File copy.

Maths

NORTH SYDNEY BOYS HIGH SCHOOL

2013 YEAR 11 YEARLY EXAMINATION

Mathematics

General Instructions

- Working time – 2 hours
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Attempt all questions
- Marks may be deducted for incomplete or illegible working

Class Teacher:
(Please tick or highlight)

○ Mr Lin

Student Name:

(To be used by the exam markers only.)

Question No	1-7	8	9	10	11	12	13	14	Total
Mark	/7	/13	/14	/13	/15	/8	/13	/17	/100

MULTIPLE CHOICE

1. What is the solution to $x^2 - 6x - 7 < 0$

(a) $-7 < x < 1$ (b) $-1 < x < 7$

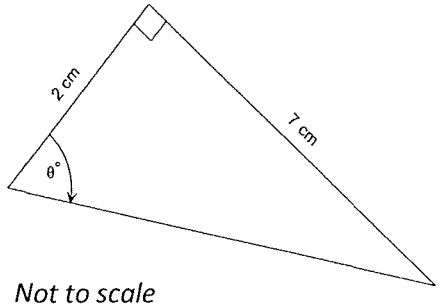
(c) $x < -7$ or $x > 1$ (d) $x < -1$ or $x > 7$

2. The function $f(x) = \frac{x}{x+1}$ is a:

(a) Exponential (b) Hyperbola

(c) Parabola (d) None of the above

3. The value of θ (to the nearest degree) is:



Not to scale

(a) 16° (b) 17°

(c) 73° (d) 74°

4. Find the perpendicular distance between $x + y - 1 = 0$ and $(2, -2)$

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{2}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{8}}$ (d) $\frac{\sqrt{2}}{4}$

5. What is the external angle sum of a regular hexagon:

(a) 60° (b) 120°

(c) 360° (d) 720°

6. If $w = z^x$, which of the following statements is true?

(a) $w = \log_z x$ (b) $x = \log_z w$

(c) $z = \log_w x$ (d) $z = \log_x w$

7. Evaluate: $\lim_{x \to 2} \frac{4 - 2x}{4 - x^2}$

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

Question 8 – Basic Arithmetic Algebra & Equations *(13 Marks)*

(a) Evaluate correct to 3 significant figures: $\frac{3.14-\sqrt{47-4\times4\times2}}{2^2-5^2}$ **2**

(b) Express $1.2\overline{34}$ as a fraction **2**

(c) Rationalise the denominator of: $\frac{3}{1-\sqrt{2}}$ **2**

(d) Solve each of the following equations for *x*

(i) $3(x-2)+x=2+2x$ **2**

(ii) $\frac{3x}{x+1}=\frac{2}{5}$ **2**

(e) Make *v* the subject of the equation: $wv-2x=v+5$ **3**

Question 9 – Functions and Graphs *(14 Marks)*

(a) Is the function $f(x)=\frac{x}{2+x^2}$ odd, even or neither? Justify your answer with working. **3**

(b) Sketch each of the following on separate number planes showing all key features

(i) $y=x^2-3x-10$ **3**

(ii) $y=-\sqrt{9-x^2}$ **2**

(c) One the same number plane clearly sketch:

(i) $f(x)=2^x$ **2**

(ii) $f(x+1)$ **2**

(iii) $f(x)-1$ **2**

Clearly showing at least two points on each graph

Question 10 - Trigonometry *(13 Marks)*

(a) In triangle ABC, AC = 8cm, BC = 10cm and $A\hat{B}C=50°$. Find the value*(s)* of $B\hat{A}C$ **3**

(b) Two boats leave a port at the same time. Boat A is travelling at 30 km/h on a bearing of $035°T$ while boat B is travelling at 35 km/h on a bearing of $140°T$.

(i) Draw a clear diagram of the situation in your answer book. **1**

(ii) How far apart are the boats after 4 hours? **3**

Question 10 continues on the next page

(c) Given that $\tan x = \frac{1}{4}$ and $\cos x > 0$. Find the exact value of $\operatorname{cosec} x$: **3**

