NORTH SYDNEY BOYS HIGH SCHOOL

2024 Year 11 Assessment Task 3

Mathematics Advanced

General Instructions

- Reading time - **5 minutes**
- Working time – **90 minutes**
- Write using blue or black pen
- Diagrams may be drawn in pencil
- NSBHS approved calculators may be used
- All necessary working should be shown in every question
- Answer the multiple choice questions **Q1-Q5** on the sheet provided
- Answer **Q6- Q27** in the space provided.
- Marks may not be awarded for carelessly or badly arranged work.

Class Teacher:

- Ms Lee
- Mr Berry
- Mr Umakanthan
- Dr Vranešević
- Mrs Moss
- Mr Ireland
- Ms Cai/Mr Lott
- Mrs Sarofim

Student Number:________________________

Question No	1-5	6-11	12-15	16-19	20-22	23-25	26-27	Total	Total %
	$\overline{5}$	$\overline{16}$	$\overline{14}$	$\overline{9}$	$\overline{7}$	$\overline{7}$	$\overline{7}$	$\overline{65}$	$\overline{100}$

SECTION I

5 marks Attempt Questions 1–5 Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5.

	Question	Mark	Outcome
1.	What is the value of x such that $\log_x 2 = \frac{1}{2}$	**1**	**MA11-6**

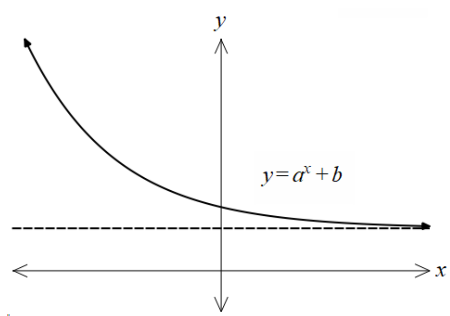
A. $\sqrt{2}$

B. 2

C. 4

D. $2\sqrt{2}$

2.	The figure shows the graph of $y = a^x + b$, where a and b are constants. Which of the following must be true?	**1**	**MA11-6**



A. $0 < a < 1$ and $b > 0$

B. $0 < a < 1$ and $b < 0$

C. $a > 1$ and $b > 0$

D. $a > 1$ and $b < 0$

3.	What is the derivative with respect to x of $\dfrac{1}{(3x-4)^5}$	**1**	**MA11-5**

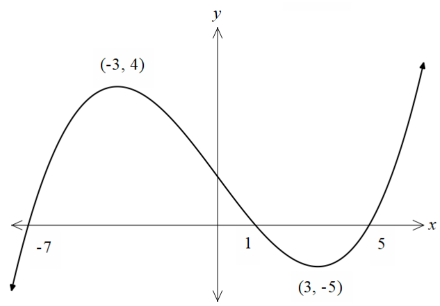
A. $\dfrac{-15x}{(3x-4)^6}$

B. $\dfrac{-15}{(3x-4)^6}$

C. $\dfrac{-5}{(3x-4)^6}$

D. $\dfrac{4}{3(3x-4)^4}$

4. The graph of $y = f(x)$ shown below. **1 MA11-5**



For what values of x is $f(x) < 0$ and $f'(x) < 0$?

A. $(-\infty, -7) \cup (1, 5)$

B. $(-\infty, -3) \cup (3, \infty)$

C. $(-3, 3)$

D. $(1, 3)$

5. **1 MA11-2**

The function $g(x)$ and $h(x)$ are defined as follows:

$$g(x) = \frac{x^2}{8}$$
$$h(x) = \sqrt{2x} + 3$$

Which is the correct expression for $h(g(x))$?

