100 Years
NORTH SYDNEY BOYS HIGH SCHOOL
1912 - 2012

MATHEMATICS

2012 Preliminary Course Final Examination
September 5, 2012

General instructions

- Working time – $2\frac{1}{2}$ hours.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets + answer sheet used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer sheet provided (numbered as page 9)

SECTION II

- Commence each new question on a new page. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NAME: ..... **# BOOKLETS USED:** .....

Class (please ✔)

◯ 12M2A – Mr Lowe

Marker's use only.

QUESTION	1-8	9	10	11	12	13	**Total**	%
MARKS	$\overline{8}$	$\overline{16}$	$\overline{14}$	$\overline{14}$	$\overline{17}$	$\overline{14}$	$\overline{83}$	

Section I: Objective response

Mark your answers on the multiple choice sheet provided. **Marks**

1. What is the value of the sum of exterior angles of a polygon? **1**

(A) $180°$ (C) $90°$

(B) $360°$ (D) None of these

2. Which of the following is *not* a condition for congruent triangles? **1**

(A) SSS (C) SAS

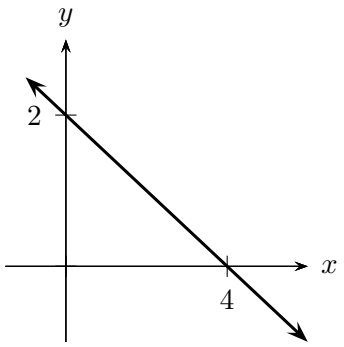
(B) AAA (D) AAS

3. Which of these conditions for the gradient make two lines parallel? **1**

(A) Gradients equal. (C) Gradients are reciprocals

(B) Gradients multiply to -1 (D) None of these

4. Which of the following is the correct equation for the line shown in the diagram? **1**



(A) $y = -2x + 2$ (C) $y = 2x + 2$

(B) $y = -\frac{1}{2}x + 2$ (D) $y = \frac{1}{2}x + 2$

5. Which of the following makes the statement true? **1**

The sum of the interior angles of an n sided polygon is ..... straight angles.

(A) $2n$ (C) $n - 1$

(B) n (D) $n - 2$

6. Which of the following makes the statement true? **1**

Each angle of ..... triangle is $60°$.

(A) an equilateral

(B) a scalene

(C) an isosceles

(D) an acute

7. What is the value of 1.9926 to two significant figures? **1**

(A) 2.0

(B) 1.9

(C) 2.09

(D) 2.01

8. What is the value of $|-8| - |10|$? **1**

(A) 2

(B) 1

(C) -1

(D) -2

End of Section I.
Examination continues overleaf.

Section II: Short answer

Question 9 (16 Marks) Commence a NEW page. **Marks**

(a) Simplify $\sqrt{12}+\sqrt{3}-\sqrt{18}$. **2**

(b) Express $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ in simplest form with a rational denominator. **2**

(c) Solve for x:

i. $9^x = 243$ **2**

ii. $x = \log_8 \dfrac{1}{64}$ **2**

iii. $\dfrac{1-x}{3}+\dfrac{x-1}{2}=1$ **3**

(d) Solve $|3x-2| \leq 4$. **2**

(e) Solve for x and y simultanously: **3**

$$\begin{cases} x+2y=1 \\ 3x+y=13 \end{cases}$$

Question 10 (14 Marks) Commence a NEW page. **Marks**

(a) Write $\dfrac{a^2-b^2}{a^{-1}+b^{-1}}$ without negative indices. **3**

(b) Simplify:

i. $1-\dfrac{m}{m-n}$ **2**

ii. 2^n+2^{n+1} **2**

iii. $\log_{12}\dfrac{2}{7}+\log_{12}\dfrac{7}{2}$ **2**

(c) For the function

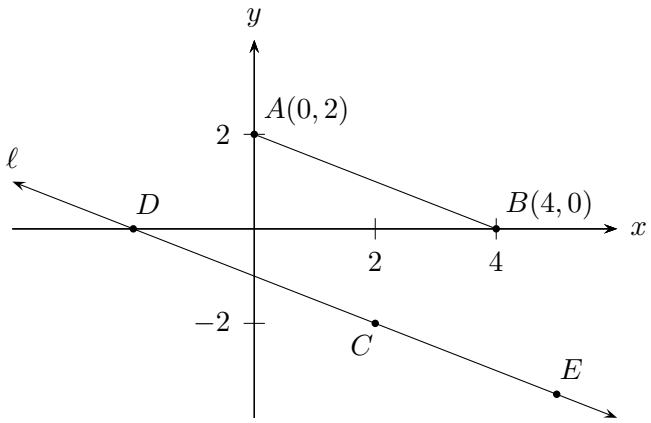
$$f(x)=\begin{cases} 1-x & -2 \leq x \leq 0 \\ x^2+2 & 0 < x \leq 2 \end{cases}$$

i. Draw a neat sketch of $f(x)$. **3**

ii. Write down the domain and range $f(x)$. **2**

Question 11 (14 Marks) Commence a NEW page. **Marks**

(a) Refer to the following diagram:



i. Find the gradient of AB. **1**

ii. Show that $AB = 2\sqrt{5}$ units. **1**

iii. Show that the equation of the line ℓ, through C parallel to AB is **1**

$$x + 2y + 2 = 0$$

iv. Find the coordinates of the point D where ℓ cuts the x axis. **1**

v. Show that the perpendicular distance from A to the ℓ is $\dfrac{6}{\sqrt{5}}$ units. **1**

vi. If the area of the trapezium $ABED$ is $18\,\text{units}^2$, find the length of DE. **3**

(b) Find the equation of the line passing through the intersection of $2x + 3y + 5 = 0$ and $3x - 4y - 3 = 0$, and through the point $(1, 2)$. **3**

(c) Sketch the region defined by **3**

$$y > 2 - x \quad \text{and} \quad y \leq x^2$$

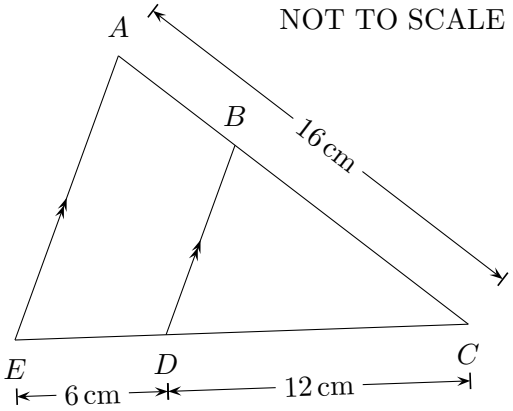
Question 12 (18 Marks) Commence a NEW page. **Marks**

(a) A quadrilateral $ABCD$ is such that $\triangle ABD$ and $\triangle BDC$ are equilateral.

i. Draw a neat diagram to represent this information. **1**

ii. Prove that $\angle BAC = 30°$. **2**

(b) The diagram shows $\triangle ACE$ with $AE \parallel BD$, $AC = 16\,\text{cm}$, $CD = 12\,\text{cm}$ and $DE = 6\,\text{cm}$.



i. Prove that $\triangle ACE$ is similar to $\triangle BCD$. **3**

ii. Hence or otherwise, find the length of AB. **2**

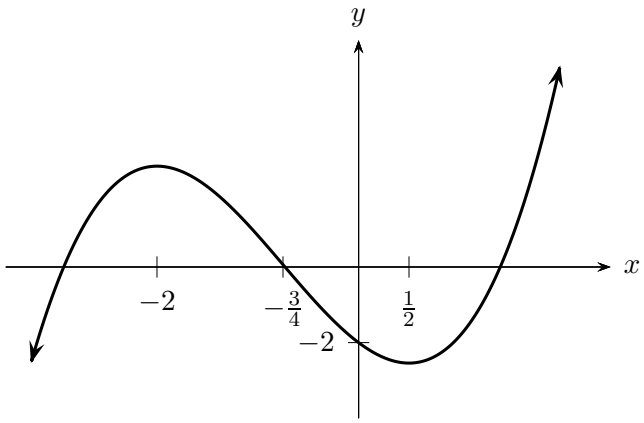
(c) A rhombus with diagonals x and $x + 2$ cm has an area of $24\,\text{cm}^2$. **3**

