**Knox Grammar School**

Name: ______________________

Teacher: ______________________

YEAR 11 FINAL EXAMINATION

**2023**

| MC | Q11 - 14 | Q15 - 16 | Q17 - 19 | Q20 - 22 | Q23 - 26 | Q27 - 31 | Q32 - 35 | TOTAL |
|---|---|---|---|---|---|---|---|---|
| /10 | /11 | /13 | /15 | /13 | /15 | /13 | /10 | /100 |

# Mathematics Advanced

**General Instructions**

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- The official NESA reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

**Total marks: 100**

**Section I – 10 marks (pages 2-6)**

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

**Section II – 90 marks (pages 7-26)**

- Attempt Questions 11 – 35
- Allow about 2 hours and 45 minutes for this section

## Section I

**10 marks**
**Attempt Questions 1–10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1–10.

---

**1** Which expression is equivalent to $\sqrt[6]{64x^{12}y^3}$ ?

A. $2x^{72}y^{36}$

B. $2x^6\sqrt{y}$

C. $4x^2y^{\frac{3}{2}}$

D. $2x^2\sqrt{y}$

**2** What is $\sqrt{48}-\sqrt{54}$ expressed in its simplest form?

A. $-\sqrt{6}$

B. $4\sqrt{3}-3\sqrt{6}$

C. $-\sqrt{3}$

D. $2\sqrt{12}-3\sqrt{6}$

**3** How many solutions are there for $\sin x = \cos x$ over the domain $0° \le x \le 720°$ ?

A. 1

B. 2

C. 3

D. 4

**4** The straight line $\ell$ is perpendicular to the straight line $7x-3y+51=0$.
The $x$ intercept of $\ell$ is $-5$. What is the equation of $\ell$ ?

A. $3x+7y+15=0$

B . $3x+7y+35=0$

C. $7x-3y+15=0$

D. $7x-3y+35=0$

**5** $f(x)=x^2$ and $g(x)=x+e.$ Which expression represents $f(g(x))$?

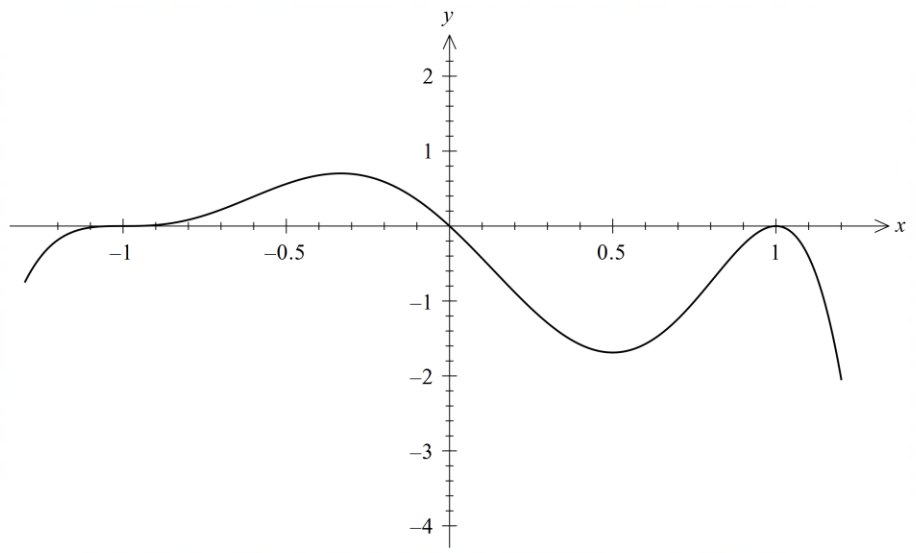
A. $f(g(x))=x^2+2ex+e^2.$

B. $f(g(x))=x^2+2e^x+e^2.$

C. $f(g(x))=x^2+2e^{x+1}+e^2.$

D. $f(g(x))=x^2+e^2.$

**6** Which polynomial is represented on the graph below?




A. $y = -4x(x+1)^3(x-1)^2$

B. $y = 4(x+1)^3(x-1)^2$

C. $y = 4x(x+1)^2(x-1)^3$

D. $y = -4x(x+1)^2(x-1)^3$

**7** What is the derivative of $-\dfrac{3}{x^2}$?

A. $-\dfrac{6}{x^3}$

B. $-\dfrac{3}{x^3}$

C. $\dfrac{3}{x^3}$

D. $\dfrac{6}{x^3}$

**8** A mathematician graphs the relation $x = \sqrt{1 - y^2}$ .

Which statement is correct?

A. This relation is many to one.

B. This relation is one to many.

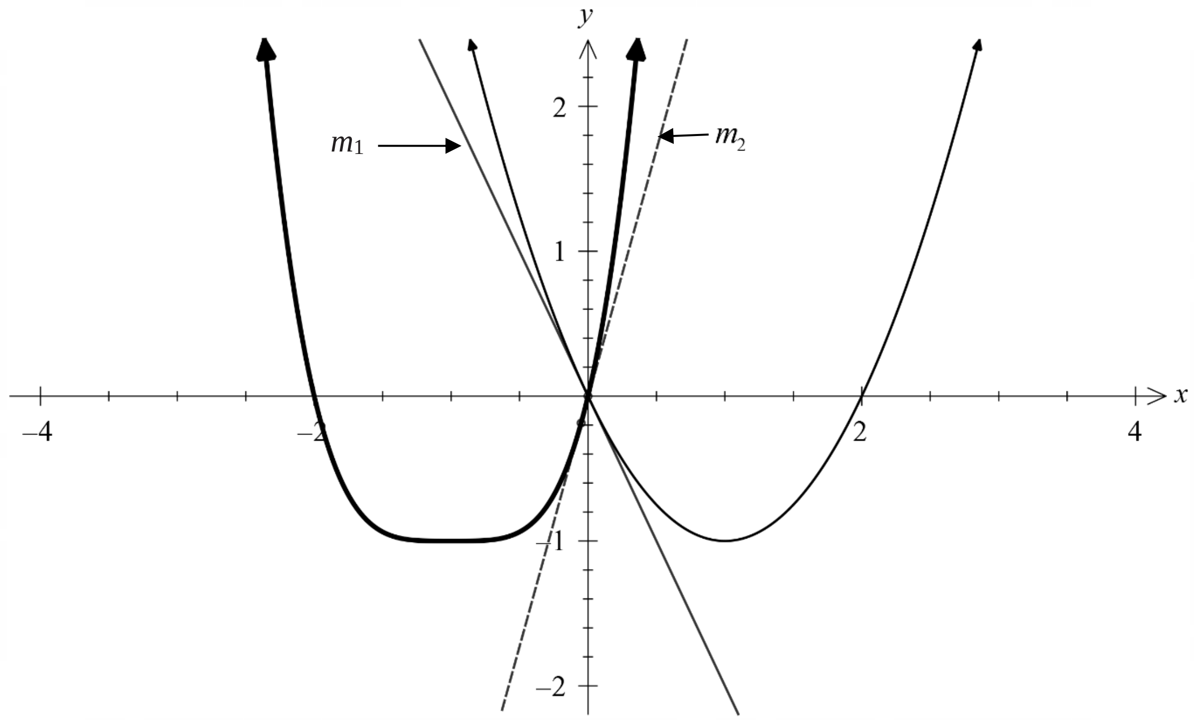
C. This relation is a function.

D. This relation is many to many.

**9** Two functions are drawn to scale below.

Their tangents at the origin are also shown.

The two tangents have gradients $m_1$ and $m_2$ , as indicated on the diagram.




Which of the following statements is true?

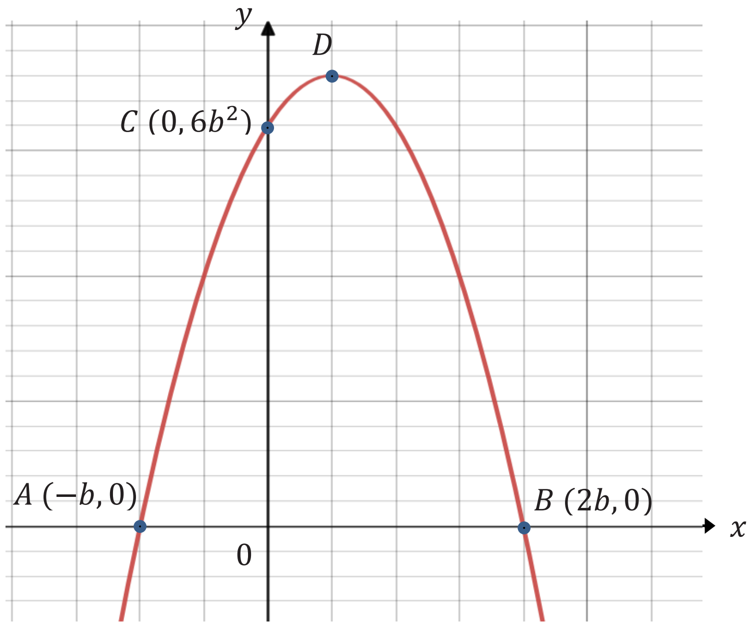
A. $m_1 > m_2$

B. $|m_1| > |m_2|$

C. $|m_2| > |m_1|$

D. The tangents are perpendicular.

**10** The parabola shown in the diagram intersects the $x$ axis at the points $A\ (-b, 0)$ and $B\ (2b, 0)$ and intersects the $y$ axis at $C(0\ , 6b^2\ )$.




