Knox Grammar School

Name: ____________________

2022 Year 11 Yearly Examination

Teacher: ____________________

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen only.
- Black pen is preferred.
- NESA-approved calculators may be used.
- In Questions 11-16, show all relevant mathematical reasoning and/or calculations.

Examiner: Mrs Dempsey

Teachers:

Mr Willcocks
Mr Menzies/Mr Vuletich
Mr Lemaire
Mr Naidoo
Mr Meli
Mr Milligan
Ms Rodriguez
Ms Ekanayake
Mr Charlier
Ms Zaidi

Section I: Pages 3 - 5

10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section
- Use the Multiple Choice Answer Sheet

Section II: Pages 6 - 29

90 marks
- Attempt Questions 11-16
- Allow about 2 hours 45 minutes for this section
- Answer in the spaces provided on the question paper
- Extra working space is provided after each 15 mark question, should you require it. Please **label your work** clearly if you make use of this space.

This paper MUST NOT be removed from the examination room.

Number of Students in Course: 249

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1 – 10.

Marks

1 Simplify fully $\dfrac{x^6}{y^6} \div x^2 y^{-3}$ **1**

(A) $\dfrac{x^3}{y^2}$ (B) $\dfrac{x^4}{y^2}$

(C) $\dfrac{x^4}{y^3}$ (D) $\dfrac{x^4}{y^9}$.

2 What is the gradient of a line perpendicular to $4x-3y+1=0$? **1**

(A) 4 (B) -4

(C) $\dfrac{4}{3}$ (D) $-\dfrac{3}{4}$

3 What is the derivative of $(1-2x)^5$? **1**

(A) $5(1-2x)^4$ (B) $-5(1-2x)^4$

(C) $10(1-2x)^4$ (D) $-10(1-2x)^4$.

Section I continues next page

Section I continued **Marks**

4 Let $f(x)=\dfrac{1}{\sqrt{x-5}}$. Which of the following is the natural domain of $f(x)$? **1**

(A) $(-\infty, 5)$ (B) $[5, \infty)$

(C) $(5, \infty)$ (D) $[-5, 5]$

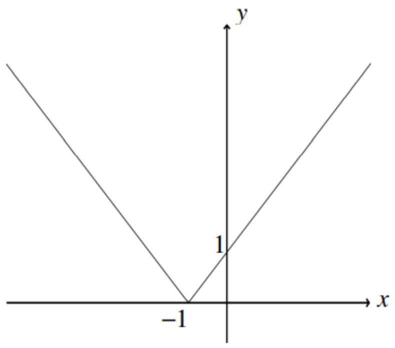
5 If $\operatorname{cosec}\theta=-\dfrac{5}{3}$ and $\cos\theta>0$, then the value of $\cot\theta$ is : **1**

(A) $-\dfrac{3}{4}$ (B) $\dfrac{3}{4}$

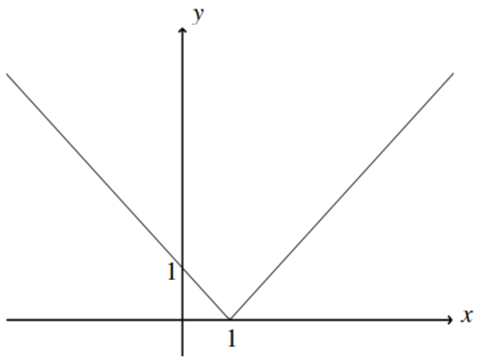
(C) $\dfrac{4}{3}$ (D) $-\dfrac{4}{3}$.

6 Which graph shows $y=|x-1|$? **1**

(A)



(B)



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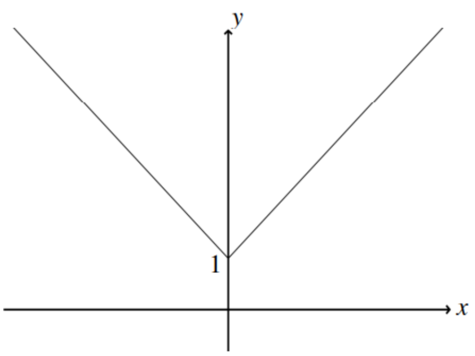

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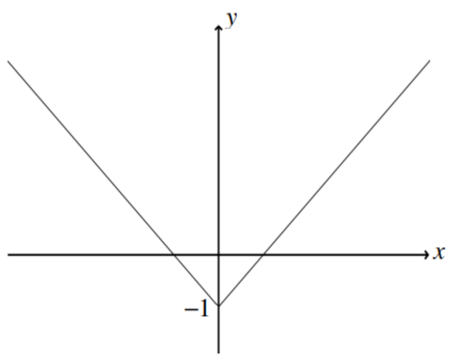

Section I continues next page

Marks

7 The expression $\dfrac{x^2-5x+6}{4-x^2}$ is equivalent to : **1**

(A) $\dfrac{6-5x}{4}$ (B) $\dfrac{x-3}{2+x}$

(C) $\dfrac{3-x}{2-x}$ (D) $\dfrac{3-x}{2+x}$.

8 If $\cos\alpha = \dfrac{a}{b}$, where α is acute, what is the value of $\cos\,(180^\circ+\alpha)$? **1**

(A) $\dfrac{\sqrt{b^2-a^2}}{b}$ (B) $\dfrac{a}{b}$

(C) $-\dfrac{a}{b}$ (D) $-\dfrac{b}{a}$.

9 The line $y = mx-2$ is a tangent to the curve $y = 3x^2-2x+1$ at the point (1, 2).
What is the value of m? **1**

(A) 4 (B) $\dfrac{1}{2}$

(C) -4 (D) 2

10 The parabola $y = x^2$ is shifted left 2 units, shifted up 1 unit and then reflected in the y-axis. What is the new equation of the parabola? **1**

(A) $(x-2)^2 = y-1$ (B) $(x+2)^2 = y+1$

(C) $(x-2)^2 = y+1$ (D) $(x+2)^2 = y-1$.

