VIRILE AGITUR

Name: _____

Teacher's Name: _____

Place Barcode Sticker here →

2025

YEAR 11 PRELIMINARY EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours 15 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided with this paper

Total marks: 75

Section I – 10 marks (pages 3–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section
- Use the Multiple Choice answer sheet provided.

Section II – 65 marks (pages 7–21)

- Attempt Questions 11–29
- Allow about 2 hours for this section
- Write your working and answers in the spaces provided in this booklet.
- Show relevant mathematical reasoning and/or calculations
- Additional writing space is provided on pages 22-23

M/C	Q11-13	Q14-15	Q16-17	Q18-19	Q20-21	Q22-23	Q24-25	Q26-27	Q28-29	TOTAL
/10	/6	/6	/8	/9	/7	/7	/8	/6	/8	/75

Blank Page

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

---

1 What is an angle of 2 radians closest to in degrees?

A. 90°

B. 103°

C. 115°

D. 127°

2 A bacteria colony's size in thousands is modelled by $N(t) = 2t^4 - 5t^3 + t^2 + 7$, where t is measured in hours.

What is the instantaneous rate of growth of the colony at $t = 2$?

A. 3

B. 8

C. 17

D. 25

3 A student is late to school if either their alarm fails or if there is traffic.

Let $P(\text{alarm fails}) = 0.3$ and $P(\text{traffic}) = 0.6$. These two events are independent.

What is the probability the student arrives on time?

A. 0.28

B. 0.42

C. 0.82

D. 0.88

7 What is the correct expression for y if $x^{3y-1}=3z$?

A. $y=\log_x(z)+\frac{1}{3}$

B. $y=3\log_3(3z)-1$

C. $y=\frac{\log_x(3z)+1}{3}$

D. $y=\frac{\log_{3z}(x)+1}{3}$

8 It is known that $f\big(g(x)\big)=4x-5$.

What could be the correct functions for $f(x)$ and $g(x)$?

A. $f(x)=2x-7,\ g(x)=2x+2$

B. $f(x)=x-2,\ g(x)=4x-2$

C. $f(x)=3x-4,\ g(x)=4x+1$

D. $f(x)=x+2,\ g(x)=4x-7$

9 What is the simplified expression for $\frac{\sec^2 x-1}{\tan x}+\frac{1-\cos^2 x}{\sin x\cos x}$?

A. $\sin x$

B. $\cot x$

C. $2\cos x$

D. $2\tan x$

4 What is $\frac{2}{x+1}+\frac{4}{x-2}$ expressed as a single fraction?

A. $\frac{6x}{x^2-x-2}$

B. $\frac{-2x-8}{x^2-x-2}$

C. $\frac{6x}{x^2+x-2}$

D. $\frac{2x-8}{x^2+x-2}$

5 A packet of six cards are labelled with the numbers 1, 2, 3, 4, 5 and 6.
One card is drawn at random, its number noted, then the card is returned to the packet.
An experiment is conducted where this process is done three times in total.

What is the probability that a number less than 5 appears on each of the cards?

A. $\frac{1}{3}$

B. $\left(\frac{2}{3}\right)^4$

C. $\left(\frac{2}{3}\right)^3$

D. $\left(\frac{1}{2}\right)^4$

6 It is given that $x<-6$.

Which one of the following is equal to $\frac{|2x^2-72|}{x-6}$?

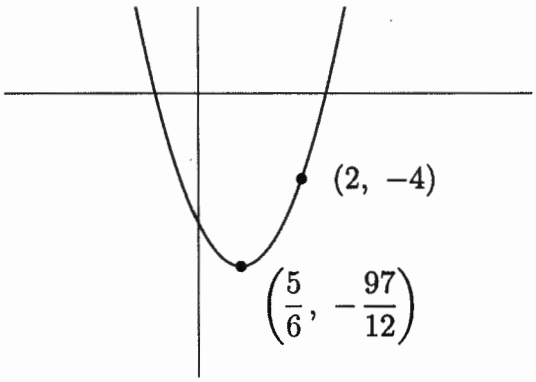
A. $2x+12$

B. $2x-12$

C. $12-2x$

D. $-2x-12$

10 In the diagram, the graph of a parabola $y = ax^2 + bx + c$ is given. Its vertex is $\left(\frac{5}{6}, -\frac{97}{12}\right)$.



What is the value of $a+b+c$?

A. -11

B. -8

C. 0

D. 4

End Of Section I

Section II

65 marks
Attempt Questions 11–29
Allow about 2 hours for this section.
Answer the questions in the spaces provided. Show all necessary working.

Question 11 (2 marks)

Solve $|1-5x|=16$. **2**

Question 12 (2 marks)

The function $f(x)=12-8x$ has the restricted domain $[-2,5)$. **2**

What is its range?

Question 13 (2 marks)

Given $p=\log_a x$ and $q=\log_a y$, find an expression in terms of p and q for $\log_a\left(\frac{x^2}{y}\right)$. **2**

Question 14 (4 marks)

Find the first derivative of each function below:

(a) $f(x)=\sqrt{x}$ **1**

(b) $f(x)=\dfrac{2x+1}{e^{3x}}$. Write your answer in simplest algebraic form. **3**

Question 15 (2 marks)

Find the value(s) of m for which the parabola $y=3x^2-2mx+5m$ has only one x-intercept. **2**

Question 16 (5 marks)

Consider the function $f(x) = 2x^3 - 6x$.

(a) State if $f(x)$ is even, odd or neither. Justify your answer. **2**

(b) Find the values of x for which $f(x) = 0$. **2**

(c) Hence, or otherwise, determined the values of x for which $f(x) > 0$. **1**

Question 17 (3 marks)

Solve the equation $\sin 2x = \frac{1}{2}$ for the domain $-\pi \leq x \leq \pi$. **3**

Question 18 (6 marks)

A fruit shop records the number of watermelon crates it sells per day.

