[Comments]

2021 | YEAR 11 PRELIMINARY EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Approved calculators may be used
- For Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks : 74

Section I – 7 marks (pages 2 – 7)

- Attempt Questions 1 – 7
- Allow about 10 minutes for this section

Section II – 67 marks (pages 8 – 25)

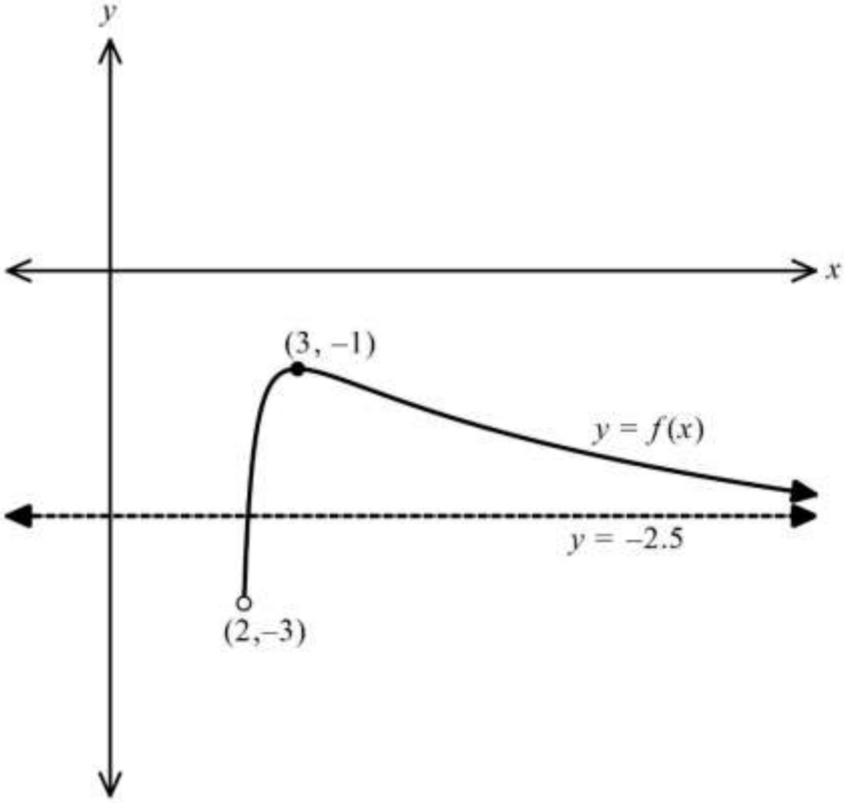
- Attempt Questions 8 – 27
- Allow about 1 hours and 50 minutes for this section

Section I

7 marks
Attempt Questions 1 – 7
Allow about 10 minutes for this section.

1. The graph of a function $y = f(x)$ is shown, below.

It has an asymptote at $y = –2.5$.



Which statement correctly describes its domain and range.

A. Domain : $[2, \infty)$; Range : $[–3, –1]$

B. Domain : $(–3, –1]$; Range : $(2, \infty)$

C. Domain : $[2, \infty]$; Range : $[–3, –1)$

D. Domain : $(2, \infty)$; Range : $(–3, –1]$

2. Which statement best describes the solutions of the equation $x^2 - 4x - 3 = 0$

A. They are both real

B. They are real and rational

C. They are real and not rational

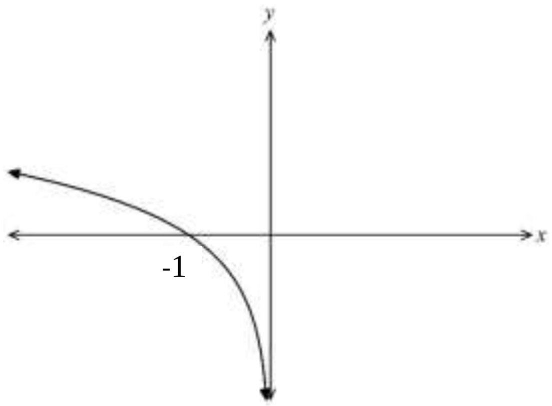
D. They are equal

3. What is the exact value of $2sin\frac{5\pi}{3}$?

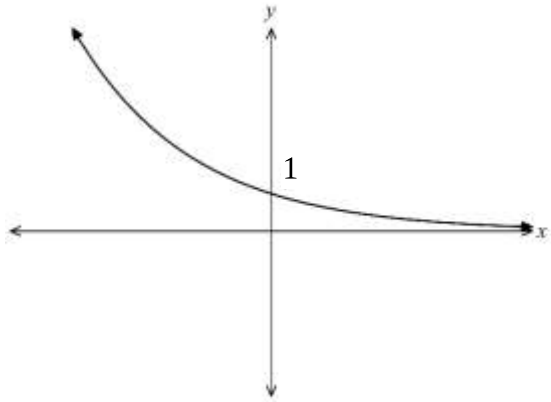
A. $\sqrt{3}$
B. 1
C. $-\sqrt{3}$
D. -1

4. Which graph shows the curve $y = \log_2 (x)$?

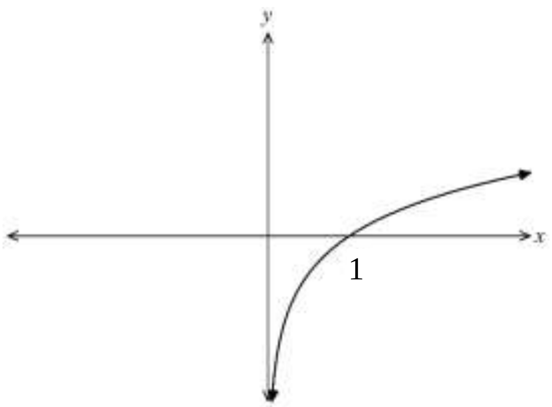
A.



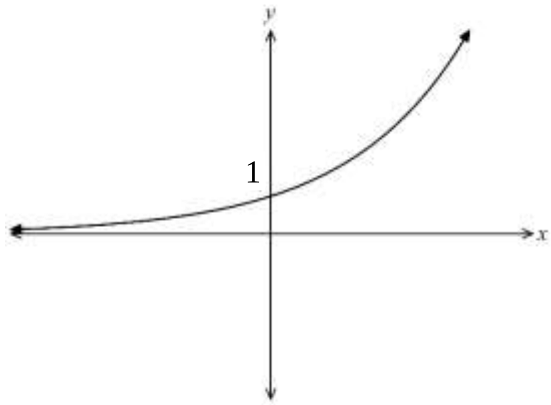
B.



