NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS ADVANCED

Year 11 Course Assessment Task 3

Wednesday September 18, 2019

General instructions

- Working time – 2 hours
  (plus 10 minutes reading time)
- Write your answers in the space provided.
- Write using a blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.
- Attempt **all** questions.
- At the conclusion of the examination, bundle any additional sheets used in the correct order within this paper and hand to examination supervisors.

Class (please ✔)

◯ 11MAT.6 – Dr Lawson

NESA STUDENT #: .....

Marker's use only.

QUESTION	1-10	11	12	13	14	15	**Total**	%
MARKS	$\overline{10}$	$\overline{18}$	$\overline{10}$	$\overline{16}$	$\overline{16}$	$\overline{13}$	$\overline{83}$	

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 10 minutes for this section

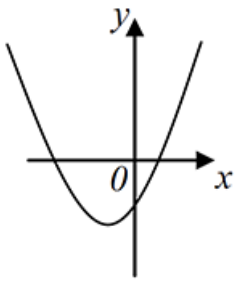
Mark your answers on the answer grid provided (labelled as page 18).

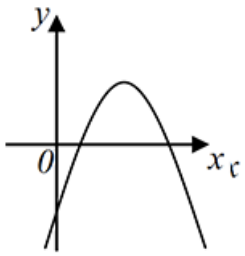
Questions **Marks**

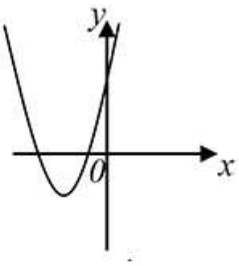
1. When the denominator is rationalised, what is $\dfrac{1}{\sqrt{5}-\sqrt{3}}$ equal to? **1**

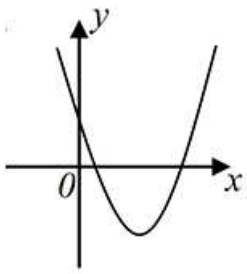
(A) $\dfrac{\sqrt{5}-\sqrt{3}}{2}$ (B) $\dfrac{\sqrt{5}-\sqrt{3}}{16}$ (C) $\dfrac{\sqrt{5}+\sqrt{3}}{2}$ (D) $\sqrt{5}+\sqrt{3}$

2. Which of the following parabolas could have the equation $y=(x-5)(x-1)$? **1**

(A)


(C)


(B)


(D)


3. What is range of the function $f(x)=\sqrt{4-x^2}$? **1**

(A) $(0,4)$ (B) $[-4,4]$ (C) $(-2,2)$ (D) $[0,2]$

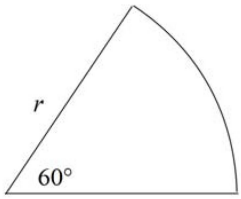
4. What is $e^{2\log_e x}$ is equal to? **1**

(A) $\sqrt{x}$ (B) x^2 (C) $2x$ (D) e^2x

5. If $4^{x-1}=32$, what is the value of x? **1**

(A) 10 (B) 3.5 (C) 3 (D) 6

6. The sector below has an area of 20π square units. **1**

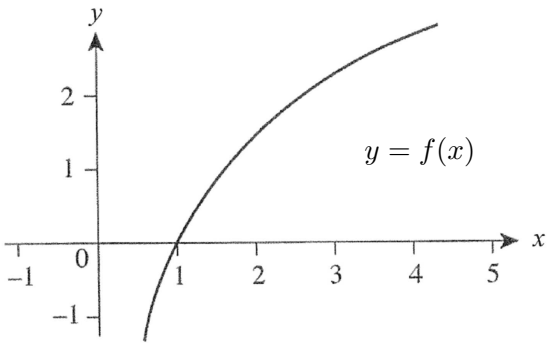


Not to scale

What is the value of r?

(A) $\sqrt{120}$ (B) $\sqrt{120}\pi$ (C) $\sqrt{\dfrac{\pi}{3}}$ (D) $\sqrt{\dfrac{1}{3}}$

7. The graph of $y = f(x)$ below has which of the following properties? **1**



(A) $f'(x) > 0$ and $f''(x) > 0$ (C) $f'(x) < 0$ and $f''(x) > 0$

(B) $f'(x) > 0$ and $f''(x) < 0$ (D) $f'(x) < 0$ and $f''(x) < 0$

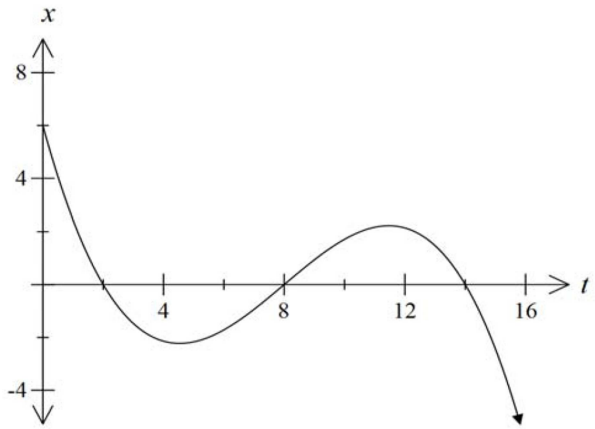
8. Evaluate $\lim\limits_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$ **1**

(A) ∞ (B) 0 (C) 4 (D) 2

9. If $f(x) = a^x + a^{-x}$, which of the following expressions is equal to $\Big[f(x)\Big]^2$? **1**

(A) $1 - f(2x)$ (B) $1 + f(2x)$ (C) $2 - f(2x)$ (D) $2 + f(2x)$

10. The displacement, x metres, from the origin of a particle moving in a straight line at any time (t seconds) is shown in the graph. **1**



When was the particle first at rest?