The probability distribution for the number of crates sold, X, is shown in the table:

Number of crates sold, X	**0**	**1**	**2**	**3**	**4**
$P(X)$	0.1	0.25	0.3	0.2	0.15

(a) Calculate the probability that at least one crate is sold on a given day. **1**

Question 18 continues over the page

Question 18 (continued)

(b) For a given day, calculate the probability the shop sells more than two crates, given at least one crate has been sold. **1**

(c) Each crate sold earns the shop \$120. **2**

Determine the expected daily revenue from watermelon crates.

(d) Calculate the variance of the random variable X, correct to two decimal places. **2**

End of Question 18

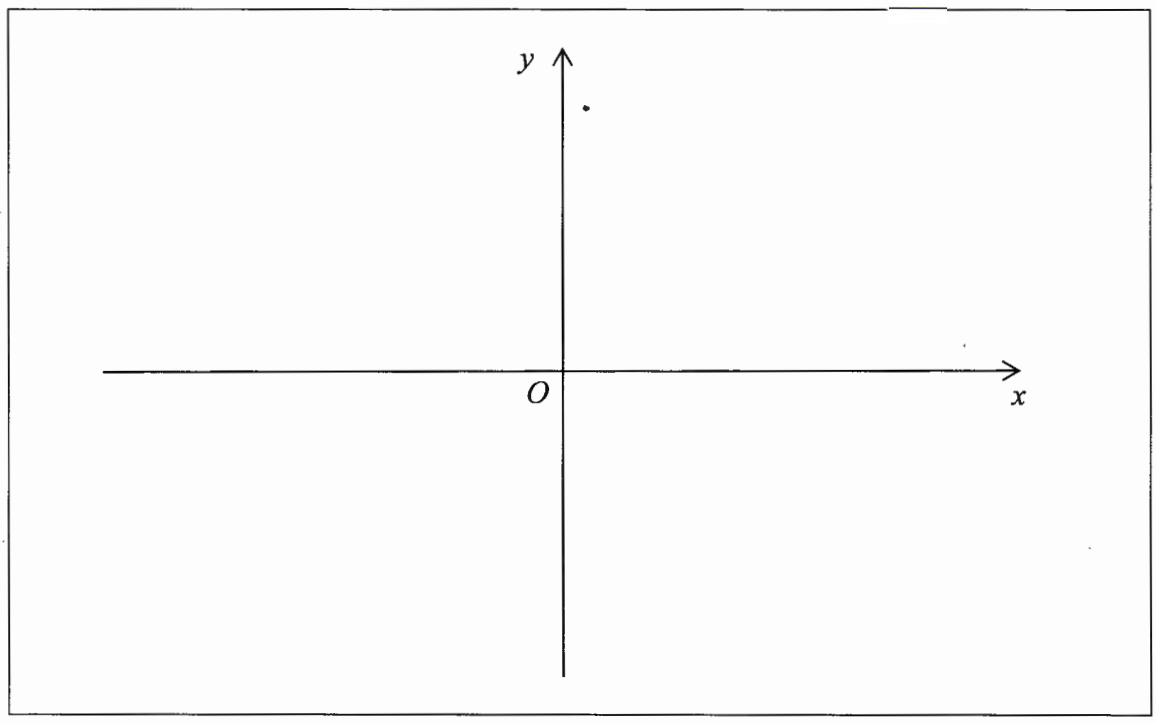
Question 19 (3 marks)

A rescue boat leaves a lighthouse at point L and travels on a bearing of $045°$ for 12 km to reach point M. The boat then changes course and travels on a bearing of $120°$ for 15 km to reach point N. **3**

By drawing a diagram representing this information, find the distance from the lighthouse at point L to point N, correct to one decimal place.

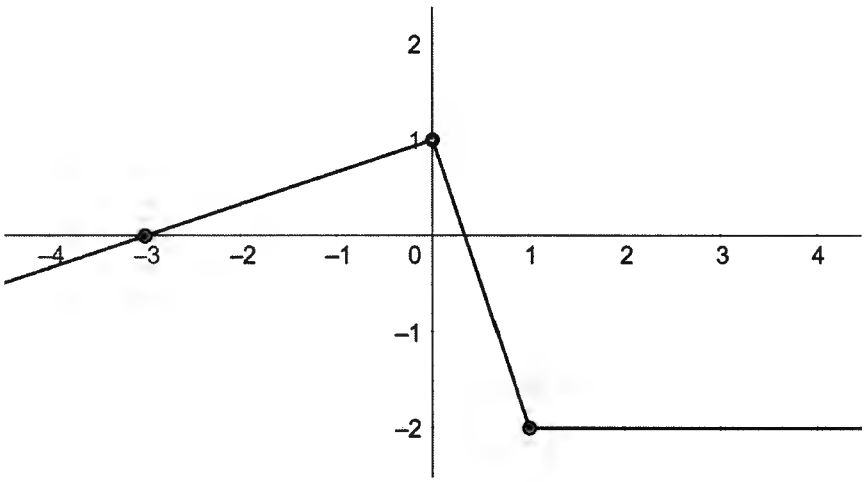
Question 20 (3 marks)

Neatly sketch the graph of $y = \log_2(x+2) - 3$ in the space below, clearly showing all relevant features. **3**



Question 21 (4 marks)

Part of the graph of $y = f(x)$ is shown.



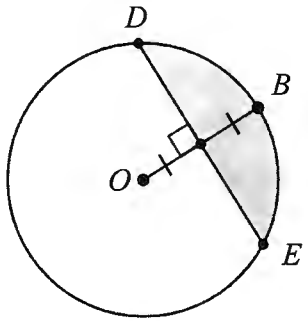
(a) Find $f'\left(\frac{1}{2}\right)$. **1**

(b) State all values of x for which $f'(x)$ is not defined. **1**

(c) Find $f\left(f'\left(-\sqrt{2}\right)\right)$. **2**

Question 22 (4 marks)

In the diagram, O is the centre of a circle, where $OB = 6$. DE is the perpendicular bisector of OB. **4**



Find the exact area of the shaded region.