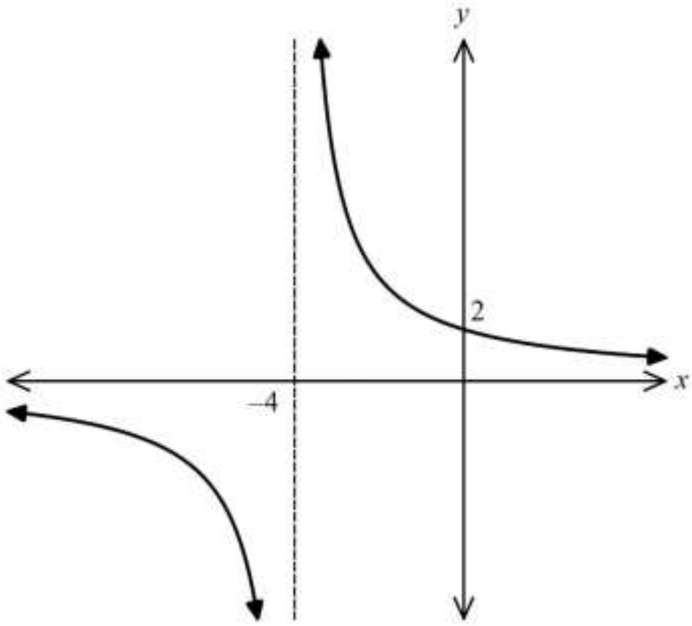
C.



D.



5. The graph below has a vertical asymptote at $x = -4$ and a horizontal asymptote at $y = 0$. The curve crosses the y-axis at $y = 2$.



Which equation could describe the curve?

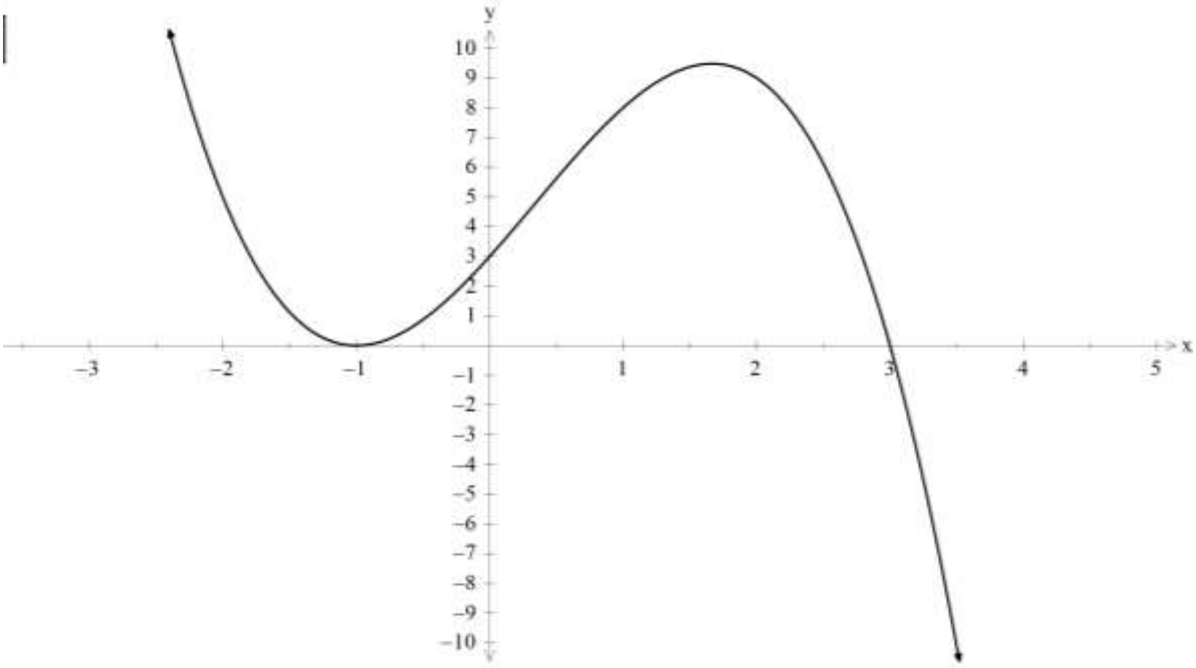
A. $y = \frac{8}{x+4}$

B. $y = \frac{2}{x+4}$

C. $y = \frac{8}{x-4}$

D. $y = \frac{2}{x-4}$

6. Choose the most appropriate equation for the graph below.



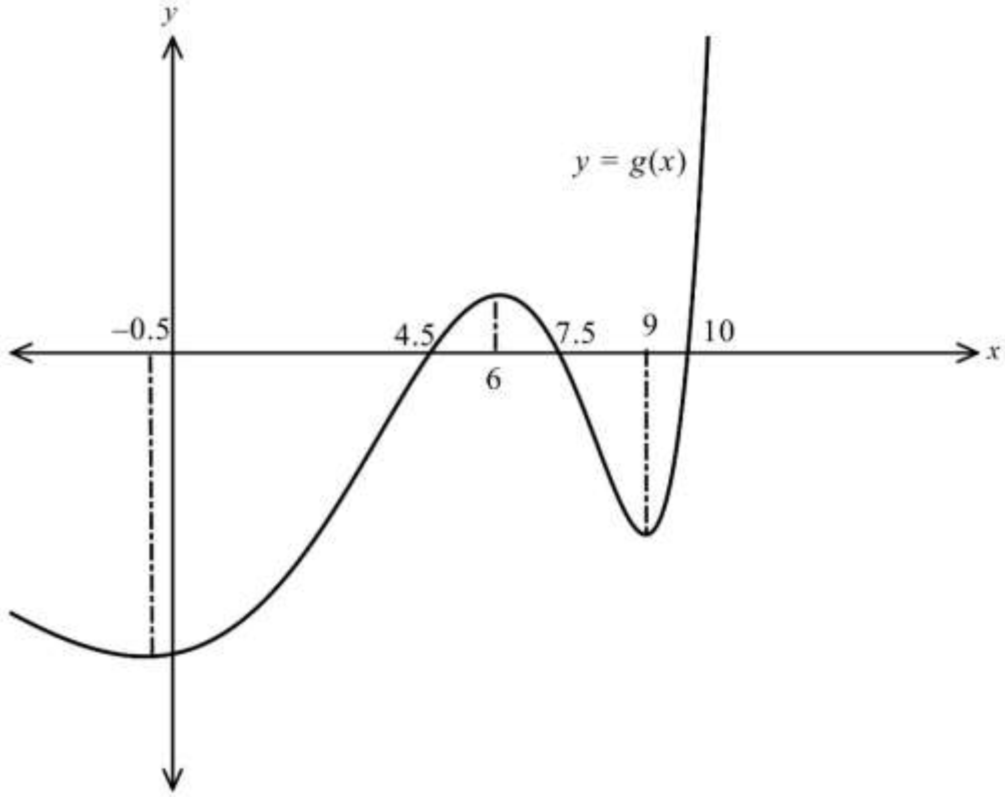
A. $y = (x+1)^2(x-3)$

B. $y = (x+1)^2(3-x)$

C. $y = (x-1)^2(x-3)$

D. $y = (x-1)^2(3-x)$

7. The graph of $y = g(x)$ is shown below.



For what values of x is $g'(x) > 0$

A. $(-0.5, 6) \cup (9, \infty)$

B. $(-0.5, 4.5) \cup (7.5, \infty)$

C. $(4.5,\ 7.5) \cup (10, \infty)$

D. $(-\infty, -0.5) \cup (6,\ 9)$

[Comments]

Class and Teacher

Student Number

Student Name

Mathematics Advanced

Section II Answer Booklet

67 marks
Attempt Questions 8 - 27
Allow about 1 hour and 50 minutes for this section

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet.
  If you use this space, clearly indicate which question you are answering.