Not to Scale

Given that $b > 0$ and the $y$ coordinate of the maximum point $D$ is $1\frac{1}{2}$ more than the $y$ coordinate of $C$, what is the value of $b$?

A. $\sqrt{2}$

B. 2

C. $\sqrt{3}$

D. 3

**2023** **Year 11 Final Examination**

# Mathematics Advanced Section II

# Answer Booklet

**90 marks**
**Attempt Questions 11 – 35**
**Allow about 2 hours and 45 minutes for this section**

**Instructions**

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 27-28. If you use this space, clearly indicate which question you are answering.
-

**Please turn over**

**Question 11** (2 marks)

Solve $t - \frac{t-4}{2} = 10$. **2**

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**Question 12** (3 marks)

Simplify completely $\frac{1}{x^2-9} - \frac{1}{x^2-2x-15}$, leaving your denominator fully factorised. **3**

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**Question 13 (**4 marks)

(a) Find the discriminant of $3x^2 - 2x + k$. **2**

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(b) If $y = 3x^2 - 2x + k$ does not intersect the $x$-axis, find the range of possible values of $k$. **2**

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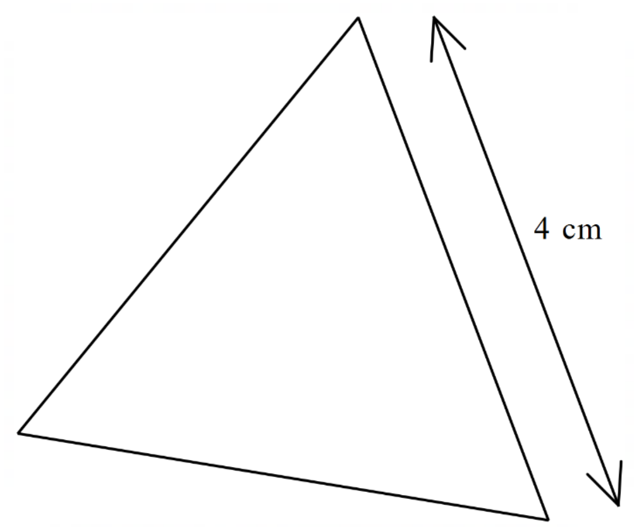
**Question 14** (2 marks)

The side length of an equilateral triangle is 4 cm. Find the exact area of the triangle. **2**

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**Question 15** (9 marks)

The polynomial function $f(x)=\dfrac{x^4}{4}-x-1$ is defined over the domain $-2\le x\le 2$.
The minimum turning point occurs when $x = 1$.

(a) What is the degree of this polynomial? **1**

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(b) Prove that $f(x)=\dfrac{x^4}{4}-x-1$ is neither an odd nor even function. **2**

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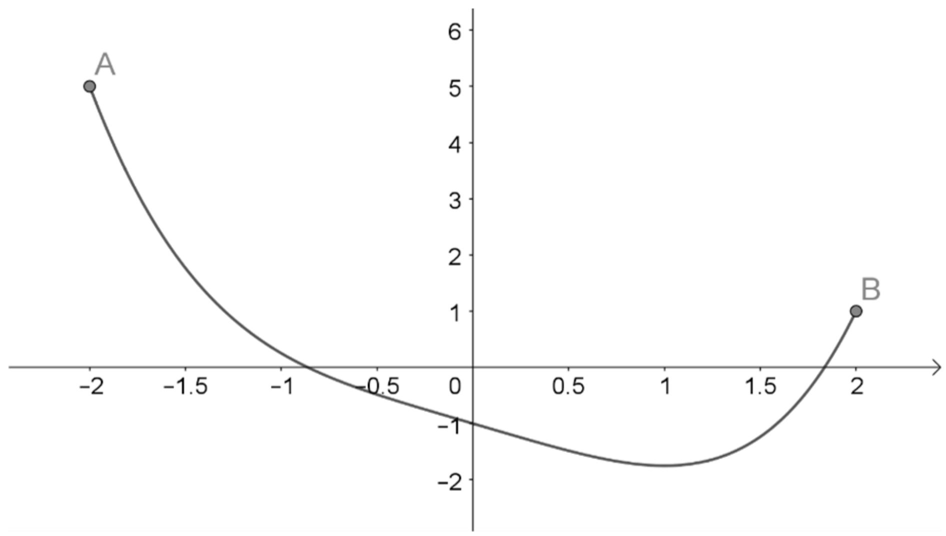
(c) $y = f(x)$ is sketched below.




Write down the range using interval notation. **2**

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(d) Draw the graph of $y = f(-x)$, showing the coordinates of the endpoints and the coordinates of the $y$-intercept. **2**

(e) Draw the graph of $y = -f(-x)$, showing the coordinates of the endpoints and the coordinates of the $y$-intercept. **2**

**Question 16** (4 marks)

A function is given by $f(x) = \dfrac{x}{x^2 + 1}$.

(a) Find $f'(x)$ and simplify it fully. **2**

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(b) Find the gradient of the normal to $y = f(x)$ at the point where $x = 2$. **2**

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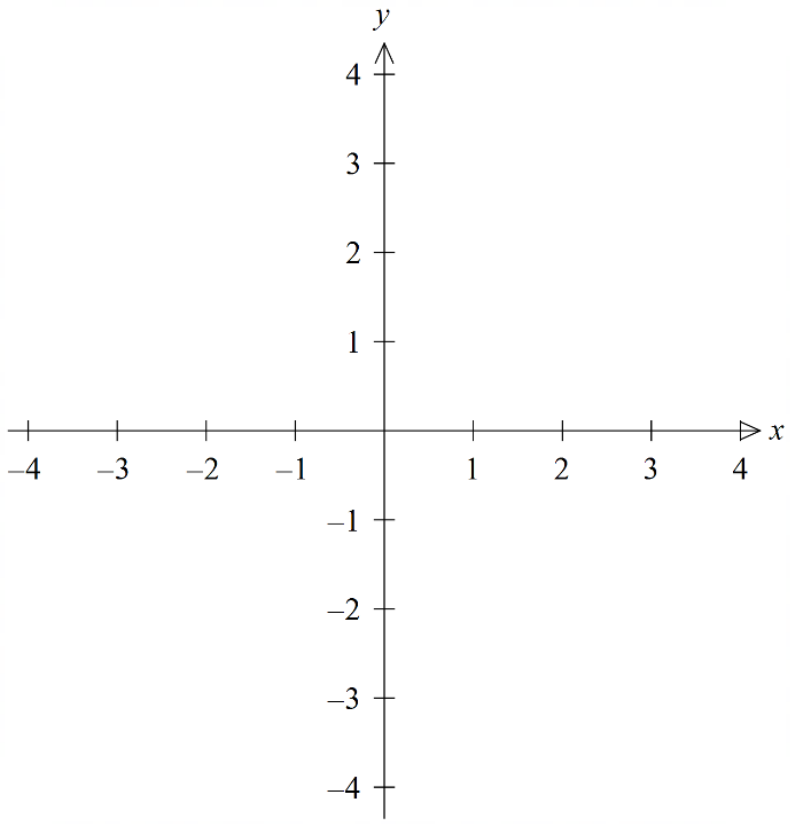
**Question 17** (5 marks)

The circle $(x-2)^2+(y+1)^2=4$ is reflected in the $y$ - axis.

(a) Sketch the new circle on the number plane below. **2**




(b) Write down the equation of your new circle. **1**

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(c) Write down the domain and range of your new circle. **2**

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**Question 18** (5 marks)

A function is defined by $f(x) = 16 - \sqrt{x}$.

(a) Find $f\left(f(9)\right)$, leaving your answer in exact form. **2**

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(b) Solve $f\left(f(x)\right) = 14$. **3**

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**Question 19** (5 marks)

A function is defined by $f(x) = 2x^2 - 7x$.

(a) Find $f(x+h)$ in expanded form. **2**

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(b) Find the derivative of $f(x) = 2x^2 - 7x$ from first principles. **3**

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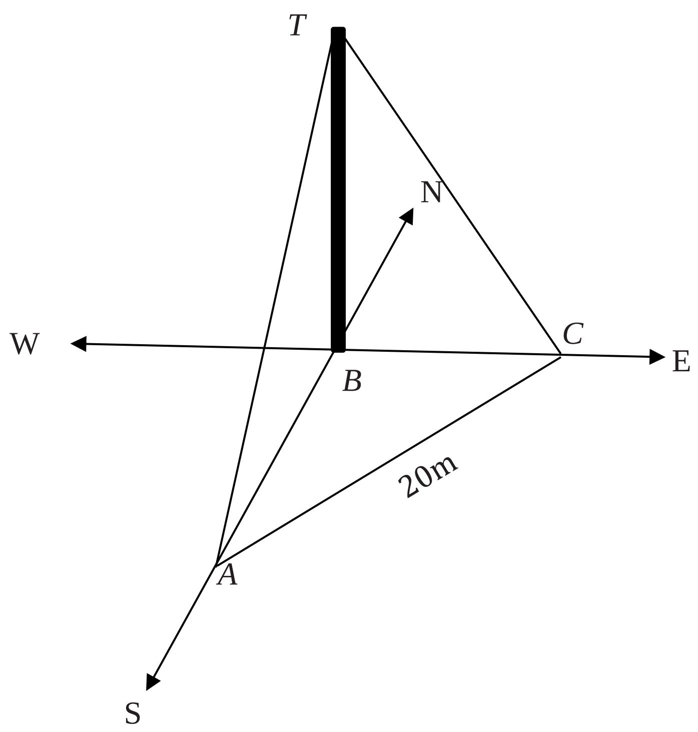
**Question 20** (5 marks)

A pillar *BT* of height *h* m is installed in a park. Alex stands due south of the pillar, and notices that the angle of elevation of the top is 45°. **5**

Chet stands due east of the pillar, and notices that the angle of elevation of the top is 60°.