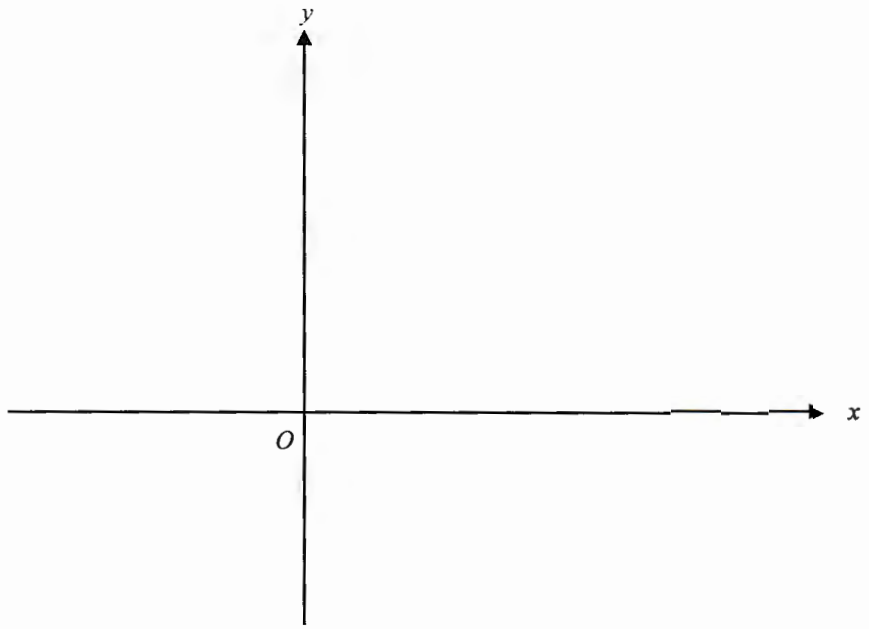
Question 23 (3 marks)

Find the equation of tangent to the curve $y = 3x^2(2x-3)^4$ at the point (1, 3). **3**

Question 24 (4 marks)

Two functions are defined such that $f(x) = |x-1|$ and $g(x) = \sqrt{x+1}$.

(a) On the axes below, draw a sketch of $y = f(x)$ and $y = g(x)$, noting intercepts. **2**



(b) Using your graph, or otherwise, find the values of x for which $f(x) = g(x)$. **2**

Question 25 (4 marks)

The number of rabbits (N) in a forest grows exponentially according to a function of the form $N = Ae^{0.805t}$, where A is a constant and t is time in years.
At $t = 0$, the forest contains 1000 rabbits.

(a) Find the value of A. **1**

(b) How long will it take for the forest to contain 10 000 rabbits? **1**
Answer in years, correct to one decimal place.

(c) Determine the rate at which the rabbit population is increasing when $t = 5$, correct to the nearest hundred. **2**

Question 26 (3 marks)

A continuous and differentiable function is defined as $f(x)=\begin{cases} x^2+ax+c & \text{for } x<3 \\ (4+b)x & \text{for } x\geq 3 \end{cases}$ **3**

find the values of a, b and c if $f(4)=28$.

Question 27 (3 marks)

Prove: $\dfrac{\sec x-\cos x}{\sec x+\cos x}+\dfrac{1-\sin^2 x}{\sin^2 x+2\cos^2 x}=\dfrac{1}{1+\cos^2 x}$. **3**

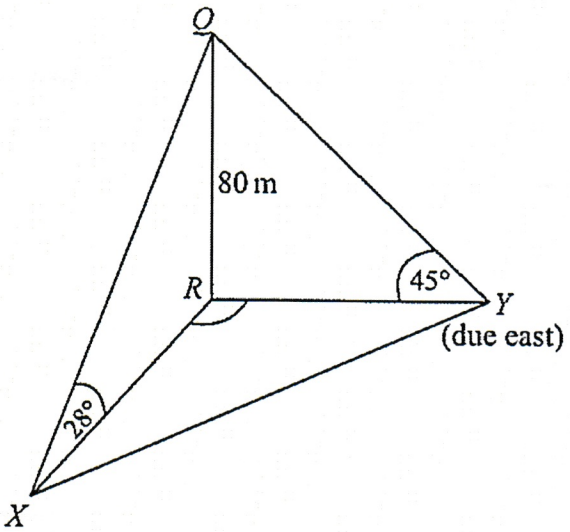
Question 28 (4 marks)

(a) Show: $\dfrac{1}{\sqrt{n}+\sqrt{n+1}}=\sqrt{n+1}-\sqrt{n}$ **2**

(b) Hence or otherwise, evaluate: **2**

$$\frac{1}{\sqrt{0}+\sqrt{1}}+\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}$$

Question 29 (4 marks)



The diagram above shows a vertical tower QR with height 80 metres. Point Y is directly due east of R and the angle of elevation of Q from Y is 45°. From another point X, the bearing of the tower is 030° and the angle of elevation is 28°. The points X, Y and R are on level ground. It can be shown that $\angle XRY = 120°$. (Do NOT prove this.)

Find the distance XY, correct to the nearest metre. **4**

End of Examination

VIRILE AGITUR

Name: _____

Teacher's Name: SOLUTIONS

Place Barcode Sticker here →

2025

YEAR 11 PRELIMINARY EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours 15 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided with this paper

Total marks: 75

Section I – 10 marks (pages 3–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section
- Use the Multiple Choice answer sheet provided.

Section II – 65 marks (pages 7–21)

- Attempt Questions 11–29
- Allow about 2 hours for this section
- Write your working and answers in the spaces provided in this booklet.
- Show relevant mathematical reasoning and/or calculations
- Additional writing space is provided on pages 22-23

M/C	Q11-13	Q14-15	Q16-17	Q18-19	Q20-21	Q22-23	Q24-25	Q26-27	Q28-29	TOTAL
/10	/6	/6	/8	/9	/7	/7	/8	/6	/8	/75

Blank Page

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

---

1 What is an angle of 2 radians closest to in degrees?

A. 90°
B. 103°
C. 115°
D. 127°

$2 \times \frac{180}{\pi} = 114.59^\circ \doteq 115^\circ$

2 A bacteria colony's size in thousands is modelled by $N(t) = 2t^4 - 5t^3 + t^2 + 7$, where t is measured in hours.

What is the instantaneous rate of growth of the colony at $t = 2$?

