



Write your student exam number in the boxes

--	--	--	--	--	--	--	--	--	--

**Northern Beaches Secondary College
Manly Campus**

2022

HIGHER SCHOOL CERTIFICATE

Year 11 Examination

Mathematics Advanced

General

Instructions

- Reading time – 10 minutes
- Working time – 2.5 hours
- Write using black/blue pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks:

90

Section I – 10 marks (pages 2-4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section
- Answer on Multiple Choice Answer Sheet provided

Section II – 80 marks (pages 5-22)

- Attempt Questions 11–33
- Allow about 2 hour and 15 minutes for this section
- Answer in the spaces provided

This paper MUST NOT be removed from the examination room

MARKS	MC	Q11-18	Q19-25	Q26-28	Q29-30	Q31-33	TOTAL
STUDENT MARK							
MAXIMUM	10	18	19	15	14	14	90



**Northern Beaches Secondary College
Manly Campus**

2022

HIGHER SCHOOL CERTIFICATE

Year 11 Examination

Mathematics Advanced

Multiple-Choice Answer Sheet

Select the alternative A, B, C, or D that best answers the question by placing a **X** in the box.

	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Section I - 10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1) Which of the following is equivalent to $(2x + y)^2 - (2x - y)^2$?

- A. 0 B. y^2 C. $4xy$ D. $8xy$

2) Which of the following is equal to $\log_2 5$?

A. $\ln 5 - \ln 2$

B. $\ln 5 + \ln 2$

C. $\frac{\ln 5}{\ln 2}$

D. $\frac{\ln 2}{\ln 5}$

3) Let f and g be two functions such that $f(3) = 6$, $f(5) = 3$, $g(6) = 3$, $g(4) = 5$.
What is the value of $f(g(4))$?

- A. 3 B. 4 C. 5 D. 6

4) Which of following is the natural domain of $y = \ln(2x - 1)$?

A. $\left[\frac{1}{2}, \infty\right)$

B. $\left[-\infty, \frac{1}{2}\right)$

C. $\left(\frac{1}{2}, \infty\right)$

D. $\left(-\infty, \frac{1}{2}\right)$

5) How many solutions does the equation $\sin^2 x + 2\sin x = 0$ have for $0 \leq x \leq 2\pi$?

- A. 1 B. 2 C. 3 D. 4

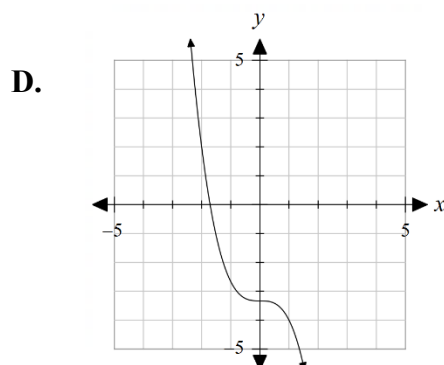
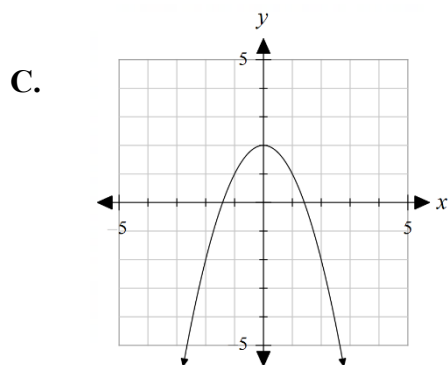
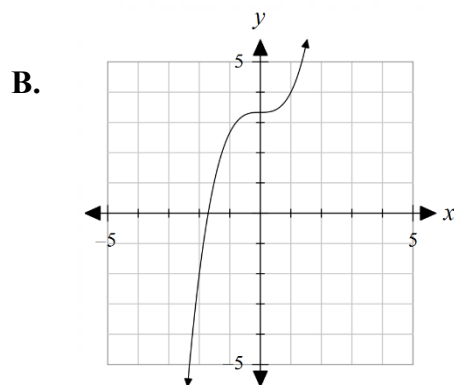
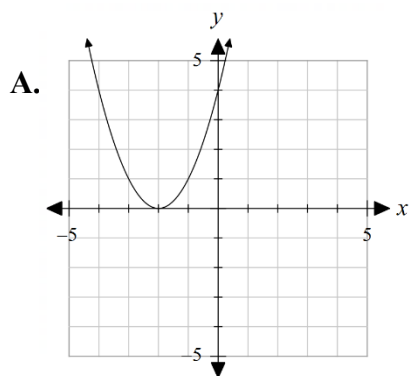
6) For the random variable X , the following probability distribution is given.

x	0	1	2	3
$P(X = x)$	$3k$	0.2	0.4	k

What is $P(X \geq 2)$?