Find the value of x, and hence the value of the other diagonal.

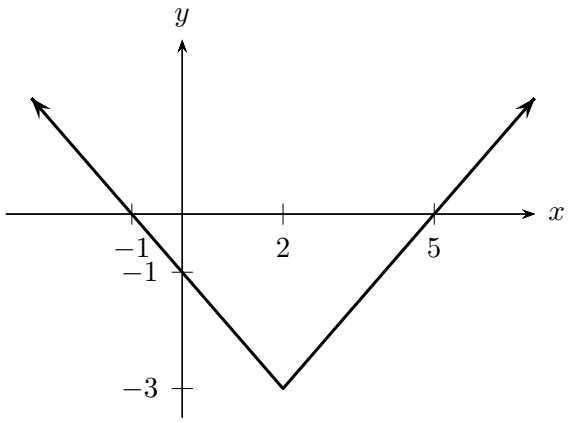
Question 12 continued on the next page ...

Question 12 continued from page 6...

(d) Copy the following diagrams into your writing booklet, and on the same set of axes, sketch the gradient function of the following:

i. 

2

ii. 

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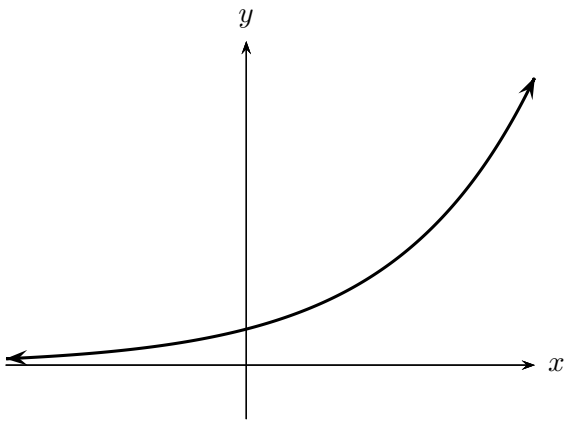
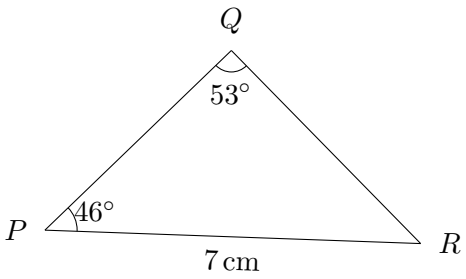
iii. 

2

Question 13 (14 Marks) Commence a NEW page. **Marks**

(a) In $\triangle PQR$, $\angle QPR = 46°$, $\angle PQR = 53°$ and $PR = 7\,\text{cm}$.



i. Find the length of QR. **2**

ii. Find the area of the triangle. **2**

(b) Solve the following equation for θ, where $0° \leq \theta \leq 360°$:

i. $\cos\theta = \dfrac{1}{\sqrt{2}}$ **2**

ii. $\tan\theta + 1 = 0$. **2**

iii. $\sin^2 x = \dfrac{3}{4}$ **2**

(c) Consider the function $y = 3\sin 2x$, where $0° \leq x \leq 360°$.

i. State the period and amplitude of the function. **2**

ii. Draw a neat sketch of the function. **2**

End of paper.

Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

NAME: .....

