T(1, 1)$.

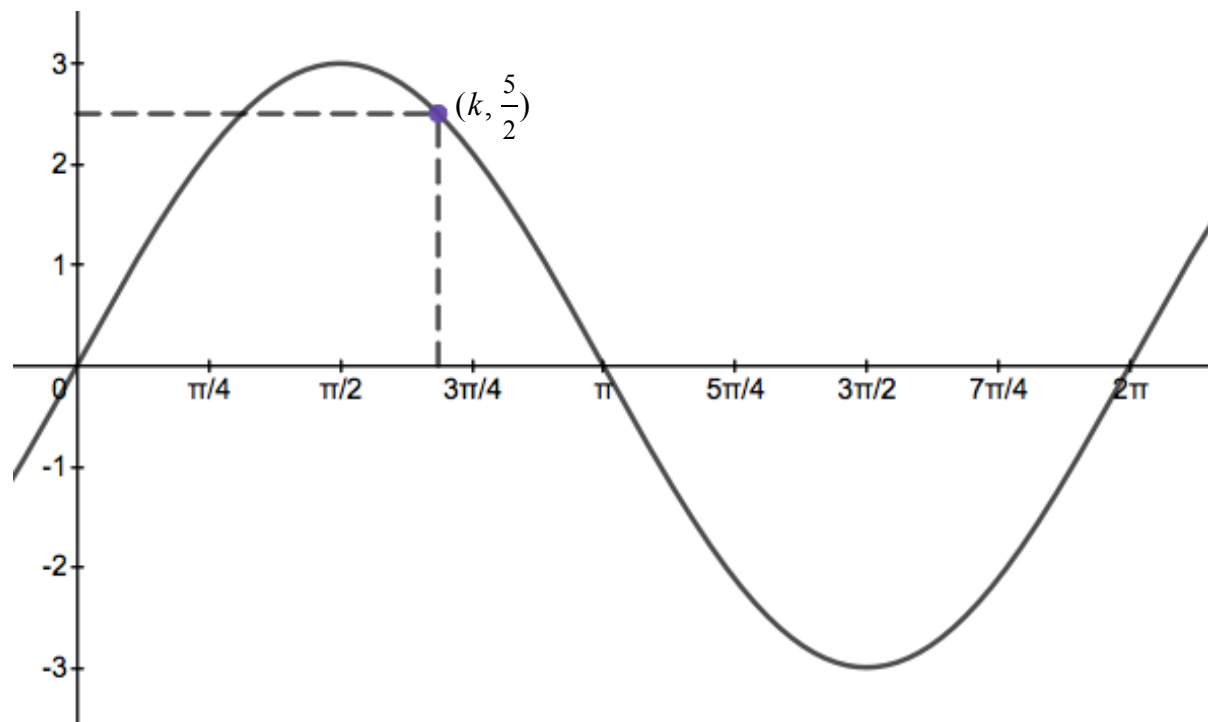


- (i) Write down the range of $y = \frac{2}{x^2 + 1}$. 1
- (ii) Explain how you know that $y = \frac{2}{x^2 + 1}$ is an even function. 1
- (iii) Use the diagram to solve $\frac{2}{x^2 + 1} \leq 1$. 1
- (b) Determine all values of t such that the roots of $t(x-1)(x-2) = x$ are real. 3

Question 16 continues on page 15

Question 16 (continued)

- (c) The graph represents the function $y = a \sin bx$.