- A. 0.2
B. 0.3
C. 0.4
D. 0.5

7) Which of the following graphs would have a tangent of $y = -2x + 3$ at the point where $x = 1$?



8) Which of the following is equivalent to $\frac{\cos 60^\circ}{1 - \cos(90^\circ - \theta)} + \frac{\cos 240^\circ}{1 - \sin(360^\circ - \theta)}$?

- A. $\sec^2 \theta$
- B. $\cot \theta \cos \theta$
- C. $\tan \theta \sec \theta$
- D. $\cot \theta \sec \theta$

9) A and B are two events such that $P(\overline{A}) = 0.3$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$.
What is $P(A \cup B)$?

- A. 0.5 B. 0.6 C. 0.7 D. 0.8

10) If $y = xf(x^2)$ which of the following is $\frac{dy}{dx}$?

- A. $xf(x^2) + f(2x)$
- B. $f(x^2) + 2x^2 f'(x^2)$
- C. $f(x^2) + 2x^2 f(x^2)$
- D. $f(x^2) + x^2 f'(2x)$

2022

HIGHER SCHOOL CERTIFICATE

Year 11 Examination

Mathematics Advanced

Section II Answer Booklet

80 marks

Attempt Questions 11–33

Allow about 2 hour and 15 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
-

Question 11 (4 marks)

a) Simplify $\frac{1}{x-2} - \frac{4}{x^2-4}$

2

b) Simplify $\frac{(x^{-3}y^2)^3}{y^7} \times \frac{x^2y^{-3}}{xy^3}$ expressing your answer with positive indices.

2

Question 12 (2 marks)

Solve $|3x - 6| = 15$

2

Question 13 (2 marks)

Find 9^x if $\log_3 7 = x$.

2

Question 14 (2 marks)

State the domain and range for the function $y = 2^{-x} - 3$.

2

Question 15 (2 marks)

Solve $\tan x = -\sqrt{3}$ for $0^\circ \leq x \leq 360^\circ$.

2

Question 16 (2 marks)

Given $\log_a 4 = x$ and $\log_a 6 = y$, find an expression for $\log_a 96$ in terms of x and y .

2

Question 17 (2 marks)

For what value of k does the equation $x^2 - 3x + k = 0$ have equal roots?

2

Question 18 (2 marks)

Solve $\log_2 x - \log_2 4 = \log_2 \left(\frac{x-3}{2} \right)$.

2

Question 19 (2 marks)

Find the values of a and b for the function below given that $g(x)$ is continuous for all x .

2

$$g(x) = \begin{cases} x^2 - 4 & x < -1 \\ ax + b & -1 \leq x \leq 2 \\ 3x + 6 & x > 2 \end{cases}$$

Question 20 (2 marks)

Given $\tan \theta = -\frac{8}{5}$ for an obtuse angle θ , find the exact value of $\sin \theta$.

2

Question 21 (4 marks)

$\angle ABC$	$\angle BAC = 0.6$
--------------	--------------------

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

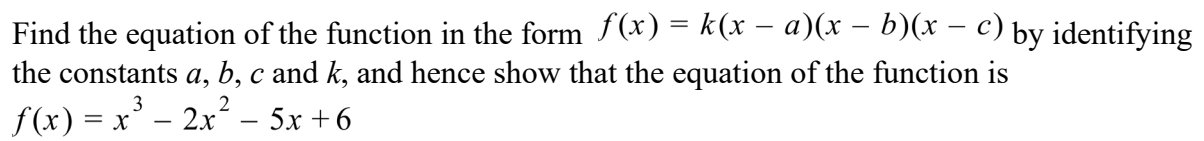
Question 22 (3 marks)

Find the equation of the tangent to the curve $y = x^2 e^{5x}$ at $x = 1$.

3

[illegible]

The graph of a cubic function $y = f(x)$ is shown below.



11

Question 24 (3 marks)

What is the average rate of change of the function

$$f(x) = \sin 2x \text{ between } x = \frac{\pi}{12} \text{ and } x = \frac{\pi}{8}?$$

Give your answer in exact form.

3

Question 25 (2 marks)

Three students sit a difficult English test. The probabilities that the three students pass the test are 25%, 40% and 55%. What is the probability that at least one student will pass?

