



NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS ADVANCED

2020 Year 11 Course Assessment Task 4

Friday, 11 September 2020

General instructions

- Working time – 2 hours.
(plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt all questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer grid provided

SECTION II

- Commence each new question on a new booklet. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT #: CLASS:

Marker's use only

QUESTION	1-5	6	7	8	9	10	11	12	13	14	Total	%
MARKS	$\overline{5}$	$\overline{5}$	$\overline{9}$	$\overline{10}$	$\overline{9}$	$\overline{7}$	$\overline{5}$	$\overline{10}$	$\overline{15}$	$\overline{10}$	$\overline{85}$	

Section I

5 marks

Attempt Questions 1 to 5

Allow approximately 10 minutes for this section

Mark your answers on the answer grid provided.

1. What is the value of $\sqrt{48} - \sqrt{12}$? 1
- (A) 2 (B) $\sqrt{12}$ (C) $\sqrt{24}$ (D) 6
2. What are the x intercepts of the parabola $y = x^2 - 2x - 3$? 1
- (A) -3 and -1 (C) -1 and 3
(B) -3 and 1 (D) 1 and 3
3. Which of the following statements is correct for the function $y = 2 \sin 2x$? 1
- (A) It has range $[-2, 2]$ and period π . (C) It has range $[-1, 1]$ and period π .
(B) It has range $[-2, 2]$ and period 2π . (D) It has range $[-1, 1]$ and period 2π .
4. If $a = e^x$, what is the value of $\log_e \frac{1}{a^2}$? 1
- (A) $-x^2$ (C) $2x$
(B) $-2x$ (D) x^2
5. If $y = \sqrt{2x+1}$, what is the expression for $\frac{dy}{dx}$? 1
- (A) $\frac{dy}{dx} = 2\sqrt{2x+1}$ (C) $\frac{dy}{dx} = \frac{1}{\sqrt{2x+1}}$
(B) $\frac{dy}{dx} = \frac{1}{2\sqrt{2x+1}}$ (D) $\frac{dy}{dx} = \frac{2}{\sqrt{2x+1}}$

Section II

75 marks

Attempt Questions 6 to 14

Allow approximately 1 hour 50 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (5 marks)

Marks

- (a) Fully simplify and leave the answer without negative index:

1

$$y^{\frac{3}{8}} \times y^{\frac{5}{8}} \times y^{-2}$$

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- (b) Given that $\log_y x = 2.75$ and $\log_y z = 0.25$, find the value of $\log_y \sqrt{xz}$.

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- (c) Fully factorise: $80x^4 - 5y^4$

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Question 7 (9 marks)

(a) Evaluate

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$$\lim_{x \rightarrow \infty} \frac{10x^2 - 5x}{2x^2 + 8x}$$

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(b) Solve for x : $3^x + 3^{x+1} - 36 = 0$

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(c) Solve by completing the square: $x^2 - 6x - 7 = 0$.

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- (d) Simplify the following, expressing your answer with a rational denominator:

3

$$\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$$

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Question 8 (10 marks)

- (a) Find the domain and the range of the following function:

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$$y = \sqrt{x^2 - 4}$$

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- (b) State whether the following function is even, odd or neither:

2

$$f(x) = x^3 - x + 1$$

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(c) Consider the following piecewise function:

$$f(x) = \begin{cases} x + 3 & \text{for } x < -1 \\ x^2 & \text{for } -1 \leq x \leq 1 \\ \sqrt{x} & \text{for } x > 1 \end{cases}$$

- i. Find the value of $3f(4) + f(-2) - f\left(-\frac{1}{2}\right)$. 3

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- ii. Sketch the above piecewise function. 3



Question 9 (9 marks)

Sketch the following, showing all important features:

