



KNOX GRAMMAR SCHOOL

2021 Preliminary HSC Assessment Task 3 – Online

Year 11 Mathematics Advanced

General Instructions

- Total Time Allowed: 70 minutes
 - 5 minutes reading time
 - 60 minutes writing time
 - 5 minutes scan and upload time
- You may be penalised if you submit your assessment after the closing time
- Write using black or blue pen only
- Write on your own single-side lined paper
- NESA approved calculators may be used
- This examination is open-book, and you may use your own textbook and notes
- You are not permitted to use any electronic device to research answers
- For Questions 1 - 10 in Section I, write down only the letter of your chosen answer
- For Questions 11 - 14 in Section II, show relevant mathematical reasoning and/or calculations
- Setter: CHG

Total Marks

40

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 30 marks (pages 5 – 8)

- Attempt Questions 11 – 13
- Allow about 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Write down only the letter of your chosen answer for Questions 1 – 10.

1 What is the derivative of e^{3x+5} .

- A. $(3x+5)e^{3x+5}$ B. $3e^{3x+5}$ C. e^{3x+5} D. $3xe^{3x+5}$

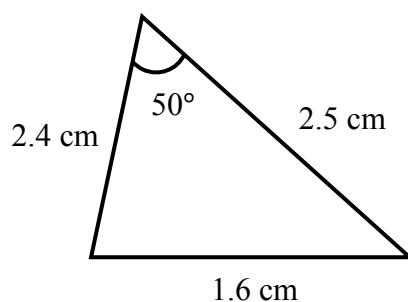
2 What is the derivative of 2^x .

- A. $2^x \ln 2$ B. e^{2x} C. $e^x \ln 2$ D. $\ln 2$

3 What is the domain of the function $f(x) = \frac{1}{\sqrt{1-x^2}}$.

- A. $(0,1)$ B. $[0,1]$ C. $(-1,1)$ D. $[-1,1]$

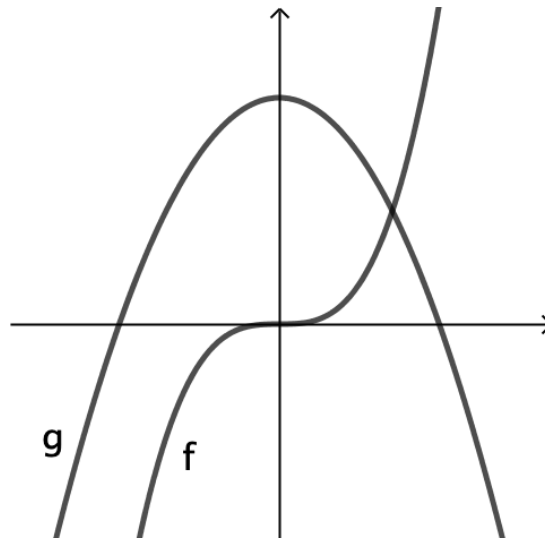
4 Which of the following would give the area of the triangle below?



- A. $\frac{1}{2} \times 1.6 \times 2.4 \times \sin 50^\circ$ B. $\frac{1}{2} \times 2.5 \times 2.4 \times \sin 50^\circ$
C. $\frac{1}{2} \times 1.6 \times 2.4 \times 50$ D. $\frac{1}{2} \times 2.5 \times 2.4 \times 50$
E. $\frac{1}{2} \times 1.6 \times 2.4 \times 50^\circ$
F. $\frac{1}{2} \times 2.4 \times 2.5 \times 50^\circ$