Question 8 (2 marks)

Simplify $\frac{\cos x-\cos^3 x}{\sin^3 x}$. **2**

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Question 9 (2 marks)

Differentiate the following:

(a) $f(x) = 2x^4 - 3x^3 + 4.$ **1**

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(b) $y = \frac{3}{x^4} + 4.$ **1**

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Question 10 (3 marks)

(a) Evaluate $\log_4(2)$. **1**

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(b) Simplify $\log_a\left(\frac{1}{a^2}\right) + \log_a\left(\sqrt[3]{a^4}\right)$. **2**

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Question 11 (6 marks)

(a) Find the value of $e^{-1.75}$ correct to 3 significant figures **1**

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(b) If $x = \frac{\sqrt{5}-2}{\sqrt{5}}$, then express $\frac{1}{x}$ with a rational denominator. **2**

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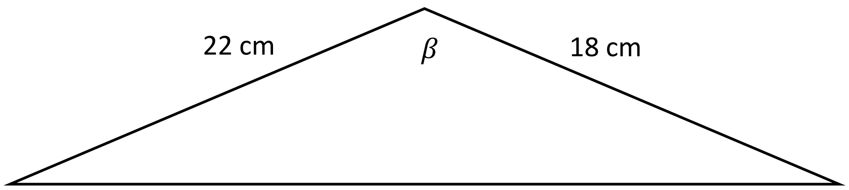
(c) Find the value of y that solves $\left(\sqrt{8}\right)^{y-3} = (64)^{4-y}$ **3**

Question 12 (2 marks)



The area of the above triangle is 99 cm^2. Find all possible values of β. **2**

Question 13 (5 marks)

Find $\frac{dy}{dx}$ for the following:

(a) $y = (5x^2 + 1)^4$. **1**

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(b) $y = \sqrt{4 - x^2}$ **2**

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(c) $y = \frac{x^2+3}{x-2}$. **2**

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Question 14 (3 marks)

For the function $f(x) = 2x^2 - 3x$, use the following formula **3**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

to find the derivative $f'(x)$ from first principles.

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Question 15 (1 mark)

(a) Draw a neat sketch of $y = 3e^{x-1}$. **1**

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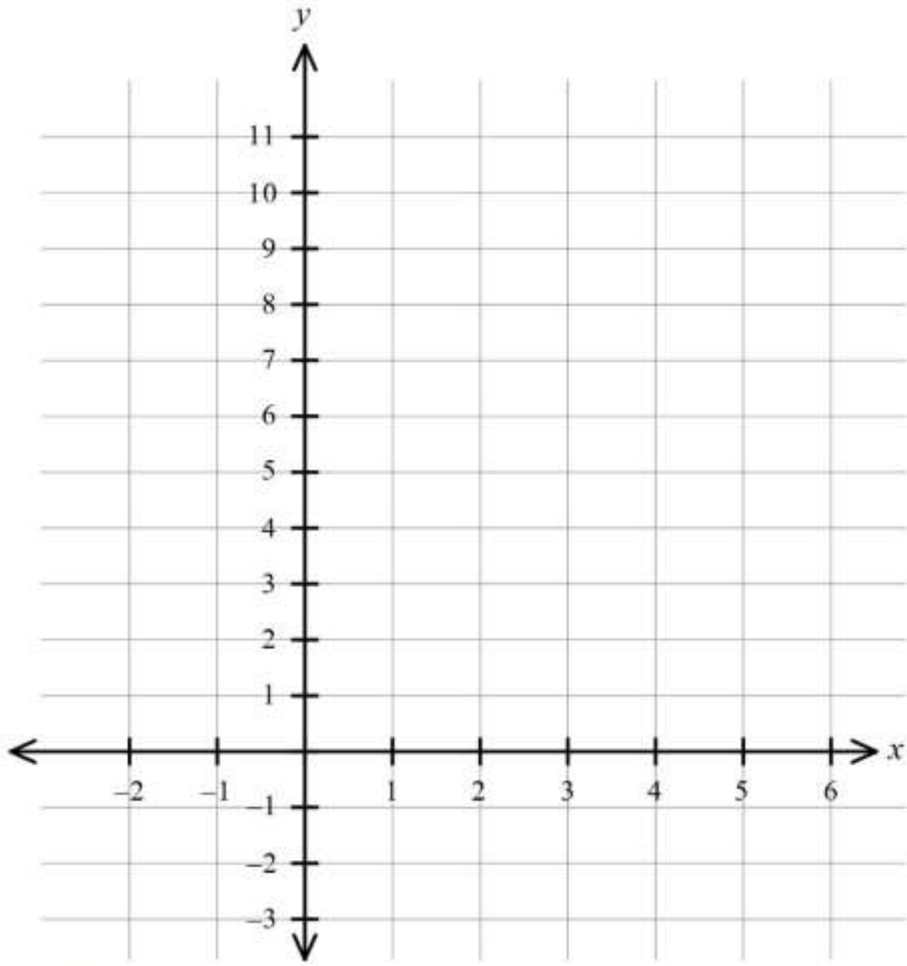

Question 16 (3 marks)

A line passes through the points $A(2, -5)$ and $B(-1,\ 1)$ on the Cartesian plane.

(a) Find the gradient of the line *AB*. **1**

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(b) Find the equation of the line which passes through *A* and is perpendicular to *AB*. **2**

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Question 17 (4 marks)

(a) Use the product rule to find $\frac{dy}{dx}$ where $y = x^3(3x-2)^5$ **2**

(b) By fully factorising your answer to part (a), find all values of x such that $\frac{dy}{dx} = 0$ **2**

Question 18 (7 marks)

Sophie is breeding mice for a laboratory. The number of mice M she has after t months is given by the equation:

$$M = 12 + 5t + 4t^2$$

(a) How many mice does she have initially? **1**

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(b) Find the average rate of increase in her mouse population in the first six months. **2**

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(c) How fast is her population of mice increasing after 6 months? **2**

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(d) Her parents have only allowed her to keep a maximum of 100 mice. After how many months will she reach this number? **2**

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Question 19 (2 marks)

Given that $f(x) = 2x^2 - x$ and $g(x) = 2x - 3$, **2**

write an expression for $f\big(g(x)\big)$.

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Question 20 (4 marks)

(a) Solve the equation $(1.035)^k = 3.46$ correct to 2 decimal places. **2**

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(b) $f(x) = e^{-3x}$. Find the value of $f'(2)$ correct to two decimal places. **2**

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Question 21 (2 marks)

Solve $|2x - 3| = 15$. **2**

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Question 22 (3 marks)

A tangent is drawn to the curve $y = x^5 - 3x^2 - 60x$, at the point where $x = -2$.