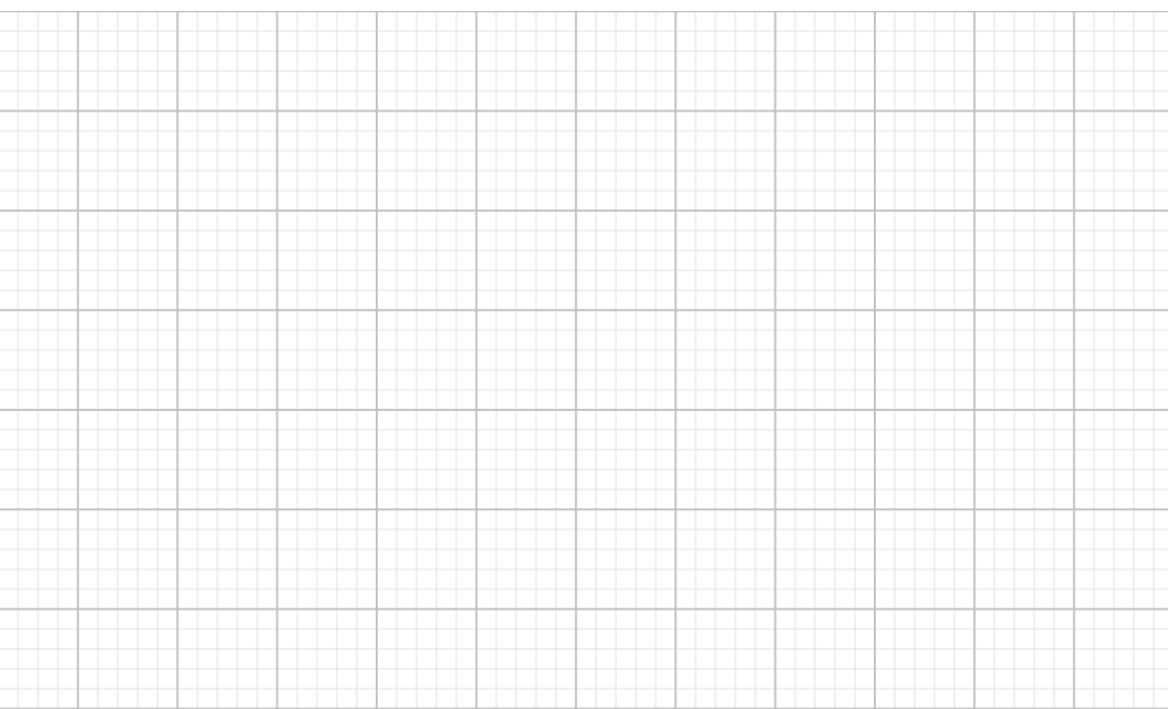
(a) $f(x) = (x - 1)(x - 2)(x + 2)$

2



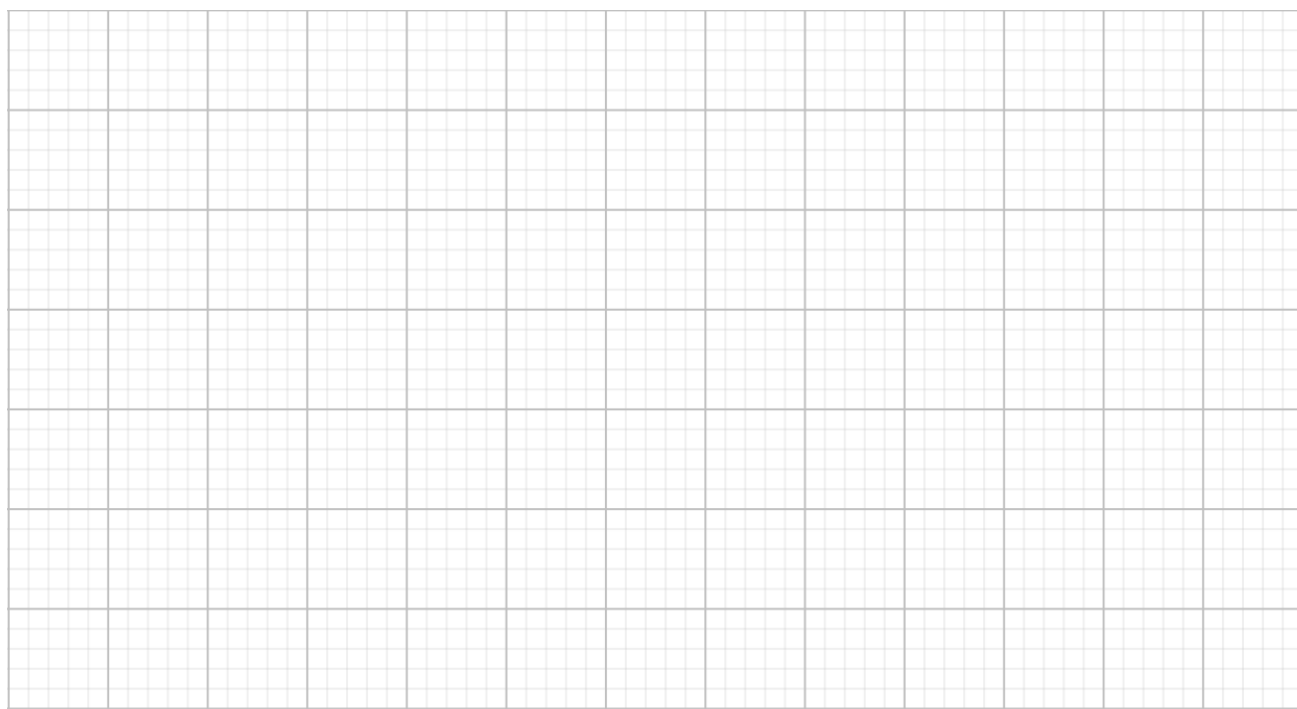
(b) $y = |x + 3| - 2$

2



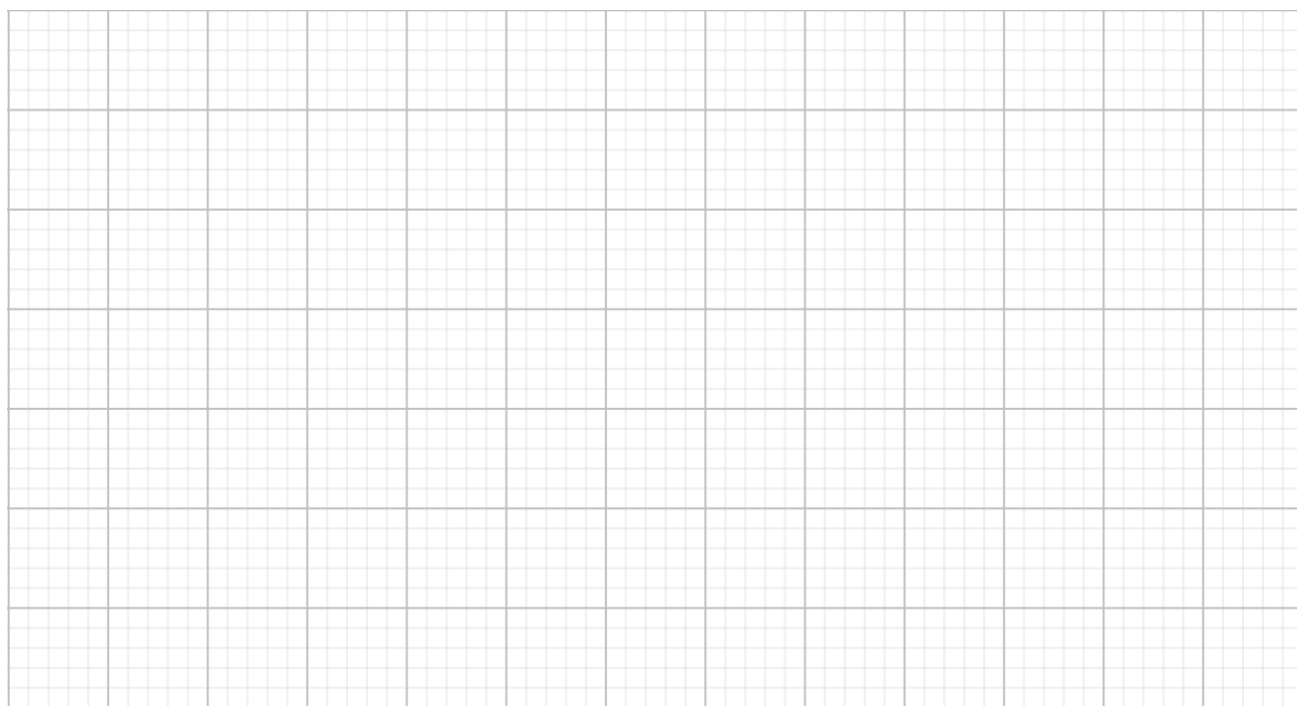
(c) $y = 2 \sin 3x$ for $0 \leq x \leq 2\pi$

3



(d) $y = -e^x + 2$

2



Question 10 (7 marks)

- (a) Simplify the following: $\tan^2 \theta \cos^2 \theta + \cot^2 \theta \sin^2 \theta$ 2

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- (b) Prove the following trigonometric identity: 3

$$\frac{\cos \theta}{1 + \sin \theta} + \frac{1 + \sin \theta}{\cos \theta} = 2 \sec \theta$$

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- (c) Solve $\sqrt{3} \tan x = -1$ for $0^\circ \leq \theta \leq 360^\circ$. 2

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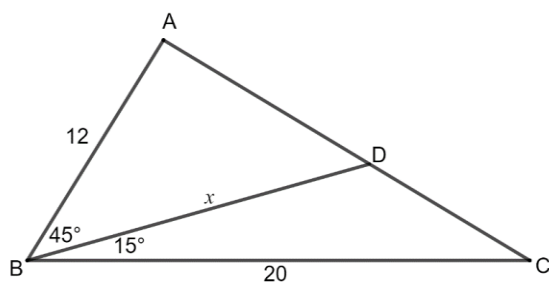
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Question 11 (5 marks)

Consider the diagram given below:



- (a) Find the exact area of $\triangle ABC$.

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- (b) Hence, find the value of x .

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Question 12 (10 marks)

For the function $y = x^3 - 6x^2 + 9x$

- (a) Find the stationary points and determine their nature.

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- (b) Find the coordinates of any points of inflexion.