If Alex and Chet are 20 m apart, find the height of the pillar to the nearest cm.

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**Question 21** (5 marks)

(a) Find the derivative of $f\left(x\right)=x^3\sqrt{x}$ . **2**

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(b) If $y=e^x\left(4x-1\right)^3$ find $\dfrac{dy}{dx}$ in fully factorised form. **3**

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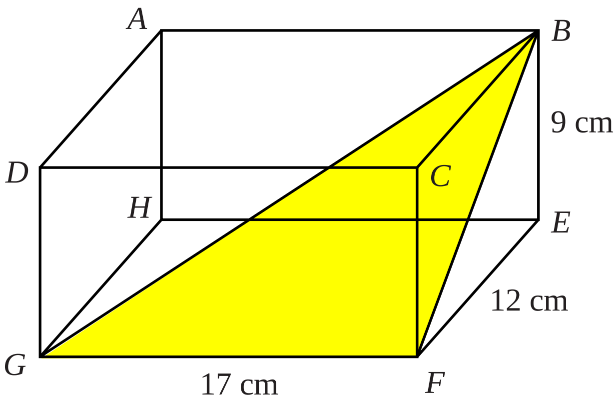
**Question 22** (3 marks)

The diagram shows a rectangular prism with dimensions as shown. **3**




Find the size of $\angle GBF$ to the nearest degree.

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**Question 23** (5 marks)

The revenue at the Parallel Café can be modelled by the function:

$$R(t) = 73\cos(0.0175t - 0.14) + 2040$$

where $t$ is the number of days after midnight on December 31.

In a similar way the operating costs of the café can be modelled by:

$$C(t) = 34\cos(0.0175t - 0.14) + 850$$

Both $R(t)$ and $C(t)$ are measured in dollars.

(a) Find, to the nearest cent, the revenue of the café after one day on January 1. **1**

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(b) Show that the profits of the cafe can be modelled by the function: **1**

$$P(t) = 39\cos(0.0175t - 0.14) + 1190$$

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(c) What is the maximum profit that the café can make? **1**

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(d) Find how many days will elapse before the café makes the maximum profit. **2**

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**Question 24** (3 marks)

Solve $4^x + 4^x + 2^x = 1$ using the substitution $u = 2^x$. **4**

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**Question 25** (3marks)

The line $L$ with equation $3x + 4y - 24 = 0$ crosses the $x$ axis at $A$ and the $y$ axis at $B$.
Find the area of triangle $OAB$, where $O$ is the origin. **2**

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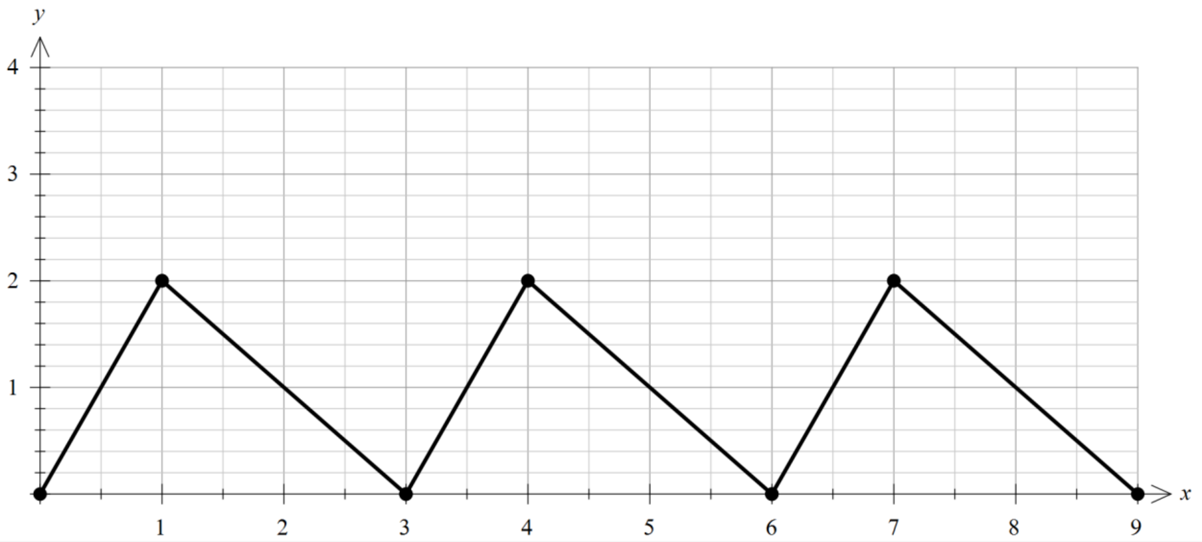
**Question 26** (4 marks)

Part of the periodic function $f(x)$ is shown below. The domain of $f(x)$ is $x \geq 0$ and the period is 3.




(a) Find $f(2)$. **1**

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(b) Find $f'(8)$. **1**

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(c) Find $f'(18.5)$. **2**

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**Question 27** (3 marks)

(a) Convert $240^\circ$ to exact radians. **1**

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(b) When evaluating $\sin 3$, a student wrote down 0.0523 to 4 decimal places.

Explain the error made by the student and write down the correct answer. **2**

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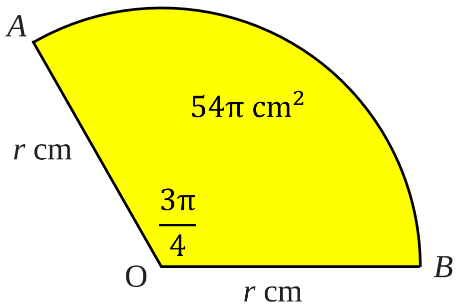
**Question 28** (2 marks)

In the diagram, *AOB* is a sector of a circle with centre at *O* and radius *r* cm.
The angle *AOB* at the centre *O* is $\frac{3\pi}{4}$ and the area of the sector is $54\pi$ cm$^2$.




Find the value of $r$, the radius of the sector. **2**

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**Question 29** (2 marks)

Simplify completely $\cos(\pi + \theta)\ \cos(\pi - \theta) - 1$. **2**

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**Question 30** (2 marks)

Given that $\sin\theta = \frac{4}{5}$, where $\theta$ is an obtuse angle, find the exact value of $\cos\theta$. **2**

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**Question 31** (4 marks)

(a) Sketch $y = \tan x$ for $0 \leq x \leq 2\pi$ in the space below, indicating $x$-intercepts and asymptotes. **2**

(b) On the same diagram sketch $y = 1 - x$. **1**

(c) Determine the number of solutions to the equation $\tan x = 1 - x$ for $0 \leq x \leq 2\pi$. **1**

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**Question 32** (2 marks)

State the domain and range for the function $y = 2^{-x} - 3$. **2**

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**Question 33** (3 marks)

Solve for $x$ the logarithmic equation $2\log_2 x - \log_2 4 = \log_2\left(\frac{4-x}{2}\right)$. **3**

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**Question 34** (2 marks)

A container of water, originally observed at $96^\circ C$, is placed in a refrigerator whose temperature is $0^\circ C$. The container is known to cool in such a way that its temperature halves every 20 minutes.

Find a formula $T(n)$ for the water's temperature in degrees Celsius after $n$ hours. **2**

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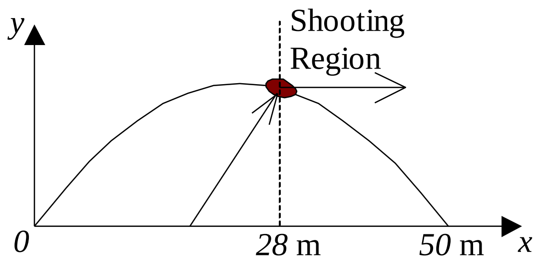
**Question 35** (3 marks)

In a rifle club, discs are projected and follow the path of the parabola with equation:

$$y = -\frac{x^2}{25} + 2x.$$

The rules of competition include the following:

1. Competitors must only shoot at the disc when it is in the "shooting region", as shown in the figure below.
2. Competitors must shoot from a position no further than 23 m from the origin.
3. The number of points awarded for each shot is equal to the height of the disc when first struck by a bullet.
4. Bonus points are awarded if the disc is completely destroyed.
   (*NB*. To guarantee complete destruction, the direction of the bullet must be normal to the path of the disc).
5. You may assume the direction of motion for the disc is always tangential to its parabolic path.




What advice about shooting position (relative to *0*) and shooting angle (relative to the *x*-axis) would you give a competitor wishing to achieve maximum points?

Justify your answer with calculations. **3**

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**End of examination**

## Section II extra writing space

**If you use this space, clearly indicate which question you are answering.**

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# Knox Grammar School

**Name:** ______________________

**YEAR 11 FINAL EXAMINATION**

**Teacher:** ______________________

## 2023

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# Mathematics Advanced Solutions

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A. 1

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**D.** 4

**4** The straight line $\ell$ is perpendicular to the straight line $7x-3y+51=0$. The $x$ intercept of $\ell$ is $-5$. What is the equation of $\ell$ ?