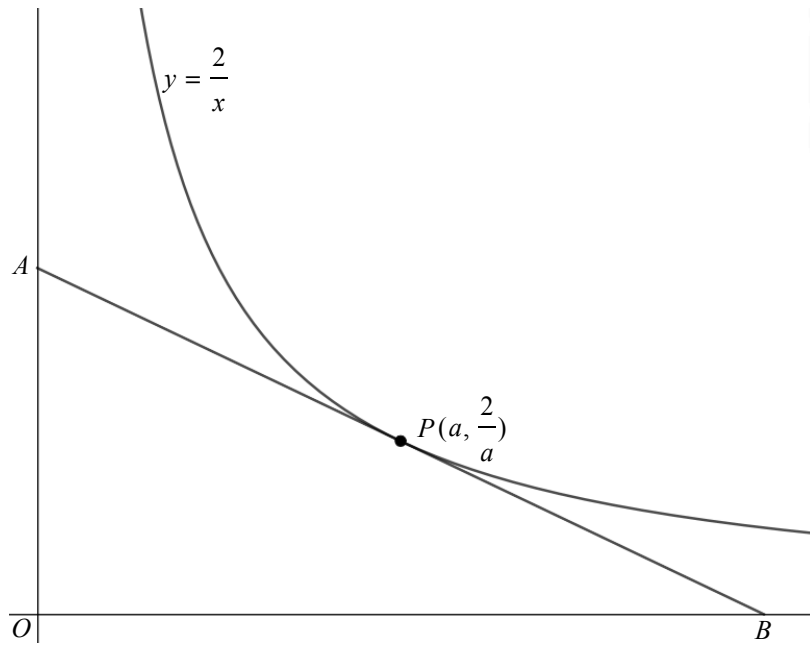
- (i) Find the values of a and b . 2
- (ii) The point $\left(k, \frac{5}{2}\right)$ lies on the graph as shown. 2

Find the exact value of $\tan k$.

Question 16 continues on page 16

Question 16 (continued)

(d)



The diagram above shows part of the hyperbola $y = \frac{2}{x}$ and the tangent to the hyperbola drawn at an arbitrary point $P\left(a, \frac{2}{a}\right)$. The tangent intersects the y -axis at A and the x -axis at B as shown.

- (i) Find the gradient of the tangent to the curve at the point $P\left(a, \frac{2}{a}\right)$. 1
- (ii) Hence show that the equation of the tangent to the curve $y = \frac{2}{x}$ at the point $P\left(a, \frac{2}{a}\right)$ is $2x + a^2y - 4a = 0$. 2
- (iii) Show that the area of $\triangle AOB$ is constant. 2

END OF EXAMINATION



2020 Year 11 Mathematics Advanced – EOPC Exam - SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
<u>Section I: Multiple Choice</u> Q1. $\sqrt{128} = \sqrt{64} \sqrt{2}$ $= 8\sqrt{2} \quad \therefore B$ Q2. $3x - 2y = 0$ $2y = 3x$ $y = \frac{3}{2}x$ $\therefore$ gradient is $\frac{3}{2} \quad \therefore A$ Q3. $\frac{a^2 a^{-3}}{\sqrt{a}} = \frac{a^{-1}}{a^{1/2}}$ $= a^{-3/2}$ $\therefore p = -\frac{3}{2} \quad \therefore D$ Q4. One to many B Q5. $(x-3)(x-6) - (x-3)(x+3)$ $= x^2 - 9x + 18 - (x^2 - 9)$ $= -9x + 27 \quad \therefore D$ Q6. Reflect in the y-axis by replacing x with $-x$. $y = 2(-x)^2 - (-x) + 1$ $= 2x^2 + x + 1 \quad \therefore C$		Q7. $\log_3 15 + \log_3 18 - \log_3 10$ $= \log_3 \left(\frac{15 \times 18}{10} \right)$ $= \log_3 27$ $= 3 \quad \therefore C$ Q8. Due to rotational symmetry $g(-2) = 4 \quad \therefore D$ Q9. $e^x (\cos x + 1) = 0$ $\therefore e^x = 0$ or $\cos x + 1 = 0$ no solution $\cos x = -1$ $x = \pi$ $\therefore 1$ solution in total $\therefore B$ Q10. Graph will pass through the origin if translated 3 units right $\Rightarrow f(x-3)$ or 2 units down $\Rightarrow f(x) - 2$ $\therefore A$	