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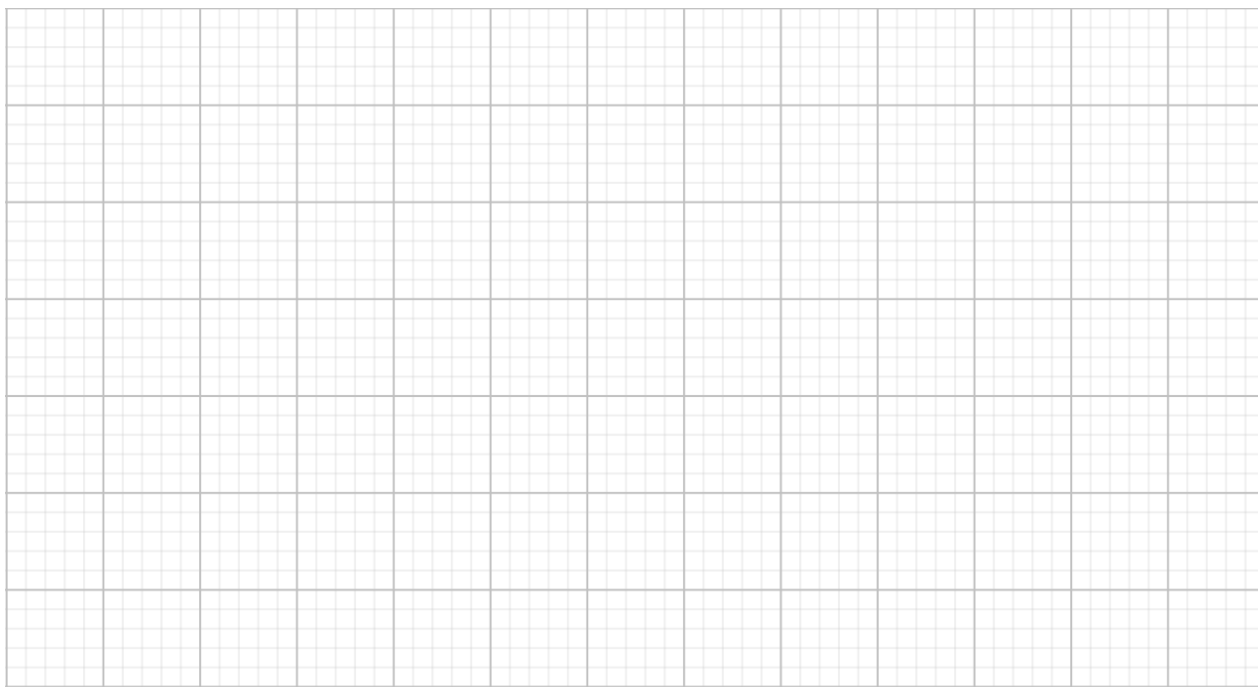
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(c) Sketch the curve in the domain $0 \leq x \leq 4$.

3



(d) For what values of x is the function decreasing?

1

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Question 13 (15 marks)

- (a) Differentiate $f(x) = x^2 - 3x + 5$ using first principles.

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- (b) Differentiate the following:

i. $x^2\sqrt{2x+1}$

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ii. $\frac{e^{5x-x^3}}{1-x}$

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- (c) Find the equation of the tangent to the curve $y = 3x^3 - 2x^2 + 1$ at $x = 1$.

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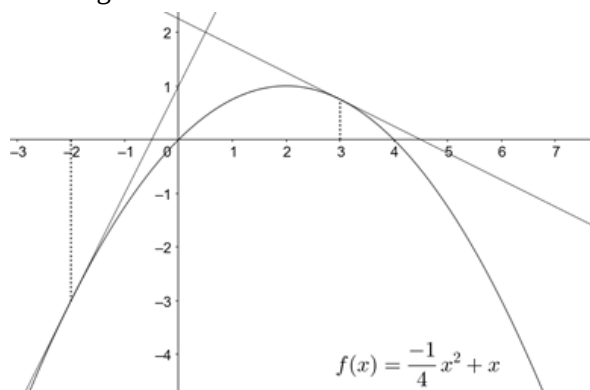
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- (d) The graph of $f(x) = -\frac{1}{4}x^2 + x$ is given below:



- i. Find the expression for $f'(x)$.

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- ii. Hence, show that the illustrated tangents are perpendicular.

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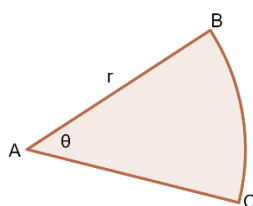
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Question 14 (10 marks)

- (a)
- ABC
- is a sector of a circle with radius
- r
- as shown



It is given that the perimeter of the sector is equal in magnitude to the area of the sector.

- i. Show that
- $\theta = \frac{4}{r-2}$
- radians.

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- ii. Hence, show that the area of the sector
- ABC
- is equal to
- $\frac{2r^2}{r-2}$
- .

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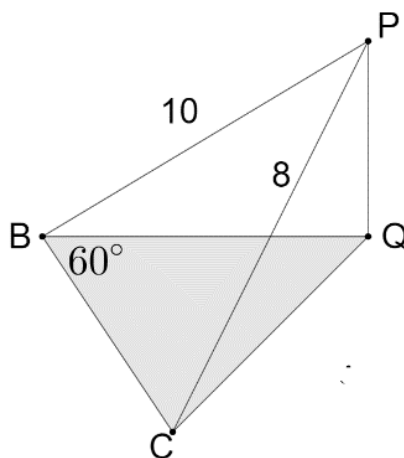
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- (b) A vertical post PQ with height h metres is held by two ropes PB due west and PC due south of it.

It is given that $\angle QBC = 60^\circ$, $PB = 10$ m, $PC = 8$ m and $\angle BCQ = 30^\circ$.



- i. Show that $\frac{CQ}{BQ} = \sqrt{3}$

1

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- ii. Hence, show that

3

$$\frac{64 - h^2}{100 - h^2} = 3$$

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- iii. Find the value of h .

2

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Answer Sheet for Section I

Mark answers to Section I by fully blackening the correct circle

NESA STUDENT #:

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|-----|-------------------------|-------------------------|-------------------------|-------------------------|
| 1 – | ☐ A | ☐ B | ☐ C | ☐ D |
| 2 – | ☐ A | ☐ B | ☐ C | ☐ D |
| 3 – | ☐ A | ☐ B | ☐ C | ☐ D |
| 4 – | ☐ A | ☐ B | ☐ C | ☐ D |
| 5 – | ☐ A | ☐ B | ☐ C | ☐ D |

2018 Mathematics Extension 1 HSC Course Assessment Task 1

Student Self Reflection

1. In hindsight, did I do the best I can? Why or why not?

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2. Which topics did I need more help with, and what parts specifically?

- Q1 - Locus

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- Q2 - Locus

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- Q3 – Series

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- Q4 – Euclidean Geometry

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- Q5 -

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3. What other parts from the feedback session can I take away to refine my solutions for future reference?

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(B) $\frac{dy}{dx} = \frac{1}{2\sqrt{2x+1}}$ (D) $\frac{dy}{dx} = \frac{2}{\sqrt{2x+1}}$

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Question 6 (5 marks)

Marks

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1

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$$y^{-1}$$

- (b) Given that $\log_y x = 2.75$ and $\log_y z = 0.25$, find the value of $\log_y \sqrt{xz}$.