Class (please ✔)

◯ 11M2A – Mr Lowe

1 – Ⓐ Ⓑ Ⓒ Ⓓ
2 – Ⓐ Ⓑ Ⓒ Ⓓ
3 – Ⓐ Ⓑ Ⓒ Ⓓ
4 – Ⓐ Ⓑ Ⓒ Ⓓ
5 – Ⓐ Ⓑ Ⓒ Ⓓ
6 – Ⓐ Ⓑ Ⓒ Ⓓ
7 – Ⓐ Ⓑ Ⓒ Ⓓ
8 – Ⓐ Ⓑ Ⓒ Ⓓ

HR 11 YEARLY Suggested Soln's

1. B
2. B
3. ~~B~~ A
4. B
5. D
6. A
7. A
8. D

9. a) $2\sqrt{3}+\sqrt{3}-3\sqrt{2}$ 2
$= 3\sqrt{3}-3\sqrt{2}$

b) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ 2

$= (\sqrt{3}+\sqrt{2})^2$

c) i) $3^{2x} = 243$

$3^{2x} = 3^5$

$2x = 5$

$x = \frac{5}{2}$ 2

ii) $x = \log_8 8^{-2}$

$= -2$ 2

iii) $2-2x+3x-3 = 6$

$x - 1 = 6$

$x = 7$ 3

d) $-4 \leq 3x-2 \leq 4$

$-2 \leq 3x \leq 6$

$-\frac{2}{3} \leq x \leq 2$ 2

e) $x+2y = 1$

$6x+2y = 26$

$-5x = -26$

$x = 5$

$y = -2$ 3

$(x,y) = (5,-2)$

10. a) $\frac{(a-b)(a+b)}{\frac{1}{a}+\frac{1}{b}}$ 1

$\frac{(a-b)(a+b)}{\frac{(a+b)}{ab}}$ 1

$= ab(a-b)$ 1

b) i) $\frac{m-n-m}{m-n}$

$= \frac{-n}{m-n} = \frac{n}{n-m}$ 2

ii) $2^n + 2 \times 2^n$ 1

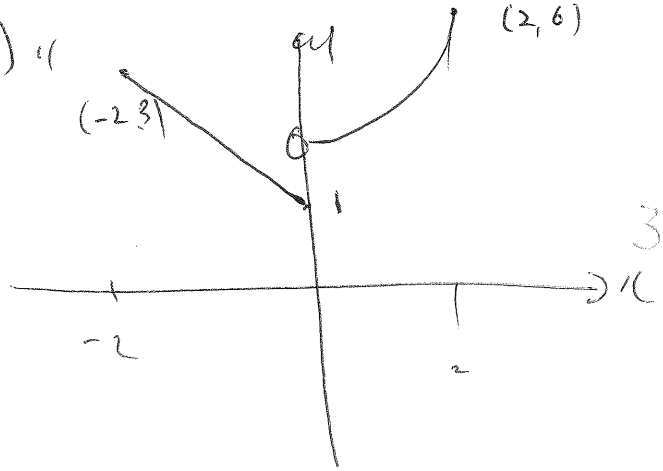
$= 2^n(3)$

$= 3 \times 2^n$ 1

iii) $\log_{12} 1$!

$= 0$ 1

c) (i)



(ii) Domain $-2 \le x \le 2$

Range $1 \le y \le 6$

(11) a) (i) $M_{AB} = -\frac{1}{2}$

(ii) $AB = \sqrt{2^2 + 4^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

(iii) $y - (-4) = \frac{-1}{2}(x - 2)$

$2y + 8 = -x + 2$

$x + 2y + 6 = 0$

(iv) $y = 0$

$x = -2$

D is $(-2, 0)$

(v) $d = \frac{|1 \cdot 0 + 2 \times 2 + 2|}{\sqrt{1^2 + 2^2}}$

$= \frac{6}{\sqrt{5}}$

(vi)