(a) Find the gradient of this tangent. **1**

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(b) Find the equation of the normal to the curve through the same point. **2**

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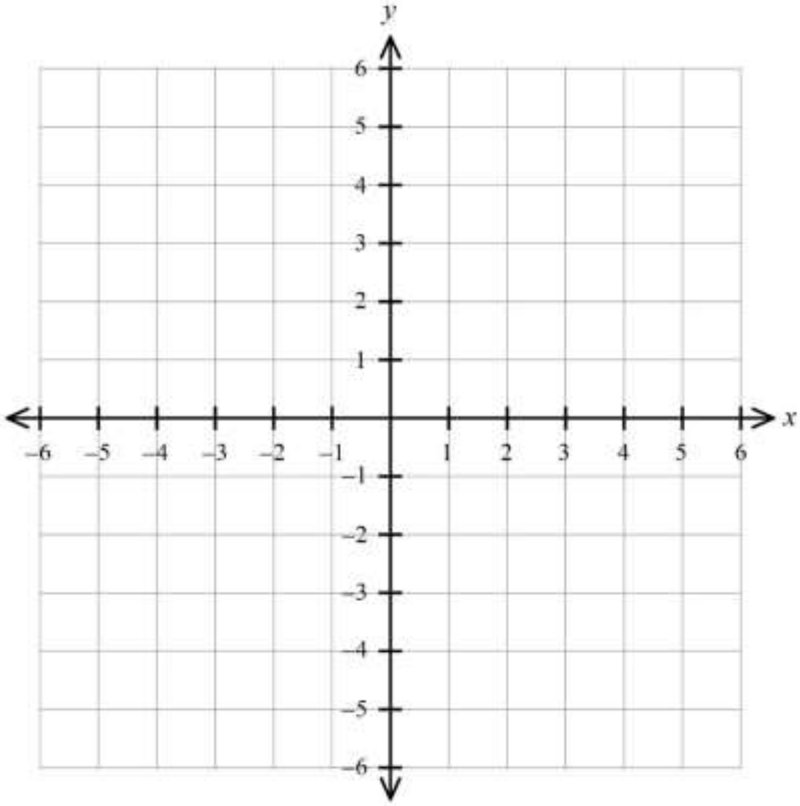
Question 23 (2 marks)

Draw a sketch of the circle with equation $x^2 - 2x + y^2 + 6y + 6 = 0$. **2**



Question 24 (3 marks)

Solve the following equation for $0° \leq x \leq 360°$:

$3cot^2x \; - \; 1 = 0$ **3**

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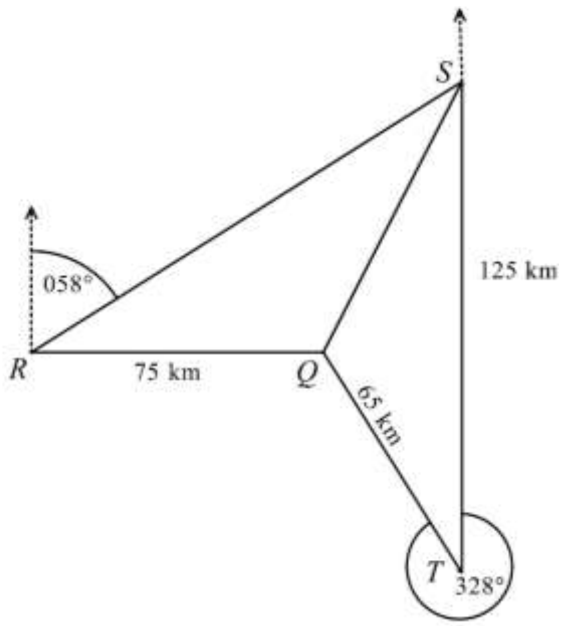
Question 25 (4 marks)

The diagram below shows the locations of four towns Q, R, S and T.



T is due south of S and Q is due east of R.

The bearing of S from R is 058° and the bearing of Q from T is 328°.

(a) Calculate the distance SQ, correct to the nearest km. **2**

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(b) Find the size of $\angle RSQ$ to the nearest degree, and hence give the bearing of Q from S. **2**

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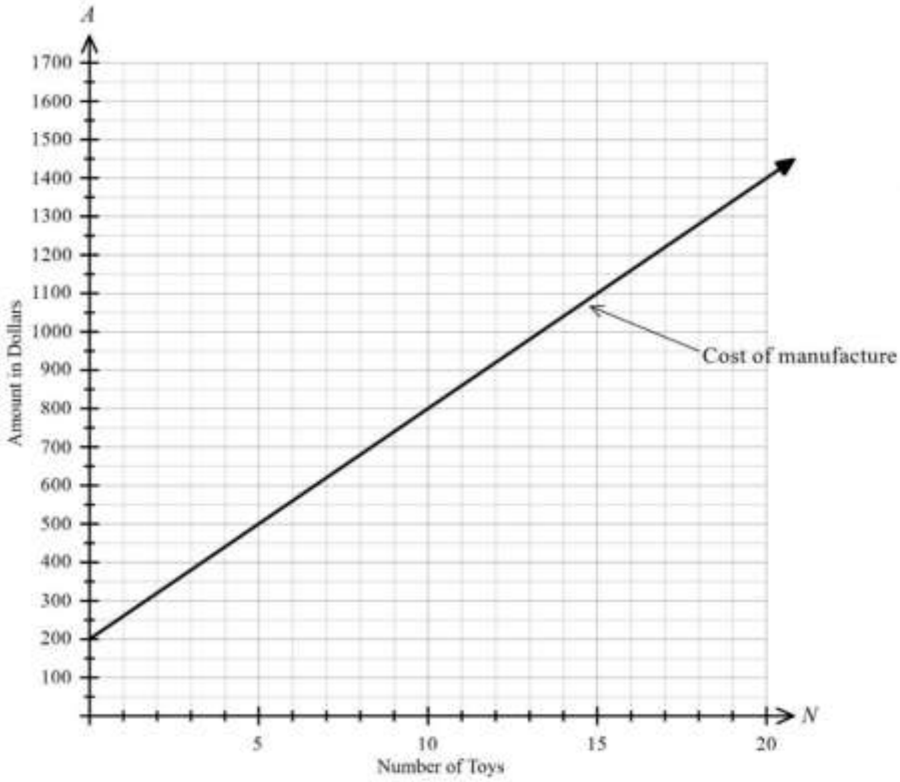
Question 26 (4 marks)

A small business manufactures toy cars, which it sells for $80 each.

The graph below shows the line which represents the cost of manufacturing the toy cars in each day's production run.

There are fixed costs each day which amount to $200, and a cost associated with producing each toy car.