2

$$\begin{aligned} \log_y \sqrt{xz} &= \frac{1}{2} \log_y (xz) \\ &= \frac{1}{2} [\log_y x + \log_y z] \\ &= \frac{1}{2} [2.75 + 0.25] \\ &= 1.5 \end{aligned}$$

- (c) Fully factorise: $80x^4 - 5y^4$

2

$$\begin{aligned} &= 5 (16x^4 - y^4) \\ &= 5 ((4x^2)^2 - (y^2)^2) \\ &= 5 (4x^2 - y^2) (4x^2 + y^2) \\ &= 5 (2x - y) (2x + y) (4x^2 + y^2) \end{aligned}$$

Question 7 (9 marks)

(a) Evaluate

2

$$\lim_{x \rightarrow \infty} \frac{10x^2 - 5x}{2x^2 + 8x}$$

$$= \lim_{x \rightarrow \infty} \frac{10 - \frac{5}{x}}{2 - \frac{8}{x}}$$

$$\text{as } x \rightarrow \infty, \frac{1}{x} \rightarrow 0$$

$$\therefore \lim_{x \rightarrow \infty} \frac{10 - \frac{5}{x}}{2 - \frac{8}{x}} = 5$$

(b) Solve for x : $3^x + 3^{x+1} - 36 = 0$

2

$$3^x + 3(3^x) - 36 = 0$$

$$\text{let } 3^x = y$$

$$\therefore y + 3y - 36 = 0$$

$$4y - 36 = 0$$

$$y = 9$$

$$\therefore 3^x = 9$$

$$3^x = 3^2 \quad \therefore x = 2$$

(c) Solve by completing the square: $x^2 - 6x - 7 = 0$.

2

$$x^2 - 6x = 7$$

$$x^2 - 6x + 9 = 16$$

$$(x-3)^2 = 16$$

$$x-3 = 4, \quad x-3 = -4$$

$$x = 7, \quad x = -1$$

- (d) Simplify the following, expressing your answer with a rational denominator:

3

$$\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$$

$$\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$$

$$= \frac{5 - 2\sqrt{15} + 3}{2}$$

$$= \frac{8 - 2\sqrt{15}}{2} = 4 - \sqrt{15}$$

Question 8 (10 marks)

- (a) Find the domain and the range of the following function:

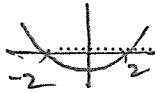
2

$$y = \sqrt{x^2 - 4}$$

$$D: x^2 - 4 \geq 0$$

$$\text{Range: } y \geq 0$$

$$(x+2)(x-2) \geq 0$$



$$x \leq -2, x \geq 2$$

- (b) State whether the following function is even, odd or neither:

2

$$f(x) = x^3 - x + 1$$

$$f(-x) = (-x)^3 - (-x) + 1$$

$$= -x^3 + x + 1$$

$$f(x) \neq f(-x)$$

Not an even function

$$f(-x) = -(x^3 - x - 1)$$

$$f(x) \neq -f(-x)$$

Not an odd function

∴ $f(x)$ is neither even nor odd.

(c) Consider the following piecewise function:

$$f(x) = \begin{cases} x+3 & \text{for } x < -1 \\ x^2 & \text{for } -1 \leq x \leq 1 \\ \sqrt{x} & \text{for } x > 1 \end{cases}$$

i. Find the value of $3f(4) + f(-2) - f(-\frac{1}{2})$.

3

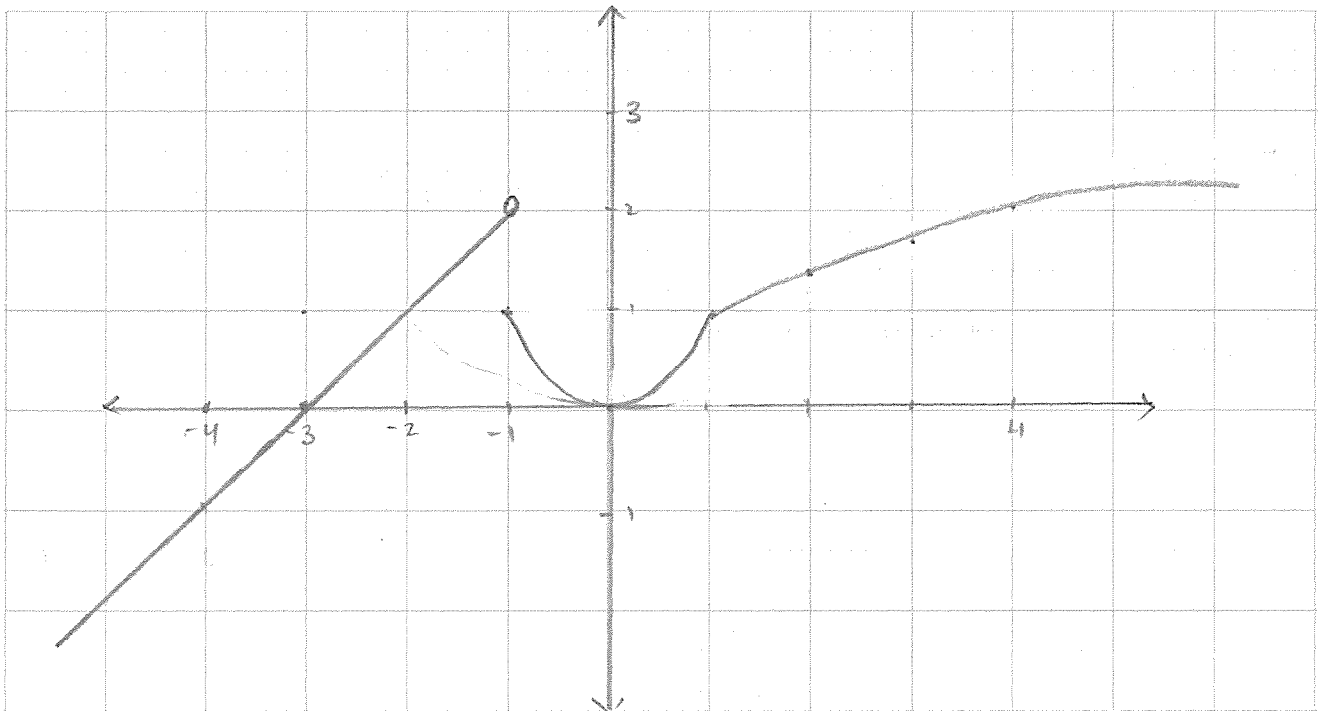
$$= 3\sqrt{4} + (-2+3) - (-\frac{1}{2})^2$$

$$= 6 + 1 - \frac{1}{4}$$

$$= \frac{27}{4}$$

ii. Sketch the above piecewise function.