(A) $t = 0$ (B) $t = 2$ (C) $t = 4.5$ (D) Never

Examination continues overleaf...

Section II

70 marks
Attempt Questions 11 to 15
Allow approximately 1 hour and 50 minutes for this section.

Question 11 (18 Marks) **Marks**

(a) Solve for x: $\dfrac{x+5}{3} - 2x = 7$ **2**

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(b) Factorise $3x^2 + 5x - 2$. **2**

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(c) Simplify $\dfrac{a^2 \times a^{x-4}}{a^{1-2x}}$ **2**

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(d) Consider the function $f(x) = \dfrac{3}{x+1}$.

i. Show that $f(x)$ is neither an even nor an odd function. **1**

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ii. State the domain of $y = f(x)$ in interval notation. **1**

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(e) Given

$$f(x) = \begin{cases} -4 & x \leq -2 \\ x & -2 < x \leq 1 \\ x^2 - 1 & x > 1 \end{cases}$$

i. Find $f(-1)$. **1**

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ii. Find $f(a^2 + 1)$. Simplify your answer. **2**

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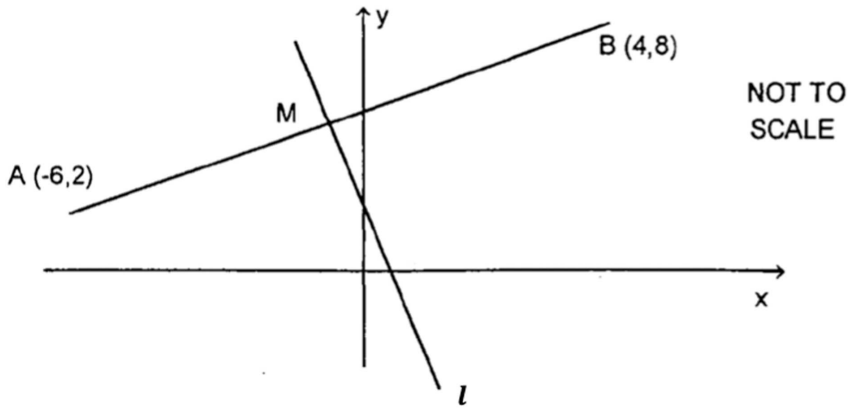
iii. Sketch the graph of $y = f(x)$ **3**

(f) The diagram below show $A(-6, 2)$ and $B(4, 8)$.



i. Find M, the midpoint of AB. **1**

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ii. Show that the gradient of AB is $\frac{3}{5}$. **1**

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iii. Line l is the perpendicular bisector of AB. Show that the equation of line l is $5x + 3y - 10 = 0$. **2**

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Question 12 (10 Marks) **Marks**

(a) Find the derivative of the followings, leaving your answer in simplest form.

i. $y = -3x^4$ **1**

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ii. $f(x) = \left(3x^2 - 5\right)^6$ **2**

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iii. $f(x) = \dfrac{2x-1}{x+4}$ **2**

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iv. $y = 3x^2\sqrt{x-2}$. (No need to fully simplify) **2**

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(b) Differentiate $f(x) = x^2$ from first principles. **2**

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(c) Find the x coordinate of the point on the curve $y = x^2 + 5$ where the tangent has the gradient of -2. **1**

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Examination continues overleaf...

Question 13 (16 Marks) **Marks**

(a) Show that $f(x) = \dfrac{1}{x^3} - x^3$ is always decreasing. **2**

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(b) Consider $f(x) = x^3 + x^2 - x - 1$.

i. Given that $x^3 + x^2 - x - 1 = (x-1)(x+1)^2$, find the x and y-intercepts. **2**

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ii. Find the stationary points and determine their nature. **4**

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iii. Find the point of inflexion. **2**

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iv. Sketch $y = x^3 + x^2 - x - 1$, showing all important features. **2**

v. How many solutions does the equation $x^3 + x^2 - x - 1 = 1$ have? **1**

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(c) The volume, in litres, of water in a bathtub, t minutes after it has started to drain, is given by the equation

$$V = 50(14 - t)^2$$

i. Calculate the initial volume of water in the bathtub. **1**

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ii. At what rate does the water drain out of the bathtub after 6 minutes? **2**

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Examination continues overleaf...

Question 14 (16 Marks) **Marks**

(a) The angle of a sector in a circle of radius 9 cm is $\frac{\pi}{5}$ radians. Find the exact value of the perimeter of the sector. **2**

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(b) i. On the same number plane, graph $y = \cos x$ and $y = \sin 2x$ for $0 \leq x \leq \pi$, showing all intercepts. **2**

ii. Show that the curves first intersect at $x = \frac{\pi}{6}$. **1**

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(c) Show that the exact value of $\cos \frac{\pi}{4} + \sin \frac{2\pi}{3}$ is $\frac{\sqrt{2}+\sqrt{3}}{2}$ **1**

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(d) i. Show that **2**

$$\frac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta = \tan\theta$$

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ii. Hence, or otherwise, solve **2**

$$\frac{\sin^3\theta}{\cos\theta} = 1 - \sin\theta\cos\theta$$

where $0 \leq x \leq 2\pi$.