(a) What equation would describe the line shown? **1**

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(b) On the graph above, draw the line which would represent the income from sales of the toy cars. **1**

(c) How many toy cars would need to be manufactured in the day for the business to break even? **1**

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(d) If 15 toy cars were made in a day, what profit (or loss) would the business make? **1**

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Question 27 (5 marks)

The displacement from the origin (x metres) of a particle travelling in a straight line at a time (t seconds) is given by:

$$x = 3t^4 - 16t^3 + 18t^2 \text{ , where } t \geq 0.$$

(a) Show that the velocity $\frac{dx}{dt} = 0$ at time t = 1 second. **1**

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(b) Find any other the time(s) when the particle is stationary. **2**

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(c) Find the distance travelled by the particle in the first 2 seconds. **2**

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Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \ r \neq 1$$

$$S = \frac{a}{1-r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

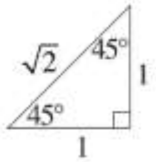
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$



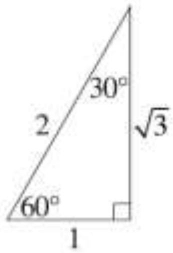
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

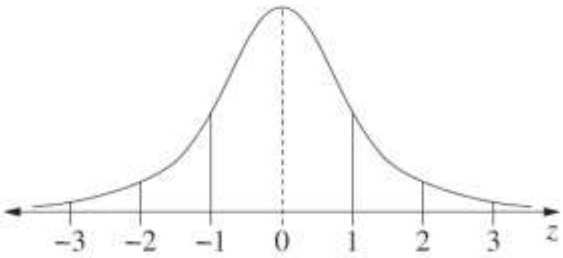
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x-\mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

Preliminary Exam Solutions 2021

Saturday, 28 August 2021 8:26 PM

SOLUTIONS

Western Mathematics

2021 | YEAR 11 FINAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Approved calculators may be used
- For Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks : 74

Section I – 7 marks (pages 2 – 7)

- Attempt Questions 1 – 7
- Allow about 10 minutes for this section

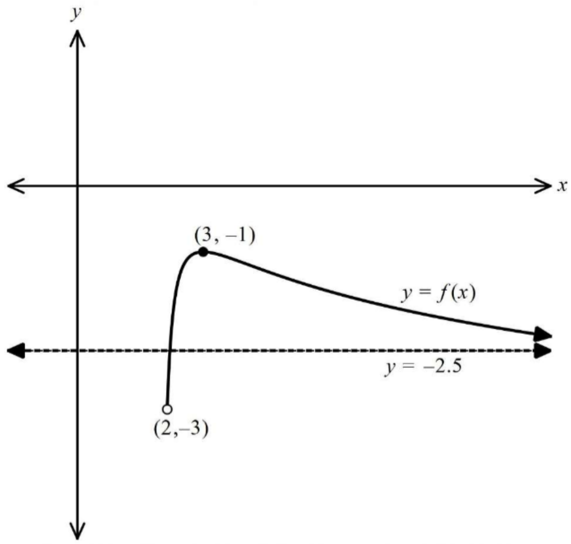
Section II – 67 marks (pages 8 – 25)

- Attempt Questions 8 – 27
- Allow about 1 hours and 50 minutes for this section

Section I

7 marks
Attempt Questions 1 – 7
Allow about 10 minutes for this section.

1. The graph of a function $y = f(x)$ is shown, below.
It has an asymptote at $y = -2.5$.



Which statement correctly describes its domain and range.

A. Domain : $[2, \infty)$; Range : $[-3, -1]$

B. Domain : $(-3, -1]$; Range : $(2, \infty)$

C. Domain : $[2, \infty]$; Range : $[-3, -1)$

D. Domain : $(2, \infty)$; Range : $(-3, -1]$

2. Which statement best describes the solutions of the equation $x^2 - 4x - 3 = 0$

A. They are both real

B. They are real and rational

C. They are real and not rational

D. They are equal

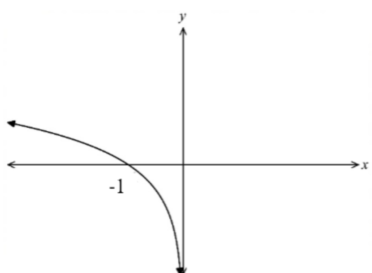
3. What is the exact value of $2sin\frac{5\pi}{3}$?

A. $\sqrt{3}$
B. 1
C. $-\sqrt{3}$
D. -1

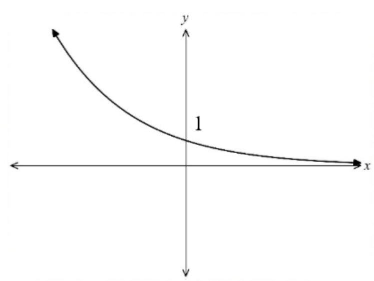
- 3 -

4. Which graph shows the curve $y = \log_2 (x)$?

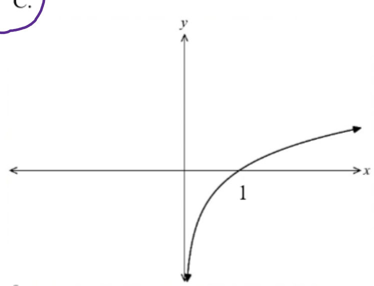
A.



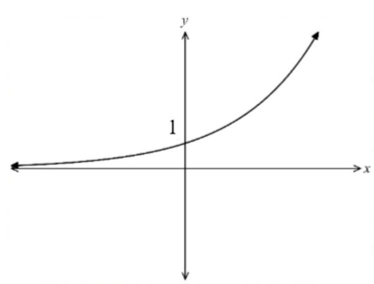
B.



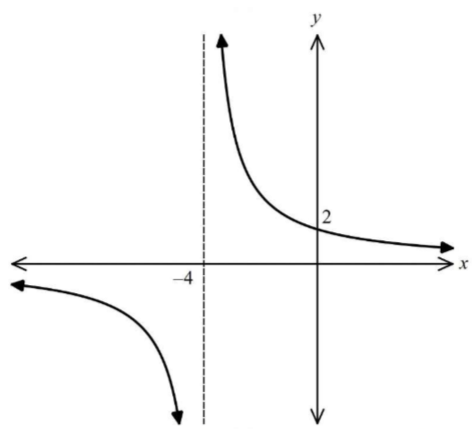
C.



D.



5. The graph below has a vertical asymptote at $x = -4$ and a horizontal asymptote at $y = 0$.
The curve crosses the y-axis at $y = 2$.



Which equation could describe the curve?