3

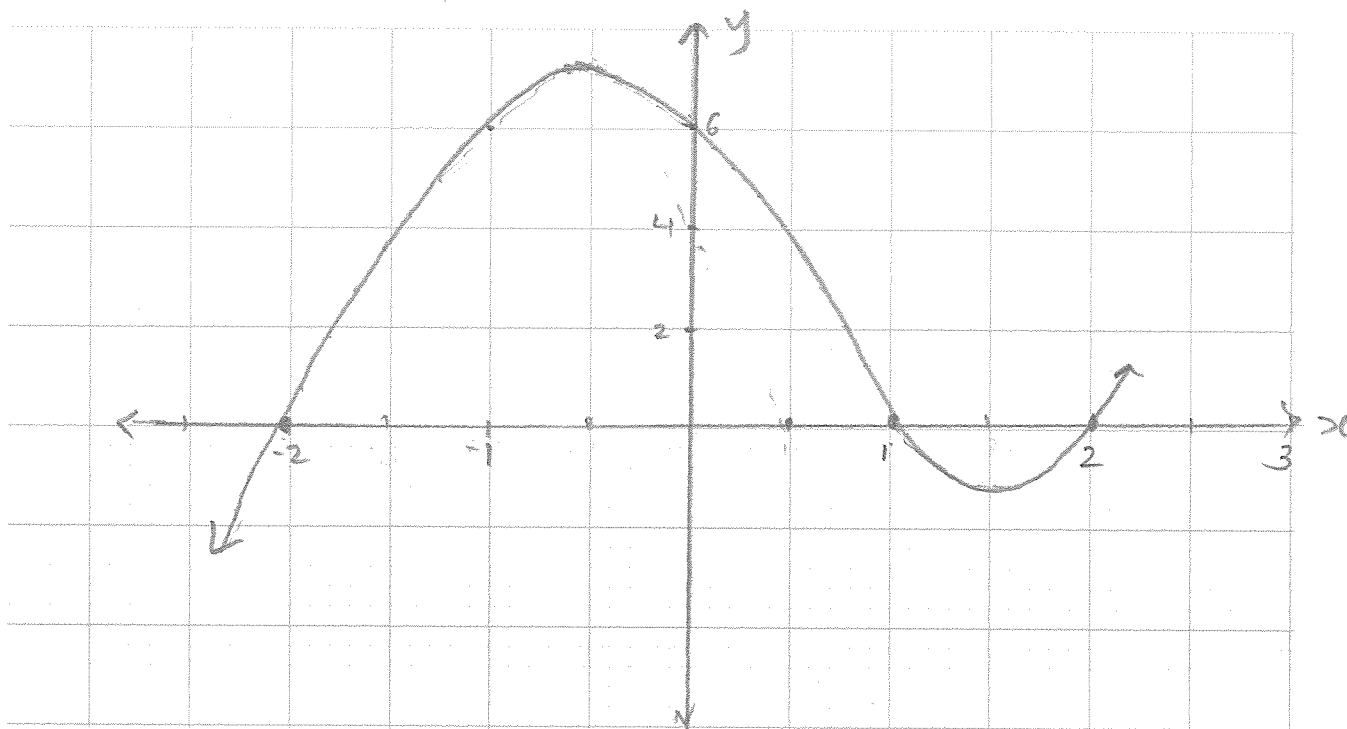


Question 9 (9 marks)

Sketch the following, showing all important features:

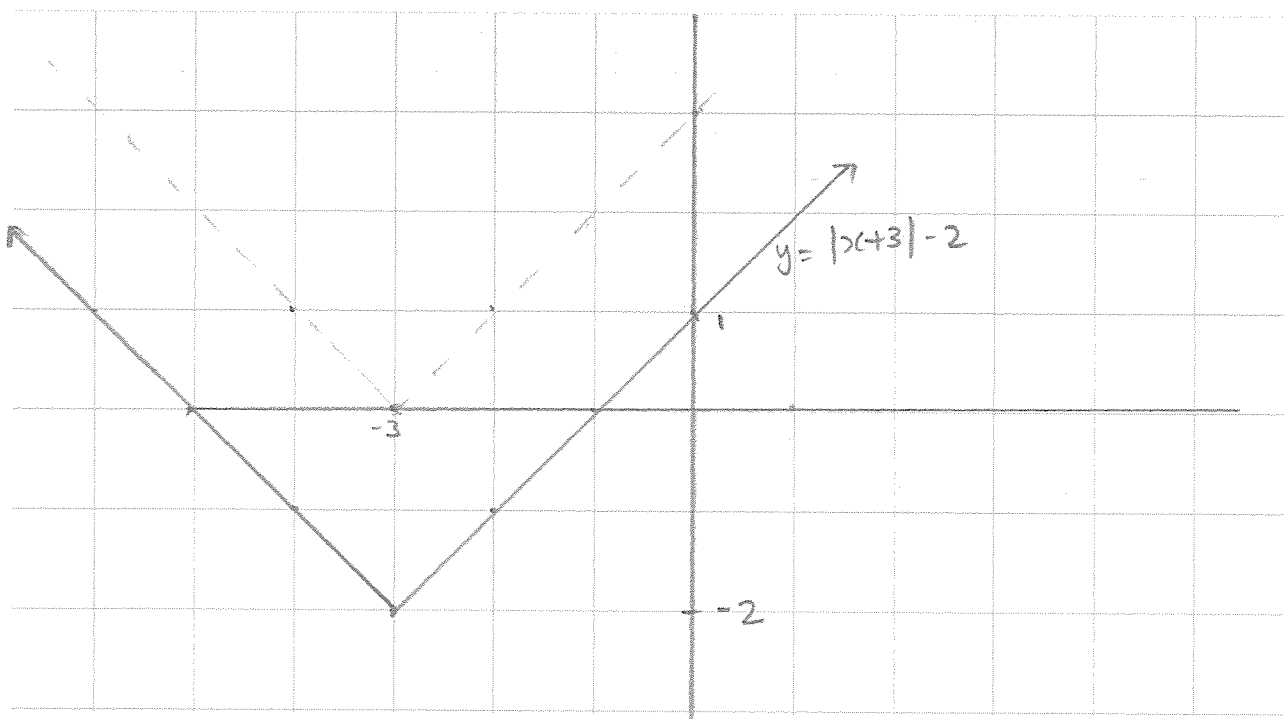
(a) $f(x) = (x - 1)(x - 2)(x + 2)$

2



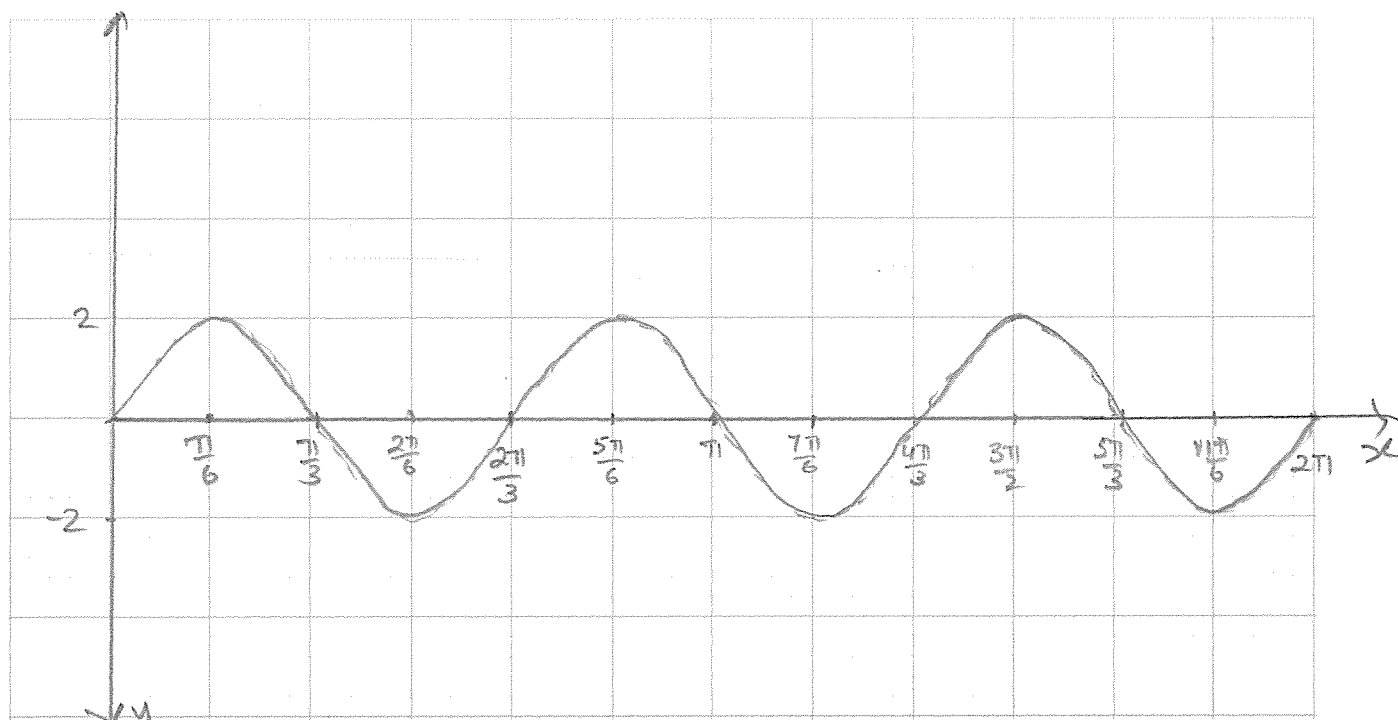
(b) $y = |x + 3| - 2$

2



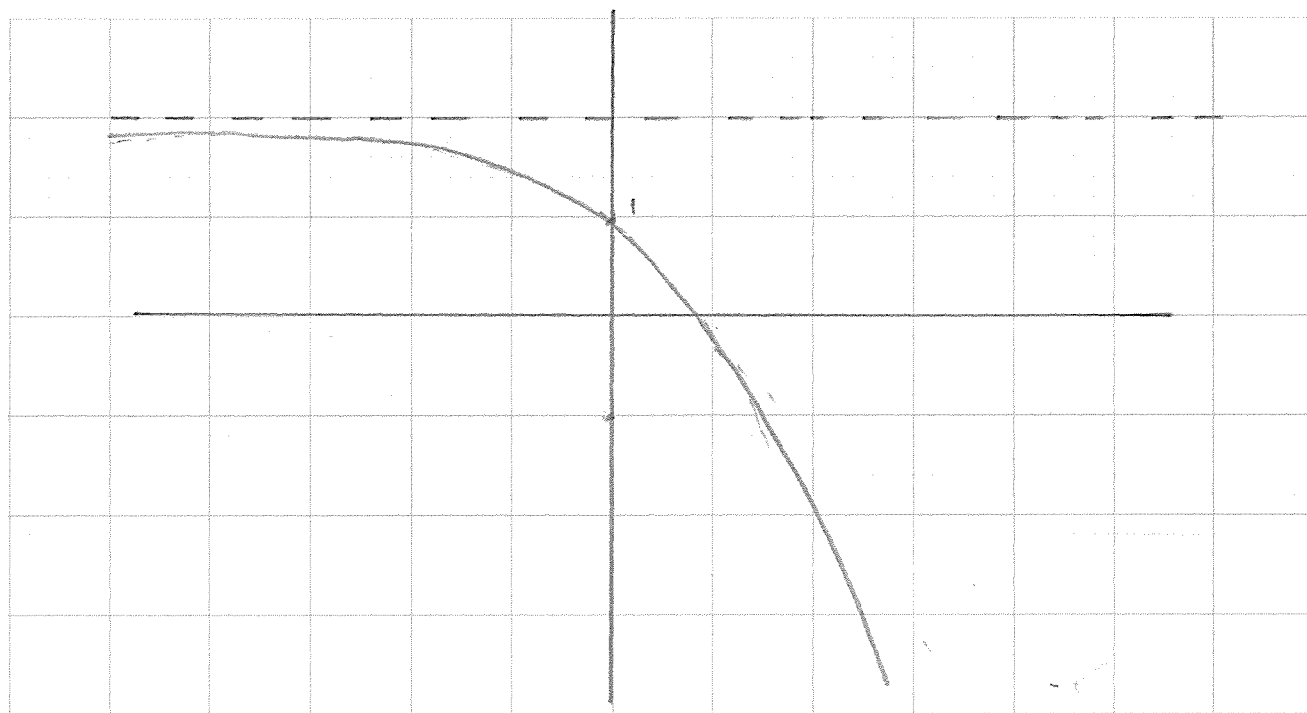
(c) $y = 2 \sin 3x$ for $0 \leq x \leq 2\pi$

3



(d) $y = -e^x + 2$

2



$$\begin{aligned} -e^x &= -2 \\ e^x &= 2 \end{aligned}$$

Question 10 (7 marks)

- (a) Simplify the following:
- $\tan^2 \theta \cos^2 \theta + \cot^2 \theta \sin^2 \theta$

2

$$\frac{\sin^2 \theta}{\cos^2 \theta} \cos^2 \theta + \frac{\cos^2 \theta}{\sin^2 \theta} \sin^2 \theta$$

$$= \sin^2 \theta + \cos^2 \theta$$

$$= 1$$

- (b) Prove the following trigonometric identity:

3

$$\frac{\cos \theta}{1 + \sin \theta} + \frac{1 + \sin \theta}{\cos \theta} = 2 \sec \theta$$

$$\text{LHS} \quad \frac{\cos \theta}{1 + \sin \theta} + \frac{1 + \sin \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta + 1 + 2 \sin \theta + \sin^2 \theta}{(1 + \sin \theta) \cos \theta}$$

$$= \frac{2 + 2 \sin \theta}{(1 + \sin \theta) \cos \theta} = \frac{2(1 + \sin \theta)}{(1 + \sin \theta) \cos \theta}$$

$$= \frac{2}{\cos \theta} = 2 \sec \theta \quad \text{RHS}$$

- (c) Solve
- $\sqrt{3} \tan x = -1$
- for
- $0^\circ \leq \theta \leq 360^\circ$
- .