A. 3
B. 8
C. 17
D. 25

$N(2) = 2(2)^4 - 5(2)^3 + 2^2 + 7$
$= 32 - 40 + 4 + 7$
$= 3$

3 A student is late to school if either their alarm fails or if there is traffic.

Let $P(\text{alarm fails}) = 0.3$ and $P(\text{traffic}) = 0.6$. These two events are independent.

What is the probability the student arrives on time?

A. 0.28
B. 0.42
C. 0.82
D. 0.88

$P(\text{Alarm fails \& Traffic}) = P(\text{Alarm fails}) \cdot P(\text{Traffic})$
$= 0.3 \times 0.6$

$\therefore P(\text{Alarm fails or Traffic}) = P(\text{Alarm fails}) + P(\text{Traffic}) - P(\text{Alarm fails \& Traffic})$
$= 0.3 + 0.6 - 0.3 \times 0.6$
$= 0.72$

$\therefore P(\text{arrives on time}) = 1 - 0.72$
$= 0.28$

4 What is $\frac{2}{x+1}+\frac{4}{x-2}$ expressed as a single fraction?

A. $\frac{6x}{x^2-x-2}$

B. $\frac{-2x-8}{x^2-x-2}$

C. $\frac{6x}{x^2+x-2}$

D. $\frac{2x-8}{x^2+x-2}$

$\frac{2(x-2)+4(x+1)}{(x+1)(x-2)} = \frac{2x-4+4x+4}{(x+1)(x-2)}$

$= \frac{6x}{x^2-x-2}$

5 A packet of six cards are labelled with the numbers 1, 2, 3, 4, 5 and 6.
One card is drawn at random, its number noted, then the card is returned to the packet.
An experiment is conducted where this process is done three times in total.

What is the probability that a number less than 5 appears on each of the cards?

A. $\frac{1}{3}$

B. $\left(\frac{2}{3}\right)^4$

C. $\left(\frac{2}{3}\right)^3$

D. $\left(\frac{1}{2}\right)^4$

$P(<5) = \frac{4}{6} = \frac{2}{3}$

$P(<5)^3 = \left(\frac{2}{3}\right)^3$

6 It is given that $x<-6$.

Which one of the following is equal to $\frac{|2x^2-72|}{x-6}$?

A. $2x+12$

B. $2x-12$

C. $12-2x$

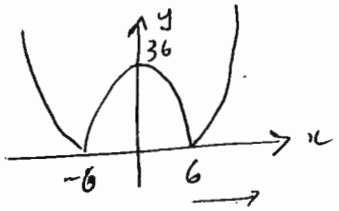
D. $-2x-12$

$\frac{2|x^2-36|}{x-6}$

$\frac{2(x-6)(x+6)}{x-6}$

$2(x+6)$

$2x+12$



7 What is the correct expression for y if $x^{3y-1}=3z$?

A. $y=\log_x(z)+\frac{1}{3}$

B. $y=3\log_3(3z)-1$

C. $y=\frac{\log_x(3z)+1}{3}$

D. $y=\frac{\log_{3z}(x)+1}{3}$

$\log_x x^{3y-1} = \log_x 3z$

$(3y-1)\log_x x = \log_x 3z$

$3y-1 = \log_x 3z$

$y = \frac{1}{3}(1+\log_x 3z)$

8 It is known that $f(g(x))=4x-5$.

What could be the correct functions for $f(x)$ and $g(x)$?

A. $f(x)=2x-7,\ g(x)=2x+2$

B. $f(x)=x-2,\ g(x)=4x-2$

C. $f(x)=3x-4,\ g(x)=4x+1$

D. $f(x)=x+2,\ g(x)=4x-7$

$f(2x+2) = 2(2x+2)-7 = 4x-3$ ×

$f(4x-2) = 4x-2-2 = 4x-4$ ×

$f(4x+1) = 3(4x+1)-4 = 12x-1$ ×

$f(4x-7) = 4x-7+2 = 4x-5$ ✓

9 What is the simplified expression for $\frac{\sec^2 x-1}{\tan x}+\frac{1-\cos^2 x}{\sin x\cos x}$?

A. $\sin x$

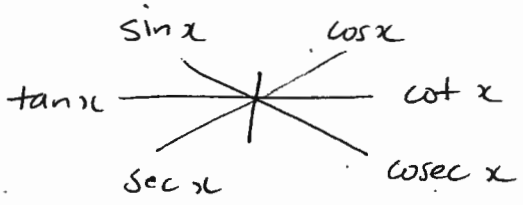
B. $\cot x$

C. $2\cos x$

D. $2\tan x$

$\frac{\tan^2 x}{\tan x} + \frac{\sin^2 x}{\sin x\cos x} = \tan x + \tan x$

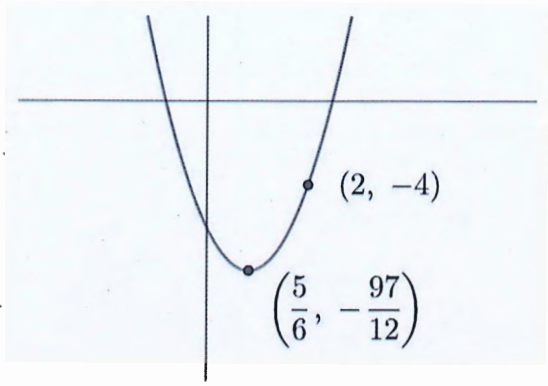
$= 2\tan x$



$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

10 In the diagram, the graph of a parabola $y=ax^2+bx+c$ is given. Its vertex is $\left(\frac{5}{6},-\frac{97}{12}\right)$.



What is the value of $a+b+c$?