- 5 Solve $\cos x = -\frac{1}{\sqrt{2}}$ for $0 \leq x \leq 2\pi$.
- A. $\frac{3\pi}{4}, \frac{5\pi}{4}$ B. $\frac{\pi}{4}, \frac{3\pi}{4}$ C. $\frac{3\pi}{4}, \frac{7\pi}{4}$ D. $\frac{\pi}{4}, \frac{7\pi}{4}$
- 6 Three swimmers compete in a race. The probabilities that each of the swimmers finish in under 30 seconds are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. Find the probability at least one of the swimmers will finish the race in under 30 seconds.
- A. $\frac{1}{60}$ B. $\frac{2}{5}$ C. $\frac{47}{60}$ D. $\frac{3}{5}$
- 7 If $\cos \theta = -\frac{12}{13}$ and $180^\circ \leq \theta \leq 360^\circ$, find $\cot \theta$.
- A. $-\frac{12}{5}$ B. $-\frac{5}{12}$ C. $\frac{5}{12}$ D. $\frac{12}{5}$
- 8 The radius and centre of a circle with equation $x^2 - 6x + y^2 + 10y + 18 = 0$ is:
- A. Radius $= 3\sqrt{2}$, Centre $(-3, 5)$
- B. Radius $= 3\sqrt{2}$, Centre $(3, -5)$
- C. Radius $= 4$, Centre $(-3, 5)$
- D. Radius $= 4$, Centre $(3, -5)$
- 9 Which relation is many-to-one?
- A. $x^2 + y^2 = 4$ B. $y = \sqrt{4 - x^2}$ C. $y = x^3 + 3$ D. $y = 2x - 4$

- 10 The graph shows two functions $y = f(x)$ and $y = g(x)$.



Define $h(x) = f(g(x))$.

How many stationary points does $y = h(x)$ have?

- A. 0 B. 1 C. 2 D. 3

Section II Answer Booklet

30 marks

Attempt Questions 11 – 13

Allow about 45 minutes for this section

Instructions

Answer the questions on your own single-side lined paper.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (10 marks)

Marks

- | | |
|--|----------|
| (a) Solve $\sin 2x = 0.5$ for $0 \leq x \leq 2\pi$. | 2 |
| | |
| (b) Solve $4\sin x = 3\operatorname{cosec} x$ for $0 \leq x \leq 2\pi$. | 2 |
| | |
| (c) Solve $2\sin^2 x - \sin x - 1 = 0$ for $0 \leq x \leq 2\pi$. | 2 |
| | |
| (d) The area of the sector of a circle is 4π units ² and the arc length of this sector is $\frac{\pi}{8}$ units. Find the radius of the circle and the angle that is subtended at the centre. | 2 |
| | |
| (e) For the graph $y = e^{-2x}$. Find the equation of the normal at (0,1) | 2 |

End of question 11

Question 12 (10 marks)**Marks**

(a) Differentiate:

(i) $y = e^x(3x + 2)$ 1

(ii) $y = \frac{2x+3}{e^x}$ 2

(b) The share price P after t hours of an exchange opening was $P = -t^3 + 6t^2$ for $0 \leq t \leq 6$.

(i) Sketch a graph of P vs t . 1

(ii) Find $\frac{dP}{dt}$ and sketch $\frac{dP}{dt}$ on the same graph as part i) above. 1

(iii) When is the share price a maximum? 1

(iv) By finding the axis of symmetry of the parabola defined by $\frac{dP}{dt}$, determine when the share price is increasing most rapidly? 1

(c) A bag contains 5 black and 7 white marbles. Two are chosen at random from the bag without replacement.

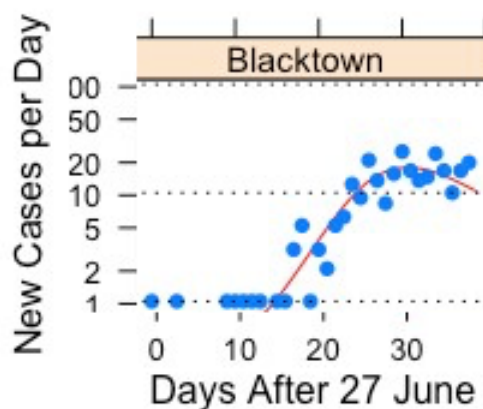
(i) Draw a probability tree labeling the probabilities on the branches. 1

(ii) Find the probability of getting a black and a white marble in either order. 2

End of Questions 12

Question 13 (10 marks)**Marks**

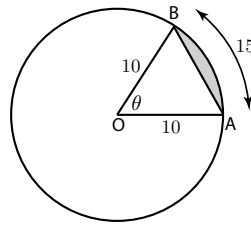
- (a) Recently the daily number of cases of Covid-19 in lockdown compliant suburbs began to decay and halve every 14 days. The decay of the number of daily cases, P can be modelled by $P = P_0 e^{-kt}$, where P_0 is the initial number of daily cases and t is the time in days.