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the space provided. There is extra writing space at the end of each 15 mark question, should you require it.
Your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Express $\dfrac{2x}{x^2+2x-3}-\dfrac{3}{x-1}$ as a single fraction in simplest form. **2**

(b) If $8^{x+3}\times 2^{x-2}=2^x\times 4^{3x-1}$, find the value of x. **3**

Question 11 continues next page

Question 11 continued

Marks

(c) Given $x = \sqrt{48}$, find $\frac{1}{x}$, giving your answer in simplest surd form, with a rational denominator. **1**

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(d) The function $y = f(x)$ is defined as :

$$f(x) = \begin{cases} x^2 + 1 & \text{for } x < 0 \\ -1 - x^2 & \text{for } x \geq 0 \end{cases}$$

What is the range of $f(x)$? **2**

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(e) Given $f(x) = 1 - 3x + x^2$ and $g(x) = \frac{1-x}{2}$, solve $g(f(x)) = 0$ for x. **2**

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Question 11 continues next page

Question 11 continued

Marks

(g) Given the function $y = 2 + \dfrac{2}{1-x}$,

(i) Determine the equation of any asymptotes and axis intercepts. **3**

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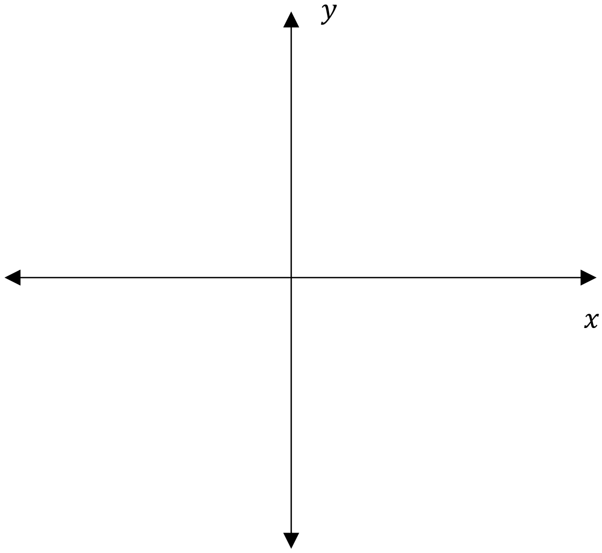
(ii) Sketch the curve on the axes below, showing clearly features indicated in part (i). **2**



End of Question 11

Extra Working Space for Question 11

Question 12 (15 marks) **Marks**

(a) Differentiate $f(x)=\dfrac{2}{x\sqrt{x}}$. **2**

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(b) Find the equation of the normal to the curve $y=3-4x^2-x^3$, at the point where $x=-1$. Leave your final answer in general form. **3**

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Question 12 continues next page

Marks

(c) A yacht sails 6.5 nautical miles from point P to point A on a bearing of 150º.
It then turns and sails 8.5 nautical miles to point B on a bearing of 220º.

(i) Draw a clear diagram illustrating the above information. **2**

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(ii) Find, correct to three significant figures, the distance the yacht must sail to return from point B to point P. **3**

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(iii) Determine, correct to the nearest degree, the bearing on which the yacht must sail in order to reach P from B. **2**

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Question 12 continues next page

Question 12 continued

Marks

(d) Given that the lines $2x-y+1=0$, $y=1$ and $x=-2$ form a triangle whose vertices are A, B and C, find the area of $\triangle ABC$. You may use the axes below to draw a sketch to help. **3**

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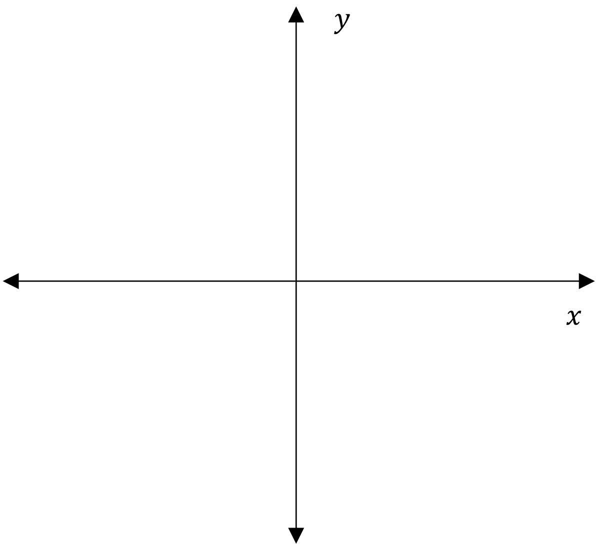

End of Question 12

Extra Working Space for Question 12

Question 13 (15 marks) **Marks**

(a) Use the formula $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to differentiate $f(x) = 3x^2 - 2x + 1$ from **first principles**. **3**

(b) Given that θ is **obtuse** and $\sin\theta = \dfrac{5\sin 24^\circ}{6}$, find θ correct to the nearest minute. **1**

Question 13 continues next page

Marks

(c) Solve for x : $\log_e(\frac{1}{x^2}) = -4$ **2**

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(d) Simplify $\dfrac{\sin(2\pi-\alpha)}{\sin(\frac{\pi}{2}-\alpha)}$. **1**

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(e) Solve the following equation for θ in the domain $0 \le \theta \le 2\pi$ **3**

$$2\cos^2\theta - \sin\theta - 1 = 0 .$$

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Question 13 continues next page

Question 13 continued

Marks

(f) Given $f(x) = 3x(2x-1)^4$, find $f'(x)$ in fully factored form. **2**

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(g) Given the curve $f(x) = \dfrac{1}{x} + 2x^3 - 2$ determine the x-value(s) of the point(s) at which the gradient of the tangent to the curve is horizontal. **3**

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End of Question 13

Extra Working Space for Question 13

Question 14 (15 marks) **Marks**

(a) Solve each of the following equations for x.