2020 Year 11 Mathematics Advanced – EOPC Exam - SOLUTIONS

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Question 11:			
(a)(i) $4x^2 + 4x - 15$ $= (2x+5)(2x-3)$	1mk	(e) $\sqrt{a+b} = \frac{2}{\sqrt{5+2}} \times \frac{\sqrt{5-2}}{\sqrt{5-2}}$ $= 2\sqrt{5} - 4$ $= \sqrt{20} - 4$ $\therefore a = 20, b = -4$	1mk 1mk for a 1mk for b
(ii) $9x - xy^2$ $= x(9 - y^2)$ $= x(3-y)(3+y)$	1mk 1mk		
(b) $\frac{1}{n-1} - \frac{1}{n}$ $= \frac{n - (n-1)}{n(n-1)}$ $= \frac{1}{n(n-1)}$	1mk for common denominator 1mk answer	(f) $\operatorname{cosec} 225$ $= -\operatorname{cosec} 45$ $= \frac{-1}{\sin 45}$ $= -\sqrt{2}$ or $\operatorname{cosec} 225$ $= \frac{1}{\sin 225}$ $= -\sqrt{2}$ Award 1 for bald answer	1mk 1mk 1mk
(c) $\frac{x}{\sin 70^\circ} = \frac{7.8}{\sin 37^\circ}$ $x = \frac{7.8 \sin 70}{\sin 37}$ $\approx 12.2 \text{ cm}$	1mk for correct substitution 1mk answer (ignore rounding)		
(d) $\frac{1}{2}(y-3) - \frac{1}{3}(3+2y) \geq -2$ $3(y-3) - 2(3+2y) \geq -12$ $3y - 9 - 6 - 4y \geq -12$ $-y - 15 \geq -12$ $-y \geq 3$ $y \leq -3$	1mk for eliminating fractions 1mk 1mk		



2020 Year 11 Mathematics Advanced – EOPC Exam - SOLUTIONS

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Question 12 (a) $3x-1 = 7$ $3x-1 = 7$ or $3x-1 = -7$ $3x = 8$ $3x = -6$ $x = \frac{8}{3}$ $x = -2$	Aw1 for • 2 correct equations - or • Solving 1 equation Award 2 for correct solⁿ	(c) (i) $3x + 2y + 7 = 0$ $2y = -3x - 7$ $y = -\frac{3}{2}x - \frac{7}{2}$ $\therefore$ gradient of L is $-\frac{3}{2}$ $\therefore$ gradient_{NP} = $\frac{2}{3}$ $y + 4 = \frac{2}{3}(x + 4)$ $3y + 12 = 2x + 8$ $0 = 2x - 3y - 4$	1 mk for $m_L = -\frac{3}{2}$ 1 mk for establishing NP
(b) (i) $f(x) = \frac{x^4 - 1}{x}$ $\therefore f(-2) = \frac{(-2)^4 - 1}{-2}$ $= -7\frac{1}{2}$	Aw1 for correct answer.	(ii) $3x + 2y + 7 = 0$ $2x - 3y - 4 = 0$ $\therefore 6x + 4y + 14 = 0$ $6x - 9y - 12 = 0$ $13y + 26 = 0$ $y = -2$ $P(-1, -2)$ $\therefore x = -1$	Aw1 for an attempt to use simultaneous equations Aw 2 for P(-1, -2)
(ii) $0 = \frac{x^4 - 1}{x}$ $x^4 - 1 = 0$ $x^4 = 1$ $x = \pm 1$	1 mk for • $x^4 - 1 = 0$ or • $x = 1$ only 2 mk • $x = \pm 1$	(iii) $r^2 = (-1 + 4)^2 + (-2 + 4)^2$ $= 9 + 4$ $= 13$ $\therefore r = \sqrt{13}$ or $r^2 = 3^2 + 2^2$ $\therefore r = \sqrt{13}$	1 mk for correct expression using either distance formula or Pythag. 2 mk for $\sqrt{13}$
(iii) $f(-x) = \frac{(-x)^4 - 1}{-x}$ $= \frac{x^4 - 1}{-x}$ $= \frac{1 - x^4}{x}$ $-f(x) = -\left(\frac{x^4 - 1}{x}\right)$ $= \frac{-x^4 + 1}{x}$ $= \frac{1 - x^4}{x}$ $\therefore$ since $f(-x) = -f(x)$ $f(x)$ is odd	Aw1 for solution with evidence of $f(-x) = -f(x)$	(iv) $(x+4)^2 + (y+4)^2 = 13$	1 mk