- (i) Show that the decay constant is $k \approx 0.0495$. **1**
- (ii) From an initial number of daily cases of $P_0 = 20$.
Find the number of daily cases after 30 days to 1 decimal place. **1**
- (iii) Show $\frac{dP}{dt} = -kP$. **1**
- (iv) From an initial number of daily cases of $P_0 = 20$. Find the number of days to reach less than 1 new case per day to the nearest day. **1**

Question 13 continues on the next page

- (b) A circle has a radius of 10cm and an arc length of 15cm between A and B.



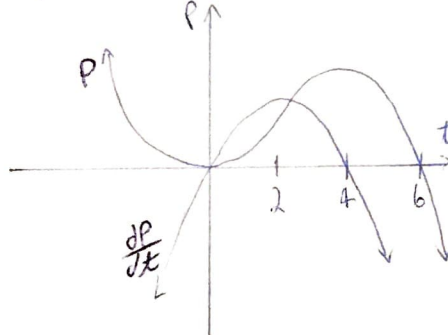
- (i) Show the angle θ is 1.5 radians. 1
- (ii) Show the area of the sector OAB is 75cm^2 . 1
- (iii) Find the area of shaded minor segment to 1 decimal place . 2
- (c) A game involves rolling two six-sided dice, followed by rolling a third six-sided die. To win the game, the number rolled on the third die must lie between the two numbers rolled previously. For example, if the first two dice show 1 and 4, the game can only be won by rolling a 2 or 3 with the third die. 2

End of Examination

2021 Year 11 Mathematics Advanced AT3 Solutions

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
① $y = e^{3x+5}$ $y' = 3e^{3x+5}$ B. ② $y = (e^{2x})^2$ $y' = 2 \cdot 2e^{2x} = 4e^{2x}$ A. ③ C. ④ B. ⑤ $\angle_{\text{ref}} = \frac{\pi}{4}$ A $x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$ $= \frac{3\pi}{4}, \frac{5\pi}{4}$ A. ⑥ $P(\text{at least } 1 < 30 \text{ sec}) =$ $= 1 - P(\text{all } > 30 \text{ sec})$ $= 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$ $= \frac{3}{5}$ D. ⑦ 12 $\frac{12}{13}$ A. ⑧ $x^2 - 6x + 9 + y^2 + 10y + 25 = -18$ $(x-3)^2 + (y+5)^2 = 16$ D. ⑨ B. ⑩ $h'(x) = f'(g(x))g'(x) = 0$ start pt. where $g'(x) = 0$ or $f'(g(x)) = 0$ ∴ 3 pts. D. i.e. $g(x) = 0$		⑪ a) $\sin 2x = \frac{1}{2}$ $\frac{5\pi}{6}, \frac{7\pi}{6}$ $\angle_{\text{ref}} 2x = \frac{\pi}{6}$ $2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$ ✓✓ b) $4\sin x = \frac{3}{\sin x}$ $\sin^2 x = \frac{3}{4}$ $\frac{5\pi}{6}, \frac{7\pi}{6}$ $\sin x = \pm \frac{\sqrt{3}}{2}$ ① $\angle_{\text{ref}} = \frac{\pi}{3}$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ ✓ c) $(\sin x - 1)(2\sin x + 1) = 0$ ① $\sin x = 1$ or $2\sin x + 1 = 0$ $x = \frac{\pi}{2}$ $\sin x = -\frac{1}{2}$ $\angle_{\text{ref}} = \frac{\pi}{6}$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ ✓✓ $\frac{5\pi}{6}, \frac{11\pi}{6}$ d) $A = \frac{1}{2} r^2 \theta = 4\pi$ * $l = r\theta = \frac{\pi}{8}$ $\theta = \frac{\pi}{8r}$ sub into * $\frac{1}{2} r^2 \frac{\pi}{8r} = 4\pi$ ① $\frac{r}{16} = 4$ $r = 48 \text{ units}$ ✓✓ $\theta = \frac{\pi}{8(48)} = \frac{\pi}{884}$	① for both $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