2

Question 26 (7 marks)

Differentiate

a) $y = 3x^3 - 5x^2 + 2$

1

b) $y = \sqrt[3]{x} - \sqrt{x}$

1

c) $y = \frac{2+x^2}{x}$

2

d) $y = \frac{3x}{2x-1}$

1

e) $y = 5(4x+7)^3$

2

Question 27 (6 marks)

A biased die has a probability distribution for the outcome of the die being rolled as follows.

x	1	2	3	4	5	6
$P(X=x)$	0.2	0.2	0.1	0.25	0.1	0.15

- a) Calculate the expected value.

1

- b)** Calculate $E(X^2)$ and hence find the variance.

2

- c)** Find $P(\mu - \sigma < X < \mu + \sigma)$.

2

- d)** Find $P(X \text{ is odd} \mid X > 3)$.

1

Question 28 (2 marks)

Sound intensity is measured in watts per square metre, W/m^2 . A sound that is just barely audible to the human ear is taken as a threshold intensity, $I_0 = 10^{-12} W/m^2$.

The loudness of a sound, B , in decibels (dB) is measured on a logarithmic scale and can be calculated with the formula $B = 10 \log_{10} \frac{I}{I_0}$

- a)** Show that the loudness of the threshold sound is 0 dB

1

Occupational health and safety regulations state that hearing protection must be worn if loudness levels in an area exceed 85 dB.

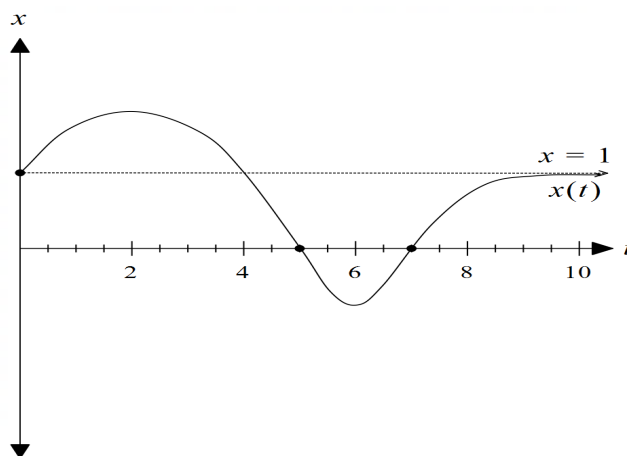
- b)** An industrial machine has a sound intensity of $0.05 W/m^2$. Determine if workers should be required to wear hearing protection when working near this machine.

1

Question 29 (5 marks)

A particle is moving along the x -axis, and its displacement from the origin is given by $x(t)$, where t is the time in seconds. $x(t)$ has a horizontal asymptote along the line $x = 1$.

The graph of $x(t)$ is shown below.



a) When is the particle at rest?

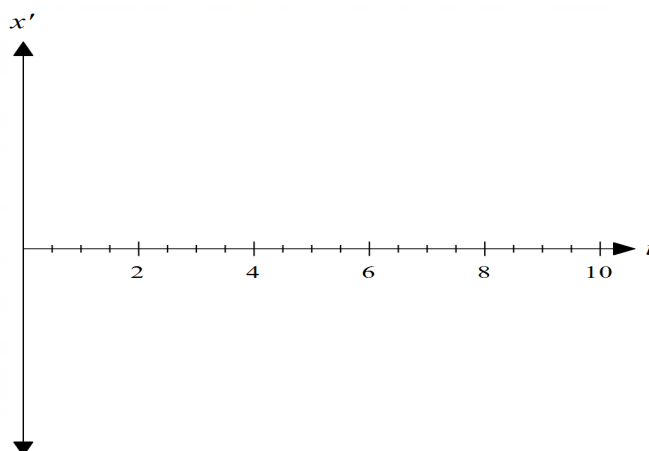
1

b) When is the particle to the left of the origin?

1

c) Sketch a graph of $x'(t)$. Mark any intercepts and asymptotes.

3



Question 30 (9 marks)

The koala population, P , in a sanctuary is modelled by the formula $P = 150 - 60e^{-0.02t}$, where t is time in weeks after the observations commence.

- a) Use differentiation to explain why the population is increasing.

2

- b) What is the initial koala population?

1

- c) What will the koala population be after 50 weeks?
Give your answer correct to the nearest whole number.