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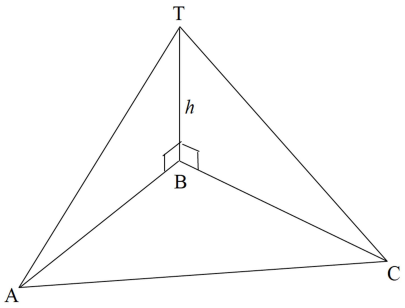
(e) The angle of elevation of a vertical tower BT of height h metres, at a point A due west of it, is $14°$. From the point C, the bearing of the tower is $343°$ and the angle of elevation is $11°$. The points A and C are 1 000 m apart and on the same level as the base B of the tower.



i. By drawing a diagram showing the bearings, explain why $\angle ABC = 107°$. **1**

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ii. Show that $AB = h \tan 76°$ **1**

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iii. Find a similar expression for CB. **1**

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iv. Show that **2**

$$h^2 = \frac{1000^2}{\tan^2 76° + \tan^2 79° - 2 \tan 76° \tan 79° \cos 107°}$$

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v. Hence, find the height of the tower, correct to the nearest metre. **1**

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Examination continues overleaf...

Question 15 (13 Marks) **Marks**

(a) Solve the equation $2\log_e(x-5) = \log_e(2x-7)$ **3**

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(b) Differentiate with respect to x:

i. $y = e^{7x}$ **1**

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ii. $y = \dfrac{e^x}{x^2}$ **2**

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(c) Find the equation of the normal to the curve $y = e^x(x^2+1)$ at $(0,1)$. **3**

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(d) i. Solve for x: $x^2 + 2x + 1 = 0$. **1**

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ii. The displacement of an object at time (t) seconds is given by **3**

$$x = e^{-2t} + 4e^{-t} - 2t$$

Hence, or otherwise, show that the object never comes to rest.
(Hint: Let $m = e^{-t}$)

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End of paper.

Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

NESA STUDENT #: .....

Class (please ✔)

Directions for multiple choice answers

- Read each question and its suggested answers.
- Select the alternative Ⓐ, Ⓑ, Ⓒ, or Ⓓ that best answers the question.
- Mark only one circle per question. There is only *one* correct choice per question.
- Fill in the response circle completely, using blue or black pen, e.g.

Ⓐ Ⓑ ● Ⓓ

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

Ⓐ Ⓑ ⊗ ●

- If you continue to change your mind, write the word `correct` and clearly indicate your final choice with an arrow as shown below:

Ⓐ Ⓑ ⊗ ● ← correct

1 – Ⓐ Ⓑ Ⓒ Ⓓ
2 – Ⓐ Ⓑ Ⓒ Ⓓ
3 – Ⓐ Ⓑ Ⓒ Ⓓ
4 – Ⓐ Ⓑ Ⓒ Ⓓ
5 – Ⓐ Ⓑ Ⓒ Ⓓ

6 – Ⓐ Ⓑ Ⓒ Ⓓ
7 – Ⓐ Ⓑ Ⓒ Ⓓ
8 – Ⓐ Ⓑ Ⓒ Ⓓ
9 – Ⓐ Ⓑ Ⓒ Ⓓ
10 – Ⓐ Ⓑ Ⓒ Ⓓ

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◯ 11MAT.6 – Dr Lawson

NAME .....

Marker's use only.

QUESTION	1-10	11	12	13	14	15	**Total**	%
MARKS	/10	/18	/10	/16	/16	/13	/83	

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 10 minutes for this section

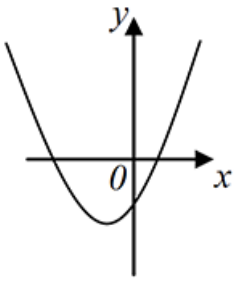
Mark your answers on the answer grid provided (labelled as page 18).

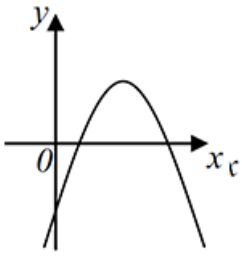
Questions **Marks**

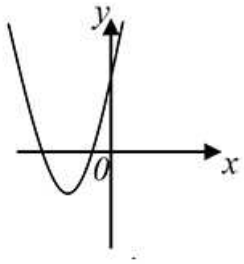
1. When the denominator is rationalised, what is $\dfrac{1}{\sqrt{5}-\sqrt{3}}$ equal to? **1**

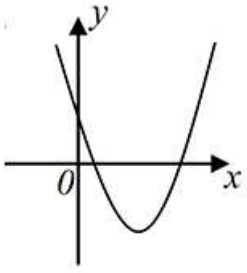
(A) $\dfrac{\sqrt{5}-\sqrt{3}}{2}$ (B) $\dfrac{\sqrt{5}-\sqrt{3}}{16}$ (C) $\dfrac{\sqrt{5}+\sqrt{3}}{2}$ (D) $\sqrt{5}+\sqrt{3}$

2. Which of the following parabolas could have the equation $y=(x-5)(x-1)$? **1**

(A) 

(C) 

(B) 

(D) 