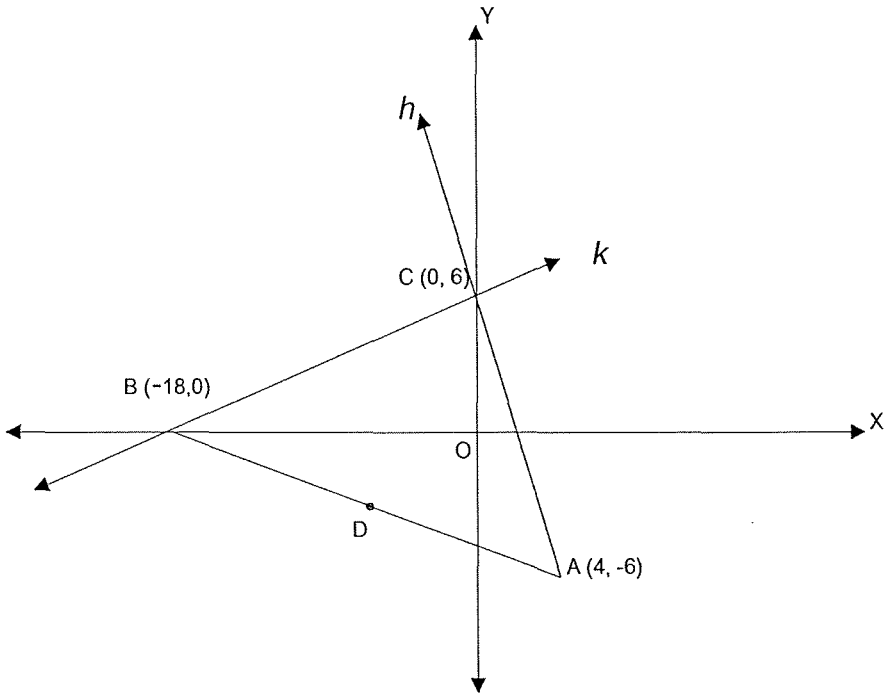
(d) Find all possible solutions for $\sin\theta - \sqrt{3}\cos\theta = 0$ for $0° \leq \theta \leq 90°$ **2**

(e) Show that $\dfrac{\sin^3 x}{\cos x} + \sin x \cos x = \tan x$ **3**

Question 11 - Linear Functions *(15 Marks)*

(a) In the diagram the lines *h* and *k* are drawn. The coordinates of A, B and C are $(4, -6)$, $(-18, 0)$ and $(0, 6)$ respectively. *D* is the midpoint of *AB*.

Not to scale



(i) Show that D has coordinates $(-7, -3)$ **1**

(ii) Calculate the length of *DC*. Leave your answer in exact form **1**

(iii) Show that the equation of the line *h* is $3x + y - 6 = 0$ **2**

(iv) Show that line *h* is perpendicular to line *k*. **2**

(v) *AB* is the diameter of a circle which passes through the points *A*, *B*, and *C*. Find the equation of this circle. **2**

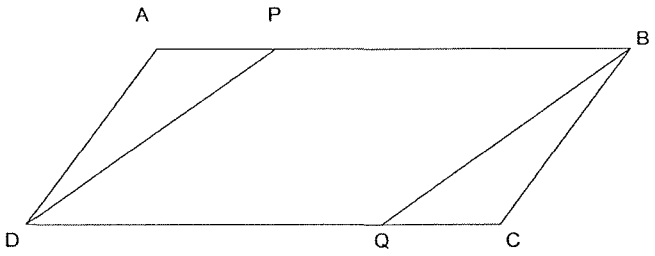
(vi) Hence, or otherwise, find the area of the circle through points *A*, *B* and *C*. Leave your answer in terms of π. **1**

(b) (i) Find the point*(s)* of intersection of: $y = x^2$ and $y = 5x - 4$. **3**

(ii) Hence, indicate on the number plane the region defined by: $y \geq x^2$ and $y < 5x - 4$. **3**

Question 12 – Plane (Euclidean) Geometry *(8 Marks)*

(a) ABCD is a parallelogram and AP = QC.



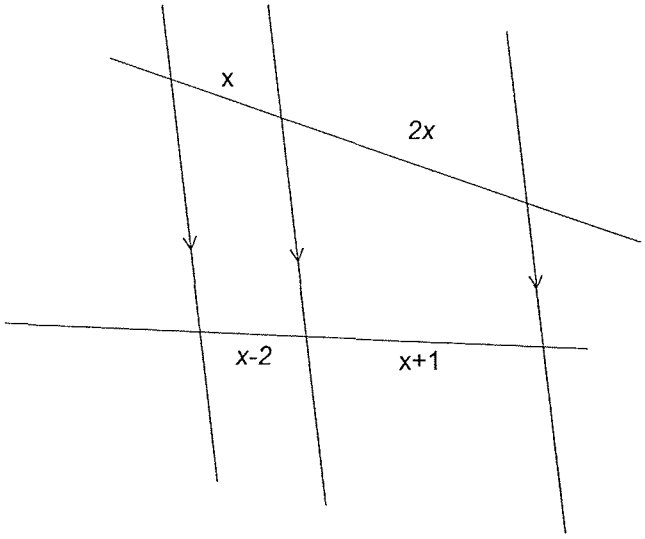
Prove:

(i) $\Delta APD \equiv \Delta BQC$ **3**

(ii) PD = QB **1**

(iii) $PD||QB$ **2**

(b) Find the value of *x* (give reasons)



Not to scale

2

Question 13 – Indices & Logarithms *(13 Marks)*

(a)

Fully simplify $\dfrac{(x^2 y^{\frac{1}{2}})^2}{y\sqrt{x^3}}$ **2**

(b) Find the value of the *x* when $3^{x-1} = \frac{1}{27}$ **2**

Question 13 continues on the next page

(c) If $\log_a x = 1.8$ and $\log_a y = 2.4$, evaluate each of the following:

(i) $\log_a xy$ **1**

(ii) $\log_a \sqrt[3]{y}$ **1**

(iii) $\log_a \frac{a}{x}$ **2**

(d) Solve for *x*: $\log_2(x-1) - \log_2(x-2) = 1$ **3**

(e) Is the following working correct? Explain your answer.

$$\left(\frac{1}{2}\right)^x < 7$$

$$\log\left(\frac{1}{2}\right)^x < \log 7$$

$$x \log\left(\frac{1}{2}\right) < \log 7$$

$$x < \frac{\log 7}{\log \frac{1}{2}}$$

$\therefore \quad x < -2.8$ (1 *d.p.*) **2**

Question 14 – Differential Calculus *(17 Marks)*

(a) Is this function, *f(x)*, continuous at $x = 1$. Justify your answer with working.

$$f(x) = \begin{cases} x^2 + x + 5 & \text{for } x \leq 1 \\ 2x + 3 & \text{for } x > 1 \end{cases}$$ **3**

(b) Differentiate each of the following with respect to *x*.

(i) $3x^5 + x^3 - 5x^2 + 20$ **1**

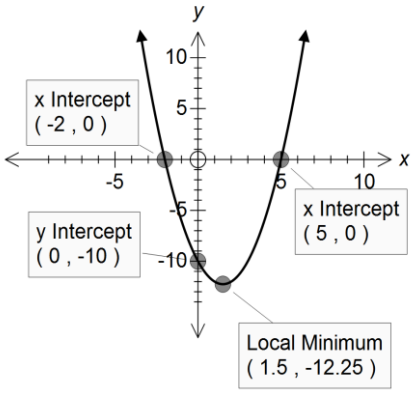
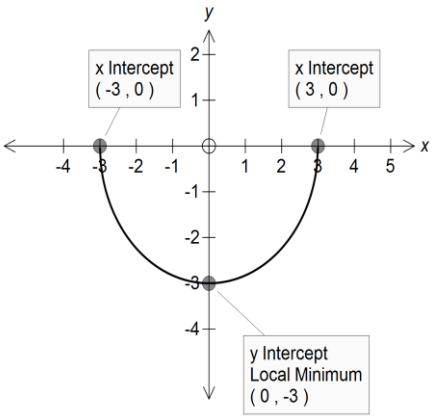
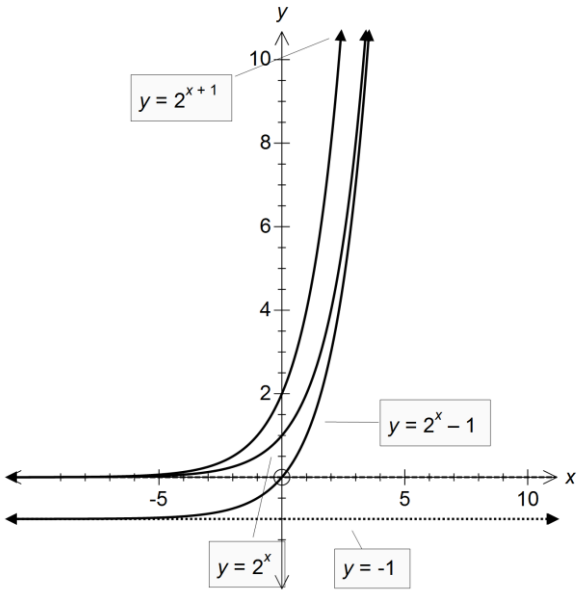
(ii) $x^2\sqrt{1+2x}$ **3**