1

Question 30 continues on page 18

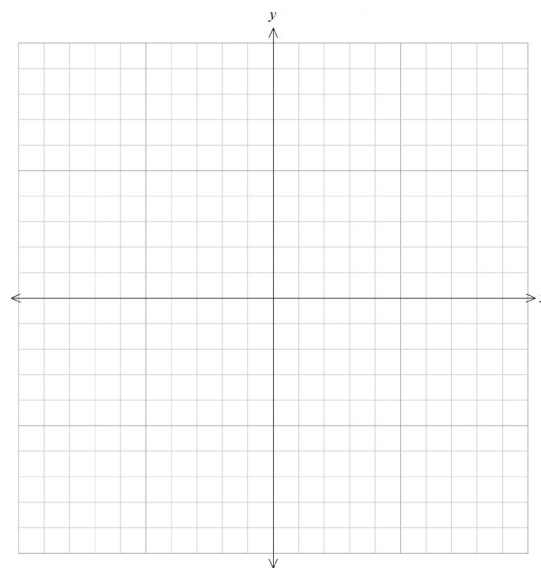
Question 30 (continued)

- d)** How long does it take for the population to reach 140?
Give your answer correct to the nearest week.

2

- e)** Sketch a graph of the model on the axis below.

2

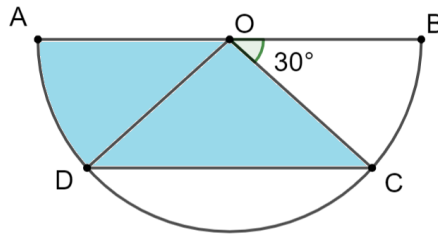


- f)** Explain why the population cannot exceed 150.

1

Question 31 (4 marks)

In the diagram, O is the centre of a semicircle, where $AB = 12$, $\angle BOC = 30^\circ$ and $AB \parallel CD$.



Find the exact area of the shaded region.

4

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 32 (5 marks)

a) Prove the following identity: $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta + 1} = \sec\theta$

2

b) Hence solve the equation $\frac{\sin 2\theta}{\cos 2\theta} + \frac{\cos 2\theta}{\sin 2\theta + 1} = 2$ for $0 \leq \theta \leq 2\pi$.
Give your answer(s) in radians.

3

Question 33 (5 marks)

In a group of 35 students, it is known that:

- 16 students study Biology
- 18 students study Chemistry
- 9 students study neither Biology nor Chemistry

- a) By using a Venn diagram, find the probability that a randomly selected student studies both Biology and Chemistry.

2

- b) Given that a randomly selected student studies Chemistry, what is the probability that they do NOT study Biology?

1

Question 33 continues on page 22

Question 33 (continued)

- c) Two students are randomly selected, one after the other.

Given that the first student studies Chemistry, what is the probability that the second student studies Biology?

2

End of paper

	Solutions	Marking Guidelines
1	$= ((2x + y) + (2x - y))((2x + y) - (2x - y))$ $= (4x)(2y)$ $= 8xy$	D
2	USE CHANGE OF BASE FORMULA ON REF SHEET	C
3	$f(g(4)) = f(5) = 3$	A
4		C
5	$\sin^2 x + 2\sin x = 0, 0 \leq x \leq 2\pi$ $\sin x(\sin x + 2) = 0$ $\sin x = 0, x = 0, \pi, 2\pi$ $\sin x = -2, \text{ no solution}$	C
6	$4k + 0.6 = 1$ $4k = 0.4$ $k = 0.1$ $P(X \geq 2)$ $= P(X = 2) + P(X = 3)$ $= 0.4 + 0.1$ $= 0.5$	D
7	From $y = -2x + 1$ the gradient is negative at $x=1$	C