**A.** $3x+7y+15=0$

B . $3x+7y+35=0$

C. $7x-3y+15=0$

D. $7x-3y+35=0$

**5** $f(x)=x^2$ and $g(x)=x+e$. Which expression represents $f(g(x))$?

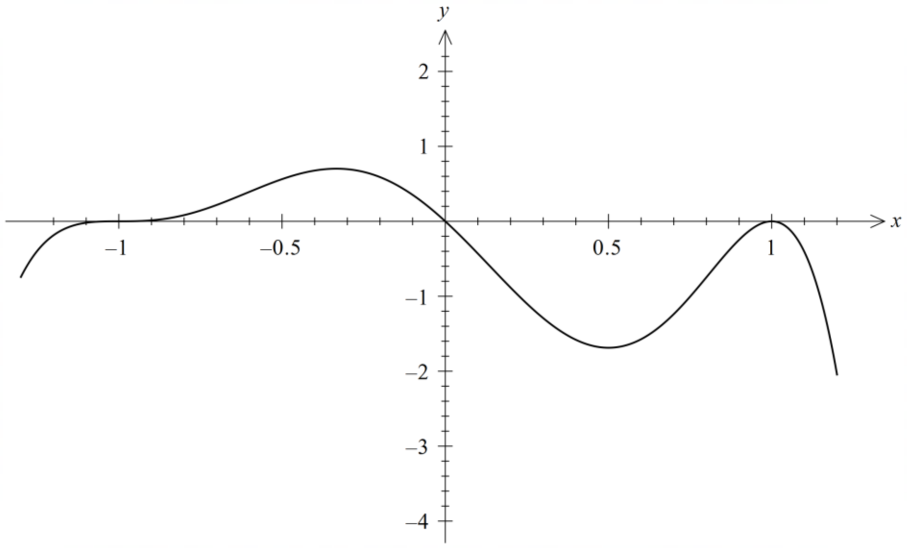
**A.** $f(g(x))=x^2+2ex+e^2$.

B. $f(g(x))=x^2+2e^x+e^2$.

C. $f(g(x))=x^2+2e^{x+1}+e^2$.

D. $f(g(x))=x^2+e^2$.

**6** Which polynomial is represented on the graph below?




**A.** $y = -4x(x+1)^3(x-1)^2$

B. $y = 4(x+1)^3(x-1)^2$

C. $y = 4x(x+1)^2(x-1)^3$

D. $y = -4x(x+1)^2(x-1)^3$

**7** What is the derivative of $-\dfrac{3}{x^2}$?

A. $-\dfrac{6}{x^3}$

B. $-\dfrac{3}{x^3}$

C. $\dfrac{3}{x^3}$

**D.** $\dfrac{6}{x^3}$

**8** A mathematician graphs the relation $x = \sqrt{1-y^2}$ .

Which statement is correct?

A. This relation is many to one.

**B.** This relation is one to many.

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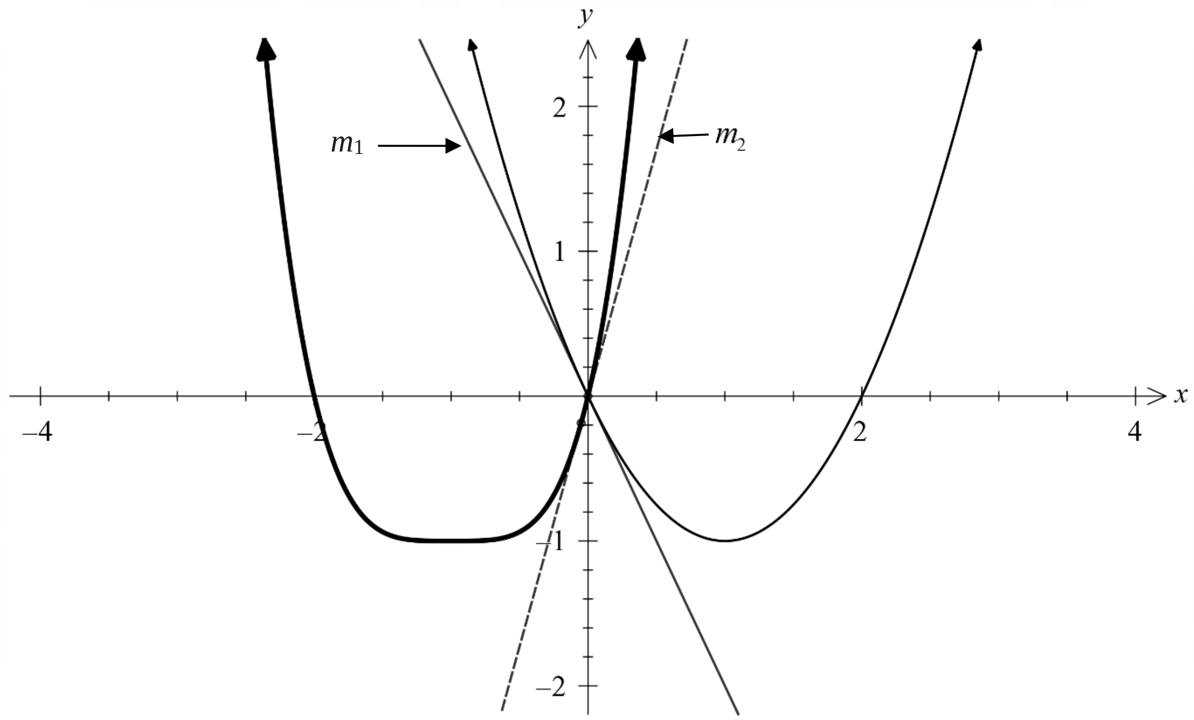
**9** Two functions are drawn to scale below.

Their tangents at the origin are also shown.

The two tangents have gradients $m_1$ and $m_2$ , as indicated on the diagram.




Which of the following statements is true?

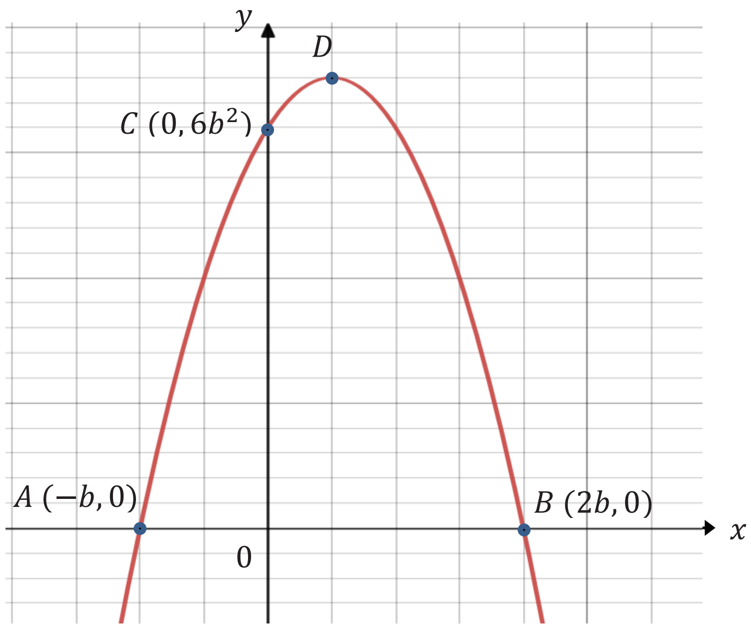
A. $m_1 > m_2$

B. $|m_1| > |m_2|$

**C.** $|m_2| > |m_1|$

D. The tangents are perpendicular.

**10** The parabola shown in the diagram intersects the $x$ axis at the points $A\ (-b, 0)$ and $B\ (2b, 0)$ and intersects the $y$ axis at $C(0\ , 6b^2\ )$.




Not to Scale

Given that $b > 0$ and the $y$ coordinate of the maximum point $D$ is $1\frac{1}{2}$ more than the $y$ coordinate of $C$, what is the value of $b$?

**A. $\sqrt{2}$**

B. 2

C. $\sqrt{3}$

D. 3

The parabola has roots $x = -b$ and $x = 2b$. This means $(x + b)\ and\ (x - 2b)$ are factors. So, the equation of the parabola can be written as $y = -a(x + b)(x - 2b)$ where $a$ is a constant. Now, by substituting $x = 0\ we\ get\ y = -a\ (b)\ (-2b) = 2ab^2$ and we are given that $x = 0, y = 6b^2$. This indicates that $2ab^2 = 6b^2$. As $b\ > 0$ we can divide by $b^2$ and we get $2a = 6$. Therefore, $a = 3$.
Hence, the eq'n of the parabola is $y = -3(x + b)(x - 2b)$ (1)
Also, the axis of symmetry of the parabola, which is the $x$ coordinate of $D$, is:
$x_D = \frac{-b\ +\ 2b}{2} = \frac{b}{2}$ and as $y$ coordinate of $D$ is $1\frac{1}{2}$ more than the $y$ coordinate of $C$ so $y_D = 6b^2 + \frac{3}{2}$. By substituting the coordinates of $D$ in (1), we get $6b^2 + \frac{3}{2} = -3\left(\frac{b}{2} + b\right)\left(\frac{b}{2} - 2b\right) \Rightarrow 6b^2 + \frac{3}{2} = -3\left(\frac{3b}{2}\right)\left(\frac{-3b}{2}\right) \Rightarrow 6b^2 + \frac{3}{2} = \frac{27b^2}{4}$
Multiplying by 4 on both sides, we get $24b^2 + 6 = 27b^2$. This means $b^2 = 2$ So, $b = \sqrt{2}$, as $b > 0$
Hence the correct option is A.