3. What is range of the function $f(x)=\sqrt{4-x^2}$? **1**

(A) $(0,4)$ (B) $[-4,4]$ (C) $(-2,2)$ (D) $[0,2]$

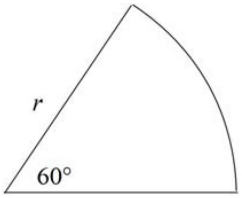
4. What is $e^{2\log_e x}$ is equal to? **1**

(A) $\sqrt{x}$ (B) x^2 (C) $2x$ (D) e^2x

5. If $4^{x-1}=32$, what is the value of x? **1**

(A) 10 (B) 3.5 (C) 3 (D) 6

6. The sector below has an area of 20π square units. **1**

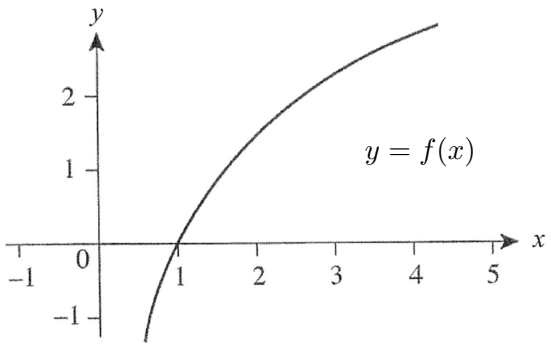


Not to scale

What is the value of r?

(A) $\sqrt{120}$ (B) $\sqrt{120}\pi$ (C) $\sqrt{\frac{\pi}{3}}$ (D) $\sqrt{\frac{1}{3}}$

7. The graph of $y = f(x)$ below has which of the following properties? **1**



(A) $f'(x) > 0$ and $f''(x) > 0$ (C) $f'(x) < 0$ and $f''(x) > 0$

(B) $f'(x) > 0$ and $f''(x) < 0$ (D) $f'(x) < 0$ and $f''(x) < 0$

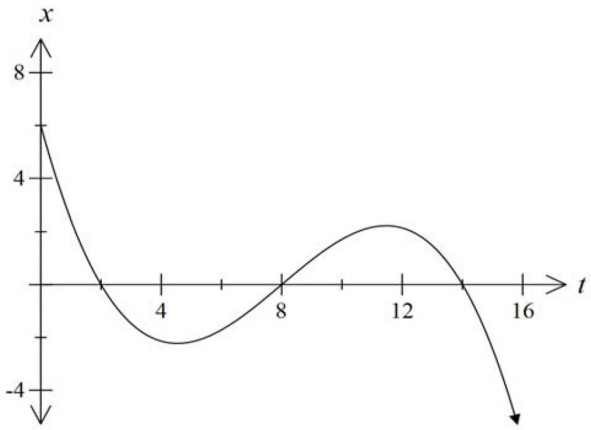
8. Evaluate $\lim\limits_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$ **1**

(A) ∞ (B) 0 (C) 4 (D) 2

9. If $f(x) = a^x + a^{-x}$, which of the following expressions is equal to $\Big[f(x)\Big]^2$? **1**

(A) $1 - f(2x)$ (B) $1 + f(2x)$ (C) $2 - f(2x)$ (D) $2 + f(2x)$

10. The displacement, x metres, from the origin of a particle moving in a straight line at any time (t seconds) is shown in the graph. **1**



When was the particle first at rest?

(A) $t = 0$ (B) $t = 2$ (C) $t = 4.5$ (D) Never

Examination continues overleaf...

Section II

70 marks
Attempt Questions 11 to 15
Allow approximately 1 hour and 50 minutes for this section.

Question 11 (18 Marks) **Marks**

(a) Solve for x: $\frac{x+5}{3} - 2x = 7$ **2**

$x + 5 - 6x = 21$

$-5x = 16$

$\therefore x = \frac{-16}{5}$

(b) Factorise $3x^2 + 5x - 2$. **2**

$3 \quad -1 \quad 2$

$= (x+2)(3x-1)$

(c) Simplify $\frac{a^2 \times a^{x-4}}{a^{1-2x}}$ **2**

$= \frac{a^{x-2}}{a^{1-2x}}$

$= a^{x-2-(1-2x)}$

$= a^{x-2-1+2x}$

$= a^{3x-3}$

(d) Consider the function $f(x) = \frac{3}{x+1}$.

i. Show that $f(x)$ is neither an even nor an odd function. **1**

$f(-x) = \frac{3}{-x+1}$

$= -\frac{3}{x-1}$

$\neq -f(x)$ $\therefore$ neither even nor odd

ii. State the domain of $y = f(x)$ in interval notation. **1**

$x \in (-\infty, -1) \cup (-1, \infty)$

(e) Given

$$f(x) = \begin{cases} -4 & x \leq -2 \\ x & -2 < x \leq 1 \\ x^2 - 1 & x > 1 \end{cases}$$