A. $\frac{x}{2}$+3

B. $\frac{x}{2}$-3

C. $\sqrt{\frac{2x}{3}}$

D. $\sqrt{\frac{x}{2} + 3}$

END OF SECTION I

Section II

65 marks

Attempt Questions 6-27

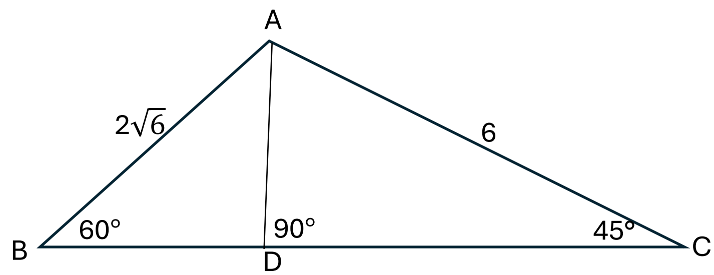
Allow about 1 hours and 22 minutes for this section

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet on pages 21 - 24.
  If you use this space, clearly indicate which question you are answering.

	Question	Mark	Outcome
6.	Solve the equation $\left\|\frac{x}{2} - 3\right\| = 1$	**2**	**MA11-1**
7.	Simplify $\dfrac{12^x \times 18^x}{3^x \times 2^x}$	**2**	**MA11-1**

8. Find the centre and radius of the circle represented by the Equation: $x^2 - 6x + y^2 + 10y + 18 = 0$ **2** **MA11-2**

9. **MA11-3**



In the diagram, ABC is triangle in which $AB = 2\sqrt{6}$, $AC = 6$, $\angle ABC = 60°$ and $\angle ACB = 45°$. AD is perpendicular from A to BC.

a. Show that $BC = \sqrt{6} + 3\sqrt{2}$ **2**

b. Hence show that $sin 75° = \frac{\sqrt{6}+\sqrt{2}}{4}$ **2**

10.

a. In the space below, sketch the graph of the function $y = \sqrt{4-x}$ showing the intercepts on the axis. **2** **MA11-2**

b. State the domain and the range of the function $y = \sqrt{4-x}$. **2** **MA11-2**

11. ABC is a triangle in which $\tan B = \frac{1}{3}$. Find the exact value of $secB$. **2** **MA11-3**

12. Differentiate the following expressions with respect to *x*. **MA11-5**

a. $(6x^2 - 5x)^5$ **1**

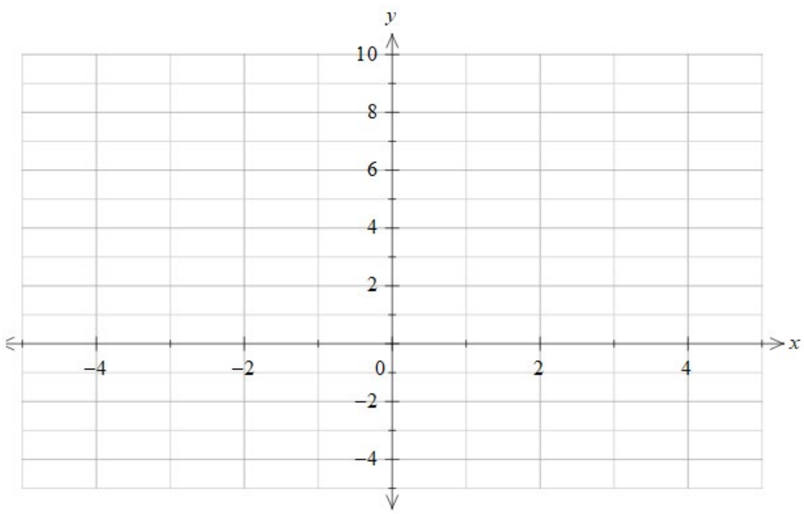
b. $7x^2 + 4 - \frac{7}{x}$ **1**

c. $\frac{3x^3+5}{x-9}$ **2**

13. Find the coordinates of the points on the curve $y = (x^2 - 1)^3$ where the tangent to the curve is parallel to the $x - axis$. **2** **MA11-5**

14. Sketch $y = |2x - 6|$ below. **1** **MA11-2**

a.



b. Hence, determine the values of k for which $x + k = |2x - 6|$ has exactly two solutions. **2** **MA11-2**