2023 **Year 11 Final Examination**

# Mathematics Advanced Section II

# Answer Booklet

**90 marks**
**Attempt Questions 11 – 35**
**Allow about 2 hours and 45 minutes for this section**

**Instructions**

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 27-28. If you use this space, clearly indicate which question you are answering.
- 

**Please turn over**

**Question 11** (2 marks)

Solve $t-\frac{t-4}{2}=10$. **2**

$$t-\frac{t-4}{2}=10$$
$$2t-t+4=20 \quad \boxed{1}$$
$$t=16 \quad \boxed{1}$$

**Question 12** (3 marks)

Simplify completely $\frac{1}{x^2-9}-\frac{1}{x^2-2x-15}$, leaving your denominator fully factorised. **3**

$$\frac{1}{x^2-9}-\frac{1}{x^2-2x-15}=\frac{1}{(x-3)(x+3)}-\frac{1}{(x-5)(x+3)} \quad \boxed{1}$$
$$=\frac{x-5-(x-3)}{(x-5)(x+3)(x-3)} \quad \boxed{1}$$
$$=\frac{-2}{(x-5)(x+3)(x-3)} \quad \boxed{1}$$

**Question 13 (**4 marks)

(a) Find the discriminant of $3x^2-2x+k$. **2**

$3x^2-2x+k \rightarrow \Delta=4-4\times3\times k=4-12k$ $\boxed{1}\boxed{1}$

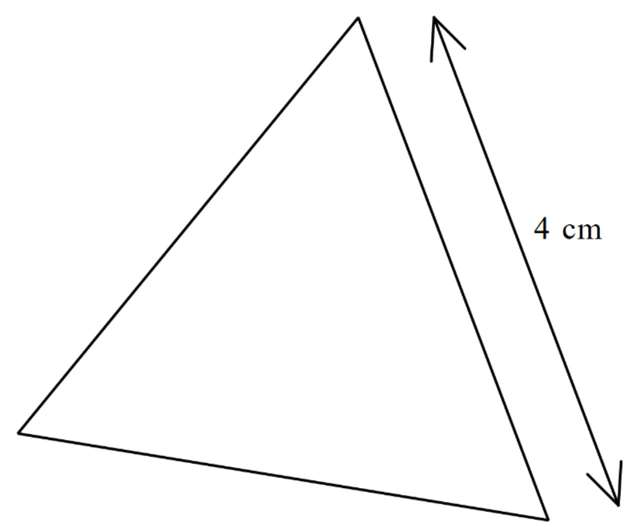
(b) If $y=3x^2-2x+k$ does not intersect the $x$-axis, find the range of possible values of $k$. **2**

It will not cut the $x$-axis if the discriminant is negative $\boxed{1}$

$4-12k<0 \rightarrow 4<12k \rightarrow k>\frac{1}{3}$ $\boxed{1}$

**Question 14** (2 marks)

The side length of an equilateral triangle is 4 cm. Find the exact area of the triangle. **2**




All the angles are 60° since the triangle is equilateral.

$$A = \frac{1}{2} \times 4^2 \times \sin 60° \quad \boxed{1}$$

$$= 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}\, cm^2 \quad \boxed{1}$$

**Question 15** (9 marks)

The polynomial function $f(x)=\dfrac{x^4}{4}-x-1$ is defined over the domain $-2\le x\le 2$.
The minimum turning point occurs when $x$ = 1.

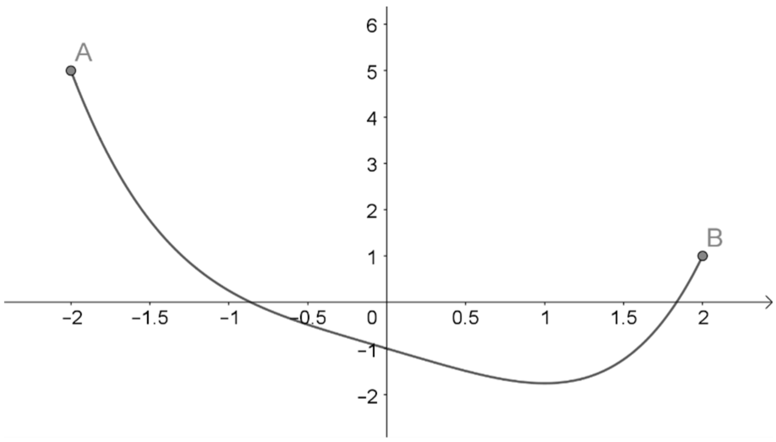
(a) What is the degree of this polynomial? **1**

Degree = 4 $\boxed{1}$

(b) Prove that $f(x)=\dfrac{x^4}{4}-x-1$ is neither an odd nor even function. **2**

$$f(x)=\frac{x^4}{4}-x-1$$
$$f(-x)=\frac{(-x)^4}{4}-(-x)-1$$
$$=\frac{x^4}{4}+x-1\neq f(x)\text{ not even} \quad \boxed{1}$$
$$-f(x)=-\frac{x^4}{4}+x+1\neq f(-x)\text{ not odd} \quad \boxed{1}$$
$$f(x)\text{ is neither odd nor even}$$

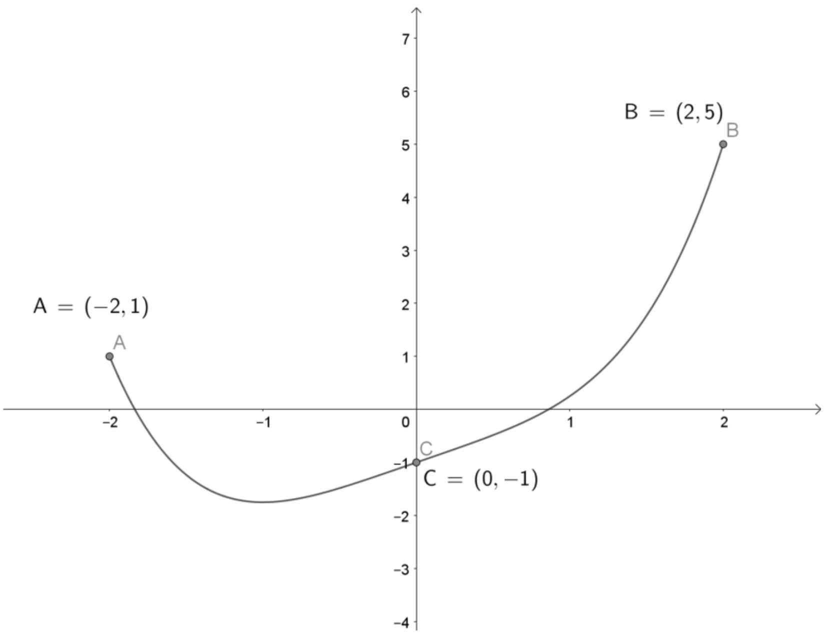
(c) $y=f(x)$ is sketched below.




Write down the range using interval notation. **2**

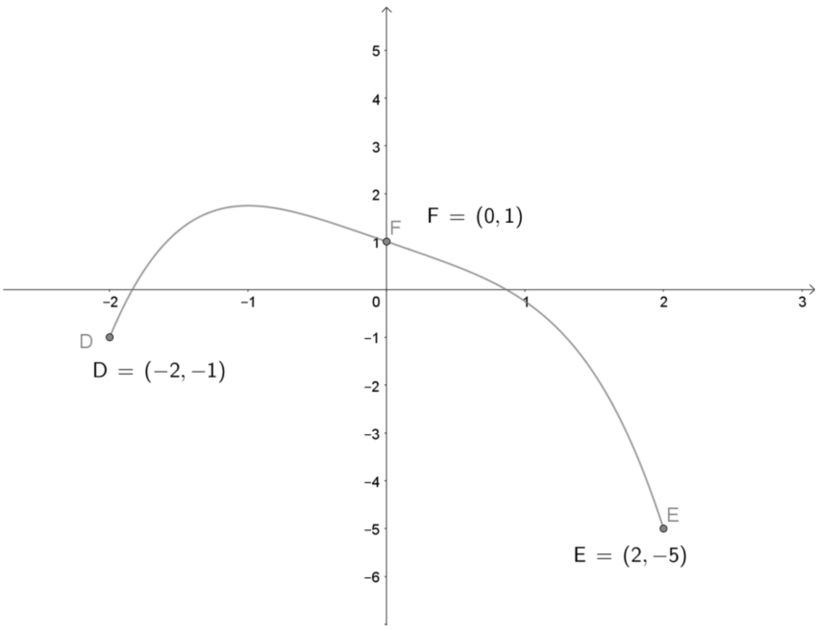
$$f(x)=\frac{x^4}{4}-x-1$$
$$f(-2)=\frac{(-2)^4}{4}-(-2)-1=5$$
$$f(1)=\frac{(1)^4}{4}-1-1=-1.75 \quad \boxed{1}$$
$$\text{Range}[-1.75,5] \quad \boxed{1}$$

(d) Draw the graph of $y = f(-x)$, showing the coordinates of the endpoints and the coordinates of the $y$-intercept. **2**




**1 mark for coordinates & 1 mark for shape**

(e) Draw the graph of $y = -f(-x)$, showing the coordinates of the endpoints and the coordinates of the $y$-intercept. **2**




**1 mark for coordinates & 1 mark for shape**

**Question 16** (4 marks)

A function is given by $f\left(x\right)=\dfrac{x}{x^2+1}$.