2020 Year 11 Mathematics Advanced – EOPC Exam - SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 12 (continued) (d) $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{3(x+h) - (x+h)^2 - (3x - x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{3x + 3h - x^2 - 2xh - h^2 - 3x + x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{3h - 2xh - h^2}{h}$ $= \lim_{h \rightarrow 0} 3 - 2x - h$ $= 3 - 2x$	Link for this line Link for correct answer		

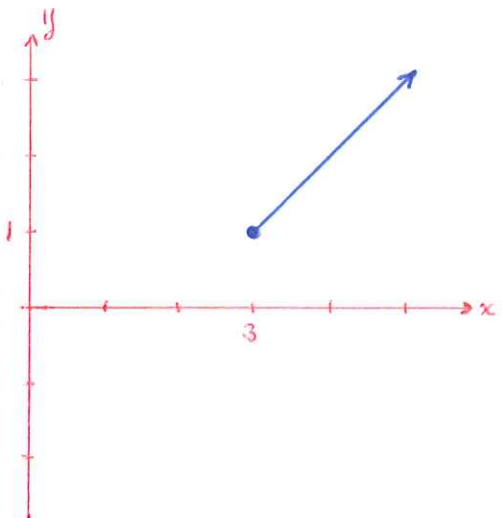


2020 Year 11 Mathematics Advanced – EOPC Exam - SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 13: (a) (i) $y = 3x^2 - \frac{1}{x}$ $= 3x^2 - x^{-1}$ $\therefore y' = 6x + x^{-2}$	1mk	(c) Domain $x \geq -1, x \neq 0$ or $[-1, 0) \cup (0, \infty)$ Award 1 for • $x \geq -1 \quad [-1, \infty)$ • $x \leq 1, x \neq 0$	2marks
(ii) $y = (2 - x^2)^5$ $\therefore y' = 5(2 - x^2)^4 \cdot -2x$ $= -10x(2 - x^2)^4$	1mk for this line	(d) (i) Axis of symmetry: $x = 2$ $y = 2^2 - 4(2) - 2$ $= -6$ $\therefore V$ is $(2, -6)$	1mk for x 1mk for y
(iii) $y = \frac{x^2}{x-1}$ $\therefore y' = \frac{(x-1)2x - x^2 \cdot 1}{(x-1)^2}$ $= \frac{x^2 - 2x}{(x-1)^2}$	(The "1" is not required) ↑ 2mks Aw 1 for y' with an error somewhere	OR $y = x^2 - 4x - 2$ $\therefore y' = 2x - 4$ $0 = 2x - 4$ $x = 2$ etc	
(iv) $y = x^3 e^{2x}$ $\therefore y' = x^3 \cdot 2e^{2x} + e^{2x} \cdot 3x^2$ $= x^2 e^{2x} (2x + 3)$	Aw 2. Aw 1 for y' with an error Aw 0 - 2 errors	(ii) $x^2 - 4x - 2 = 0$ $\therefore x = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot -2}}{2}$ $= \frac{4 \pm \sqrt{24}}{2}$ $= 2 \pm \sqrt{6}$ But x co-ordinate of A < 0 $\therefore x = 2 - \sqrt{6}$	1mk (unsimplified)
(b) (i) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ $= \frac{82^2 + 118^2 - 189^2}{2 \cdot 82 \cdot 118}$ $\approx -0.778...$ $\therefore \angle C \approx 141^\circ$	1mk 1mk	Accept $\frac{4 - \sqrt{24}}{2}$	2mks
(ii) $039^\circ / 39^\circ / N 39^\circ E$			