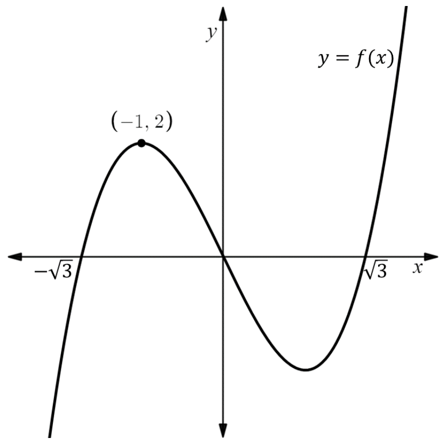
15. Given $y = 3x^2 + 6x + 1$. **2** **MA11-5**

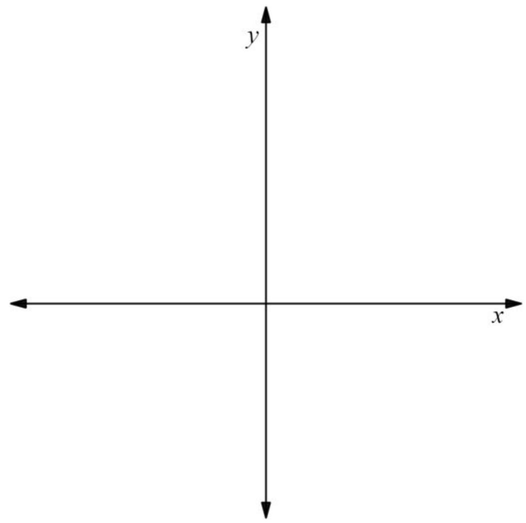
a. Find the equation of the normal at the point (2,25)

b. Find the point A where the normal cuts the x-axis and the point B where the normal cuts y-axis. **2** **MA11-5**

c. If O is the origin find the area of $\Delta\, AOB$. **1** **MA11-5**

16. **2** **MA11-5**





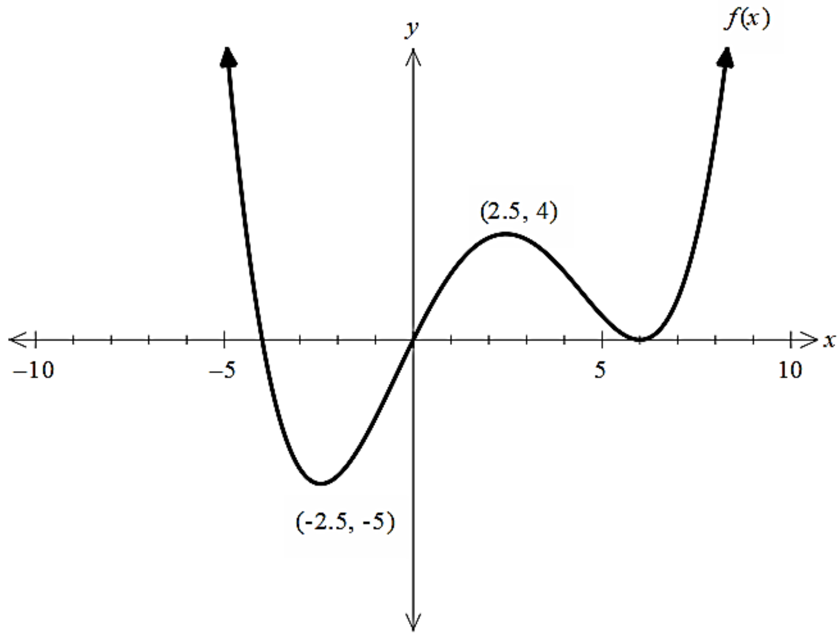
The diagram above on the left shows the graph of the curve $y = f(x)$. On the set of axes above on the right, sketch the graph of the curve $y = f'(x)$ showing the shape of the curve and its x intercepts.