2021 Year 11 Mathematics Advanced AT3 Solutions

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
11e) $y' = -2e^{-2x}$ $m_T = -2e^{-2(0)}$ $= -2$ $m_N = \frac{1}{2}$ ① $y - 1 = \frac{1}{2}(x - 0)$ $y = -\frac{1}{2}x + 1$ ✓✓ 12a)i) $y' = e^x(3x+2) + 3e^x$ $= e^x(3x+5)$ ✓		iii) 4 hours ✓ iv) 2 hours ✓ c) $\frac{5}{12}$ B $\frac{4}{11}$ B $\frac{7}{11}$ W $\frac{7}{12}$ W $\frac{5}{11}$ B $\frac{6}{11}$ W ✓	
b) ii) $y' = \frac{2e^x - (2x+3)e^x}{e^{2x}}$ ① $= \frac{-2xe^x - e^x}{e^{2x}}$ $= \frac{-2x-1}{e^x}$ ✓✓		ii) $P(BW + WB) = \frac{5}{12} \times \frac{7}{11} + \frac{7}{12} \times \frac{5}{11}$ ① for $P(BW)$ $= \frac{35}{66}$ ✓✓ $or P(WB) = \frac{35}{66}$	
b) i) $\frac{dP}{dt} = -3t^2 + 12t$ ii) $\frac{dP}{dt} = -3t(t-4)$ $P = -t^2(t-6)$ 	① correct shapes & intercepts for P ① correct shape & intercepts for $\frac{dP}{dt}$	13a)i) $\frac{1}{2}P_0 = P_0 e^{-14k}$ $0.5 = e^{-14k}$ $\ln 0.5 = -14k$ $k = \frac{\ln 0.5}{-14}$ ✓ ≈ 0.0495 ii) $P = 20e^{-0.0495(30)}$ ≈ 4.5 ✓ iii) $P = 20e^{-0.0495t}$ $\frac{dP}{dt} = -0.0495(20e^{-0.0495t})$ $= -kP$ $or P = P_0 e^{-kt}$ $\frac{dP}{dt} = -kP_0 e^{-kt}$ $= -kP$ ✓	

intercepts
or
 $\frac{dP}{dt}$

2021 Year 11 Mathematics Advanced AT3 Solutions

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments																																																	
3a) iv) $I = 20e^{-0.0495t}$ $\frac{1}{20} = e^{-0.0495t}$ $\ln \frac{1}{20} = -0.0495t$ ① $t = \frac{\ln \frac{1}{20}}{-0.0495}$ ≈ 60.5 $\approx 61 \text{ days}$ → ① accept either		c) <table border="1"><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>1</td><td>x</td><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>2</td><td>x</td><td>x</td><td>x</td><td>1</td><td>2</td><td>3</td></tr><tr><td>3</td><td>1</td><td>x</td><td>x</td><td>x</td><td>1</td><td>2</td></tr><tr><td>4</td><td>2</td><td>1</td><td>x</td><td>x</td><td>x</td><td>1</td></tr><tr><td>5</td><td>3</td><td>2</td><td>1</td><td>x</td><td>x</td><td>x</td></tr><tr><td>6</td><td>4</td><td>3</td><td>2</td><td>1</td><td>x</td><td>x</td></tr></table> $P(\text{win}) = 8 \times \frac{1}{6} \times \frac{1}{36} +$ $6 \times \frac{2}{6} \times \frac{1}{36} +$ $4 \times \frac{3}{6} \times \frac{1}{36} +$ $2 \times \frac{4}{6} \times \frac{1}{36}$ $= \frac{1}{216} (8 + 12 + 12 + 8)$ $= \frac{5}{27}$ ✓✓ ① or		1	2	3	4	5	6	1	x	x	1	2	3	4	2	x	x	x	1	2	3	3	1	x	x	x	1	2	4	2	1	x	x	x	1	5	3	2	1	x	x	x	6	4	3	2	1	x	x	
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b) i) $l = r\theta$ $15 = 10\theta$ ① or $\theta = 1.5 \times 10$ $\theta = \frac{15}{10}$ ✓ ii) $A = \frac{1}{2} \times 10^2 \times 1.5$ ① $= 75 \text{ cm}^2$ iii) $A = 75 - \frac{1}{2} \times 10 \times 10 \sin 1.5$ $\approx 25.1 \text{ units}^2$ ✓																																																				