(i) $|x+1| = |x-2|$ **2**

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(ii) $\log_e x - \log_e(5-x) = 1$ **2**

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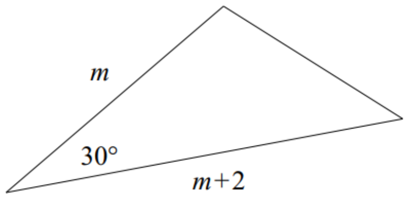
Question 14 continues next page

Question 14 continued

Marks

(b) The area of the triangle below is 12 square units. Find the value of m. **3**



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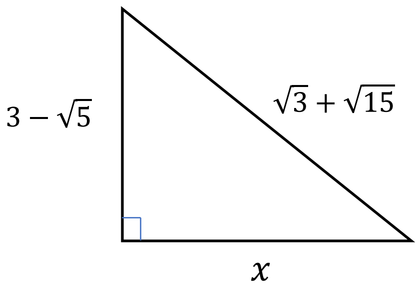
(c) Find the exact length of the side marked x : **2**



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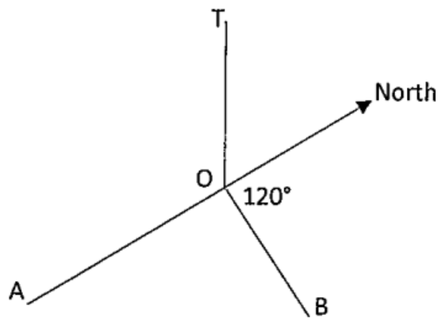
Question 14 continues next page

Question 14 continued

Marks

(d) From a point, A due south of a tower, OT, the angle of elevation of the top of a tower, T is 23º. From another point B, on a bearing of 120º from the tower, the angle of elevation of T is 32º. The distance AB is 200 metres.



(i) Add the given information to the diagram above. **1**

(ii) Let the height of the tower be h.

Show that $OA = h\tan 67^\circ$ and similarly that $OB = h\tan 58^\circ$ **2**

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(iii) Find angle AOB and hence find the height of the tower, correct to the nearest metre. **3**

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End of Question 14

Extra Working Space for Question 14

Extra Working Space for Question 14

Question 15 (15 marks) **Marks**

(a) Given that the equation $x^2 - 6x + y^2 + 3y = 1$ defines a circle, find its centre and radius. **2**

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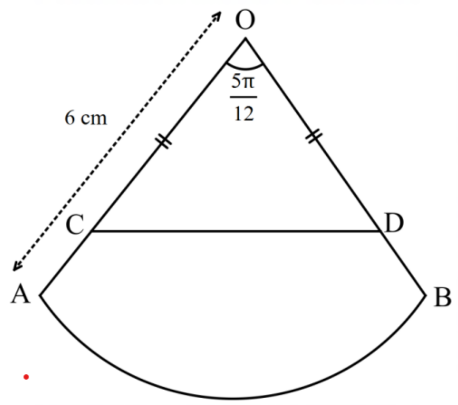
(b) The diagram shows a sector AOB. The length OA is 6 cm and $\angle AOB = \frac{5\pi}{12}$.

The points C and D lie on OA and OB respectively and OC = OD, as shown in the diagram below.



(i) Find the exact area of sector AOB **1**

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(ii) If the area of triangle *COD* is half the area of sector AOB, find the length of OC correct to two significant figures. **2**

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Question 15 continues next page

Marks

(d) The amount of caffeine, C in the human body, t hours after it has been consumed is given by the formula $C = C_0 e^{-0.14t}$

(i) Show that $\dfrac{dC}{dt} = -0.14C$ **1**

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(ii) Given that, for a particular person, the initial concentration of caffeine, C_0 is 130 mg, find, correct to one decimal place, the rate at which the concentration is being reduced after 3 hours. **2**

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(iii) Giving your answer correct to one decimal place, after how many hours has the amount of caffeine been reduced to half the original concentration? **3**

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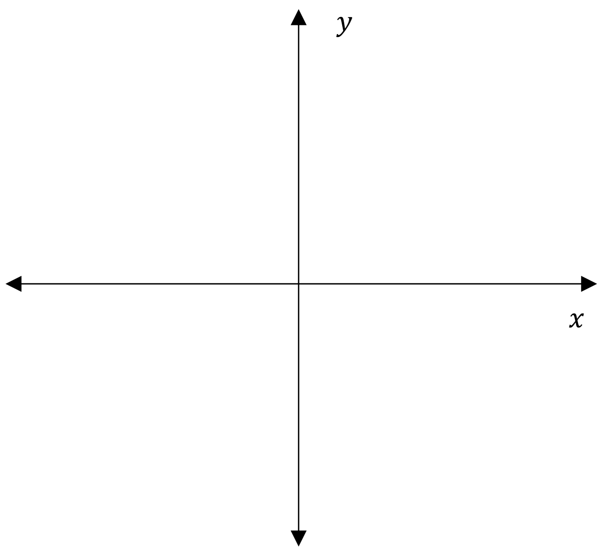
Question 15 continues next page

(e) The perpendicular bisector of the interval *CD* has equation $4x-3y+16=0$. **4**

If the point *C* has coordinates (-9, 10), find the coordinates of *D*.

You may use the axes provided below to draw a sketch to assist.



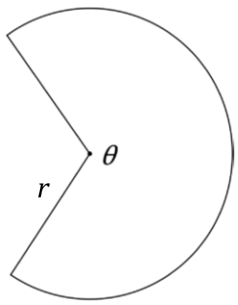
End of Question 15

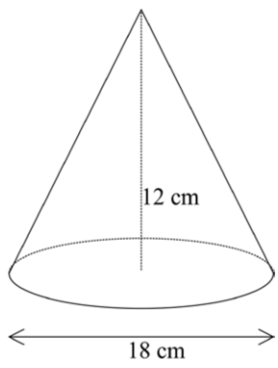
Extra Working Space for Question 15

Question 16 (15 marks) **Marks**

(a) Find, correct to the nearest degree, the angle of inclination of the tangent to the curve $y = e^{\frac{x}{2}}$ at the point where $y = e$. **3**

(b) An open cone is made by rolling a piece of paper in the shape of a sector, as shown in the diagrams below. **3**





The cone is to have height 12 cm and base diameter 18 cm.
Find the size of the angle θ in radians, leaving your answer exact.