17. Prove: **2 MA11-4**

$$\left(1+\cot^2\theta\right)\sqrt{1-\cos^2\theta}=\operatorname{cosec}\theta$$

18. Solve $\left(\sqrt{3}\tan\theta-1\right)\left(\sqrt{2}\sin\theta+1\right)=0$ for $-180^\circ\leq\theta\leq180^\circ$. **2 MA11-4**

19. The graph of $f(x)$ is shown. **3** **MA11-2**



Graph $y = 2 - f(x-1)$ on the same set of axes showing all essential points.

20. The observed brightness of a star can be measured by its energy flux and can be transformed by a logarithmic scale to be more easily measured by an apparent stellar magnitude. **3** **MA11-6**

It is defined to be $m = -\sqrt[5]{100}\log\left(\dfrac{f}{f_v}\right)$.

Where m is the magnitude, f is the energy flux of the star and f_V is the reference energy flux.

The star Betelgeuse has a magnitude of 0.45 while the star Deneb has a magnitude of 1.26.

Which star is brighter and by how much brighter is it?

21. Solve the equation $(\log_3 x)^2 = 1$ **2** **MA11-6**

22. Solve the equation $\log_2(x+4) + \log_2(x-2) = 4$. **3** **MA11-6**

23. In the space below, sketch the curve $y = \log_e(x+1)$ showing any intercepts on the axes and the equation of any asymptote. **2** **MA11-6**

24. ABC is a triangle in which $\sin(A+B) = 1$ and $\cos(A-B) = \frac{1}{2}$. Find the sizes of angles A and B in radian given that $A > B$. **2** **MA11-4**

25. Three points E, F and G lie on a horizontal plain. **3** **MA11-3**
The distance $EF = 1.1$ km and $FG = 520$ m.
A vertical tower HG is constructed so that
$\angle HFG = 28°$ and $\angle HEG = 25°$.
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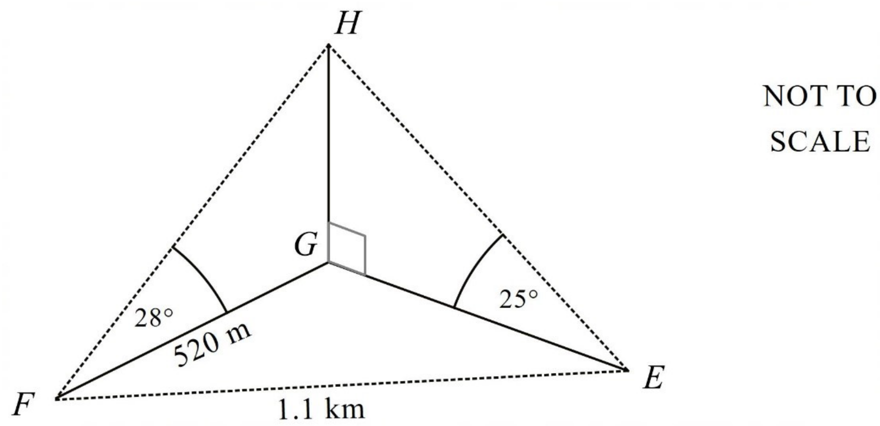

Calculate the height of the tower HG and hence find the distance EG and the size of $\angle EGF$, rounding each appropriately.

26. A function $f(x)$ with range $(0, \infty)$ is such that $f(a) \times f(b) = f(a+b)$ for any real numbers a and b. **2** **MA11-1**

Show that $f\left(\frac{a}{2}\right) = \sqrt{f(a)}$ for any real number a.

27. Nick constructs a swimming pool in the shape below.
Nick's pool design is made up of a sector and a semi-circle,
the radius(r) of the sector is equal to the diameter of the semi-circle.