i. Find $f(-1)$. **1**

$f(-1) = -1$

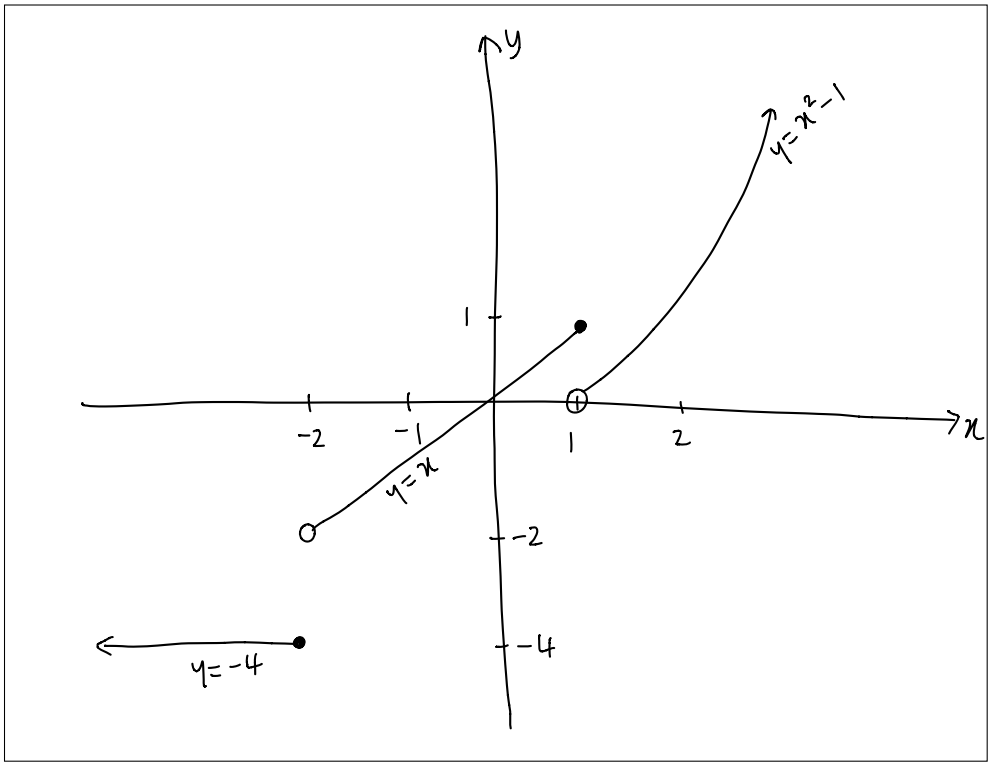
ii. Find $f(a^2+1)$. Simplify your answer. **2**

$f(a^2+1) = (a^2+1)^2 - 1$

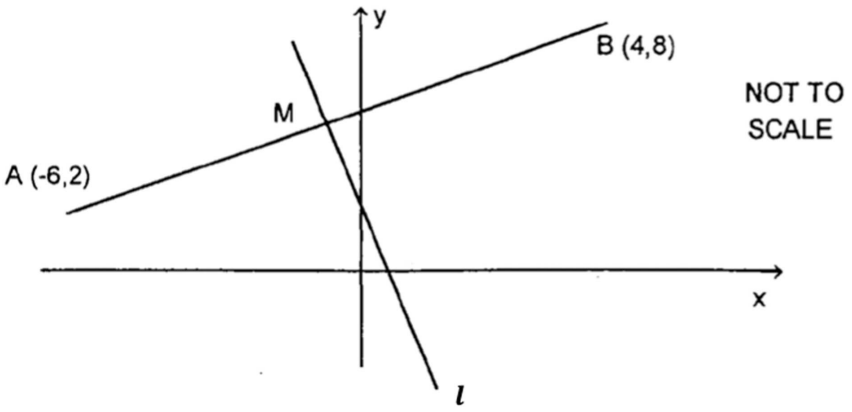
$= a^4 + 2a^2 + 1 - 1$

$= a^4 + 2a^2$

iii. Sketch the graph of $y = f(x)$ **3**



(f) The diagram below show $A(-6, 2)$ and $B(4, 8)$.



i. Find M, the midpoint of AB. **1**

$M\left(\frac{-2}{2}, \frac{10}{2}\right)$

$= M(-1, 5)$

ii. Show that the gradient of AB is $\frac{3}{5}$. **1**

$m_{AB} = \frac{8-2}{4-(-6)}$

$= \frac{6}{10} = \frac{3}{5}$

iii. Line l is the perpendicular bisector of AB. Show that the equation of line l is $5x + 3y - 10 = 0$. **2**

$m_l = -\frac{5}{3}$, $(-1, 5)$

$\therefore$ Eqn: $y - 5 = -\frac{5}{3}(x + 1)$

$3y - 15 = -5x - 5$

$5x + 3y - 10 = 0$

Question 12 (10 Marks) **Marks**

(a) Find the derivative of the followings, leaving your answer in simplest form.

i. $y = -3x^4$ **1**

$y' = -12x^3$

ii. $f(x) = (3x^2 - 5)^6$ **2**

$f'(x) = 6(3x^2-5)^5 \cdot 6x$

$= 36x(3x^2-5)^5$

iii. $f(x) = \dfrac{2x-1}{x+4}$ **2**

$f'(x) = \dfrac{2(x+4) - (2x-1)}{(x+4)^2}$

$= \dfrac{2x+8-2x+1}{(x+4)^2}$

$= \dfrac{9}{(x+4)^2}$

iv. $y = 3x^2\sqrt{x-2}$. (No need to fully simplify) **2**

$= 3x^2(x-2)^{1/2}$

$y' = 6x(x-2)^{1/2} + 3x^2 \cdot \frac{1}{2}(x-2)^{-1/2}$

$= 6x\sqrt{x-2} + \dfrac{3x^2}{2\sqrt{x-2}}$

(b) Differentiate $f(x) = x^2$ from first principles. **2**

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \dfrac{(x+h)^2 - x^2}{h}$

$= \lim_{h\to 0} \dfrac{x^2+2xh+h^2-x^2}{h}$

$= \lim_{h\to 0} 2x + h$

$= 2x$

(c) Find the x coordinate of the point on the curve $y = x^2 + 5$ where the tangent has the gradient of -2. **1**

$y' = 2x$

$\therefore 2x = -2$

$x = -1$

Examination continues overleaf...