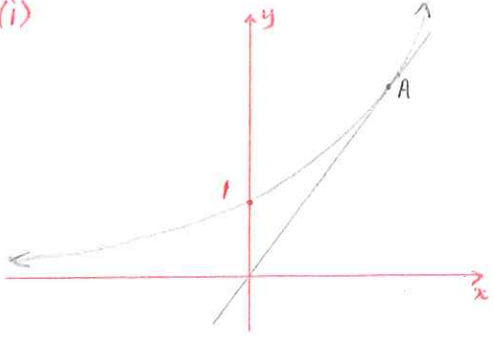
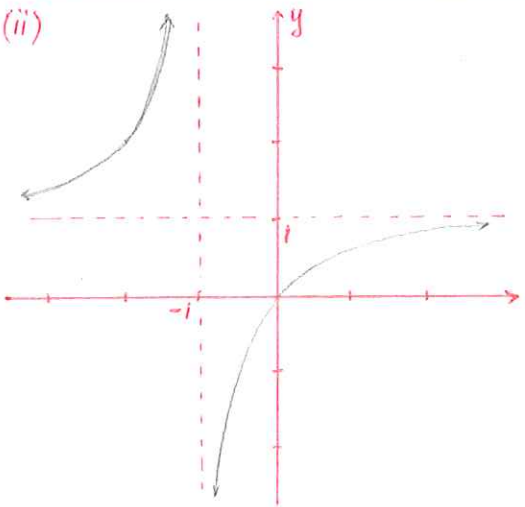


2020 Year 11 Mathematics Advanced – EOPC Exam - SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 14: (a) $y = \sqrt{3x-2}$ $= (3x-2)^{1/2}$ $\therefore y' = \frac{1}{2}(3x-2)^{-1/2} \cdot 3$ $= \frac{3}{2\sqrt{3x-2}}$ when $x = 2$ $y' = \frac{3}{2\sqrt{3 \cdot 2 - 2}}$ $= \frac{3}{4}$ $\therefore$ gradient of normal $= -\frac{4}{3}$ when $x = 2, y = 2$ $y - 2 = -\frac{4}{3}(x - 2)$ $3y - 6 = -4x + 8$ $4x + 3y - 14 = 0$	1mk for correct derivative 1mk 1mk	(ii) Area $= \frac{1}{2} \cdot 12^2 \cdot \frac{2\pi}{3} - 18\sqrt{3}$ $= 48\pi - 18\sqrt{3}$ (c) (i) $P = 2500e^{-0.036 \times 20}$ ≈ 1217 (or 1216) (ii) $\frac{dP}{dt} = 90e^{-0.036t}$ when $t = 20$ $= 90e^{-0.036 \times 20}$ ≈ 44 parrots/year (iii) $250 = 2500e^{-0.036t}$ $0.1 = e^{-0.036t}$ $-0.036t = \ln 0.1$ $t \approx 64$ years (d) $f(g(x)) = (\sqrt{x-3})^2 + 1$ $= x - 2$ But domain of $g(x)$ is $x \geq 3$ $\therefore$ this is the domain of $f(g(x))$ 	Aw 1 for use of $\frac{1}{2}r^2\theta$ subtraction of $18\sqrt{3}$ Aw 2 correct solution 1mk 1mk for derivative 1mk for sub 1mk 1mk 1mk
(b) (i) $A = \frac{1}{2}ac \sin O$ $18\sqrt{3} = \frac{1}{2} \cdot 6 \cdot 12 \sin O$ $\sin \angle AOC = \frac{\sqrt{3}}{2}$ $\therefore \angle AOC = \frac{2\pi}{3}$	1mk 1mk		
(ii) $L = r\theta$ $= 12 \cdot \frac{2\pi}{3}$ $= 8\pi$ Award 1 for students who wrongly identified the size of $\angle AOC$.	1mk		1mk