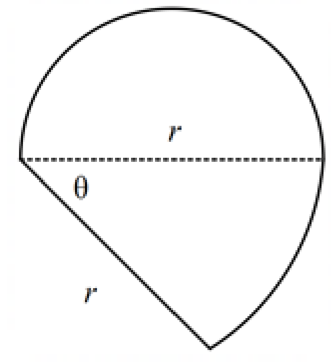

Figure not to scale

a. Find the perimeter of the pool in terms of r and θ. **1** **MA11-3**

Nick wants to be able to use this design in several different properties over the next year, so he decides to let the area of the pool equal 1 square unit for future ease in scaling the pool sizes. **3** **MA11-3**

b. Show that, if this is the case, the perimeter can be given by:

$$P = \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right).$$

END OF EXAM

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A. $\sqrt{2}$

B. 2

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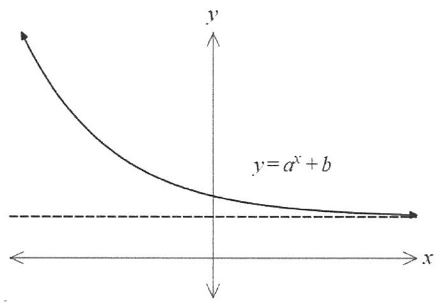
2. The figure shows the graph of $y = a^x + b$, where a and b are constants. Which of the following must be true? **1** **MA11-6**



A. $0 < a < 1$ and $b > 0$

B. $0 < a < 1$ and $b < 0$

C. $a > 1$ and $b > 0$

D. $a > 1$ and $b < 0$

3. What is the derivative with respect to x of $\dfrac{1}{(3x-4)^5}$ **1** **MA11-5**

A. $\dfrac{-15x}{(3x-4)^6}$

B. $\dfrac{-15}{(3x-4)^6}$

C. $\dfrac{-5}{(3x-4)^6}$

D. $\dfrac{4}{3(3x-4)^4}$

Section II

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	Question	**Mark**	**Outcome**
6.	Solve the equation $\left\|\frac{x}{2} - 3\right\| = 1$	2	MA11-1

$\frac{x}{2} - 3 = 1$

$\frac{x}{2} = 4$

$\boxed{x = 8}$

$\frac{x}{2} - 3 = -1$

$\frac{x}{2} = 2$

$\boxed{x = 4}$

7.	Simplify $\dfrac{12^x \times 18^x}{3^x \times 2^x}$	2	MA11-1

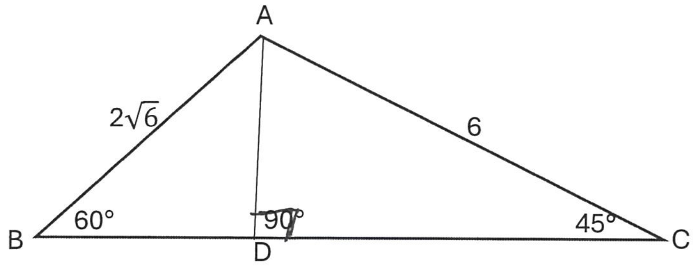
$= \dfrac{(2^2 \times 3)^x \times (3^2 \times 2)^x}{3^x \times 2^x}$

$= \dfrac{2^{2x} \times \cancel{3^x} \times 3^{2x} \times \cancel{2^x}}{\cancel{3^x} \times \cancel{2^x}}$

$= 2^{2x} \times 3^{2x}$

$= (6)^{2x}$

9. **MA11-3**



In the diagram, ABC is triangle in which $AB = 2\sqrt{6}$, $AC = 6$, $\angle ABC = 60°$ and $\angle ACB = 45°$. AD is perpendicular from A to BC.

a. Show that $BC = \sqrt{6} + 3\sqrt{2}$ **2**

$\cos 60 = \frac{BD}{2\sqrt{6}} \therefore BD = 2\sqrt{6} \times \frac{1}{2}$

$= \sqrt{6}$

$\frac{DC}{6} = \cos 45$ $\quad DC = 6\cos 45$

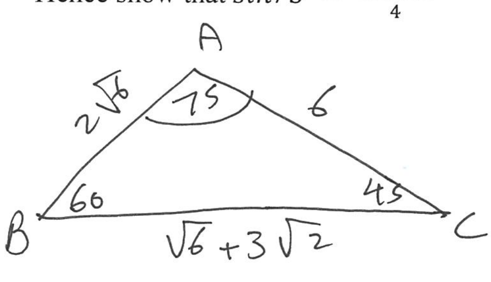
$DC = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$