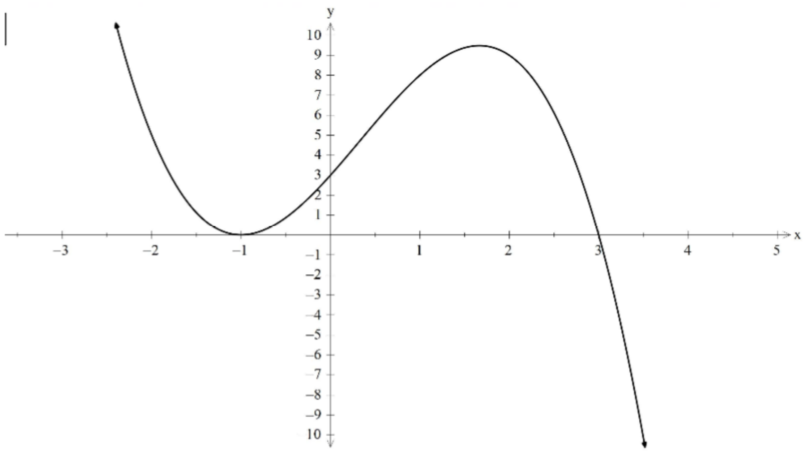
A. $y = \frac{8}{x+4}$

B. $y = \frac{2}{x+4}$

C. $y = \frac{8}{x-4}$

D. $y = \frac{2}{x-4}$

6. Choose the most appropriate equation for the graph below.



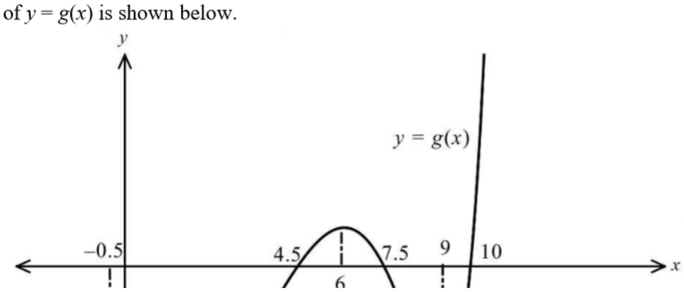
A. $y = (x + 1)^2(x - 3)$

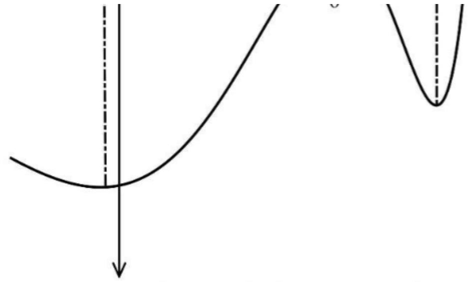
B. $y = (x + 1)^2(3 - x)$

C. $y = (x - 1)^2(x - 3)$

D. $y = (x - 1)^2(3 - x)$

7. The graph of $y = g(x)$ is shown below.





For what values of x is $g'(x) > 0$

A. $(-0.5, 6) \cup (9, \infty)$

B. $(-0.5, 4.5) \cup (7.5, \infty)$

C. $(4.5,\ 7.5) \cup (10, \infty)$

D. $(-\infty, -0.5) \cup (6,\ 9)$

MC:

D C C C A B A

Western Mathematics

2021 YEAR 11 FINAL EXAMINATION

Class and Teacher

Mathematics Advanced

Student Number

Section II Answer Booklet

Student Name

67 marks
Attempt Questions 8 - 27
Allow about 1 hour and 50 minutes for this section

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or

- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet.
  If you use this space, clearly indicate which question you are answering.

Question 8 (2 marks)

Simplify $\frac{\cos x - \cos^3 x}{\sin^3 x}$. **2**

$= \frac{\cos x (1 - \cos^2 x)}{\sin^3 x}$

$= \frac{\cos x \cdot \cancel{\sin^2 x}}{\cancel{\sin^3 x}_{\sin x}} = \cot x$

Question 9 (2 marks)

Differentiate the following:

(a) $f(x) = 2x^4 - 3x^3 + 4.$ **1**

$f'(x) = 8x^3 - 9x^2$

(b) $y = \frac{3}{x^4} + 4.$ **1**

$y = 3x^{-4} + 4$

$y' = -12x^{-5}$

$= \frac{-12}{x^5}$

Question 10 (3 marks)

(a) Evaluate $\log_4(2)$. **1**

$\frac{1}{2}$

(b) Simplify $\log_a\left(\frac{1}{a^2}\right) + \log_a\left(\sqrt[3]{a^4}\right)$. $= \log_a a^{-2} + \log_a a^{4/3}$ **2**

$-2 + \frac{4}{3}$

$-\frac{2}{3}$

Question 11 (6 marks)

(a) Find the value of $e^{-1.75}$ correct to 3 significant figures **1**

0.174

(b) If $x = \frac{\sqrt{5}-2}{\sqrt{5}}$, then express $\frac{1}{x}$ with a rational denominator. **2**

$\frac{1}{x} = \frac{\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$

$= \frac{\sqrt{5}(\sqrt{5}+2)}{5-4}$

$= 5 + 2\sqrt{5}$

(c) Find the value of y that solves $\left(\sqrt{8}\right)^{y-3} = (64)^{4-y}$ **3**

$\left(8^{\frac{1}{2}}\right)^{y-3} = (64)^{4-y}$

$\left(\left(2^3\right)^{\frac{1}{2}}\right)^{y-3} = \left(2^6\right)^{4-y}$

$\left(2^{\frac{3}{2}}\right)^{y-3} = 2^{24-6y}$

$\Rightarrow \frac{3}{2}y - \frac{9}{2} = 24 - 6y$

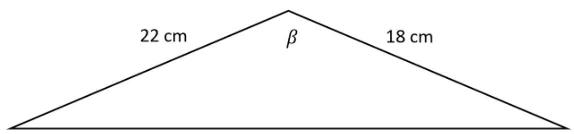
$7.5y = 2$

$y = \frac{28.5}{7.5}$

$= \frac{57}{15}$

$= \boxed{3.8}$

Question 12 (2 marks)



The area of the above triangle is 99 cm^2. Find all possible values of β. **2**

$A = \frac{1}{2} \times 22 \times 18 \times \sin\theta = 99$

$198 \sin\theta = 99$

$\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ$ or 150°

Question 13 (5 marks)

Find $\frac{dy}{dx}$ for the following:

(a) $y = (5x^2 + 1)^4$. **1**

$y' = 4 \times 10x\,(5x^2+1)^3$

$= 40x(5x^2+1)^3$

(b) $y = \sqrt{4 - x^2}$ **2**

$y = (4 - x^2)^{\frac{1}{2}}$

$y = \frac{1}{2}x - 4x(4 - x^2)$

$y' = \frac{-x}{\sqrt{4-x^2}}$

(c) $y = \frac{x^2+3}{x-2}$. $\frac{u}{v}$ **2**

$u = x^2+3 \qquad v = x-2$

$u' = 2x \qquad v' = 1$

$y' = \frac{u'v - uv'}{v^2}$

$= \frac{2x(x-2) - (x^2+3)}{(x-2)^2}$

$= \frac{x^2 - 4x - 3}{(x-2)^2}$

- 12 -

Question 14 (3 marks)

For the function $f(x) = 2x^2 - 3x$, use the following formula **3**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

to find the derivative $f'(x)$ from first principles.