Question 16 continues next page

Marks

(c) Given that $f(x)=\dfrac{(2x-5)^2}{3-2x}$, find $f'(2)$. **2**

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(d) The size, y of a population at time, t is given by $y=\dfrac{N}{1+Ke^{-2t}}$ for $t \geq 0$, where $N > 0$ and $K > 1$ are constants.

(i) What limiting value does the population size approach for large values of t? **1**

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(ii) Find $\dfrac{dy}{dt}$, the rate of change of y, and hence show that the population is always increasing. **2**

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Question 16 (d) continues next page

Question 16(d) continued

Marks

(iv) Show that $\frac{dy}{dt} = \frac{2}{N}y(N-y)$ **2**

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(v) Given that the population is increasing most rapidly when $\frac{dy}{dt} = 0$ and using part (iv) or otherwise, show that the population is increasing most rapidly when $y = \frac{N}{2}$. **2**

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End of Examination

Extra Working Space for Question 16

Knox Grammar School

Name: SOLUTIONS

2022 Year 11 Yearly Examination

Teacher: L DEMPSEY

Mathematics Advanced

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10 marks

- Attempt Questions 1-10
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Section II: Pages 6 - 29

90 marks

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Number of Students in Course: 249

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1 – 10.

Marks

1 Simplify fully $\frac{x^6}{y^6} + x^2y^{-3}$ **1**

(A) $\frac{x^3}{y^2}$ (B) $\frac{x^4}{y^2}$

(C) $\frac{x^4}{y^3}$ (D) $\frac{x^4}{y^9}$.

$\frac{x^6}{y^6} \times \frac{1}{x^2 y^{-3}}$

$= \frac{x^4}{y^3}$

2 What is the gradient of a line perpendicular to $4x-3y+1=0$? **1**

(A) 4 (B) −4

(C) $\frac{4}{3}$ (D) $-\frac{3}{4}$

$3y = 4x+1$

$y = \frac{4x+1}{3}$

$m = \frac{4}{3}$

$\therefore m_\perp = \frac{-3}{4}$

3 What is the derivative of $(1-2x)^5$? **1**

(A) $5(1-2x)^4$ (B) $-5(1-2x)^4$

(C) $10(1-2x)^4$ (D) $-10(1-2x)^4$.

$\frac{d}{dx}(1-2x)^5$

$= 5(1-2x)^4(-2)$

$= -10(1-2x)^4$

Section I continues next page

Section I continued **Marks**

4 Let $f(x) = \frac{1}{\sqrt{x-5}}$. Which of the following is the natural domain of $f(x)$? **1**

(A) $(-\infty, 5)$ (B) $[5, \infty)$

(C) $(5, \infty)$ (D) $[-5, 5]$

D: $x-5>0$

$x>5$

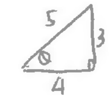
5 If cosec $\theta = -\frac{5}{3}$ and $\cos\theta > 0$, then the value of $\cot\theta$ is : **1**

(A) $-\frac{3}{4}$ (B) $\frac{3}{4}$

(C) $\frac{4}{3}$ (D) $-\frac{4}{3}$.

$\sin\theta = \frac{-3}{5}$, $\cos\theta > 0$

$\therefore$ Q4

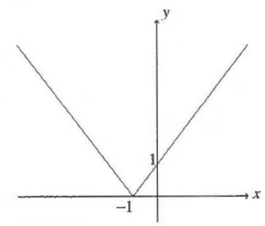


$\tan\theta = -\frac{3}{4}$

$\therefore \cot\theta = -\frac{4}{3}$

6 Which graph shows $y = |x-1|$? **1**

(A)



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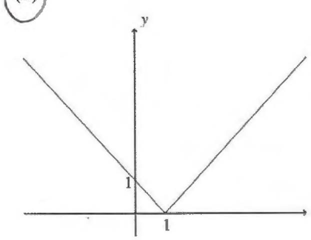

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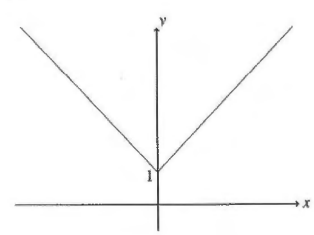

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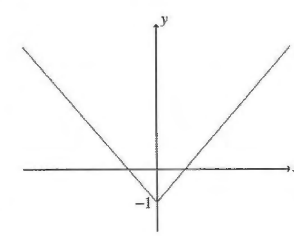


Section I continues next page

Marks

7 The expression $\dfrac{x^2-5x+6}{4-x^2}$ is equivalent to : **1**

(A) $\dfrac{6-5x}{4}$ (B) $\dfrac{x-3}{2+x}$

(C) $\dfrac{3-x}{2-x}$ (D) $\dfrac{3-x}{2+x}$.

$\dfrac{(x-3)(x-2)}{(2-x)(2+x)}$

$= -\dfrac{(x-3)(2-x)}{(2-x)(2+x)}$

$= \dfrac{3-x}{2+x}$

8 If $\cos\alpha = \dfrac{a}{b}$, where α is acute, what is the value of $\cos(180^\circ+\alpha)$? **1**

(A) $\dfrac{\sqrt{b^2-a^2}}{b}$ (B) $\dfrac{a}{b}$

(C) $-\dfrac{a}{b}$ (D) $-\dfrac{b}{a}$.