2

$$\tan x = \frac{-1}{\sqrt{3}}$$

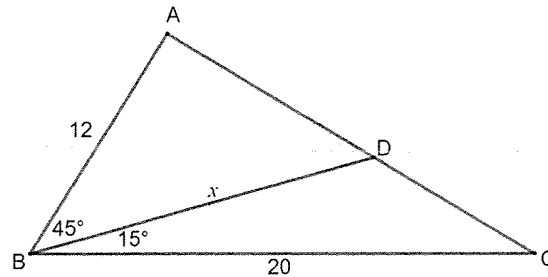
$$\text{acute angle} = 30^\circ$$

$$x = (180 - 30^\circ), (360 - 30^\circ)$$

$$= 150^\circ, 330^\circ$$

Question 11 (5 marks)

Consider the diagram given below:



- (a) Find the exact area of $\triangle ABC$.

2

$$\begin{aligned} \text{Area}_{\triangle ABC} &= \frac{1}{2} \times 12 \times 20 \times \sin 60^\circ \\ &= \frac{240\sqrt{3}}{4} \\ &= 60\sqrt{3} \text{ units}^2 \end{aligned}$$

- (b) Hence, find the value of x .

3

$$\begin{aligned} \frac{1}{2} \times 12 \times x \sin 45^\circ + \frac{1}{2} \times 20 \times x \sin 15^\circ &= 60\sqrt{3} \\ 2x (3 \sin 45^\circ + 5 \sin 15^\circ) &= 60\sqrt{3} \\ x &= \frac{60\sqrt{3}}{2(3 \sin 45^\circ + 5 \sin 15^\circ)} \end{aligned}$$

Question 12 (10 marks)For the function $y = x^3 - 6x^2 + 9x$

- (a) Find the stationary points and determine their nature.

4

$$y = x^3 - 6x^2 + 9x$$

$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

$$\text{for st points, } \frac{dy}{dx} = 0$$

$$3x^2 - 12x + 9 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3, x = 1$$

$$\text{when } x = 1, y = 1 - 6 + 9 = 4$$

$$x = 3, y = 27 - 54 + 27 = 0$$

$$\therefore \text{st pts: } (1, 4), (3, 0)$$

$$y'' = 6x - 12$$

$$\text{when } x = 1, y'' = -6$$

$$y'' < 0 \therefore (1, 4) \text{ a maximum}$$

$$\text{when } x = 3$$

$$y'' = 6 \times 3 - 12 = 6$$

$$y'' > 0$$

$$\therefore (3, 0) \text{ is a minimum}$$

- (b) Find the coordinates of any points of inflexion.

2

for point of inflexion

$$y' = 0$$

$$6x - 12 = 0$$

$$x = 2$$

$$\therefore y = 8 - 6 \times 4 + 18$$

$$= 26 - 24 = 2$$

$$\therefore \text{POI: } (2, 2)$$

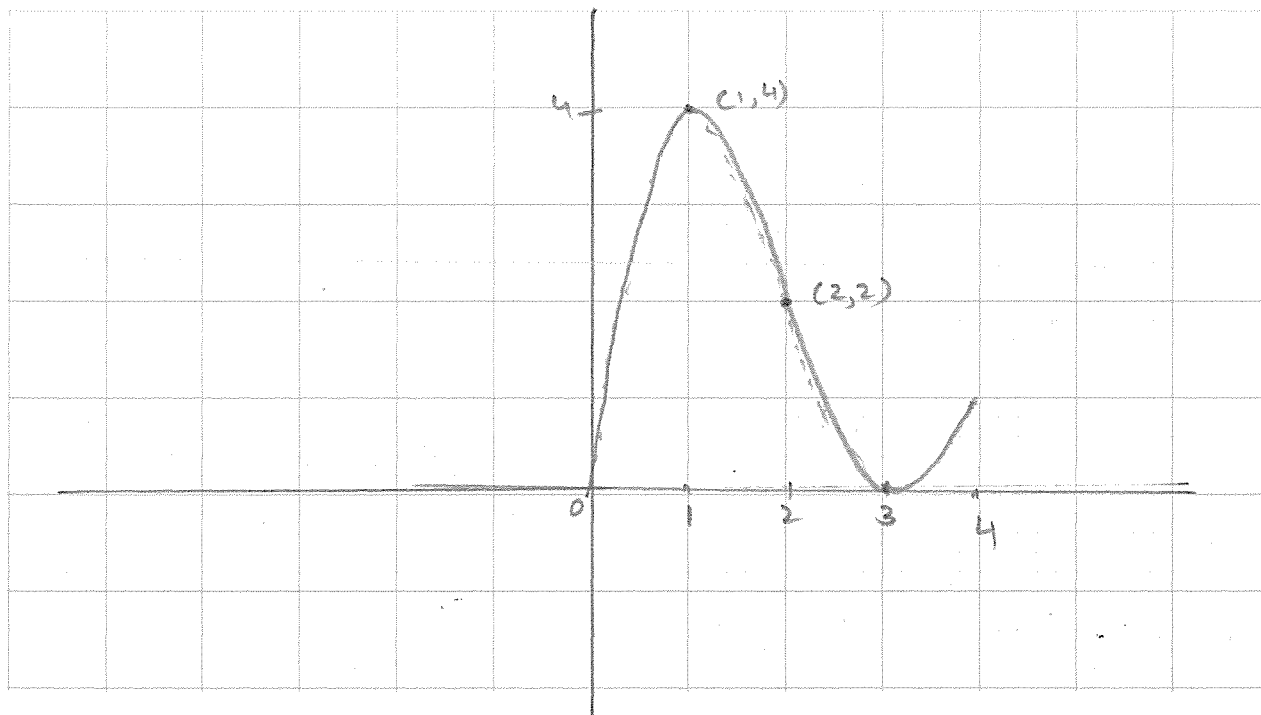
Test the nature

x	1	2	3
y''	-6	0	6

at $x = 2$ change of concavity $(2, 2)$ is a point of inflexion.

- (c) Sketch the curve in the domain $0 \leq x \leq 4$.

3



- (d) For what values of x is the function decreasing?

1

..... function is decreasing for $1 < x < 3$

.....

.....

Question 13 (15 marks)

- (a) Differentiate
- $f(x) = x^2 - 3x + 5$
- using first principles.

3

$$\begin{aligned}
 f'(x+h) &= (x+h)^2 - 3(x+h) + 5 \\
 \frac{f(x+h) - f(x)}{x+h-x} &= \frac{x^2 + 2hx + h^2 - 3x - 3h + 5 - x^2 + 3x - 5}{h} \\
 \frac{dy}{dx} \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2hx + h^2 - 3h}{h} \\
 &= \lim_{h \rightarrow 0} 2x - 3 + h \\
 f'(x) &= 2x - 3
 \end{aligned}$$

- (b) Differentiate the following:

i. $x^2\sqrt{2x+1}$

$$\begin{aligned}
 u &= x^2 & v &= (2x+1)^{\frac{1}{2}} \\
 u' &= 2x & v' &= \frac{1}{2}(2x+1)^{-\frac{1}{2}} \times 2 = \frac{1}{\sqrt{2x+1}}
 \end{aligned}$$

3

$$\begin{aligned}
 y' &= uv' + vu' \\
 &= \frac{x^2}{\sqrt{2x+1}} + \frac{2x\sqrt{2x+1}}{\sqrt{2x+1}} = \frac{x^2 + 4x^2 + 2x}{\sqrt{2x+1}} = \frac{5x^2 + 2x}{\sqrt{2x+1}}
 \end{aligned}$$

ii. $\frac{e^{5x-x^3}}{1-x}$

$$\begin{aligned}
 u &= e^{5x-x^3} & v &= 1-x \\
 u' &= (5-3x^2)e^{5x-x^3} & v' &= -1
 \end{aligned}$$

3

$$\begin{aligned}
 y &= \frac{v'u - u'v}{v^2} = \frac{(5-3x^2)(1-x)e^{5x-x^3} + e^{5x-x^3}}{(1-x)^2} \\
 &= \frac{e^{5x-x^3} [(5-3x^2)(1-x) + 1]}{(1-x)^2} \\
 &= \frac{e^{5x-x^3} (5-3x^2)}{1-x}
 \end{aligned}$$

- (c) Find the equation of the tangent to the curve $y = 3x^3 - 2x^2 + 1$ at $x = 1$.