$f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$

$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h}$

$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h - \cancel{2x^2} + \cancel{3x}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}}$

$= 4x - 3$

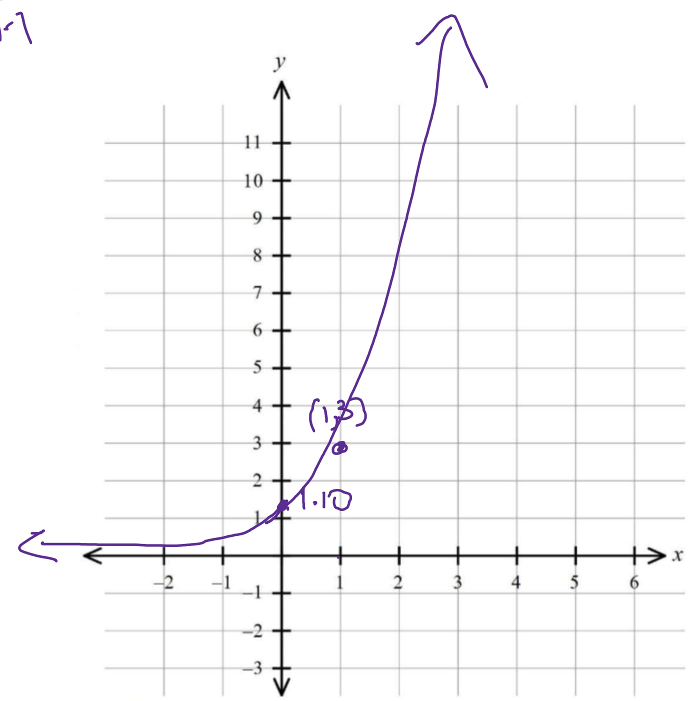
Question 15 (1 mark)

(a) Draw a neat sketch of $y = 3e^{x-1}$. **1**

$x = 0$

$y = 3/e$

$= 1.1$



Question 16 (3 marks)

A line passes through the points $A(2,-5)$ and $B(-1,\ 1)$ on the Cartesian plane.

(a) Find the gradient of the line *AB*. **1**

$m = \frac{1+5}{-1-2} = -2$

(b) Find the equation of the line which passes through *A* and is perpendicular to *AB*. **2**

$A(2-5)$ $m = \frac{1}{2}$

Equation: $y+5 = \frac{1}{2}(x-2)$

$2y+10 = x-5$

$x-2y-15=0$

Question 17 (4 marks)

(a) Use the product rule to find $\frac{dy}{dx}$ where $y = x^3(3x-2)^5$ **2**

$u = x^3$ $v = (3x-2)^5$

$u' = 3x^2$ $v' = 15(3x-2)^4$

$\frac{dy}{dx} = 15x^3(3x-2)^4 + 3x^2(3x-2)^5$

$= 3x^2(3x-2)^4(5x + 3x - 2)$

$= 3x^2(3x-2)^4(8x-2)$

$= 12x^2(3x-2)^4(4x-1)$

(b) By fully factorising your answer to part (a), find all values of x such that $\frac{dy}{dx} = 0$ **2**

$y' = 12x^2(3x-2)^4(4x-1) = 0$

$x = 0$, $x = 2/3$, $\frac{1}{4}$

Question 18 (7 marks)

Sophie is breeding mice for a laboratory. The number of mice M she has after t months is given by the equation:

$$M = 12 + 5t + 4t^2$$

(a) How many mice does she have initially? **1**

$t=0$ $M=12$

(b) Find the average rate of increase in her mouse population in the first six months. **2**

$t=0$ $M=12$ $t=6$ $M = 12+30+4\times36$

$= 186$

Avge rate: $\frac{186-12}{6} = \frac{174}{6} = 29$ mice/month.

(c) How fast is her population of mice increasing after 6 months? 2

rate $= \frac{dM}{dt} = 5 + 8t$
$= 53$ mice/month when $t = 6$

(d) Her parents have only allowed her to keep a maximum of 100 mice. After how many months will she reach this number? 2

$100 = 12 + 5t + 4t^2$
$4t^2 + 5t - 88 = 0$
$t = \frac{-5 + \sqrt{25 + 16 \times 88}}{8}$
$= 4.11$ months

Question 19 (2 marks)

Given that $f(x) = 2x^2 - x$ and $g(x) = 2x - 3$, 2

write an expression for $f(g(x))$.

$f(g(x))$
$= f(2x-3)$
$= 2(2x-3)^2 - (2x-3)$
$= 2(4x^2 - 12x + 9) - 2x + 3$
$8x^2 - 24x + 18 - 2x + 3$
$8x^2 - 26x + 21$

Question 20 (4 marks)

(a) Solve the equation $(1.035)^k = 3.46$ correct to 2 decimal places. 2

$k = \frac{\log(3.46)}{\log(1.035)}$ 36.08

(b) $f(x) = e^{-3x}$. Find the value of $f'(2)$ correct to two decimal places. 2

$f'(x) = -3e^{-3x}$
$f'(2) = -3e^{-6}$

- 18 -

-1.01

Question 21 (2 marks)

Solve $|2x - 3| = 15$. **2**

$2x-3=15$ $\quad$ $2x-3=-15$

$x=9$ $\quad$ $x=-6$

Question 22 (3 marks)

A tangent is drawn to the curve $y = x^5 - 3x^2 - 60x$, at the point where $x = -2$.

(a) Find the gradient of this tangent. **1**

$\frac{dy}{dx} = 5x^4 - 6x - 60$

$x=-2$ $\quad m_t = 80+12-60$

$= 32$

(b) Find the equation of the normal to the curve through the same point. **2**

$\boxed{m_n = -\frac{1}{32}}$ $\quad x=-2$

$y = -32-12+120$

$=76$

So equation: $y-76 = -\frac{1}{32}(x+2)$

$32y - 32\times76 = -x-2$

$x + 32y = 32\times76-2$

- 19 -

Question 23 (2 marks)

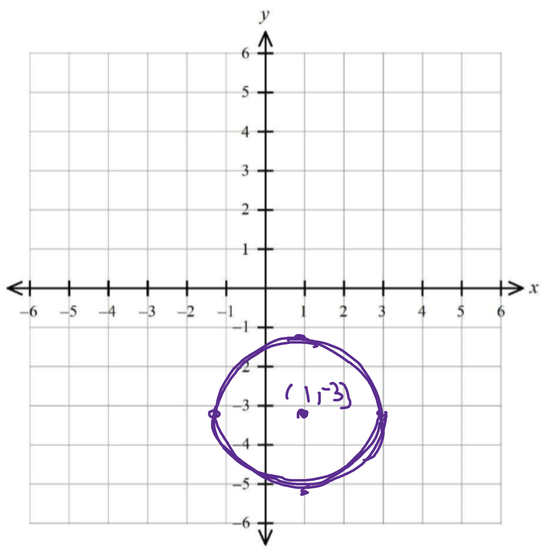
Draw a sketch of the circle with equation $x^2 - 2x + y^2 + 6y + 6 = 0$. **2**

$x^2 - 2x + \quad y^2 + 6y = -6$

$+1 \qquad\qquad +9 \qquad +10$

$(x-1)^2 + (y+3)^2 = 4$

centre $(1,-3)$ $\quad r = 2$



Question 24 (3 marks)

Solve the following equation for $0° \leq x \leq 360°$:

$3cot^2x - 1 = 0$ **3**

$\cot^2 x = \frac{1}{3}$

$\tan^2 x = 3$

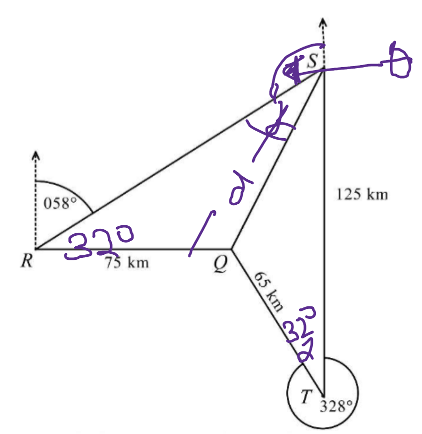
$\tan x = \pm\sqrt{3}$

rel acute $\angle$: $60°$

1 2 3 4

$\Rightarrow x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

The diagram below shows the locations of four towns Q, R, S and T.