$\cos(180^\circ+\alpha)$

$= -\cos\alpha$

$= -\dfrac{a}{b}$

9 The line $y=mx-2$ is a tangent to the curve $y=3x^2-2x+1$ at the point (1, 2).
What is the value of m? **1**

(A) 4 (B) $\dfrac{1}{2}$

(C) -4 (D) 2

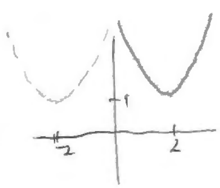
$\dfrac{dy}{dx}=6x-2$

when $x=1$, $m=6(1)-2$
$=4$

10 The parabola $y=x^2$ is shifted left 2 units, shifted up 1 unit and then reflected in the y-axis. What is the new equation of the parabola? **1**

(A) $(x-2)^2=y-1$ (B) $(x+2)^2=y+1$

(C) $(x-2)^2=y+1$ (D) $(x+2)^2=y-1$.



$y=(x-2)^2+1$

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the space provided. There is extra writing space at the end of each 15 mark question, should you require it.
Your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Express $\dfrac{2x}{x^2+2x-3}-\dfrac{3}{x-1}$ as a single fraction in simplest form. **2**

$\dfrac{2x}{(x+3)(x-1)} - \dfrac{3}{x-1}$

$= \dfrac{2x-3(x+3)}{(x+3)(x-1)}$

$= \dfrac{-x-9}{(x+3)(x-1)}$

(b) If $8^{x+3}\times 2^{x-2} = 2^x\times 4^{3x-1}$, find the value of x. **3**

$2^{3(x+3)}\times 2^{x-2} = 2^x\times 2^{2(3x-1)}$

$2^{3x+9+x-2} = 2^{x+6x-2}$

$\therefore 4x+7=7x-2$

$3x=9$

$x=3$

Question 11 continues next page

Question 11 continued

Marks

(c) Given $x = \sqrt{48}$, find $\frac{1}{x}$, giving your answer in simplest surd form, with a rational denominator. **1**

$\frac{1}{x} = \frac{1}{\sqrt{48}}$

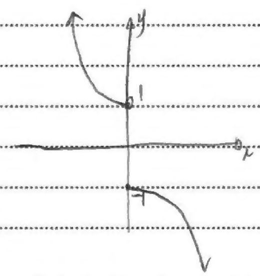
$= \frac{1}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{12}$

(d) The function $y = f(x)$ is defined as :

$$f(x) = \begin{cases} x^2+1 & \text{for } x<0 \\ -1-x^2 & \text{for } x \geq 0 \end{cases}$$

What is the range of $f(x)$? **2**



Range: $(-\infty,-1] \cup (1,\infty)$

(e) Given $f(x) = 1-3x+x^2$ and $g(x) = \frac{1-x}{2}$, solve $g(f(x)) = 0$ for x. **2**

$\frac{1-(1-3x+x^2)}{2} = 0$

$3x - x^2 = 0$

$x(3-x) = 0$

$x = 0$ or $x = 3$

Question 11 continues next page

Question 11 continued

Marks

(g) Given the function $y = 2 + \frac{2}{1-x}$,

(i) Determine the equation of any asymptotes and axis intercepts. **3**

vertical asymptote : $x=1$

horizontal asymptote : $y=2$

$y\text{-int} = 2 + \frac{2}{1}$

$= 4$

$x\text{-int}: y=0$

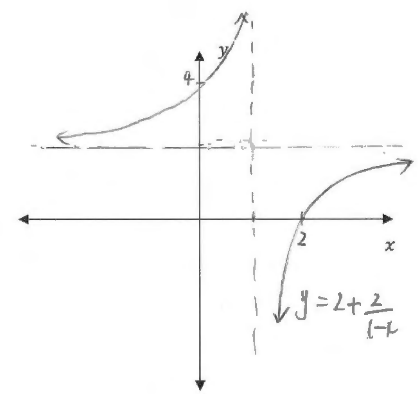
$0 = 2 + \frac{2}{1-x}$

$2(1-x) = -2$

$-2x = -4$

$x = 2$

(ii) Sketch the curve on the axes below, showing clearly features indicated in part (i). **2**



End of Question 11

Extra Working Space for Question 11

Question 12 (15 marks) **Marks**

(a) Differentiate $f(x)=\dfrac{2}{x\sqrt{x}}$. **2**

$f(x) = 2x^{-\frac{3}{2}}$

$f'(x) = -3x^{-\frac{5}{2}}$

$= \dfrac{-3}{x^2\sqrt{x}}$

(b) Find the equation of the normal to the curve $y=3-4x^2-x^3$, at the point where $x=-1$. Leave your final answer in general form. **3**

gradient fn, $\dfrac{dy}{dx} = -8x-3x^2$

when $x=-1$, $m = -8(-1)-3(-1)^2$

$= 5$

$m_\perp = -\frac{1}{5}$

$y = 3-4(-1)^2-(-1)^3$

$= 0$

eqn : $y = -\frac{1}{5}(x+1)$

$5y = -x-1$

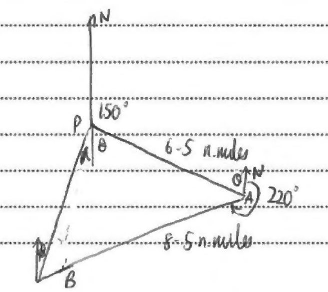
$x+5y+1=0$

Question 12 continues next page

Marks

(c) A yacht sails 6.5 nautical miles from point P to point A on a bearing of 150°. It then turns and sails 8.5 nautical miles to point B on a bearing of 220°.