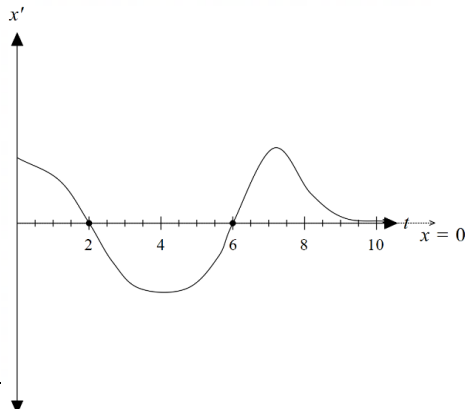
8	$\frac{\cos 60^\circ}{1 - \cos(90 - \theta)} + \frac{\cos 240^\circ}{1 - \sin(360 - \theta)}$ $= \frac{\cos 60^\circ}{1 - \sin\theta} + \frac{\cos 240^\circ}{1 - (-\sin\theta)}$ $= \frac{1}{2} \left(\frac{1}{1 - \sin\theta} \right) - \frac{1}{2} \left(\frac{1}{1 + \sin\theta} \right)$ $= \frac{1}{2} \left(\frac{1}{1 - \sin\theta} - \frac{1}{1 + \sin\theta} \right)$ $\frac{1}{2} \left(\frac{1 + \sin\theta - (1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)} \right)$ $\frac{1}{2} \left(\frac{2\sin\theta}{1 - \sin^2\theta} \right)$ $\frac{1}{2} \left(\frac{2\sin\theta}{\cos^2\theta} \right) = \frac{(\sin\theta)}{(\cos\theta)} \left(\frac{1}{\cos\theta} \right)$ $\tan\theta \cdot \sec\theta$	C
9	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= 0.7 + 0.5 - 0.4$ $= 0.8$	D
10	$u = x$ $u' = 1$ $v = f(x^2)$ $v' = 2x f'(x^2) \text{ (chain rule)}$ $\therefore y' = f(x^2) + 2x^2 f'(x^2)$	B
11a	$\frac{1}{x-2} - \frac{4}{(x+2)(x-2)}$ $= \frac{(x+2) - 4}{(x+2)(x-2)}$ $= \frac{x-2}{(x+2)(x-2)}$ $= \frac{1}{x+2}$ Students who wrote (x-2)(x²-4) as the common denominator are encouraged to always look for the lowest common denominator.	2 marks – correct solution 1 mark – writes with common denominator
11b	$\frac{(x^{-3}y^2)^3}{y^7} \times \frac{x^2y^{-3}}{xy^3} = \frac{x^{-9}y^6}{y^7} \times xy^{-6}$ $= \frac{x^{-8}}{y^7}$ $= \frac{1}{x^8y^7}$	2 marks – correct solution 1 mark – some correct simplification

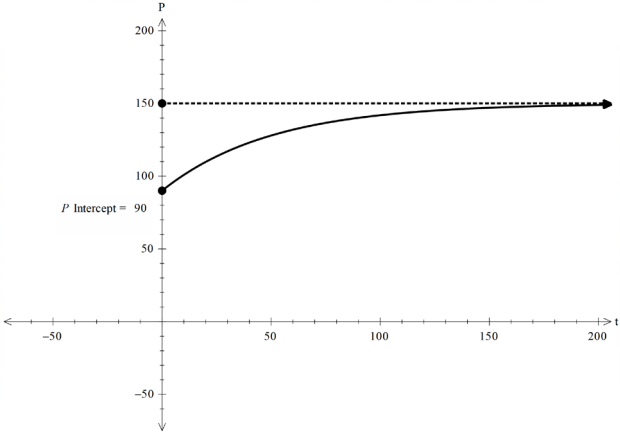
12	$3x - 6 = \pm 15$ $x = 7, x = -3$	2marks – correct solutions
13	$7 = 3^x$ $9^x = (3^2)^x = (3^x)^2 = 7^2 = 49$ Students who used the calculator and change of base law are required to demonstrate use of change of base for full marks.	2 marks – correct solution 1 mark writes $3^x=7$
14	Domain $(-\infty, \infty)$ Range $(-3, \infty)$	1 mark for each
15	$\tan x = -\sqrt{3}$ related angle $x = 60^\circ$ tan negative in Q 2 and Q 4 so $x = 120^\circ, 300^\circ$	2 marks – correct solution 1 mark – correct related angle but wrong quadrant(s)
16	$\log_a 96 = \log_a (4^2 \times 6)$ $= \log_a 4^2 + \log_a 6$ $= 2\log_a 4 + \log_a 6$ $= 2x + y$	2 marks – correct solution 1 mark – some correct use of log law
17	Equal roots $\Delta = 0$ $b^2 - 4ac = 0$ $(-3)^2 - 4 \times 1 \times k = 0$ $9 - 4k = 0$ $4k = 9$ $k = \frac{9}{4}$	2 marks – correct solution 1 mark – finds discriminant correctly, or solves incorrect discriminant=0
18	$\text{LHS} = \log_2 \left(\frac{x}{4} \right)$ $\therefore \frac{x}{4} = \frac{x-3}{2}$ $\frac{x}{4} = \frac{2(x-3)}{4}$ $x = 2(x-3)$ $x = 2x - 6$ $x = 6$	2 marks – correct solution 1 mark – simplifies LHS using log law
19	For $x = -1$ $g(-1) = (-1)^2 - 4 = -3$ $-a + b = -3 \Rightarrow b = a - 3$ ① For $x = 2$ $g(2) = 3(2) + 6 = 12$ $2a + b = 12$ ② sub ① into ② $2a + a - 3 = 12 \Rightarrow a = 5$ sub $a = 5$ into ① $b = 2$	2 marks for correct solution 1 mark for finding $g(-1)$ and $g(2)$