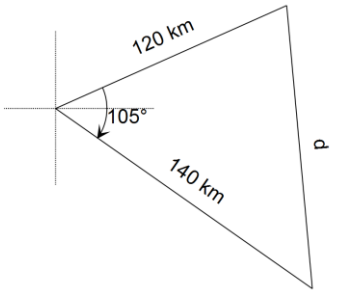
(iii) $\frac{x+2}{2x-1}$ **3**

(c) Differentiate $\frac{2}{x}$ using first principles. **3**

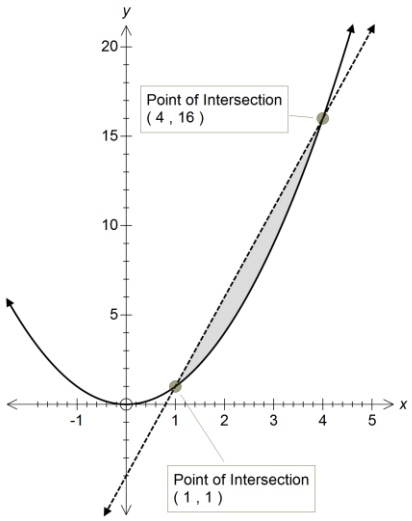
(d) Calculate the area that the tangent to the curve $y = x^3 - 1$ that that passes through the point $(-2, -9)$ makes with the coordinate axes. **4**

	Multiple Choice	
	1. B **2.** B **3.** D **4.** A **5.** C **6.** B **7.** B	✓ 1 mark each
	Question 8	
(a)	0.0349	✓ Answer ✓ Rounding
(b)	$let\ x = 1.234343434\ldots$ ① $\therefore 100x = 123.434343\ldots$ ② (2) – (1) $99x = 122.2$ $x = \frac{122.2}{99}$ $= \frac{611}{495}$ *or* $1\frac{116}{49}$	✓ Multiplying by 100 ✓ Correct answer
(c)	$\frac{3}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{3+3\sqrt{2}}{1-2}$ $= -3-3\sqrt{2}$	✓ Multiplying by conjugate ✓ Correct answer
(d) (i)	$3x - 6 + x = 2 + 2x$ $4x - 6 = 2 + 2x$ $2x = 8$ $x = 4$	✓ Expanding brackets and gathering like terms ✓ Correct answer
(d) (ii)	$\frac{3x}{x+1} = \frac{2}{5}$ $15x = 2x + 2$ $13x = 2$ $x = \frac{2}{13}$	✓ Cross multiplying ✓Correct answer
(e)	$wv - 2x = v + 5$ $wv - v = 2x + 5$ $v(w-1) = 2x + 5$ $v = \frac{2x+5}{w-1}$	✓ All v*'s* to one side ✓ Factorising ✓ Correct answer
	Question 9	
(a)	$f(-x) = \frac{(-x)}{2+(-x)^2}$ $= \frac{-x}{2+x^2}$ $= -\frac{x}{2+x^2}$ $= -f(x)$ $\therefore$ *f(x) is odd*	✓ Substituting in $-x$ ✓ Working ✓ Final answer

(b) (i)		✓ X-intercepts ✓ Y-intercept ✓ Vertex
(b) (ii)		✓ Correct shape ✓ Correct intercepts
(c)		✓ Correct shape/(including translation) ✓ At least 2 points clearly shown *(times 3 – 2 marks each graph)*