(i) Draw a clear diagram illustrating the above information. **2**



(ii) Find, correct to three significant figures, the distance the yacht must sail to return from point B to point P. **3**

$\theta = 30°$ (cointerior ∠s ∥ lines)

$\angle PAB = 360° - (30° + 220°)$

$= 110°$

Using Cosine Rule

$a^2 = b^2 + p^2 - 2bp\cos A$

$BP^2 = 6.5^2 + 8.5^2 - 2 \times 6.5 \times 8.5 \cos 110°$

$= 152.293...$

$\therefore BP = 12.3407...$

$= 12.3$ n. miles (to 3sf)

(iii) Determine, correct to the nearest degree, the bearing on which the yacht must sail in order to reach P from B. **2**

Using Sine Rule, $\frac{\sin P}{p} = \frac{\sin A}{a}$

$\sin P = \frac{8.5 \sin 110°}{12.3}$

$= 0.649...$

$\angle BPA = 40.49...$

$= 40°$ (to nearest deg)

$\theta = 30°$ (alternate ∠s ∥ lines)

$\therefore \alpha = 10°$

$\therefore$ bearing $= 010°$ T

Marks

(d) Given that the lines $2x - y + 1 = 0$, $y = 1$ and $x = -2$ form a triangle whose vertices are A, B and C, find the area of $\triangle ABC$. You may use the axes below to draw a sketch to help. **3**

$2x - y + 1 = 0$

$y = 2x + 1$

when $x = -2$, $y = 2(-2) + 1$

$= -3$

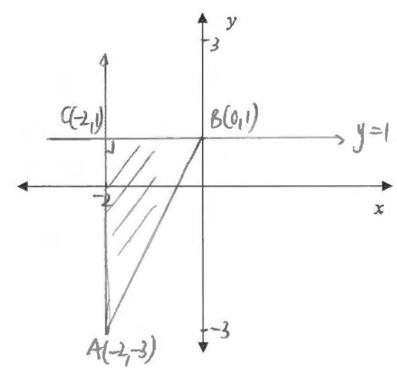
$\therefore A(-2, 3)$

$B(0, 1)$

$C = (-2, 1)$

Area $= \frac{1}{2} \times 4 \times 2$

$= 4\ u^2$



End of Question 12

Extra Working Space for Question 12

Question 13 (15 marks) **Marks**

(a) Use the formula $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ to differentiate $f(x)=3x^2-2x+1$ from **first principles**. 3

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{3(x+h)^2-2(x+h)+1-(3x^2-2x+1)}{h}$$

$$=\lim_{h\to 0}\frac{3(x^2+2xh+h^2)-2x-2h+1-3x^2+2x-1}{h}$$

$$=\lim_{h\to 0}\frac{3x^2+6xh+3h^2-2h-3x^2}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(6x+3h-2)}{\cancel{h}}$$

$$=\lim_{h\to 0}(6x+3h-2)$$

$$=6x-2$$

(b) Given that θ is **obtuse** and $\sin\theta=\dfrac{5\sin 24^\circ}{6}$, find θ correct to the nearest minute. 1

$\sin\theta = 0.3389\ldots$

$\therefore \theta = 19.81\ldots$ or $160.187\ldots$

but θ is obtuse $\therefore \theta = 160^\circ 11'$ (to nearest minute)

Question 13 continues next page

Marks

(c) Solve for x: $\log_e(\frac{1}{x^2}) = -4$ **2**

$e^{-4} = \frac{1}{x^2}$

$\frac{1}{e^4} = \frac{1}{x^2}$

$x^2 = e^4$

$\therefore x = \pm e^2$

(d) Simplify $\dfrac{\sin(2\pi - \alpha)}{\sin(\frac{\pi}{2} - \alpha)}$. **1**

$= \frac{-\sin\alpha}{\cos\alpha}$

$= -\tan\alpha$

(e) Solve the following equation for θ in the domain $0 \le \theta \le 2\pi$ **3**

$$2\cos^2\theta - \sin\theta - 1 = 0.$$

$2(1-\sin^2\theta) - \sin\theta - 1 = 0$

$2 - 2\sin^2\theta - \sin\theta - 1 = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\theta = \frac{3\pi}{2}$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Question 13 continues next page

Marks

(f) Given $f(x) = 3x(2x-1)^4$, find $f'(x)$ in fully factored form. **2**

$f'(x) = 3x.4(2x-1)^3.2 + 3(2x-1)^4$

$= 3(2x-1)^3[4x + 2x - 1]$

$= 3(2x-1)^3(6x-1)$

(g) Given the curve $f(x) = \frac{1}{x} + 2x^3 - 2$ determine the x-value(s) of the point(s) at which the gradient of the tangent to the curve is horizontal. **3**

$f(x) = x^{-1} + 2x^3 - 2$

$f'(x) = -x^{-2} + 6x^2$

$0 = -\frac{1}{x^2} + 6x^2$

$0 = -1 + 6x^4$

$\therefore x^4 = \frac{1}{6}$

$x = \pm \frac{1}{\sqrt[4]{6}}$

End of Question 13

Extra Working Space for Question 13

Question 14 (15 marks) **Marks**

(a) Solve each of the following equations for x.

(i) $|x+1|=|x-2|$ **2**

$x+1 = x-2$ or $x+1 = 2-x$

no sol'ns $\quad 2x = 1$

$x = \frac{1}{2}$

(ii) $\log_e x - \log_e(5-x) = 1$ **2**

$\log_e \frac{x}{5-x} = 1$

$\frac{x}{5-x} = e$

$x = 5e - ex$

$\therefore x(1+e) = 5e$

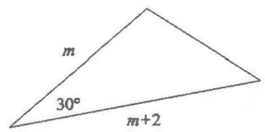
$x = \frac{5e}{1+e}$

Question 14 continues next page

Question 14 continued

Marks

(b) The area of the triangle below is 12 square units. Find the value of m. **3**



$A = \frac{1}{2}ab\sin C$

$12 = \frac{1}{2}m(m+2)\sin 30°$

$12 = \frac{m(m+2)}{4}$

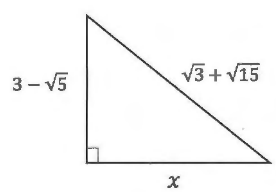
$48 = m^2+2m$

$m^2+2m-48=0$

$\therefore (m+8)(m-6)=0$

$m=-8$ invalid, $m=6$.