	Alternative method: For $x = -1$, $g(-1) = (-1)^2 - 4 = -3 \therefore (-1, -3)$ For $x = 2$, $g(2) = 3(2) + 6 = 12 \therefore (2, 12)$ $m = \frac{12 - (-3)}{2 - (-1)} = \frac{15}{3} = 5$ $y + 3 = 5(x + 1) \Rightarrow y = 5x + 2$ $\therefore a = 5 \text{ and } b = 2$	
20	hyp = $\sqrt{8^2 + 5^2} = \sqrt{89}$ $\sin\theta = \frac{8}{\sqrt{89}}$ (obtuse angle is in second quadrant where sine is positive) <i>Marker's comments:</i> <i>Many did not explain why value was positive and received only 1 mark</i>	2 marks for correct solution 1 mark for finding hyp
21	$\frac{\sin \angle ACB}{5} = \frac{\sin 0.6}{4}$ $\angle ACB = \sin^{-1}\left(\frac{5\sin 0.6}{4}\right) = 0.783556\dots$ OR $\pi - \sin^{-1}\left(\frac{5\sin 0.6}{4}\right) = 2.35803\dots$ (ambiguous case) $\angle ACB = 2.35803\dots$ (given AB is longest side, $\angle ABC$ is obtuse) $\angle ABC = \pi - 0.6 - \left(\pi - \sin^{-1}\left(\frac{5\sin 0.6}{4}\right)\right) = 0.184 \text{ rad}$	4 marks for correct solution 3 marks for correct solution without consideration to ambiguous case (ans 1.758 rad) 2 marks for finding correct angACB 1 mark for writing sine rule with correct substitutions
21	Alternate: $4^2 = AC^2 + 5^2 - 2(AC)(5)\cos 0.6$ $AC^2 - 10\cos 0.6 AC + 9 = 0$ $AC = \frac{10\cos 0.6 \pm \sqrt{100\cos^2 0.6 - 4(9)}}{2}$ $AC = 6.960 \text{ or } 1.293$ But AB is longest side so $AC = 1.293$ $\frac{\sin \angle ABC}{1.293} = \frac{\sin 0.6}{4} \Rightarrow \angle ABC = 0.184$ OR $\cos \angle ABC = \frac{5^2 + 4^2 - 1.293^2}{2 \times 5 \times 4} \Rightarrow \angle ABC = 0.184$	4 marks for correct solution 3 marks for correct solution without noting the different cases 2 marks for correctly writing expression to find AC 1 mark for writing cosine

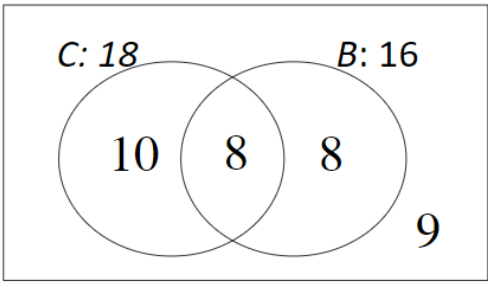
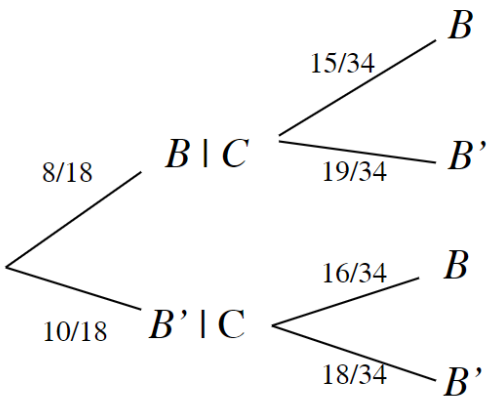
		rule with correct substitutions
21	<i>Marker's comments:</i> - Many students converted to degrees which made more work to solve question, and introduced potential for more errors in converting between radians and degrees - Conversions to degrees also resulted in less accurate final answers (not penalised as long as adequate working shown throughout process) - Error in finding missing length using Pythagoras' Theorem - Error in finding $\sin C = 0.7058$ versus angle C, and using that result in subsequent work	
22	$U = x^2 \qquad V = e^{5x}$ $U' = 2x \qquad V' = 5e^{5x}$ $\frac{dy}{dx} = 2xe^{5x} + 5x^2e^{5x}$ at $x = 1$, $\frac{dy}{dx} = 2e^5 + 5e^5 = 7e^5$ and $y = e^5$ $\therefore$ equation of tangent $y - e^5 = 7e^5(x - 1)$ $y = 7e^5x - 6e^5$	3 marks for correct solution 2 marks correctly finding gradient at $x=1$ and y value 1 mark for correctly differentiating
22	<i>Marker's comments:</i> Keep values as exact. i.e. students wrote $7e^5$ as 1038.89 (or full calculator display). Full marks still awarded in both cases.	
23	$f(x) = k(x+2)(x-1)(x-3)$ $(0,6) \quad 6 = k(2)(-1)(-3) \Rightarrow k = 1$ $\therefore f(x) = (x+2)(x-1)(x-3)$ $= (x^2 + x - 2)(x - 3)$ $= x^3 - 3x^2 + x^2 - 3x - 2x + 6$ $= x^3 - 2x^2 - 5x + 6$	3 marks for correct solution 2 marks for showing why $k=1$ 2 marks for using intercepts to find $f(x)$ and showing all steps to expansion 1 mark for using intercepts to write $f(x)$
24	$\frac{f\left(\frac{\pi}{8}\right) - f\left(\frac{\pi}{12}\right)}{\frac{\pi}{8} - \frac{\pi}{12}} = \frac{\sin\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)}{\frac{\pi}{24}} = \frac{\frac{\sqrt{2}-1}{2}}{\frac{\pi}{24}} = \frac{12(\sqrt{2}-1)}{\pi}$	3 marks for correct solution 1 mark for finding values for