(a) Find $f'\left(x\right)$ and simplify it fully. **2**

$$f\left(x\right)=\frac{x}{x^2+1}\rightarrow u=x, u'=1 \quad v=x^2+1, v'=2x$$

$$f'\left(x\right)=\frac{vu'-uv'}{v^2}$$

$$=\frac{x^2+1-x\left(2x\right)}{\left(x^2+1\right)^2} \qquad \boxed{1}$$

$$=\frac{1-x^2}{\left(x^2+1\right)^2} \qquad \boxed{1}$$

(b) Find the gradient of the normal to $y=f\left(x\right)$ at the point where $x = 2$. **2**

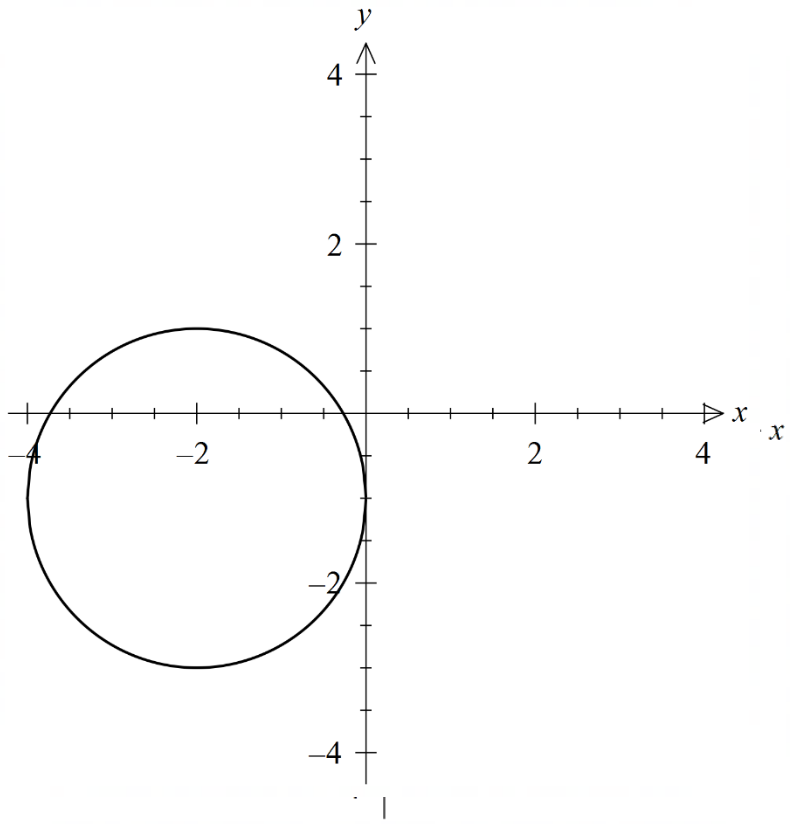
$$m(\text{tangent})=\frac{1-4}{\left(4+1\right)^2}=\frac{-3}{25}\boxed{1}$$

$$m(\text{normal})=\frac{25}{3} \qquad \boxed{1}$$

**Question 17** (5 marks)

The circle $(x-2)^2+(y+1)^2=4$ is reflected in the $y$ - axis.

(a) Sketch the new circle on the number plane below. **2**




**1 mark for centre at (-2, -1) and 1 mark for *y*-axis tangential to circle.**

(b) Write down the equation of your new circle. **1**

$(x+2)^2+(y+1)^2=4$ $\boxed{1}$

(c) Write down the domain and range of your new circle. **2**

Domain: [-4,0] **1 mark**

Range: [-3,1] **1 mark**

**Question 18** (5 marks)

A function is defined by $f(x) = 16 - \sqrt{x}$.

(a) Find $f(f(9))$, leaving your answer in exact form. **2**

$f(9) = 16 - \sqrt{9} = 13$ $\boxed{1}$

$f(f(9)) = f(13) = 16 - \sqrt{13}$ $\boxed{1}$

(b) Solve $f(f(x)) = 14$. **3**

$f(f(x)) = 14$

$f(16 - \sqrt{x}) = 14$ $\boxed{1}$

$\therefore 16 - \sqrt{16 - \sqrt{x}} = 14$

$\therefore \sqrt{16 - \sqrt{x}} = 2$ $\boxed{1}$

$\therefore 16 - \sqrt{x} = 4$

$\therefore \sqrt{x} = 12$

$\therefore x = 144$ $\boxed{1}$

**Question 19** (5 marks)

A function is defined by $f(x) = 2x^2 - 7x$ .

(a) Find $f(x+h)$ in expanded form. **2**

$$\begin{aligned} f(x) &= 2x^2 - 7x \\ f(x+h) &= 2(x+h)^2 - 7(x+h) \boxed{1} \\ &= 2x^2 + 4xh + 2h^2 - 7x - 7h \quad \boxed{1} \end{aligned}$$

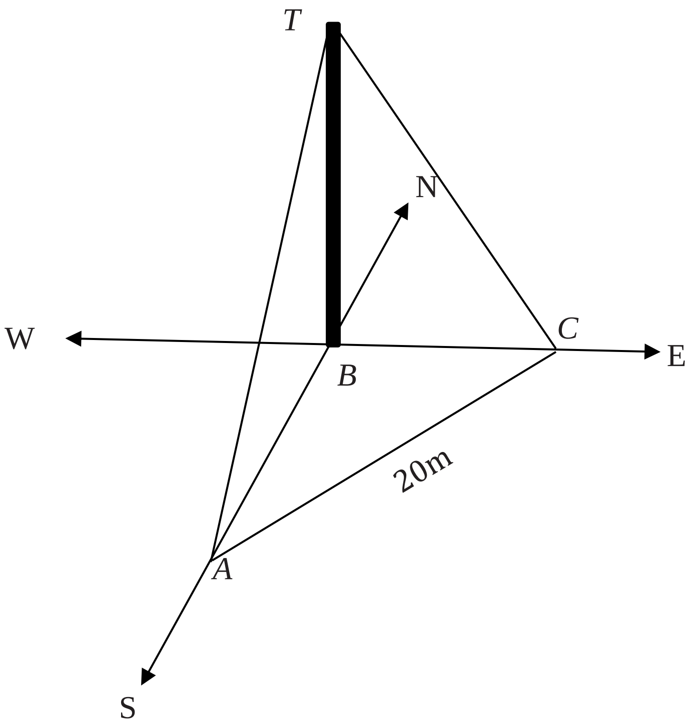
(b) Find the derivative of $f(x) = 2x^2 - 7x$ from first principles. **3**

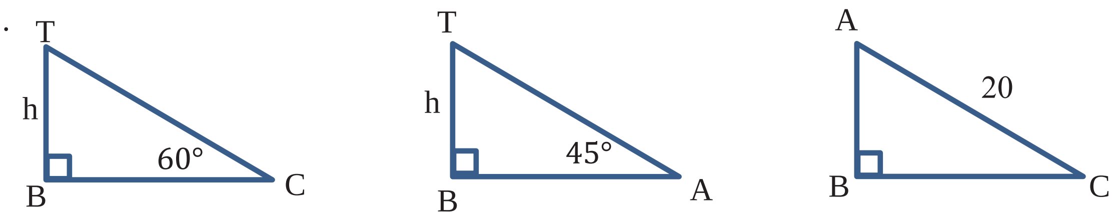
$$\begin{aligned} f(x) &= 2x^2 - 7x \\ f(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 7x - 7h - \left(2x^2 - 7x\right)}{h} \quad \boxed{1} \\ &= \lim_{h \to 0} \frac{4xh + 2h^2 - 7h}{h} \quad \boxed{1} \\ &= \lim_{h \to 0} 4x + 2h - 7 = 4x - 7 \quad \boxed{1} \end{aligned}$$

**Question 20** (5 marks)

A pillar *BT* of height *h* m is installed in a park. Alex stands due south of the pillar, and notices that the angle of elevation of the top is 45°. **5**
Chet stands due east of the pillar, and notices that the angle of elevation of the top is 60°.

If Alex and Chet are 20 m apart, find the height of the pillar to the nearest cm.