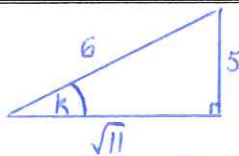


2020 Year 11 Mathematics Advanced – EOPC Exam - SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 15: (a) (i) $LHS = \cos^2\theta(\sec^2\theta - 1)$ $= \cos^2\theta(\tan^2\theta)$ $= \cos^2\theta \cdot \frac{\sin^2\theta}{\cos^2\theta}$ $= \sin^2\theta$ $= RHS$	1mk for $\sec^2\theta - 1 = \tan^2\theta$ 1mk for $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$	(d)(i)  Aw 1 - exponential passes thru (0,1) - tangent thru origin Aw 2 - correct solⁿ	
(ii) $4\cos^2\theta(\sec^2\theta - 1) = 1$ But $\cos^2\theta(\sec^2\theta - 1) = \sin^2\theta$ $\therefore 4\sin^2\theta = 1$ $\sin^2\theta = \frac{1}{4}$ $\sin\theta = \pm \frac{1}{2}$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	1mk 1mk 1mk	(ii) $y = e^{0.5x}$ $y = mx$ Since they meet at A $mx = e^{0.5x}$ — ① Now, $y' = 0.5e^{0.5x}$ At A, $y' = m$ so sub in ① $0.5e^{0.5x} \cdot x = e^{0.5x}$ $0.5x = 1$ $x = 2$ $\therefore y = e^{0.5(2)} = e$ $\therefore A$ is (2, e)	Aw 1 for using pt of intersection Aw 2 for answer
(b) $k = -2, b = -3, c = -1$	1mk each		
(c) (i) $1 - \frac{1}{x+1}$ $= \frac{x+1}{x+1} - \frac{1}{x+1}$ $= \frac{x}{x+1}$	1mk		
(ii) 	Award 1mk - hyperbola Award 2 Correct graph + asymptotes + passes thru (0, 0)	(iii) $m = 0.5e^{0.5x}$ When $x = 2$ $= 0.5e^1$ $= \frac{e}{2}$	1mk



2020 Year 11 Mathematics Advanced – EOPC Exam - SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 16:			
(a) (i) $(0, 2]$ or $0 < x \leq 2$	1mk	$\therefore \tan k = \frac{5}{\sqrt{11}}$	1mk
(ii) The graph is symmetrical about the y-axis	1mk	But from graph, $\frac{\pi}{2} < k < \pi$	
(iii) $(\infty, -1] \cup [1, \infty)$	1mk	$\therefore \tan k = \frac{-5}{\sqrt{11}}$	1mk
(b) $t(x-1)(x-2) = x$ $t(x^2 - 3x + 2) = x$ $tx^2 - 3tx + 2t = x$ $tx^2 - 3tx - x + 2t = 0$ $tx^2 - (3t+1)x + 2t = 0$	1mk	(d) (i) $y = \frac{2}{x}$ $= 2x^{-1}$ $\therefore y' = -2x^{-2}$ $= \frac{-2}{x^2}$ when $x = a$ $= \frac{-2}{a^2}$	1mk
For real roots, $\Delta \geq 0$ $\Delta = (3t+1)^2 - 4 \cdot t \cdot 2t = 0$ $= 9t^2 + 6t + 1 - 8t^2$ $= t^2 + 6t + 1$	1mk	(ii) $y - \frac{2}{a} = -\frac{2}{a^2}(x-a)$ $a^2y - 2a = -2(x-a)$ $a^2y - 2a = -2x + 2a$ $2x + a^2y - 4a = 0$, as reqd.	1mk
Solving $\Delta = 0$ $t = \frac{-6 \pm \sqrt{36 - 4 \cdot 1 \cdot 1}}{2}$ $= \frac{-6 \pm \sqrt{32}}{2}$ $= -6 \pm 2\sqrt{2}$	1mk	(iii) $2x + a^2y - 4a = 0$ when $x = 0$ when $y = 0$ $a^2y - 4a = 0$ $2x - 4a = 0$ $ay - 4 = 0$ $2x = 4a$ $y = \frac{4}{a}$ $x = 2a$	1mk for A or B
$\therefore$ roots are real if $t \leq -6 - 2\sqrt{2}$ or $t \geq -6 + 2\sqrt{2}$	1mk	$\therefore \text{Area } \Delta AOB = \frac{1}{2} \cdot 2a \cdot \frac{4}{a}$ $= 4 \text{ sq units}$ $\therefore \text{area is constant.}$	2mks for ΔAOB 's area.
c) (i) $a = 3, b = 1$	1mk each		
(ii) $y = 3 \sin x$ sub $(k, \frac{5}{2})$ $\frac{5}{2} = 3 \sin k$ $\sin k = \frac{5}{6}$			