(c) Find the exact length of the side marked x : **2**



$x^2+(3-\sqrt{5})^2 = (\sqrt{3}+\sqrt{15})^2$

$x^2+9-6\sqrt{5}+5 = 3+2\sqrt{45}+15$

$x^2+14-6\sqrt{5} = 18+6\sqrt{5}$

$x^2 = 4+12\sqrt{5}$

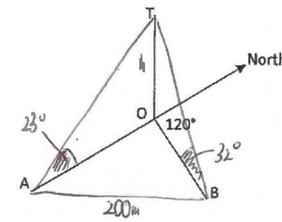
$\therefore x = \sqrt{4+12\sqrt{5}}$, $x>0$.

Question 14 continues next page

Question 14 continued

Marks

(d) From a point, A due south of a tower, OT, the angle of elevation of the top of a tower, T is 23°. From another point B, on a bearing of 120° from the tower, the angle of elevation of T is 32°. The distance AB is 200 metres.



(i) Add the given information to the diagram above. **1**

(ii) Let the height of the tower be h.

Show that $OA = h\tan 67°$ and similarly that $OB = h\tan 58°$ **2**

In ΔTAO, $\angle ATO = 67°$

$\therefore \tan 67° = \frac{OA}{h}$

$OA = h\tan 67°$

In ΔTBO, $\angle BTO = 58°$

$\therefore \tan 58° = \frac{OB}{h}$

$\therefore OB = h\tan 58°$

as req'd

(iii) Find angle AOB and hence find the height of the tower, correct to the nearest metre. **3**

$\angle AOB = 180°-120°$ (straight $\angle$)

$= 60°$

Using Cosine Rule in ΔAOB, $AB^2 = a^2+b^2-2ab\cos\angle AOB$

$200^2 = (h\tan 67°)^2+(h\tan 58°)^2-2h\tan 67°\, h\tan 58° \cos 60°$

$40000 = h^2\tan^2 67°+h^2\tan^2 58°-2h^2\tan 67°\tan 58° \times \frac{1}{2}$

$= h^2(\tan^2 67°+\tan^2 58°-\tan 67°\tan 58°)$

$\therefore h^2 = \frac{40000}{4.3409...}$

$\therefore h = \sqrt{\frac{40000}{4.3409...}}$

$= 95.99...$

$= 96$ m (to nearest m)

End of Question 14

Extra Working Space for Question 14

Question 15 (15 marks) **Marks**

(a) Given that the equation $x^2-6x+y^2+3y=1$ defines a circle, find its centre and radius. **2**

$x^2-6x+(-3)^2+y^2+3y+(\frac{3}{2})^2=1+9+\frac{9}{4}$

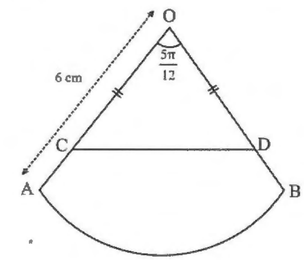
$(x-3)^2+(y+\frac{3}{2})^2=\frac{49}{4}$

$\therefore$ centre $(3,-\frac{3}{2})$

radius $=\frac{7}{2}$

(b) The diagram shows a sector AOB. The length OA is 6 cm and $\angle AOB=\frac{5\pi}{12}$.

The points C and D lie on OA and OB respectively and OC = OD, as shown in the diagram below.



(i) Find the exact area of sector AOB **1**

$A=\frac{1}{2}r^2\theta$

$=\frac{1}{2}\times6^2\times\frac{5\pi}{12}$

$=\frac{15\pi}{2}$

(ii) If the area of triangle *COD* is half the area of sector AOB, find the length of OC correct to two significant figures. **2**

Let $OC=x$ $\quad\therefore 2\times\frac{1}{2}ab\sin C=\frac{15\pi}{2}$

$x^2\sin\frac{5\pi}{12}=\frac{15\pi}{2}$

$x=\sqrt{\frac{15\pi}{2\sin\frac{5\pi}{12}}}$

$=4.938\ldots \quad, x>0$

$\therefore OC=4.9$ cm (to 2sf)

Question 15 continues next page

Marks

(d) The amount of caffeine, C in the human body, t hours after it has been consumed is given by the formula $C = C_0 e^{-0.14t}$

(i) Show that $\frac{dC}{dt} = -0.14C$ **1**

$C = C_0 e^{-0.14t}$

$\frac{dC}{dt} = -0.14 C_0 e^{-0.14t}$

$= -0.14C$

as req'd

(ii) Given that, for a particular person, the initial concentration of caffeine, C_0 is 130 mg, find, correct to one decimal place, the rate at which the concentration is being reduced after 3 hours. **2**

$\frac{dC}{dt} = -0.14 C_0 e^{-0.14t}$

when $t=3$, $\frac{dC}{dt} = -0.14 \times 130 e^{-0.14 \times 3}$

$= -11.95\ldots$

ie. concentration is being reduced by 12.0 mg/h (to 1dp)

(iii) Giving your answer correct to one decimal place, after how many hours has the amount of caffeine been reduced to half the original concentration? **3**

$\frac{C}{C_0} = e^{-0.14t}$

$0.5 = e^{-0.14t}$

$-0.14t = \ln 0.5$

$t = \frac{\ln 0.5}{-0.14}$

$= 4.951\ldots$

$\therefore$ after 5 hours

Question 15 continues next page

Marks

(e) The perpendicular bisector of the interval CD has equation $4x - 3y + 16 = 0$. **4**
If the point C has coordinates (-9, 10), find the coordinates of D.
You may use the axes provided below to draw a sketch to assist.