	Question 10	
(a)	*let* $\angle BAC = \theta$ $\frac{\sin\theta}{10} = \frac{\sin 50}{8}$ $\sin\theta = \frac{10 \sin 50}{8}$ $\theta = \sin^{-1}\left(\frac{10 \sin 50}{8}\right)$ $\therefore \quad \theta = 73^\circ$ *or* $\theta = 180^\circ - 73^\circ$ $= 107^\circ$	✓ Applying sine rule ✓ Acute answer ✓ Obtuse answer
(b) (i)		✓ diagram
(b) (ii)	$d^2 = 120^2 + 140^2 - 2 \times 120 \times 140 \times \cos 105^\circ$ $\therefore d = 207$ ***km*** (*nearest* ***km***)	✓ cosine rule ✓ correct substitution ✓ correct answer
(c)	*hypotenuse* $= \sqrt{4^2 + 1^2}$ $= \sqrt{17}$ $\text{cosec } x = \frac{\textit{hypotenuse}}{\textit{opposite}}$ $= + \frac{\sqrt{17}}{1}$ *(first quadrant)* $= \sqrt{17}$	✓ Pythagoras theorem ✓ Correct ration for cosec ✓ Correct sign/final answer
(d)	$\sin\theta = \sqrt{3} \cos\theta$ $\tan\theta = \sqrt{3}$ $\theta = \tan^{-1}\sqrt{3}$ $\theta = 60^\circ$	✓ Equation for tan ✓ Correct answer

(e)	$LHS = \frac{\sin^3 x}{\cos x} + \sin x \cos x$ $= \frac{\sin^3 x}{\cos x} + \frac{\sin x \cos^2 x}{\cos x}$ $= \frac{\sin x(\sin^2 x + \cos^2 x)}{\cos x}$ $= \frac{\sin x}{\cos x}$ $= RHS$	✓ Common denominator ✓ sinx+cosx =1 ✓ final answer
	Question 11	
(a) (i)	$D = \left(\frac{-18+4}{2}, \frac{0-6}{2}\right)$ $= (-7, -3)$	✓ Correct working
(a)(ii)	$DC = \sqrt{(-7-0)^2 + (-3-6)^2}$ $= \sqrt{49+81}$ $= \sqrt{130}$	✓ Correct working
(a)(iii)	$\frac{y-6}{x-0} = \frac{-6-6}{4-0}$ $\frac{y-6}{x} = -\frac{12}{4}$ $\frac{y-6}{x} = -3$ $y-6 = -3x$ $\therefore 3x + y - 6 = 0$	✓ Correct substitution into formula ✓ Adequate working
(a)(iv)	$m_h \times m_k = \frac{-6-6}{4-0} \times \frac{6-0}{0+18}$ $= -\frac{12}{4} \times \frac{6}{18}$ $= -\frac{72}{72}$ $= -1$ $\therefore$ *the lines are perpendicular*	✓ Calculating gradient ✓ Showing gradients multiple to -1
(a) (v)	$(x+7) + (y+3) = 130$	✓ Centre ✓ radius
(a) (vi)	$= 130\pi$	✓ Correct answer

(b)(i)	$x^2 = 5x - 4$ $x^2 - 5x + 4 = 0$ $(x-1)(x-4) = 0$ $x = 1$ *or* $x = 4$ $y = 1^2$ *or* $y = 4^2$ *(1,1) and (4,16)*	✓ Solving simultaneously ✓ x coordinates ✓ y coordinates
(b) (ii)		✓ graphs correct ✓ shaded region ✓ open circles at points of intersection
	Question 12	
(a) (i)	*In ΔS APD & BQC* *AD = CB (opposite sides of a parellelogram are equal)* *∠ DAP = ∠ BCP* (*opposite angles of a parallelogram are equal)* *AP = QC (given)* *∴ Δ APD ≡ ΔBQC* (*SAS*)	✓ equal sides and angle identified ✓ all reasons given ✓ final line including SAS
(a) (ii)	*PD = QB* (matching sides in congruent triangles are equal)	✓ Correct reason
(a) (iii)	*AP = QC (given)* *AB = CD* (*opposite sides of parallelogram ABCD*) *∴ AB – AP = CD – QC* *∴ PB = QC* *∴ PBQD is a parallelogram (two pairs of opposite sides equal)* *∴ PD ‖ QB (opposite sides of a parallelogram are parallel)*	✓ PB=QC ✓ Correct reason