T is due south of S and Q is due east of R.

The bearing of S from R is 058° and the bearing of Q from T is 328°.

(a) Calculate the distance SQ, correct to the nearest km. **2**

$d = 360 - 328$
$= 32^\circ$

Cosine rule: $d^2 = 65^2 + 125^2 - 2 \times 65 \times 125 \times \cos 32^\circ$

$d = 78$ km (correct to

.... new est Km)

(b) Find the size of $\angle RSQ$ to the nearest degree, and hence give the bearing of Q from S. **2**

$\angle SRQ = 90 - 58^\circ = 32^\circ$

$\theta = 180 - 58^\circ = 122^\circ$

bearing of Q from S: $360^\circ - (122 + \alpha)$

in $\triangle RSQ$: i.e. $360^\circ - (122 + \hat{RSQ})$

$\frac{\sin\alpha}{75} = \frac{\sin 32}{d}$

$\hat{RSQ} = 31^\circ 25'$

$= 31^\circ$ (to nearest deg)

$\Rightarrow$ bearing of Q from S

$360^\circ - (153^\circ)$

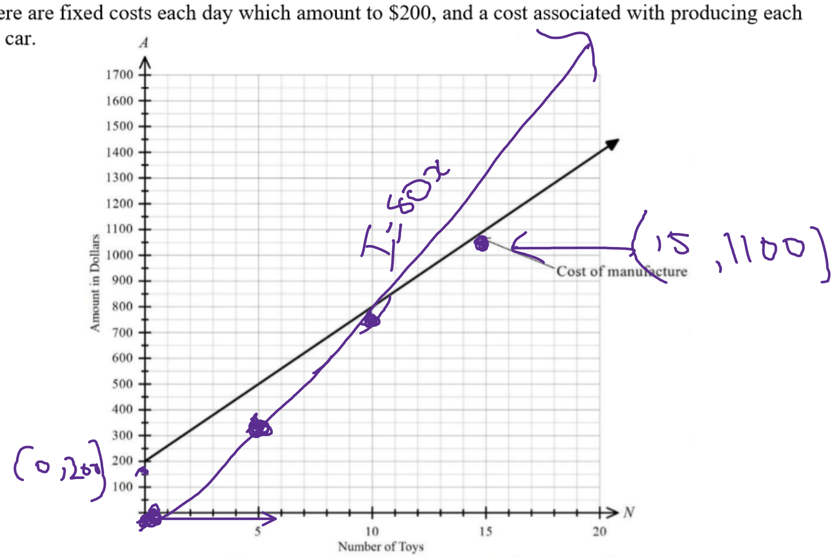
$= 207^\circ$

Question 26 (4 marks)

A small business manufactures toy cars, which it sells for $80 each.

The graph below shows the line which represents the cost of manufacturing the toy cars in each day's production run.

There are fixed costs each day which amount to $200, and a cost associated with producing each toy car.



(a) What equation would describe the line shown? **1**

$m = \frac{1100 - 200}{15 - 0} = \frac{900}{15} = 60$

$C = 60x + 200$

(b) On the graph above, draw the line which would represent the income from sales of the toy cars. **1**

$I = 80x$

(c) How many toy cars would need to be manufactured in the day for the business to break even? **1**

10

(d) If 15 toy cars were made in a day, what profit (or loss) would the business make? **1**

$I = 80 \times 15$

$C = 60 \times 15 + 200$

$P = 80 \times 15 - 60 \times 15 - 200$

$= \$100$

Question 27 (5 marks)

The displacement from the origin (x metres) of a particle travelling in a straight line at a time (t seconds) is given by:

$x = 3t^4 - 16t^3 + 18t^2$, where $t \geq 0$.

(a) Show that the velocity $\frac{dx}{dt} = 0$ at time $t = 1$ second. **1**

$\frac{dx}{dt} = 12t^3 - 48t^2 + 36t$

$t=1 \quad \frac{dx}{dt} = v = 12 - 48 + 36 = 0$

(b) Find any other the time(s) when the particle is stationary. **2**

$v = 0$

$12t(t^2 - 4t + 3) = 0$

$12t(t-3)(t-1) = 0$

$t = 1, 0, 3$

(c) Find the distance travelled by the particle in the first 2 seconds. **2**

$t=0 \quad x=0$

$t=1 \quad x=5$

$t=2 \quad x = 48 - 128 + 72$

$= -8$

distance moved

18 m

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Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

2021 Year 11 Final Examination

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$l = \frac{\theta}{360} \times 2\pi r$

Area

$A = \frac{\theta}{360} \times \pi r^2$

$A = \frac{h}{2}(a+b)$

Surface area

$A = 2\pi r^2 + 2\pi rh$

$A = 4\pi r^2$

Volume

$V = \frac{1}{3}Ah$

$V = \frac{4}{3}\pi r^3$

Functions

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

For $ax^3 + bx^2 + cx + d = 0$:

$\alpha + \beta + \gamma = -\frac{b}{a}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$(x-h)^2 + (y-k)^2 = r^2$

Financial Mathematics

$A = P(1+r)^n$

Sequences and series

$T_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$

$T_n = ar^{n-1}$

$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n - 1)}{r-1}, \ r \neq 1$

$S = \frac{a}{1-r}, \ |r| < 1$

Logarithmic and Exponential Functions

$\log_a a^x = x = a^{\log_a x}$

$\log_a x = \frac{\log_b x}{\log_b a}$

$a^x = e^{x \ln a}$

Trigonometric Functions

$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$

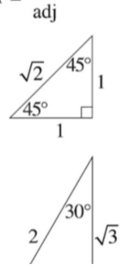
$A = \frac{1}{2}ab\sin C$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$



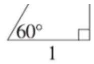
Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution

$A = \frac{1}{2}r^2\theta$



Trigonometric identities

$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

If $t = \tan \frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$\cos A = \frac{1 - t^2}{1 + t^2}$

$\tan A = \frac{2t}{1 - t^2}$

$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

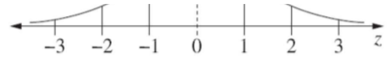
$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A \cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A \sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq r) = \int_a^r f(x)\,dx$

$P(a < X < b) = \int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

- 30 -

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$

Integral Calculus

$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$

where $n \neq -1$

$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$

$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$

$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$

$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$

$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$

$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$

$y = \ln f(x)$ $\qquad \dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

$y = a^{f(x)}$ $\qquad \dfrac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$

$y = \log_a f(x)$ $\qquad \dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a) f(x)}$

$y = \sin^{-1} f(x)$ $\qquad \dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$

$y = \cos^{-1} f(x)$ $\qquad \dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$

$y = \tan^{-1} f(x)$ $\qquad \dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$
$$\approx \frac{b-a}{2n}\left\{f(a)+f(b)+2\left[f(x_1)+\cdots+f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underset{\sim}{u}| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$