3

$$y' = 9x^2 - 4x$$

at $x=1$, $m_{\text{tangent}} = 9 - 4 = 5$

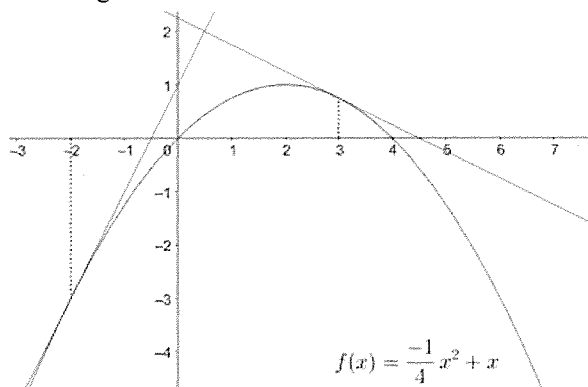
when $x=1$, $y = 3 - 2 + 1 = 2$ $(1, 2)$

$\therefore$ eq. of tangent

$$y - 2 = 5(x - 1)$$

$$y = 5x - 3$$

- (d) The graph of $f(x) = -\frac{1}{4}x^2 + x$ is given below:



- i. Find the expression for $f'(x)$.

1

$$f'(x) = -\frac{2x}{4} + 1 = -\frac{1}{2}x + 1$$

- ii. Hence, show that the illustrated tangents are perpendicular.

2

$$m_1 = f'(-2) = -\frac{1}{2} \times (-2) + 1 = 2$$

$$m_2 = f'(3) = -\frac{1}{2} \times 3 + 1$$

$$= -\frac{3}{2} + 1$$

$$= -\frac{1}{2}$$

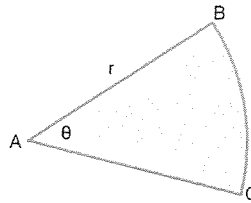
$$f'(-2) \times f'(3) = 2 \times -\frac{1}{2}$$

$$m_1 \times m_2 = -1$$

$\therefore$ tangents at $x = -2$, and $x = 3$ are perpendicular.

Question 14 (10 marks)

- (a)
- ABC
- is a sector of a circle with radius
- r
- as shown



It is given that the perimeter of the sector is equal in magnitude to the area of the sector.

- i. Show that
- $\theta = \frac{4}{r-2}$
- radians.

2

$$\begin{aligned}
 \text{Perimeter} &= \text{area of sector} \\
 2r + r\theta &= \frac{1}{2}r^2\theta \\
 2r\theta &= \frac{1}{2}r^2\theta - 2r\theta \\
 4r &= r^2\theta - 2r\theta = \cancel{r^2\theta} \\
 \therefore r\theta(r-2) &= 4r \\
 \theta &= \frac{4r}{r(r-2)} = \frac{4}{r-2}
 \end{aligned}$$

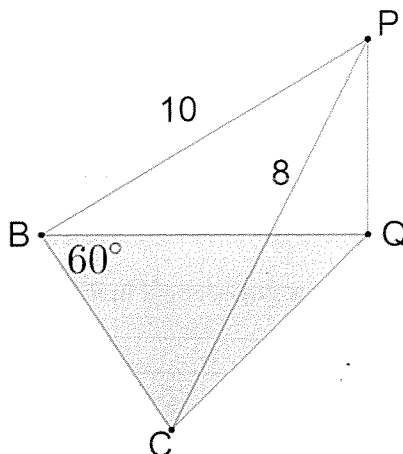
- ii. Hence, show that the area of the sector
- ABC
- is equal to
- $\frac{2r^2}{r-2}$
- .

2

$$\begin{aligned}
 \text{Area of the sector} &= \frac{1}{2}r^2\theta \\
 &= \frac{1}{2}r^2 \cdot \frac{4}{r-2} \\
 &= \frac{2r^2}{r-2}
 \end{aligned}$$

- (b) A vertical post PQ with height h metres is held by two ropes PB due west and PC due south of it.

It is given that $\angle QBC = 60^\circ$, $PB = 10$ m, $PC = 8$ m and $\angle BCQ = 30^\circ$.



- i. Show that $\frac{CQ}{BQ} = \sqrt{3}$

1

$$\frac{CQ}{BQ} = \tan 60^\circ \quad \text{as } \angle BQC = 90^\circ$$

$$\frac{CQ}{BQ} = \sqrt{3}$$

- ii. Hence, show that

3

$$\begin{aligned} \text{In } \triangle PCQ, \quad 8^2 - h^2 &= CQ^2 = 64 - h^2 \\ \text{In } \triangle PBQ, \quad 10^2 - h^2 &= BQ^2 \\ \therefore \frac{CQ^2}{BQ^2} &= \left(\frac{CQ}{BQ}\right)^2 = (\sqrt{3})^2 = \frac{64 - h^2}{100 - h^2} \quad [\text{using Part (i)}] \\ \therefore \frac{64 - h^2}{100 - h^2} &= 3 \end{aligned}$$

- iii. Find the value of h .

2

$$64 - h^2 = 300 - 3h^2$$

$$2h^2 = 300 - 64$$

$$h = \frac{236}{2} = 118$$

$$= 10.86 \text{ m}$$

Answer Sheet for Section I

Mark answers to Section I by fully blackening the correct circle

NESA STUDENT #:

- | | | | | |
|-----|------------------------------------|------------------------------------|------------------------------------|-------------------------|
| 1 – | ☐ A | ☒ B | ☐ C | ☐ D |
| 2 – | ☐ A | ☐ B | ☒ C | ☐ D |
| 3 – | ☒ A | ☐ B | ☐ C | ☐ D |
| 4 – | ☐ A | ☒ B | ☐ C | ☐ D |
| 5 – | ☐ A | ☐ B | ☒ C | ☐ D |

2018 Mathematics Extension 1 HSC Course Assessment Task 1

Student Self Reflection

1. In hindsight, did I do the best I can? Why or why not?

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2. Which topics did I need more help with, and what parts specifically?

- Q1 - Locus

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- Q2 - Locus

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- Q3 – Series

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- Q4 – Euclidean Geometry

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- Q5 -

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3. What other parts from the feedback session can I take away to refine my solutions for future reference?

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