(b)	$\frac{x}{2x} = \frac{x-2}{x+1}$ *(ratio of intercepts on parallel lines)* $\frac{1}{2} = \frac{x-2}{x+1}$ $(x \neq 0)$ $2x - 4 = x + 1$ $x = 5$	✓ Ratio of intercepts (must state reason) ✓ Correct answer
	Question 13	
(a)	$\frac{\left(x^2 y^{\frac{1}{2}}\right)^2}{y\sqrt{x^3}} = \frac{x^4 y}{x^{\frac{3}{2}} y}$ $= x^{4-\frac{3}{2}}$ $= x^{\frac{5}{2}}$ *or* $\sqrt{x^5}$	✓ Simplifiying numerator and denominator individually ✓ Final answer
(b)	$3^{x-1} = 3^{-3}$ $x - 1 = -3$ $x = -2$	✓ Expressing RHS with a negative index ✓ answer
(c) (i)	$\log_a xy = \log_a x + \log_a y$ $= 1.8 + 2.4$ $= 4.2$	✓ Correct answer
(c) (ii)	$\log_a \sqrt[3]{y} = \log_a (y)^{\frac{1}{3}}$ $= \frac{1}{3}\log_a y$ $= \frac{1}{3} \times 2.4$ $= 0.8$	✓ Correct answer
(c) (iii)	$\log_a \frac{a}{x} = \log_a a - \log_a x$ $= 1 - 1.8$ $= -0.8$	✓ Simplifying log ✓ Correct answer
(d)	$\log_2(x-1) - \log_2(x-2) = 1$ $\log_2\left(\frac{x-1}{x-2}\right) = \log_2 2$ $\therefore \quad \frac{x-1}{x-2} = 2$ $x - 1 = 2x - 4$ $x = 3$	✓ Simplifying LHS ✓ Writing RHS in terms of a log ✓ Correct answer
(e)	It is incorrect since the inequality on the 4th lines should be turned the other was as we are dividing by $\log\frac{1}{2}$, which is negative.	✓ Stating it is incorrect ✓ Identifying the reason

	Question 14	
(a)	$\lim_{x \to 1^-} f(x) = 1^2 + 1 + 5$ $= 7$ $\lim_{x \to 1^+} f(x) = 2(1) + 3$ $= 5$ $\therefore \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$ *$\therefore$ function is not continous when x=1*	✓ Evaluating limits ✓ Correct notation ✓ Stating it is discontinuous
(b) (i)	$15 \quad +3x \quad 10$	✓ Correct answer
(b) (ii)	$\frac{d}{dx}\left(x^2\sqrt{1+2x}\right) = \left(\frac{d}{dx}\left(x^2\right)\right)\left(\sqrt{1+2x}\right) + \left(\frac{d}{dx}\left(\sqrt{1+2x}\right)\right)\left(x^2\right)$ $= (2x)(1+2x)^{\frac{1}{2}} + \left((2)\left(\frac{1}{2}\right)(1+2x)^{-\frac{1}{2}}\right)\left(x^2\right)$ $= 2x\sqrt{1+2x} + \frac{x^2}{\sqrt{1+2x}}$	✓ Chain rule ✓ Product rule ✓ Answer
(b) (iii)	$\frac{d}{dx}\left(\frac{x+2}{2x-1}\right) = \frac{\left(\frac{d}{dx}(x+2)\right)(2x-1) - \left(\frac{d}{dx}(2x-1)\right)(x+2)}{(2x-1)^2}$ $= \frac{(1)(2x-1) - (2)(x+2)}{(2x-1)^2}$ $= \frac{2x-1-2x-4}{(2x-1)^2}$ $= \frac{-5}{(2x-1)^2}$	✓ Knowing quotient rule ✓ Correct substitutions into formula ✓ Correct answer
(c)	$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$ $= \lim_{h \to 0} \frac{\left(\frac{2x-2(x+h)}{x(x+h)}\right)}{h}$ $= \lim_{h \to 0} \frac{\left(\frac{2x-2x-2h}{x(x+h)}\right)}{h}$ $= \lim_{h \to 0} \frac{\left(\frac{-2\cancel{h}}{x(x+h)}\right)}{\cancel{h}}$ $= \lim_{h \to 0} \frac{-2}{x(x+h)}$ $= -\frac{2}{x^2}$	✓ Correct use of formula ✓ Cancelling the *h*'s ✓ Answer

(d)	$\frac{dy}{dx} = 3x^2$ $\therefore \quad m_{tgt} = 3(-2)^2$ $= 12$ $y - y_1 = m(x - x_1)$ $y + 9 = 12(x + 2)$ $y + 9 = 12x + 24$ $y = 12x + 15$ $\therefore \quad y\text{-}int = 15$ $when\ y = 0$ $12x = -15$ $x = -\frac{15}{12}$ $\therefore \quad x\text{-}int = -1.25$ $\therefore \quad Area = \frac{1}{2} \times 15 \times \lvert -1.25 \rvert$ $= \frac{75}{8}\ units^2$	✓ gradient of tangent ✓ Equation of tangent ✓ x & y intercepts ✓ area