A. −11

B. −8

C. 0

D. 4

End Of Section I

$y = a\left(x-\frac{5}{6}\right)^2 - \frac{97}{12}$

$(2,-4)$ lies on it, so satisfies eqn:

$-4 = a\left(2-\frac{5}{6}\right)^2 - \frac{97}{12}$

$a = 3$

also, axis of symmetry is $-\frac{b}{2a} = x$

$\frac{5}{6} = \frac{-b}{2a}$

$\frac{5}{6} = \frac{-b}{6}$

$b = -5$

$\therefore y = 3x^2 - 5x + c$

$(2,-4)$ lies on this:

$-4 = 3(2)^2 - 5(2) + c$

$c = -6$

$\therefore a+b+c = 3-5-6$

$= -8$

Section II

65 marks
Attempt Questions 11–29
Allow about 2 hours for this section.
Answer the questions in the spaces provided. Show all necessary working.

Question 11 (2 marks)

Solve $|1-5x|=16$. **2**

$1-5x=16$ OR $5x-1=16$ ✓

$5x=-15$ $5x=17$

$x=-3$ $x=\frac{17}{5}$ ✓

Question 12 (2 marks)

The function $f(x)=12-8x$ has the restricted domain $[-2,5)$. **2**

What is its range?

$f(x)$ is a line, so is continuous for $[-2,5)$

$f(-2)=12-8(-2)=28$

$f(5)=12-8(5)=-28$. } ✓

$\therefore$ Range is $(-28,28]$ or $-28<y\leq 28$ ✓

Question 13 (2 marks)

Given $p=\log_a x$ and $q=\log_a y$, find an expression in terms of p and q for $\log_a\left(\frac{x^2}{y}\right)$. **2**

$\log_a \frac{x^2}{y} = \log_a x^2 - \log_a y$

$= 2\log_a x - \log_a y$ ✓

$= 2p-q$ ✓

Question 14 (4 marks)

Find the first derivative of each function below:

(a) $f(x)=\sqrt{x}$ 1

$= x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{1}{2\sqrt{x}}$ ✓

(b) $f(x)=\frac{2x+1}{e^{3x}}$. Write your answer in simplest algebraic form. 3

$f'(x) = \frac{e^{3x} \cdot 2 - (2x+1)(3)e^{3x}}{(e^{3x})^2}$ ✓

$= \frac{2e^{3x} - 6xe^{3x} - 3e^{3x}}{(e^{3x})^2}$ ✓

$= \frac{-e^{3x}(1+6x)}{(e^{3x})^2} = \frac{-(1+6x)}{e^{3x}}$ ✓

Question 15 (2 marks)

Find the value(s) of m for which the parabola $y=3x^2-2mx+5m$ has only one x-intercept. 2

Need $\Delta = 0$

$\Delta = (2m)^2 - 4(3)(5m)$

$\Delta = 4m^2 - 60m$ ✓

$\therefore 4m^2 - 60m = 0$

$m^2 - 15m = 0$

$m(m-15) = 0$

$m = 0$ or $m = 15$ ✓

Question 16 (5 marks)

Consider the function $f(x) = 2x^3 - 6x$.

(a) State if $f(x)$ is even, odd or neither. Justify your answer. **2**

$f(x) = 2x^3 - 6x$

$-f(x) = -2x^3 + 6x$

$f(-x) = 2(-x)^3 - 6(-x)$

$= -2x^3 + 6x$ ✓

$\therefore$ odd function as $f(-x) = -f(x)$ ✓

(b) Find the values of x for which $f(x) = 0$. **2**

$2x^3 - 6x = 0$

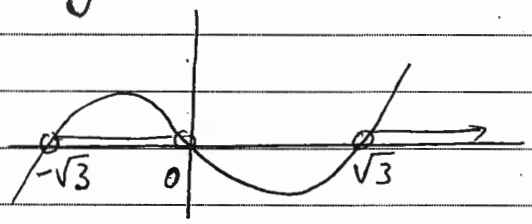
$2x(x^2 - 3) = 0$

$x = 0$ or $x^2 = 3$ ✓

$x = \pm\sqrt{3}$ ✓

(c) Hence, or otherwise, determined the values of x for which $f(x) > 0$. **1**

$y = 2x^3 - 6x$



$-\sqrt{3} < x < 0$ OR $x > \sqrt{3}$ ✓

AH $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

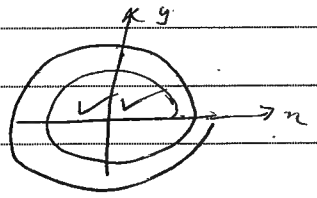
Question 17 (3 marks)

Solve the equation $\sin 2x = \frac{1}{2}$ for the domain $-\pi \leq x \leq \pi$. **3**

$-2\pi \leq 2x \leq 2\pi$

Acute $2x$: $2x = \sin^{-1}\frac{1}{2}$

$2x = \frac{\pi}{6}$



$\therefore 2x = \frac{\pi}{6}, \pi - \frac{\pi}{6} = \frac{5\pi}{6}, \frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}, \frac{5\pi}{6} - 2\pi = -\frac{7\pi}{6}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, -\frac{11\pi}{12}, -\frac{7\pi}{12}$

Question 18 (6 marks)

A fruit shop records the number of watermelon crates it sells per day.

The probability distribution for the number of crates sold, X, is shown in the table:

Number of crates sold, X	0	1	2	3	4
$P(X)$	0.1	0.25	0.3	0.2	0.15

(a) Calculate the probability that at least one crate is sold on a given day. **1**

$1 - P(0)$

$= 1 - 0.1$

$= 0.9$

Question 18 continues over the page

Question 18 (continued)

(b) For a given day, calculate the probability the shop sells more than two crates, given at least one crate has been sold. **1**

$$\frac{0.2 + 0.15}{0.9}$$

$$= \frac{0.35}{0.9}$$

$$= \frac{7}{18}$$ ✓

(c) Each crate sold earns the shop \$120. **2**

Determine the expected daily revenue from watermelon crates.