$4x - 3y + 16 = 0$

$3y = 4x + 16$

$y = \frac{4x+16}{3}$

$m = \frac{4}{3}$

$m_\perp = -\frac{3}{4}$

$CD: y - 10 = -\frac{3}{4}(x+9)$

$4(y-10) = -3(x+9)$

$4y - 40 = -3x - 27$

$3x + 4y - 13 = 0$

pt of intersection:

$4x - 3y + 16 = 0$ — ①

$3x + 4y - 13 = 0$ — ②

①×3: $12x - 9y + 48 = 0$

②×4: $12x + 16y - 52 = 0$

$25y - 100 = 0$

$y = 4$

sub into ① $4x - 3(4) + 16 = 0$

$4x = -4$

$x = -1$

$\therefore$ pt of intersection $(-1, 4)$

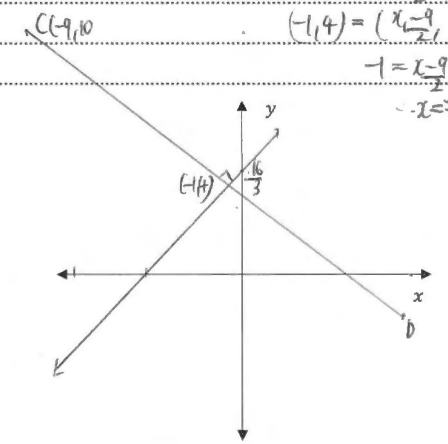
$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$(-1, 4) = \left(\frac{x-9}{2}, \frac{y+10}{2}\right)$

$-1 = \frac{x-9}{2}$ $\quad 4 = \frac{y+10}{2}$

$\therefore x = 7$ $\quad y = -2$

$\therefore D(7, -2)$



End of Question 15

Extra Working Space for Question 15

Question 16 (15 marks) **Marks**

(a) Find correct to the nearest deg the angle of inclination of the tangent to the curve $y = e^{\frac{x}{2}}$ at the point where $y = e$. **3**

$y = e^{\frac{x}{2}}$

$\frac{dy}{dx} = \frac{1}{2}e^{\frac{x}{2}}$

when $y = e$, $e = e^{\frac{x}{2}}$

$\frac{x}{2} = 1$

$x = 2$

$\therefore m = \frac{1}{2}e^{\frac{2}{2}}$

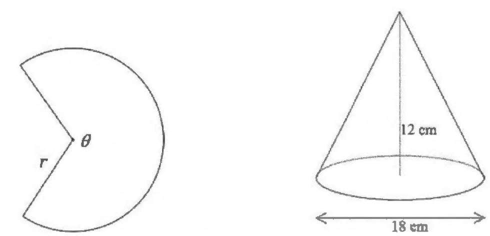
$= \frac{e}{2}$

$\tan\theta = \frac{e}{2}$

$\theta = 53.65...$

$= 54°$ (to nearest deg)

(b) An open cone is made by rolling a piece of paper in the shape of a sector, as shown in the diagrams below. **3**



The cone is to have height 12 cm and base diameter 18 cm.
Find the size of the angle θ in radians, leaving your answer exact.

length of arc, $l = \pi d$

$= \pi \times 18$

$= 18\pi$

radius of sector = slant height of cone

$\therefore r^2 = 12^2 + 9^2$

$= 225$

$r = 15$, $r > 0$

Now, $l = r\theta$

$18\pi = 15\theta$

$\therefore \theta = \frac{6\pi}{5}$

Question 16 continues next page

Marks

(c) Given that $f(x)=\frac{(2x-5)^2}{3-2x}$, find $f'(2)$. **2**

$$f'(x) = \frac{(3-2x).2(2x-5).2-(-2)(2x-5)^2}{(3-2x)^2}$$

$$= \frac{2(2x-5)(2(3-2x)+(2x-5))}{(3-2x)^2}$$

$$= \frac{2(2x-5)(1-2x)}{(3-2x)^2}$$

$$f'(2) = \frac{2(4-5)(1-2(2))}{(3-4)^2}$$

$$= 6$$

(d) The size, y of a population at time, t is given by $y=\frac{N}{1+Ke^{-2t}}$ for $t\geq 0$, where $N>0$ and $K>1$ are constants.

(i) What limiting value does the population size approach for large values of t? **1**

as $t\to\infty$, $Ke^{-2t}\to 0$

$\therefore y\to N$

(ii) Find $\frac{dy}{dt}$, the rate of change of y, and hence show that the population is always increasing. **2**

$$\frac{dy}{dt} = \frac{d}{dt}\left(N(1+Ke^{-2t})^{-1}\right)$$

$$= -N(1+Ke^{-2t})^{-2}\times -2Ke^{-2t}$$

$$= \frac{2NKe^{-2t}}{(1+Ke^{-2t})^2}$$

$$= \frac{2NK}{e^{2t}(1+Ke^{-2t})^2}$$

>0 for all t.

$\therefore y$ is increasing for all t.

Question 16 (d) continues next page

Question 16(d) continued

Marks

(iv) Show that $\frac{dy}{dt}=\frac{2}{N}y(N-y)$ **2**

$$LHS = \frac{dy}{dt} = \frac{2NK}{e^{2t}(1+Ke^{-2t})^2}$$

$$RHS = \frac{2}{N}\times\frac{N}{1+Ke^{-2t}}\left(N-\frac{N}{1+Ke^{-2t}}\right)$$

$$= \frac{2}{1+Ke^{-2t}}\cdot\frac{N(1+Ke^{-2t})-N}{1+Ke^{-2t}}$$

$$= \frac{2N(1+Ke^{-2t}-1)}{(1+Ke^{-2t})^2}$$

$$= \frac{2NKe^{-2t}}{(1+Ke^{-2t})^2}$$

$$= \frac{2NK}{e^{2t}(1+Ke^{-2t})^2}$$

$$= LHS$$

$\therefore \frac{dy}{dt} = \frac{2}{N}y(N-y)$

(v) Given that the population is increasing most rapidly when $\frac{dy}{dt}=0$ and using part (iv) or otherwise, show that the population is increasing most rapidly when $y=\frac{N}{2}$. **2**

$\frac{dy}{dt} = \frac{2}{N}y(N-y)=0$

quadratic (concave down parabola)

$\Rightarrow y(N-y)=0$

zeroes at $y=0$ and $y=N$

$\therefore$ max when $y=\frac{0+N}{2}$

ie $y=\frac{N}{2}$ as reqd

End of Examination