Question 13 (16 Marks) **Marks**

(a) Show that $f(x) = \frac{1}{x^3} - x^3$ is always decreasing. **2**

$f(x) = x^{-3} - x^3$

$f'(x) = -3x^{-4} - 3x^2$

$= \frac{-3}{x^4} - 3x^2$

For all real x, $x^2 \geq 0$ & $x^4 > 0$

$\therefore \frac{-3}{x^4} - 3x^2 < 0$ for all x.

$\therefore$ always decreasing

(b) Consider $f(x) = x^3 + x^2 - x - 1$.

i. Given that $x^3 + x^2 - x - 1 = (x-1)(x+1)^2$, find the x and y-intercepts. **2**

x-int : $x = 1, -1$

y-int : $y = -1$

ii. Find the stationary points and determine their nature. **4**

$f'(x) = 3x^2 + 2x - 1$

$3 \quad -1$

$= (x+1)(3x-1)$

$\therefore x = -1, \frac{1}{3}$

$y = 0, \frac{-32}{27}$ $\therefore (-1, 0)$ & $\left(\frac{1}{3}, \frac{-32}{27}\right)$

x	-2	-1	0	$\frac{1}{3}$	1
y'	7	0	-1	0	4
	/	—	\	—	/

$\therefore$ max @ $(-1, 0)$

min @ $\left(\frac{1}{3}, \frac{-32}{27}\right)$

iii. Find the point of inflexion. **2**

$f''(x) = 6x + 2$

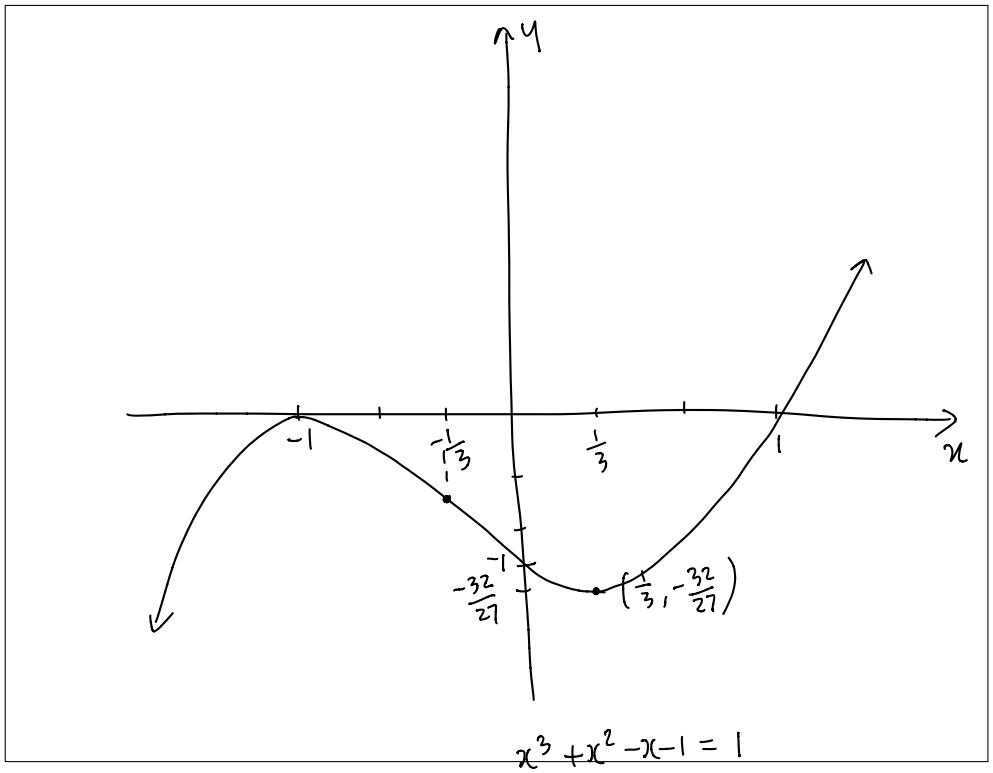
$6x + 2 = 0$

$x = -\frac{1}{3}$, $y = -\frac{16}{27}$

x	-1	$-\frac{1}{3}$	0
y''	-4	0	2

$\therefore$ pt of inflexion @ $\left(-\frac{1}{3}, -\frac{16}{27}\right)$

iv. Sketch $y = x^3 + x^2 - x - 1$, showing all important features. **2**



v. How many solutions does the equation ~~$x^3 + x^2 - x = 0$~~ have? **1**

1 solution

(c) The volume, in litres, of water in a bathtub, t minutes after it has started to drain, is given by the equation

$$V = 50(14 - t)^2$$

i. Calculate the initial volume of water in the bathtub. **1**

When $t=0$, $V = 50 \times 14^2$

$= 9800\,L$

ii. At what rate does the water drain out of the bathtub after 6 minutes? **2**

$\frac{dV}{dt} = 100(14-t)^1 \times (-1)$

$= -100(14-t)$

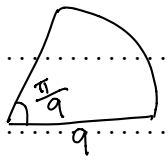
When $t=6$, $\frac{dV}{dt} = -800$ L/min

$\therefore$ The water drains out at 800L/min.