$E(X) = 1 \times 0.25 + 2 \times 0.3 + 3 \times 0.2 + 4 \times 0.15$ ✓

$= 2.05$

$\therefore 120 \times 2.05 = \$246$ ✓

(d) Calculate the variance of the random variable X, correct to two decimal places. **2**

✓

$Var(X) = 1^2 \times 0.25 + 2^2 \times 0.3 + 3^2 \times 0.2 + 4^2 \times 0.15 - 2.05^2$

$= 1.4475$

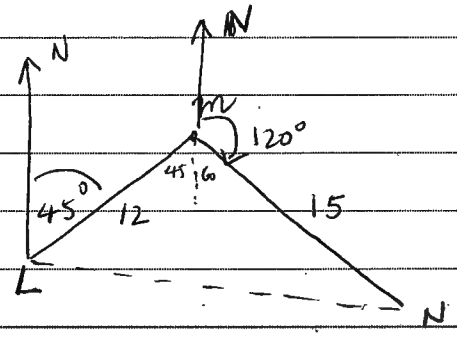
$= 1.45$ (2 dp) ✓

End of Question 18

Question 19 (3 marks)

A rescue boat leaves a lighthouse at point L and travels on a bearing of 045° for 12 km to reach point M. The boat then changes course and travels on a bearing of 120° for 15 km to reach point N. **3**

By drawing a diagram representing this information, find the distance from the lighthouse at point L to point N, correct to one decimal place.



$\angle LMN = 45^\circ + 60 = 105^\circ$ ✓

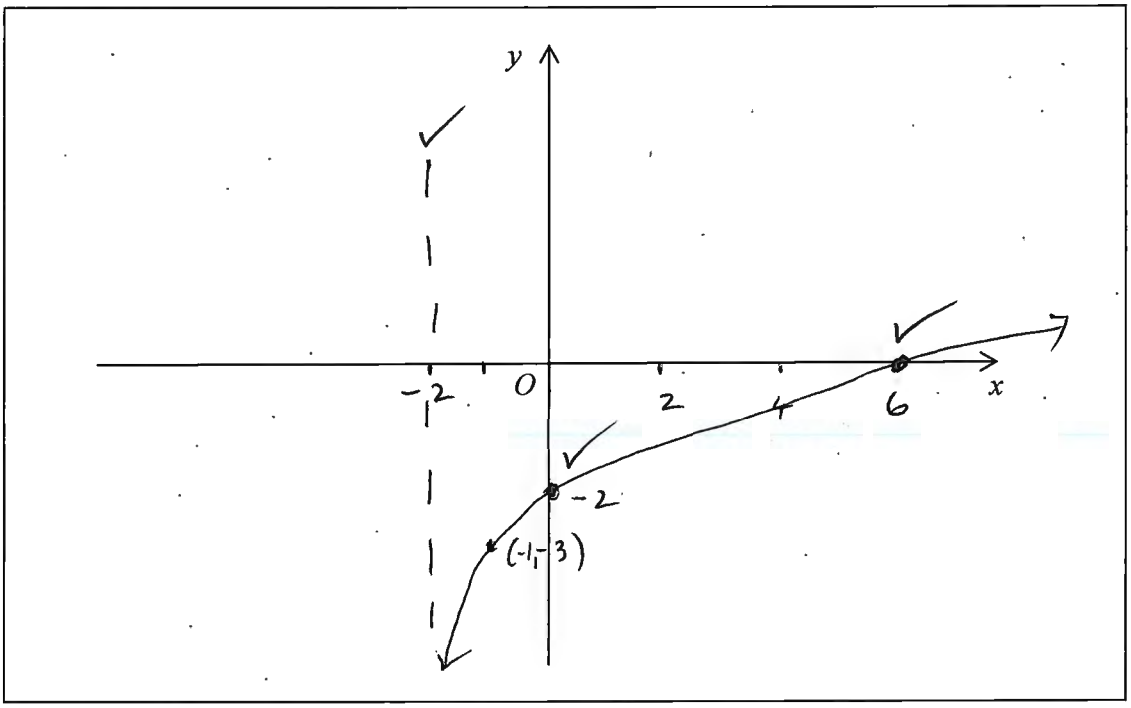
$LN^2 = 12^2 + 15^2 - 2(12)(15)\cos 105^\circ$ ✓

$\doteq 462.17$

$LN = 21.5$ km (1dp) ✓

Question 20 (3 marks)

Neatly sketch the graph of $y = \log_2(x+2) - 3$ in the space below, clearly showing all relevant features. **3**



x int ($y=0$)

$3 = \log_2(x+2)$

$2^3 = x+2$

$x = 6$

y int ($x=0$)

$y = \log_2 2 - 3$

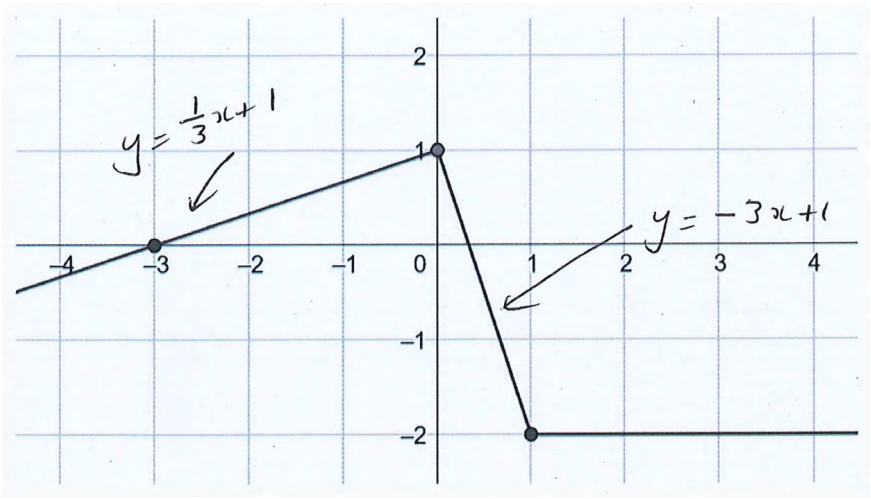
$y = -2$

VA : $x = -2$

$y = \log_2 x$ moved (L) 2 units
moved down 3 units.

Question 21 (4 marks)

Part of the graph of $y = f(x)$ is shown.