$DC = 3\sqrt{2}$

$\therefore BC = BD + DC$

$= \sqrt{6} + 3\sqrt{2}$

b. Hence show that $sin75° = \frac{\sqrt{6}+\sqrt{2}}{4}$ **2**



$\frac{\sin 75}{\sqrt{6}+3\sqrt{2}} = \frac{\sin 60}{6}$

$\sin 75 = \frac{\frac{\sqrt{3}}{2}(\sqrt{6}+3\sqrt{2})}{6}$

$= \frac{\frac{3\sqrt{2}}{2} + \frac{3\sqrt{6}}{2}}{6}$

$= \frac{3\sqrt{2} + 3\sqrt{6}}{12}$

$= \frac{3(\sqrt{3}+\sqrt{6})}{12} =$

$\frac{\sin 75}{\sqrt{6}+3\sqrt{2}} = \frac{\sin 45}{2\sqrt{6}}$

$\sin 75 = \frac{\frac{1}{\sqrt{2}}(\sqrt{6}+3\sqrt{2})}{2\sqrt{6}}$

$= \frac{\frac{\sqrt{6}}{\sqrt{2}} + 3}{2\sqrt{6}} = \frac{\sqrt{3}+3}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{2}+3\sqrt{6}}{12}$

5

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

12. Differentiate the following expressions with respect to x. **MA11-5**

a. $(6x^2 - 5x)^5$ **1**

$$\frac{dy}{dx} = 5(6x^2 - 5x)^4 \times (12x - 5)$$

$$= 5(12x-5)(6x^2-5x)^4$$

b. $7x^2 + 4 - \frac{7}{x}$ **1**

$$y = 7x^2 + 4 - 7x^{-1}$$

$$\frac{dy}{dx} = 14x + \frac{7}{x^2}$$

c. $\frac{3x^3+5}{x-9}$ **2**

$u = 3x^3 + 5$ $\quad v = x - 9$

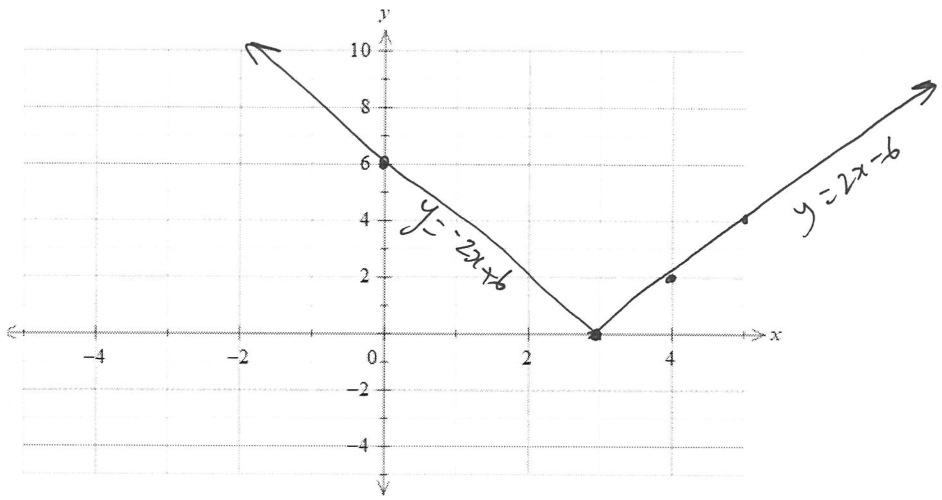
$u' = 9x^2$ $\quad v' = 1$

$$\frac{dy}{dx} = \frac{9x^2(x-9) - (3x^3+5)}{(x-9)^2}$$

$$= \frac{6x^3 - 81x^2 - 5}{(x-9)^2}$$

14. Sketch $y = |2x - 6|$ below. **1 MA11-2**

a.