Examination continues overleaf...

Question 14 (16 Marks) **Marks**

(a) The angle of a sector in a circle of radius 9 cm is $\frac{\pi}{5}$ radians. Find the exact value of the perimeter of the sector. **2**

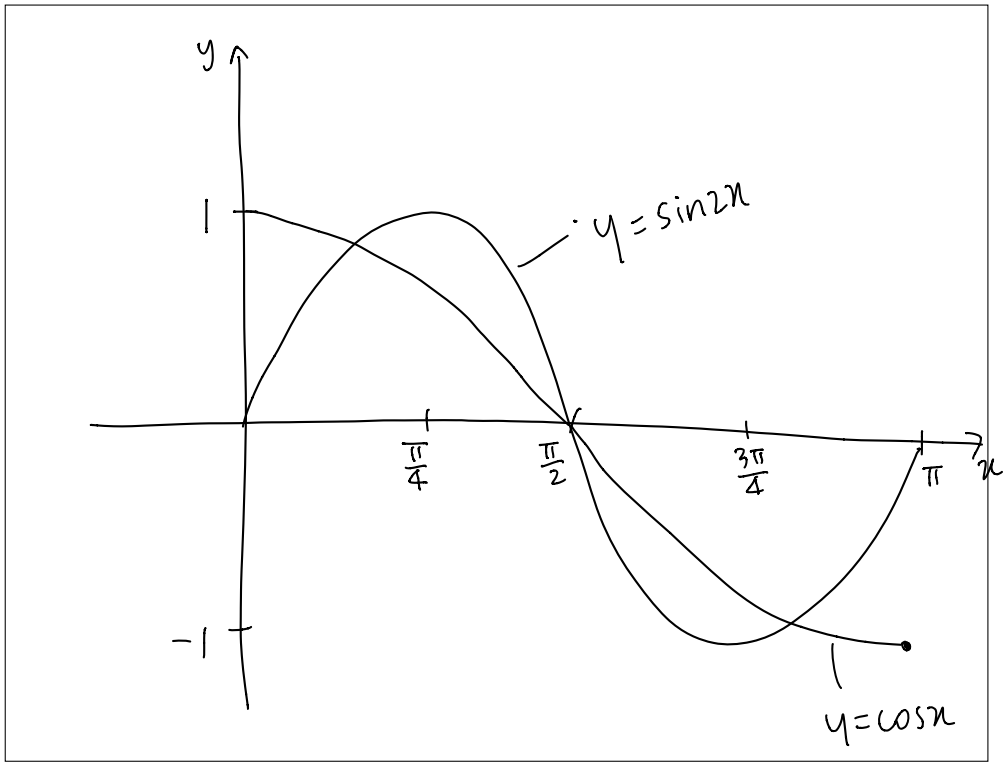


$l = r\theta = 9 \times \frac{\pi}{5} = \frac{9\pi}{5}$

$\therefore \left(\frac{9\pi}{5} + 18\right)$ cm

(b) i. On the same number plane, graph $y = \cos x$ and $y = \sin 2x$ for $0 \leq x \leq \pi$, showing all intercepts. **2**

$T = 2\pi$ $T = \pi$



ii. Show that the curves first intersect at $x = \frac{\pi}{6}$. **1**

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin 2\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\therefore$ Intersect @ $x = \frac{\pi}{6}$

(c) Show that the exact value of $\cos\frac{\pi}{4} + \sin\frac{2\pi}{3}$ is $\frac{\sqrt{2}+\sqrt{3}}{2}$ **1**

$\cos\frac{\pi}{4} + \sin\frac{\pi}{3} = \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}+\sqrt{3}}{2}$

(d) i. Show that **2**

$$\frac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta = \tan\theta$$

LHS $= \dfrac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta$

$= \dfrac{\sin^3\theta + \sin\theta\cos^2\theta}{\cos\theta}$

$= \dfrac{\sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta}$

$= \tan\theta$

$=$ RHS

ii. Hence, or otherwise, solve **2**

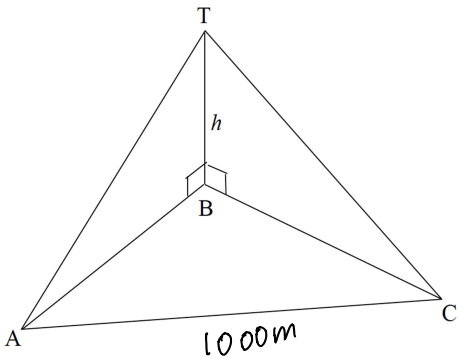
$$\frac{\sin^3\theta}{\cos\theta} = 1 - \sin\theta\cos\theta$$

where $0 \leq x \leq 2\pi$.

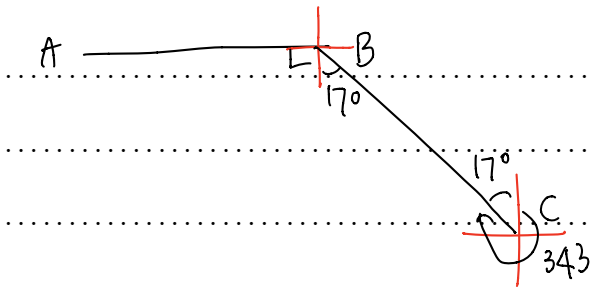
$\tan\theta - \sin\theta\cos\theta = 1 - \sin\theta\cos\theta$

$\tan\theta = 1$, $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

(e) The angle of elevation of a vertical tower BT of height h metres, at a point A due west of it, is 14°. From the point C, the bearing of the tower is 343° and the angle of elevation is 11°. The points A and C are 1 000 m apart and on the same level as the base B of the tower.