In $\Delta CBT$ $\tan 60^\circ = \dfrac{h}{BC} \rightarrow BC = \dfrac{h}{\tan 60^\circ} = \dfrac{h}{\sqrt{3}}$ $\boxed{1}$

In $\Delta ABT$ $\tan 45^\circ = \dfrac{h}{AB} \rightarrow AB = \dfrac{h}{\tan 45^\circ} = h$ $\boxed{1}$

In $\Delta ABC\ AB^2 + BC^2 = 20^2$ $\boxed{1}$

$$h^2 + \left(\frac{h}{\sqrt{3}}\right)^2 = 20^2$$

$$h^2 + \frac{h^2}{3} = 400 \quad \boxed{1}$$

$$\frac{4h^2}{3} = 400$$

$$h^2 = 300 \to h = 17.32 \text{ m nearest cm.} \quad \boxed{1}$$

**Question 21** (5 marks)

(a) Find the derivative of $f(x) = x^3\sqrt{x}$. **2**

$$f(x) = x^3\sqrt{x}$$

$$f(x) = x^3\sqrt{x} = x^{\frac{7}{2}} \boxed{1}$$

$$f'(x) = \frac{7}{2}x^{\frac{5}{2}} \quad \boxed{1}$$

(b) If $y = e^x(4x-1)^3$ find $\dfrac{dy}{dx}$ in fully factorised form. **3**

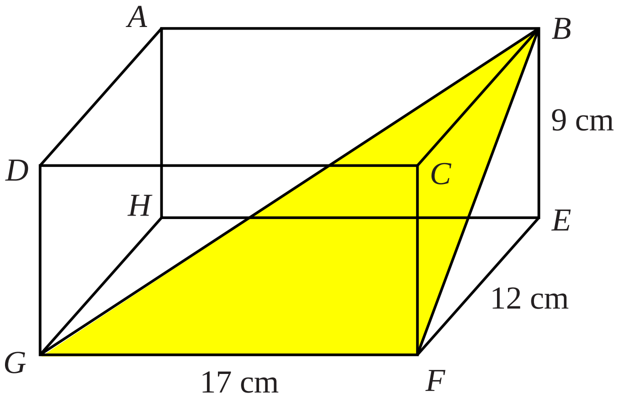
$y = e^x(4x-1)^3 \to u = e^x, u' = e^x$ and $v = (4x-1)^3 \to v' = 12(4x-1)^2$ $\boxed{1}$

$y' = vu' + uv' = (4x-1)^3 e^x + e^x \times 12(4x-1)^2$ $\boxed{1}$

$= e^x(4x-1)^2(4x+11)$ $\boxed{1}$

**Question 22** (3 marks)

The diagram shows a rectangular prism with dimensions as shown. **3**




Find the size of $\angle GBF$ to the nearest degree.

Using Pythagoras' theorem in right-angled triangle $FBE$, we get

$BF^2 = 9^2 + 12^2$; that is, $BF = 15\ cm$. $\boxed{1}$

Also, triangle $GFB$ is right angled as $\angle GFB = 90°$

so $\tan\angle \mathrm{GBF} = \frac{17}{15}$; that is, $\angle GBF = tan^{-1}\left(\frac{17}{15}\right)$. $\boxed{1}$

$\therefore \angle GBF = 48°\ 35' \approx 49°$. (to the nearest degree) $\boxed{1}$

**Question 23** (5 marks)

The revenue at the Parallel Café can be modelled by the function:

$$R(t) = 73\cos(0.0175t - 0.14) + 2040$$

where $t$ is the number of days after midnight on December 31.

In a similar way the operating costs of the café can be modelled by:

$$C(t) = 34\cos(0.0175t - 0.14) + 850$$

Both $R(t)$ and $C(t)$ are measured in dollars.

(a) Find, to the nearest cent, the revenue of the café after one day on January 1. **1**

$$R(t) = 73\cos(0.0175\times 1 - 0.14) + 2040 \approx \$2112.45 \quad \boxed{1}$$

(b) Show that the profits of the cafe can be modelled by the function: **1**

$$P(t) = 39\cos(0.0175t - 0.14) + 1190$$

Profit = Revenue – Cost

$$P(t) = [73\cos(0.0175t - 0.14) + 2040] - [34\cos(0.0175t - 0.14) + 850] \ \boxed{1}$$

$$= 39\cos(0.0175t - 0.14) + 850$$

(c) What is the maximum profit that the café can make? **1**

$$Max\,P(t) = 39\times 1 + 1190 = \$1229 \qquad \boxed{1}$$

(d) Find how many days will elapse before the café makes the maximum profit. **2**

$$P(t) = 39\cos(0.0175t - 0.14) + 850 = \$1229$$

$$\therefore 0.0175t - 0.14 = 0 \qquad \boxed{1}$$

$$\therefore t = 8 \text{ days} \qquad \boxed{1}$$

**Question 24** (3 marks)

Solve $4^x + 4^x + 2^x = 1$ using the substitution $u = 2^x$. **4**

Let $u = 2^x \rightarrow u^2 = \left(2^x\right)^2 = \left(2^2\right)^x = 4^x$ $\boxed{1}$

$$\begin{cases} 4^x + 4^x + 2^x = 1 \\ u^2 + u^2 + u - 1 = 0 \\ 2u^2 + u - 1 = 0 \\ (2u-1)(u+1) = 0 \quad \boxed{1} \end{cases}$$

$$\begin{cases} u = \frac{1}{2}, u = -1 \text{ but } 2^x > 0 \\ 2^x = \frac{1}{2} \text{ only} \quad \boxed{1} \end{cases}$$

So $x = -1$ $\boxed{1}$

**Question 25** (3marks)

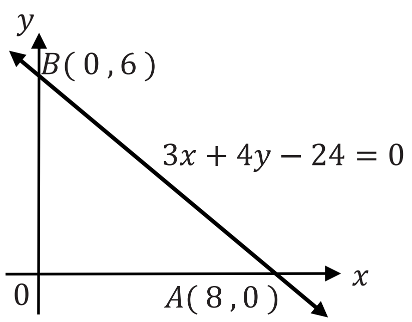
The line *L* with equation $3x + 4y - 24 = 0$ crosses the $x$ axis at *A* and the *y* axis at *B*.
Find the area of triangle *OAB*, where *O* is the origin. **2**

The line $3x + 4y - 24 = 0$ cuts the $x$ axis when $y = 0$ *that is* $3x - 24 = 0$. So $x = 8$.

Therefore, the point *A* has coordinates ( 8 , 0 ). The line $3x + 4y - 24 = 0$ cuts the $y$ axis when $x = 0$; *that is*, $4y - 24 = 0$ so $y = 6$.

Therefore, the point *B* has coordinates (0, 6 ). $\boxed{1}$

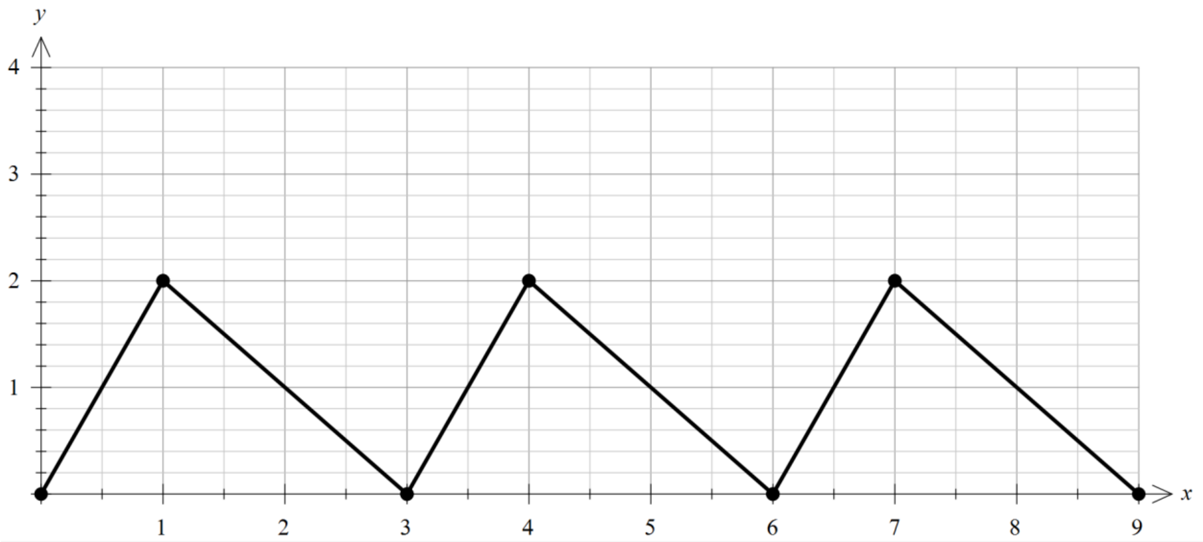
*Discussion on intercepts only is satisfactory for 1 mark.*




Hence, the area of $\Delta AOB = \frac{1}{2} \times 8 \times 6 = 24\ units^2$. $\boxed{1}$

**Question 26** (4 marks)

Part of the periodic function $f(x)$ is shown below. The domain of $f(x)$ is $x \geq 0$ and the period is 3.




(a) Find $f(2)$. **1**

$f(2) = 1$ $\boxed{1}$

(b) Find $f'(8)$. **1**

$$f'(8) = \frac{\text{rise}}{\text{run}}$$

$$f'(8) = \frac{-2}{2}$$

$$f'(8) = -1$$

(c) Find $f'(18.5)$. **2**

$f'(18.5) = f'(18.5 - 4 \times 3) = f'(6.5)$ as period $= 3$ $\boxed{1}$ *OR* $\boxed{1}$ for finding equivalent point using periodicity

$$f'(6.5) = \frac{1}{0.5}$$

$\therefore f'(18.5) = 2$ $\boxed{1}$

$\boxed{1}$ for finding equivalent point using periodicity

**Question 27** (3 marks)

(a) Convert $240^\circ$ to exact radians. **1**

$240^\circ \times \frac{\pi}{180^\circ}$

$= \frac{4\pi}{3}$ $\boxed{1}$

(b) When evaluating $\sin 3$, a student wrote down 0.0523 to 4 decimal places.