b. Hence, determine the values of k for which $x + k = |2x - 6|$ has exactly two solutions. **2 MA11-2**

at (3,0) vertex $y = |2x-6|$

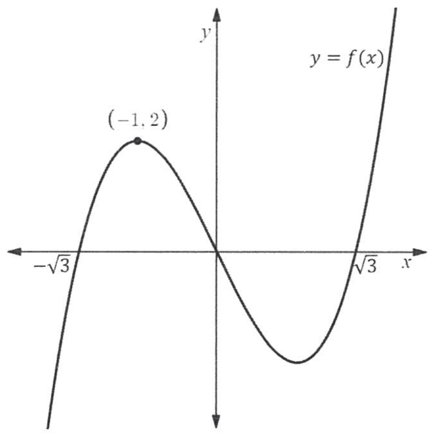
$y = x + k$

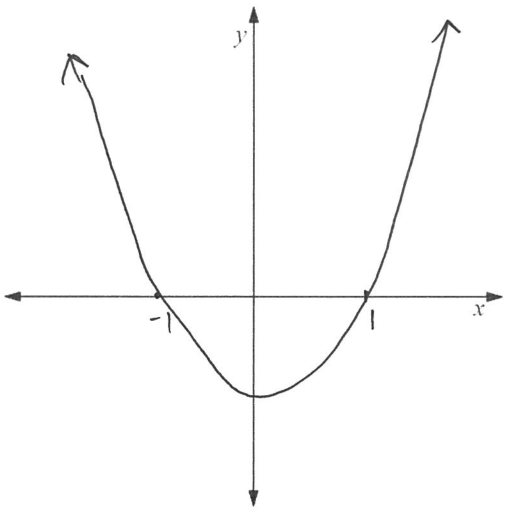
$0 = 3 + k$

$k = -3$

$y = x - 3$ will intersect $y = |2x-6|$ at one point

16. **2** **MA11-5**





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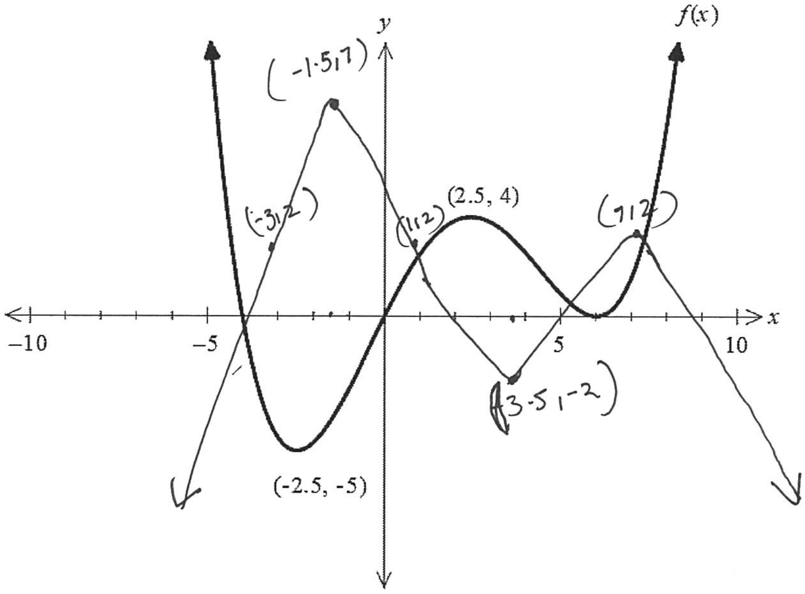
18. The graph of $f(x)$ is shown. **3** **MA11-2**



Graph $y = 2 - f(x-1)$ on the same set of axes showing all essential points.

20 The observed brightness of a star can be measured by its energy flux and can be transformed by a logarithmic scale to be more easily measured by an apparent stellar magnitude. **3** **MA11-6**

It is defined to be $m = -\sqrt[5]{100}\log\left(\frac{f}{f_v}\right)$.