	<i>Marker's comments:</i> <i>Some students differentiated function and substituted into the derivative</i> <i>Some students were finding the average of $\sin(\pi/4)$ and $\sin(\pi/6)$</i>	$\sin(\pi/4)$ and $\sin(\pi/6)$ 1 mark for showing understanding on how to calculate average rate of change
25	$P(\text{at least one pass}) = P(\text{not none})$ $= 1 - P(0 \text{ passes})$ $= 1 - 0.75 \times 0.6 \times 0.45$ $= 0.7975 = \frac{319}{400}$	2 marks for correct solution 1 mark for writing expression on how to find probability (i.e 2nd line of sample answer)
26a	$\frac{dy}{dx} = 9x^2 - 10x$	1 mark correct solution
26b	$y = x^{\frac{1}{3}} - x^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-\frac{1}{2}}$	1 mark correct solution
26c	$y = 2x^{-1} + x$ $\frac{dy}{dx} = -2x^{-2} + 1$	2 marks correct solution 1 mark partial correct with correct progress to index form
26d	$\frac{dy}{dx} = \frac{(2x-1)(3) - 3x(2)}{(2x-1)^2}$ $= -\frac{3}{(2x-1)^2}$	1 mark correct solution
26e	$\frac{dy}{dx} = 15(4x+7)^2 \times 4$ $= 60(4x+7)^2$	2 marks correct solution 1 mark partial correct use of $f'(f(x))$ rule with only one error
27a	$E(X) = 3.3$	1 mark correct solution
27b	$E(X^2) = 13.8$ $Var(X) = 13.8 - 3.3^2 = 2.91$	2 marks correct solution 1 mark partial correct with only

		one error in either part
27c	$\sigma = \sqrt{2.91} = 1.705\dots$ $\mu - \sigma \approx 1.59, \mu + \sigma \approx 5.006$ $P(\mu - \sigma < X < \mu + \sigma) = P(1.59 < X < 5.006)$ $= P(X = 2, 3, 4, 5)$ $= 0.65$	2 marks correct solution 1 mark partial correct finds bounds of prob.
27d	$\frac{P(X \text{ odd} \cap X > 3)}{P(X > 3)}$ $= \frac{P(X = 5)}{P(X = 4, 5, 6)}$ $= \frac{0.1}{0.5}$ $= \frac{1}{5}$	1 mark correct solution
28a	$B = 10 \log_{10} \frac{I_0}{I_0}$ $= 10 \log_{10} 1$ $= 10 \times 0$ $= 0$	1 mark correct solution
28b	$b = 10 \log_{10} \frac{0.05}{10^{-12}}$ $= 10 \times 10.69897$ $= 106.9897$ > 85 $\therefore$ workers MUST wear hearing protection	1 mark correct solution
29	(a) At rest where $dx/dt = 0 \rightarrow$ at $t = 2, 6$	1 Mark: Correct answer
	(b) Left of $x = 0$ when graph is below the t axis $\rightarrow 5 < t < 7$	1 Mark: Correct answer
	(c) 	3 Marks: Correct answer 2 Marks: 1 mistake 1 Mark: Correct intercepts or asymptote