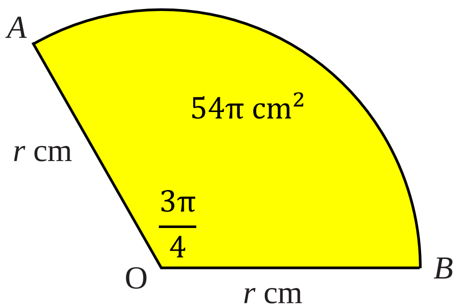
Explain the error made by the student and write down the correct answer. **2**

Student has calculated $\sin 3^\circ$ $\boxed{1}$. Correct answer $= 0.1411\,(4\text{dp})$. $\boxed{1}$

................................................................................................................................................

**Question 28** (2 marks)

In the diagram, *AOB* is a sector of a circle with centre at *O* and radius *r* cm.
The angle *AOB* at the centre *O* is $\frac{3\pi}{4}$ and the area of the sector is $54\pi\ \text{cm}^2$.




Find the value of $r$, the radius of the sector. **2**

From the reference sheet, the area of a sector is $A = \frac{1}{2}r^2\theta$.

By substituting $A = 54\pi\ \text{cm}^2$ and $\theta = \frac{3\pi}{4}$,we get $54\pi = \frac{1}{2}r^2 \times \frac{3\pi}{4}$; that is, $54\pi = \frac{3\pi}{8}r^2$ $\boxed{1}$

By dividing both sides by $3\pi, we\ get$

$18 = \frac{r^2}{8}$ this means $144 = r^2$ So, $r = 12$ cm.

The radius is positive.

Hence, the radius of this sector is 12 cm. $\boxed{1}$

**Question 29** (2 marks)

Simplify completely $\cos(\pi+\theta)\cos(\pi-\theta)-1$. **2**

$$=-\cos\theta\times-\cos\theta-1$$
$$=\cos^2\theta-1\ \boxed{1}$$
$$=-\left(1-\cos^2\theta\right)$$
$$=-\sin^2\theta\quad\boxed{1}$$

**Question 30** (2 marks)

Given that $\sin\theta=\frac{4}{5}$, where $\theta$ is an obtuse angle, find the exact value of $\cos\theta$. **2**

$$\sin^2\theta+\cos^2\theta=1\ \&\ \sin\theta=\frac{4}{5}$$
$$So\ \cos^2\theta=1-\left(\frac{4}{5}\right)^2=\frac{9}{25}\ \boxed{1}.$$
$$\Rightarrow\cos\theta=\pm\frac{3}{5}.\ \text{However, }\cos\theta<0\ \text{as }\theta\text{ is obtuse}.$$
$$\therefore\cos\theta=-\frac{3}{5}\ \text{only}\ \boxed{1}$$

*OR*

As $\theta$ is an acute angle we construct a right angle triangle and use Pythagoras' theorem to find the missing third side. So, the third side is

$\sqrt{5^2-4^2}=3.\ \boxed{1}$

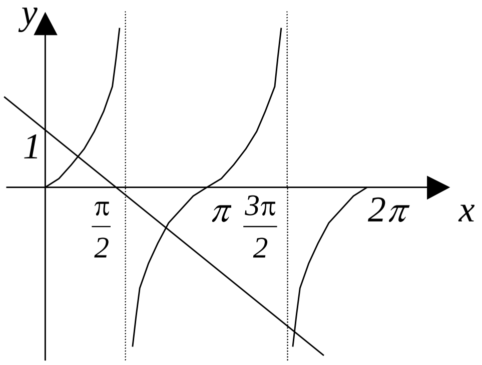
However, $\theta$ is obtuse and so $\cos\theta=-\frac{3}{5}\ \boxed{1}$

**Question 31** (4 marks)

(a) Sketch $y = \tan x$ for $0 \le x \le 2\pi$ in the space below, indicating $x$-intercepts and asymptotes. **2**

- **1 mark for shape and 1 mark for (asymptotes + intercepts) – See below**

(b) On the same diagram sketch $y = 1 - x$. **1**




**1 mark for line with correct orientation**

(c) Determine the number of solutions to the equation $\tan x = 1 - x$ for $0 \le x \le 2\pi$. **1**

**Three solutions** $\boxed{1}$

**Question 32** (2 marks)

State the domain and range for the function $y = 2^{-x} - 3$. **2**

$Domain: all\ real\ x \quad \boxed{1}$

$Range: y > -3 \quad \boxed{1}$

**Question 33** (3 marks)

Solve for $x$ the logarithmic equation $2\log_2 x - \log_2 4 = \log_2\left(\frac{4-x}{2}\right)$ . **3**

$\log_2\left(x^2\right) - \log_2 4 = \log_2\left(\frac{4-x}{2}\right)$

$\log_2\left(\frac{x^2}{4}\right) = \log_2\left(\frac{4-x}{2}\right)$ $\boxed{1}$

$\Rightarrow \frac{x^2}{4} = \frac{4-x}{2}$ noting log is one–to–one

$\Rightarrow x^2 = 8-2x$ or $x^2+2x-8=0$

$\Rightarrow (x+4)(x-2)=0$ $\boxed{1}$

$x=-4$ or $x=2$ But $0<x<4$ (Why?)

So $x=2$ only. $\boxed{1}$

**Question 34** (2 marks)

A container of water, originally observed at $96^\circ C$, is placed in a refrigerator whose temperature is $0^\circ C$. The container is known to cool in such a way that its temperature halves every 20 minutes.

Find a formula $T(n)$ for the water's temperature in degrees Celsius after $n$ hours. **2**

$T\left(1\times\frac{1}{3}\right) = \frac{1}{2}\times 96$

$T\left(2\times\frac{1}{3}\right) = \left(\frac{1}{2}\right)^2 \times 96$

$T\left(3\times\frac{1}{3}\right) = \left(\frac{1}{2}\right)^3 \times 96$

$\vdots \quad \vdots \quad = \quad \vdots$

$T\left(x\times\frac{1}{3}\right) = \left(\frac{1}{2}\right)^x \times 96$

$\boxed{1}$ for pattern or equivalent logic

Let $n = x\times\frac{1}{3}$ or $\frac{x}{3}$.

$\therefore x = 3n \quad \textit{and}$ so $T(n) = \left(\frac{1}{2}\right)^{3n}\times 96 \;\; OR \;\; 96\left(\frac{1}{2}\right)^{3n}$ $\boxed{1}$

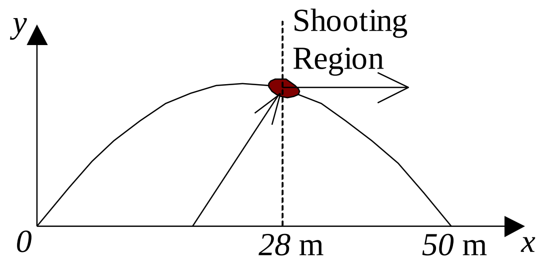
**Question 35** (3 marks)

In a rifle club, discs are projected and follow the path of the parabola with equation:

$$y = -\frac{x^2}{25} + 2x\,.$$

The rules of competition include the following:

1. Competitors must only shoot at the disc when it is in the "shooting region", as shown in the figure below.
2. Competitors must shoot from a position no further than 23 m from the origin.
3. The number of points awarded for each shot is equal to the height of the disc when first struck by a bullet.
4. Bonus points are awarded if the disc is completely destroyed.
   (*NB*. To guarantee complete destruction, the direction of the bullet must be normal to the path of the disc).
5. You may assume the direction of motion for the disc is always tangential to its parabolic path.




What advice about shooting position (relative to *0*) and shooting angle (relative to the *x*-axis) would you give a competitor wishing to achieve maximum points?
Justify your answer with calculations. **3**

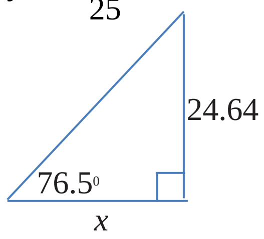
By Rule 3, you would want to hit the disc as soon as it enters the shooting region; that is, when $x = 28$ since it appears the disc is descending upon entry into the shooting region. (*This latter comment can be proven by establishing the coordinates of the turning point.*) Also, by Rule 4, you would want to hit the disc at right-angles to its path. From

$y = -\frac{x^2}{25} + 2x$ we have $\frac{dy}{dx} = -\frac{2x}{25} + 2$ and when $x = 28$, $\frac{dy}{dx} = -\frac{2 \times 28}{25} + 2$ or $-\frac{6}{25}$. This

represents the gradient of the tangent at $x = 28$. So gradient of normal = $\frac{25}{6}\left(m_1 \times m_2 = -1\right)$.

$\therefore$ shooting angle = $\tan^{-1}\left(\frac{25}{6}\right) \approx 76.5^\circ$. Additionally, the disc's height when $x = 28$ is

$$y = -\frac{28^2}{25} + 2 \times 28 = 24.64.$$




Finally, $\tan 76.5^\circ = \frac{24.64}{x}$ or $x = \frac{24.64}{\tan 76.5^\circ} \approx 5.9\,\text{m}$.

Consequently, the shooting position is approximately 28 – 5.9 = 22.1 m from *O*.

| **Marking Criteria** | **Marks** |
|---|---|
| • Provides correct solution | 3 |
| • Correct value for shooting angle | 2 |
| • Recognises bullet must penetrate disc at right-angles when $x = 28$ | 1 |

**Section I Multiple Choice Answer Key**

| Question | Answer |
|---|---|
| 1 | D |
| 2 | B |
| 3 | D |
| 4 | A |
| 5 | A |
| 6 | A |
| 7 | D |
| 8 | B |
| 9 | C |
| 10 | A |

**End of examination**