(a) Find $f'\left(\frac{1}{2}\right)$. **1**

$= -3$ $\left(m = \frac{-3}{1} = -3\right)$ ✓

(b) State all values of x for which $f'(x)$ is not defined. **1**

$x = 0$ and $x = 1$ ① for both x values.

(c) Find $f\left(f'\left(-\sqrt{2}\right)\right)$. **2**

$f\left(f'\left(-\sqrt{2}\right)\right)$

$= f\left(\frac{1}{3}\right)$ ✓

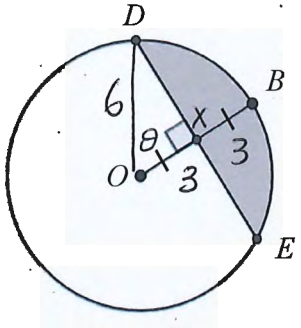
$= -3\left(\frac{1}{3}\right) + 1$

$= -1 + 1$ ✓

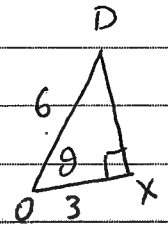
$= 0$

Question 22 (4 marks)

In the diagram, O is the centre of a circle, where $OB = 6$. DE is the perpendicular bisector of OB. **4**



Find the exact area of the shaded region.



$\cos\theta = \frac{3}{6}$

$\theta = \frac{\pi}{3}$ ✓

$A_{\Delta ODX} = \frac{1}{2} \times 3 \times 6 \sin\frac{\pi}{3}$

$= 9 \times \frac{\sqrt{3}}{2}$

$= \frac{9\sqrt{3}}{2}$ ✓

$A_{\text{sector } \angle OB} = \frac{1}{2} \times 6^2 \times \frac{\pi}{3} = 6\pi$

$\therefore A_{\text{shaded DXB}} = 6\pi - \frac{9\sqrt{3}}{2}$ ✓

$\therefore A_{\text{shaded segment}} = 2\left(6\pi - \frac{9\sqrt{3}}{2}\right)$

$= 12\pi - 9\sqrt{3} \ u^2$ ✓

Question 23 (3 marks)

Find the equation of tangent to the curve $y = 3x^2(2x-3)^4$ at the point (1, 3). **3**

$y = 3x^2(2x-3)^4$

$\frac{dy}{dx} = 3x^2 \times 4(2)(2x-3)^3 + (2x-3)^4(6x)$ ✓

$= 6x(2x-3)^3[4x + 2x - 3]$

$= 6x(2x-3)^3(6x-3)$

When $x = 1$ $m_T = 6(-1)(3) = -18$ ✓

∴ Eqⁿ of tangent: $y - 3 = -18(x-1)$

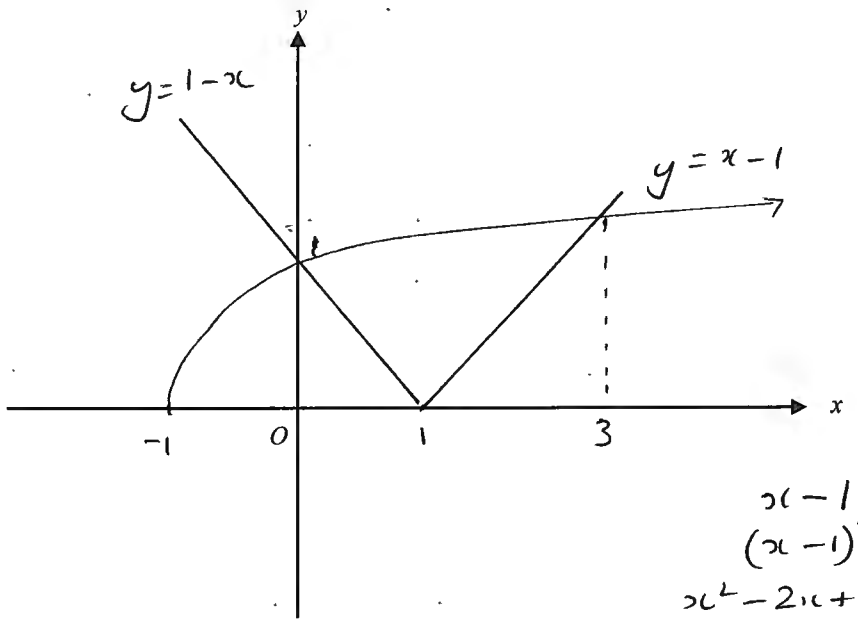
$y - 3 = -18x + 18$

$y = -18x + 21$ ✓

Question 24 (4 marks)

Two functions are defined such that $f(x)=|x-1|$ and $g(x)=\sqrt{x+1}$.

(a) On the axes below, draw a sketch of $y=f(x)$ and $y=g(x)$, noting intercepts. **2**



$x-1=\sqrt{x+1}$

$(x-1)^2=x+1$

$x^2-2x+1=x+1$

$x^2-3x=0$

$x=3 \quad (x\neq 0)$

(b) Using your graph, or otherwise, find the values of x for which $f(x)=g(x)$. **2**

$x=0$ and $x=3$

$1-x=\sqrt{x+1}$

$(1-x)^2=x+1$

$x^2-2x+1=x+1$

$x^2-3x=0$

$x(x-3)=0$

$x=0 \quad (x\neq -3)$

Question 25 (4 marks)

The number of rabbits (N) in a forest grows exponentially according to a function of the form $N = Ae^{0.805t}$, where A is a constant and t is time in years.
At $t = 0$, the forest contains 1000 rabbits.

(a) Find the value of A. **1**

When $t=0$, $N=1000$

$\therefore 1000 = Ae^0$

$1000 = A$ ✓

(b) How long will it take for the forest to contain 10 000 rabbits? **1**

Answer in years, correct to one decimal place.

$N = 1000e^{0.805t}$

$10\,000 = 1000e^{0.805t}$

$10 = e^{0.805t}$

$\ln 10 = 0.805t$ ✓

$\frac{\ln 10}{0.805} = t$

$\therefore t = 2.86$ years

$t = 2.9$ years (1 dp) ~~3/4 months~~ ✓

(c) Determine the rate at which the rabbit population is increasing when $t = 5$, correct to the nearest hundred. **2**

$\frac{dN}{dt} = 0.805 \times 1000e^{0.805t}$ ✓

When $t=5$,

$\frac{dN}{dt} = 0.805 \times 1000 \times e^{0.805\times 5}$

$= 45064.148$

$= 45100$ (nearest hundred) ✓

$\therefore$ Rate of increase is 45100 rabbits/year when $t=5$.