30a	$P = 150 - 60e^{-0.02t} \rightarrow \frac{dP}{dt} = 0 - 60e^{-0.02t} \cdot -0.02$ $= 1.2e^{-0.02t}$ $> 0 \text{ since } e^{-0.02t} > 0 \text{ for all } t$ $\therefore P \text{ is increasing.}$	2 Marks: Correct answer 1 Mark: Obtains a correct derivative OR Obtains a value from an incorrect derivative of equal merit
b	$t = 0 \rightarrow P = 150 - 60e^0 = 90$	1 Mark: Correct answer
c	$t = 50 \rightarrow P = 150 - 60e^{(-0.02) \cdot 50} = 127.927... = 128$	1 Mark: Correct answer
d	$P = 150 - 60e^{-0.02t}$ $P = 140 \rightarrow 140 = 150 - 60e^{-0.02t}$ $-10 = -60e^{-0.02t} \rightarrow \frac{1}{6} = e^{-0.02t}$ $-0.02t = \ln\left(\frac{1}{6}\right) \rightarrow t = \frac{\ln\left(\frac{1}{6}\right)}{-0.02}$ $t = 89.587... \text{ weeks}$ $t \approx 90 \text{ weeks}$	2 Marks: Correct answer 1 Mark: Obtains $e^{-0.02t} = \frac{1}{6}$
e	 The graph shows a coordinate plane with the vertical axis labeled 'P' and the horizontal axis labeled 't'. The vertical axis has tick marks at -50, 50, 100, 150, and 200. The horizontal axis has tick marks at -50, 50, 100, 150, and 200. A curve starts at a point labeled 'P Intercept = 90' on the vertical axis and increases, approaching a horizontal dashed line at P = 150 as t increases. The curve is concave down.	2 Marks: Correct answer 1 Mark: Obtains any two of correct P intercept, correct asymptote or concavity.
f	As $t \rightarrow \infty$, $e^{-0.02t} \rightarrow 0$ $\therefore P \rightarrow 150 - 0 = 150$ Since $e^{-0.02t} > 0$ for all t, P will always be less than 150.	1 Mark: Correct answer

31	$OA = OB = OC = OD = 6$ (radii) $\angle OCD = 30$ (alternate, $AB \parallel CD$) $\angle ODC = 30$ (base angles of isosceles Δ) $\angle DOC = 120$ (angle sum Δ) $\angle AOD = 30$ (straight angle) Area of ΔODC : $A = \frac{1}{2} (6)(6) \sin 120$ $A = 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$ Area of sector AOD: $A = \frac{30}{360} \times \pi(6)^2$ $A = 3\pi$ $\therefore \text{Area shaded} = 9\sqrt{3} + 3\pi$	4 marks 3 marks: finds the area of the triangle or the sector including the correct formulation of the total area 2 marks: provides correct formulation of the total area 1 mark: finds angle DOC or OD or OC
32a	$\begin{aligned} LHS &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta + 1} \\ &= \frac{\sin \theta(\sin \theta + 1) + \cos^2 \theta}{\cos \theta(\sin \theta + 1)} \\ &= \frac{\sin^2 \theta + \sin \theta + \cos^2 \theta}{\cos \theta(\sin \theta + 1)} \\ &= \frac{1 + \sin \theta}{\cos \theta(\sin \theta + 1)} \\ &= \frac{1}{\cos \theta} \\ &= \sec \theta \end{aligned}$	2 marks 1 mark: correctly forms one fraction
32b	$\frac{\sin 2\theta}{\cos 2\theta} + \frac{\cos 2\theta}{\sin 2\theta + 1} = 2$ $\sec 2\theta = 2$ $\cos 2\theta = \frac{1}{2}$ $2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	3 marks 1 st mark: uses part (a) successfully 2 nd mark: finds at least one solution to the equation with working

33a	<i>Total: 35</i>  $35 - 9 = 26$ students study at least one $18 + 16 - 26 = 8$ students study both Therefore, $8/35$	2 marks 1 mark for finding the number of students who study both Bio and Chem, or equivalent merit
33b	$10/18 = 5/9$	1 mark
33c	There are two cases: the first student studies Biology, or the first student does not study Biology.  $P(\text{2nd Bio}) = \frac{8}{18} \times \frac{15}{34} + \frac{10}{18} \times \frac{16}{34} = \frac{70}{153}$	2 marks 1 mark for some consideration of two cases / consideration of 'without replacement'
34		
35		