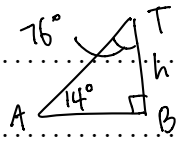
i. By drawing a diagram showing the bearings, explain why $\angle ABC = 107°$. **1**



$\therefore 90° + 17°$

$= 107°$

ii. Show that $AB = h \tan 76°$ **1**



$\tan 76° = \frac{AB}{h}$

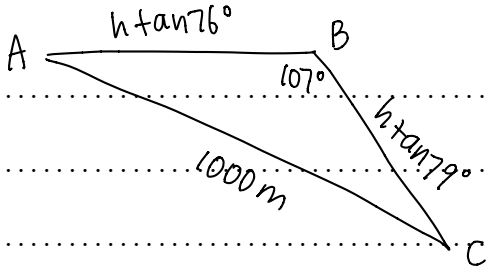
$\therefore AB = h \tan 76°$

iii. Find a similar expression for CB. **1**

$CB = h \tan 79°$

iv. Show that 2

$$h^2 = \frac{1000^2}{\tan^2 76° + \tan^2 79° - 2 \tan 76° \tan 79° \cos 107°}$$



cosine rule

$1000^2 = h^2 \tan^2 76 + h^2 \tan^2 79 - 2 \times h \tan 76 \times h \tan 79° \times \cos 107°$

$1000^2 = h^2 (\tan^2 76 + \tan^2 79 - 2 \tan 76 \tan 79 \cos 107°)$

$\therefore h^2 = \frac{1000^2}{\tan^2 76 + \tan^2 79 - 2 \tan 76 \tan 79 \cos 107°}$

v. Hence, find the height of the tower, correct to the nearest metre. **1**

$h = 135.310\ldots = 135$ m (to the nearest m)

Examination continues overleaf...

Question 15 (13 Marks) **Marks**

(a) Solve the equation $2\log_e(x-5) = \log_e(2x-7)$ **3**

$(x-5)^2 = 2x-7$

$x^2-10x+25 = 2x-7$

$x^2-12x+32 = 0$

$(x-4)(x-8) = 0$

$x = 4, 8$

As $x-5 > 0$, $x = 8$ is the only solution.

(b) Differentiate with respect to x:

i. $y = e^{7x}$ **1**

$y' = 7e^{7x}$

ii. $y = \dfrac{e^x}{x^2}$ **2**

$y' = \dfrac{e^x \cdot x^2 - 2xe^x}{x^4}$

$= \dfrac{xe^x - 2e^x}{x^3}$

$= \dfrac{e^x(x-2)}{x^3}$

(c) Find the equation of the normal to the curve $y = e^x(x^2+1)$ at $(0,1)$. **3**

$y' = e^x(2x) + e^x(x^2+1)$

$= e^x(x^2+2x+1)$

$= e^x(x+1)^2$

At $(0,1)$, $m_{tan} = 1$, $\therefore m_{nor} = -1$

$\therefore$ Eqn of normal is

$y-1 = -x$

$y = -x+1$

(d) i. Solve for x: $x^2 + 2x + 1 = 0$. **1**

$(x+1)^2 = 0$

$x = -1$

ii. The displacement of an object at time (t) seconds is given by **3**

$$x = e^{-2t} + 4e^{-t} - 2t$$

Hence, or otherwise, ~~find the time(s)~~ show that the object never comes to rest. (Hint: Let $m = e^{(} - t)$)

$\dot{x} = -2e^{-2t} - 4e^{-t} - 2$

$= -2(e^{-2t} + 2e^{-t} + 1)$

$= -2(m^2 + 2m + 1)$

$= -2(m+1)^2$

when at rest, $m = -1$

$\therefore e^{-t} = -1$

no solutions

$\therefore$ never come to rest.

End of paper.

Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

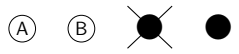
NESA STUDENT #: .....

Class (please ✔)

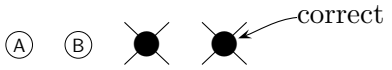
Directions for multiple choice answers

- Read each question and its suggested answers.
- Select the alternative Ⓐ, Ⓑ, Ⓒ, or Ⓓ that best answers the question.
- Mark only one circle per question. There is only *one* correct choice per question.
- Fill in the response circle completely, using blue or black pen, e.g.

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



- If you continue to change your mind, write the word `correct` and clearly indicate your final choice with an arrow as shown below:



1 – Ⓐ Ⓑ Ⓒ Ⓓ
2 – Ⓐ Ⓑ Ⓒ Ⓓ
3 – Ⓐ Ⓑ Ⓒ Ⓓ
4 – Ⓐ Ⓑ Ⓒ Ⓓ
5 – Ⓐ Ⓑ Ⓒ Ⓓ

6 – Ⓐ Ⓑ Ⓒ Ⓓ
7 – Ⓐ Ⓑ Ⓒ Ⓓ
8 – Ⓐ Ⓑ Ⓒ Ⓓ
9 – Ⓐ Ⓑ Ⓒ Ⓓ
10 – Ⓐ Ⓑ Ⓒ Ⓓ