$\text{Area} = \frac{h}{2}(AB + DE)$

$18 = \frac{3}{\sqrt{5}}\left(2\sqrt{5} + DE\right)$

$18 = 6 + \frac{3}{\sqrt{5}} DE$

$12 = \frac{3}{\sqrt{5}} DE$

$DE = 4\sqrt{5}$

b) Line

$2x + 3y + 5 + k(3x - 4y - 3) = 0$

Through $(1, 2)$

$2 + 6 + 5 + k(3 - 8 - 3) = 0$

$13 - 8k = 0$

$8k = 13$

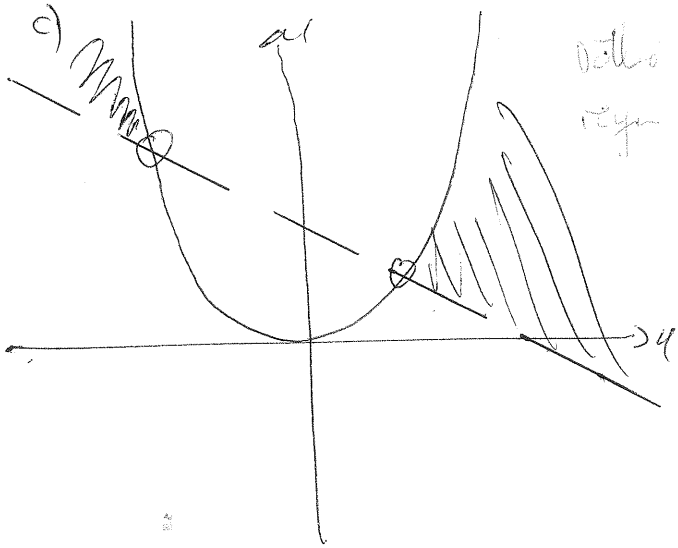
$k = \frac{13}{8}$

$\therefore 2x + 3y + 5 + \frac{13}{8}(3x - 4y - 3) = 0$

$16x + 24y + 40 + 39x - 52y - 39 = 0$

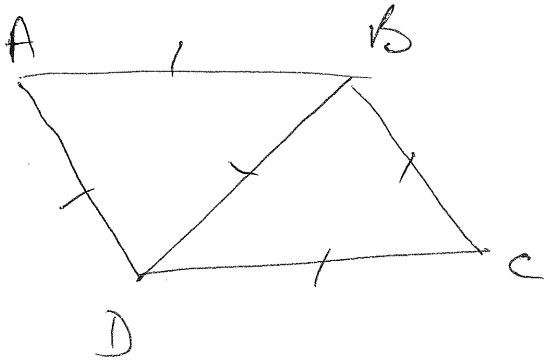
$55x - 28y + 1 = 0$

c)



(12)

a) i)



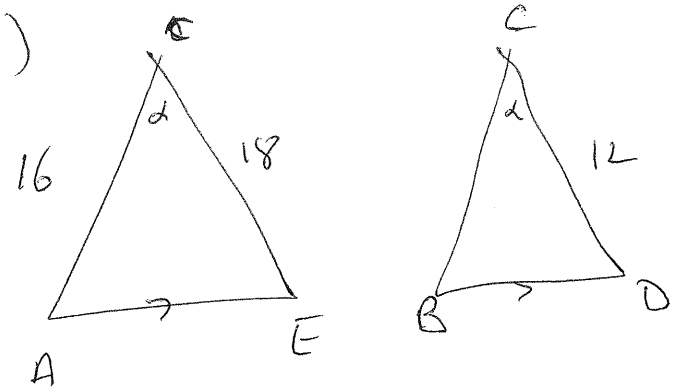
ii) $\angle BAD$ is 60° [Data]

ABCD is a rhombus as all sides equal

$\therefore$ AC Bisects $\angle BAD$

$\therefore B\hat{A}C = 30^\circ$

b)