Question 26 (3 marks)

A continuous and differentiable function is defined as $f(x)=\begin{cases} x^2+ax+c & \text{for } x<3 \\ (4+b)x & \text{for } x\geq 3 \end{cases}$ **3**

find the values of a, b and c if $f(4)=28$.

$f(4)=28 \Rightarrow (4+b)4=28$

$b+4=7$

$b=3$ ✓

Since its continuous,

$x^2+ax+c=(4+b)x$ for $x=3$

$3^2+3a+c=7\times 3$

$3a+c=12$ --- (1)

Since its differentiable then

$f'(3)=7$

$2x+a=7$

$2(3)+a=7$

$a=1$ ✓

∴ From (1) $3(1)+c=12$

$c=9$ ✓

$\therefore a=1,\ b=3,\ c=9$

Question 27 (3 marks)

Prove: $\dfrac{\sec x-\cos x}{\sec x+\cos x}+\dfrac{1-\sin^2 x}{\sin^2 x+2\cos^2 x}=\dfrac{1}{1+\cos^2 x}$. **3**

$$LHS = \frac{\frac{1}{\cos x}-\cos x}{\frac{1}{\cos x}+\cos x}+\frac{\cos^2 x}{(1-\cos^2 x)+2\cos^2 x} \quad \checkmark$$

$$= \frac{1-\cos^2 x}{1+\cos^2 x}+\frac{\cos^2 x}{1+\cos^2 x} \quad \checkmark$$

$$= \frac{1-\cos^2 x+\cos^2 x}{1+\cos^2 x} \quad \checkmark$$

$$= \frac{1}{1+\cos^2 x}$$

$$= RHS$$

Question 28 (4 marks)

(a) Show: $\dfrac{1}{\sqrt{n}+\sqrt{n+1}}=\sqrt{n+1}-\sqrt{n}$ **2**

$$LHS = \frac{1}{\sqrt{n}+\sqrt{n+1}} \times \frac{\sqrt{n}-\sqrt{n+1}}{\sqrt{n}-\sqrt{n+1}}$$

$$= \frac{\sqrt{n}-\sqrt{n+1}}{n-(n+1)} \quad \checkmark$$

$$= \frac{\sqrt{n}-\sqrt{n+1}}{-1} \quad \checkmark$$

$$= \sqrt{n+1}-\sqrt{n}$$

$$= RHS$$

(b) Hence or otherwise, evaluate: **2**

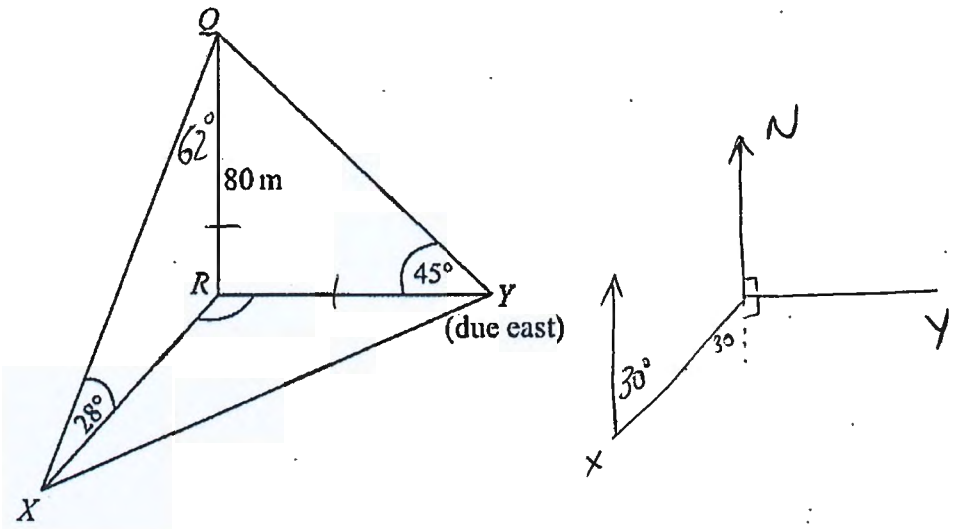
$$\left(\frac{1}{\sqrt{0}+\sqrt{1}}\right)+\left(\frac{1}{\sqrt{1}+\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}+\sqrt{3}}\right)+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}$$

$$\sqrt{1}-\sqrt{0}+\sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+\ldots+\sqrt{99}-\sqrt{98}+\sqrt{100}-\sqrt{99} \quad \checkmark$$

$$= -\sqrt{0}+\sqrt{100}$$

$$= \sqrt{100} \quad \checkmark$$

Question 29 (4 marks)



The diagram above shows a vertical tower QR with height 80 metres. Point Y is directly due east of R and the angle of elevation of Q from Y is 45°. From another point X, the bearing of the tower is 030° and the angle of elevation is 28°. The points X, Y and R are on level ground. It can be shown that $\angle XRY = 120°$. (Do NOT prove this.)

Find the distance XY, correct to the nearest metre. **4**

$\tan 62° = \frac{XR}{80}$

$XR = 80 \tan 62°$ ✓ $\quad YR = 80$

$XY^2 = XR^2 + YR^2 - 2(XR)(YR)\cos \angle XRY$

$= 80^2 \tan^2 62° + 80^2 - 2 \times 80^2 \tan 62° \times \cos 120°$ ✓

$= 80^2 \left[\tan^2 62° + 1 + \frac{2}{2} \tan 62°\right]$

$= 80^2 \left[\tan^2 62° + \tan 62° + 1\right]$

$\doteq 41074.29$ ✓

$XY = \sqrt{41074.29}$

$XY \doteq 202.668$

$XY = 203$ m (nearest metre)

✓

End of Examination