In $\triangle ACE$ & $\triangle BCD$

$\angle C$ is common

$\angle CAB = \angle CBD$ (corresponding $\angle$s $\parallel$ lines)

$\therefore \triangle ACE \;|||\; \triangle BCD$ (AAA)

ii) $\frac{BC}{16} = \frac{x}{18}\ \frac{2}{3}$

$BC = \frac{32}{3}$

$\therefore AB = 16 - \frac{32}{3}$

$= \frac{16}{3}$ u

c) $\frac{1}{2}\, x(x+2) = 24$

$x^2 + 2x - 48 = 0$

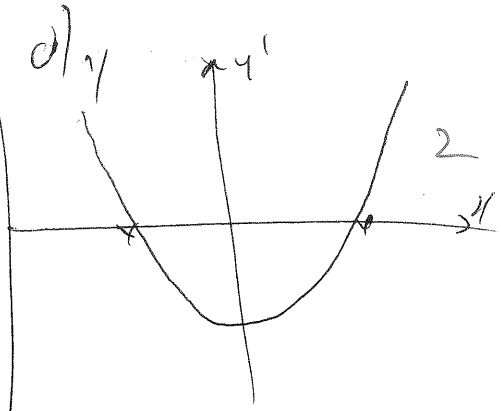
$(x+8)(x-6) = 0$

$x = 6$ or -8

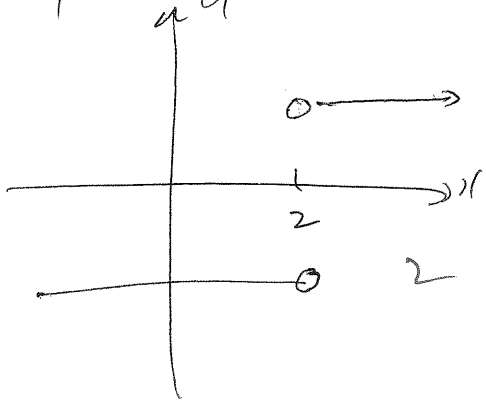
$x > 0$

$\therefore$ Diagonals are $6u + 8c$

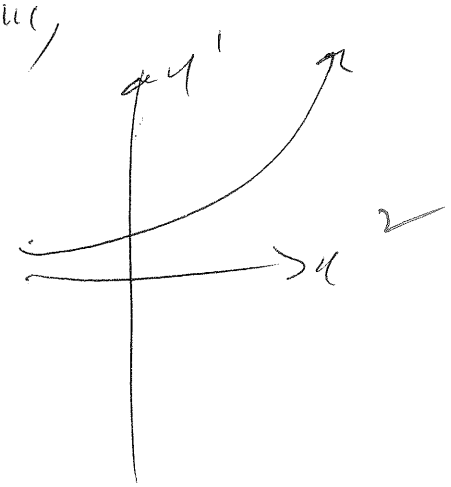
d) i)



ii)



iii)



(13) a) i) $\frac{QR}{\sin 46} = \frac{7}{\sin 53}$

$QR = \frac{7 \sin 46}{\sin 53}$ ✓

$= \underline{6.3}$ cm ✓

ii) $A = \frac{1}{2} \times QR \times RP \times \sin 81$ ✓

$= \underline{21.8 \text{ cm}^2}$ ✓

b) i) $\cos\theta = \frac{1}{\sqrt{2}}$

S	A ✓
T	C ✓

Acute $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$

$\therefore \theta = 45^\circ, 315^\circ$ 2

ii) $\tan\theta = -1$

✓S	A
T	C ✓

Acute $\tan^{-1}(1) = 45$

$\theta = 135^\circ, 315^\circ$ 2

iii) $\sin^2 x = \frac{3}{4}$

✓S	A ✓
✓T	C ✓

$\sin x = \pm\frac{\sqrt{3}}{2}$

Acute $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$ 2

$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

c) i) $p = \frac{360}{2} = 180^\circ$ ✓

$a = 3$ ✓

ii)