Where m is the magnitude, f is the energy flux of the star and f_V is the reference energy flux.

The star Betelgeuse has a magnitude of 0.45 while the star Deneb has a magnitude of 1.26.

Which star is brighter and by how much brighter is it?

(magnitude of star Betelgeuse) $m_B = 0.45$ $m_D = 1.26$ (magnitude of star Deneb)

f_B = flux of star Betelgeuse f_D = flux of star Deneb

$$1.26 - 0.45 = -\sqrt[5]{100}\log\left(\frac{f_D}{f_r}\right) + \sqrt[5]{100}\log\left(\frac{f_B}{f_r}\right)$$

$$\frac{0.81}{\sqrt[5]{100}} = \log\left(\frac{f_B}{f_r}\right) - \log\left(\frac{f_D}{f_r}\right)$$

$$\frac{0.81}{\sqrt[5]{100}} = \log\left(\frac{f_B}{\cancel{f_r}} \times \frac{\cancel{f_r}}{f_D}\right)$$

$$\frac{0.81}{\sqrt[5]{100}} = \log_{10}\left(\frac{f_B}{f_D}\right)$$

$$10^{\frac{0.8}{\sqrt[5]{100}}} = \frac{f_B}{f_D}$$

$$f_B = 10^{\frac{0.8}{\sqrt[5]{100}}} \times f_D$$

$$\doteqdot 2.1\, f_D$$

∴ Star Betelgeuse is 2.1 brighter than star Deneb

21. Solve the equation $(\log_3 x)^2 = 1$ **1** **MA11-6**

$\log_3 x = \pm 1$

$x = 3^1$ $\quad$ $x = 3^{-1}$

$\boxed{x = 3}$ $\quad$ $\boxed{x = \frac{1}{3}}$

22. Solve the equation $\log_2(x + 4) + \log_2(x - 2) = 4$. **3** **MA11-6**

$\log_2 (x+4)(x-2) = 4$

$x^2 + 2x - 8 = 2^4$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = -6$ $\quad$ $\boxed{x = 4}$

$x > 0$

Not a solution

25. Three points E, F and G lie on a horizontal plain. **3** **MA11-3**
The distance $EF = 1.1$ km and $FG = 520$ m.
A vertical tower HG is constructed so that
$\angle HFG = 28°$ and $\angle HEG = 25°$.
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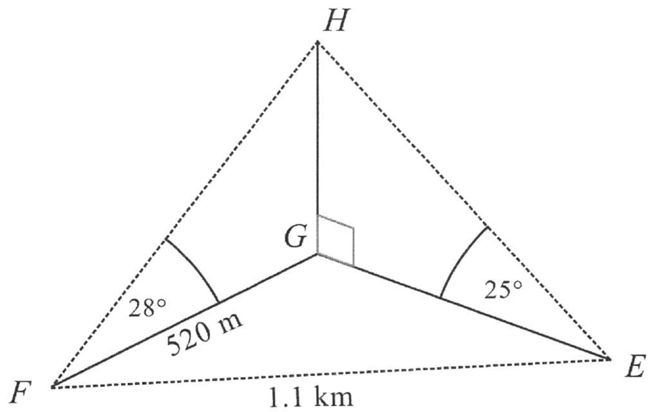

NOT TO SCALE

Calculate the height of the tower HG and hence find the distance EG and the size of $\angle EGF$, rounding each appropriately.

$$\tan 28 = \frac{HG}{520}$$

$$HG = 520\tan 28° = 276 \text{ m}$$

$$\tan 25 = \frac{HG}{EG}$$

$$EG = \frac{HG}{\tan 25} = \frac{520\tan 28}{\tan 25}$$

$$EG = 593 \text{ m}$$

$$\cos\angle EGF = \frac{520^2 + 593^2 - 1100^2}{2(520)(593)}$$

$$\angle EGF = 162°$$

26. A function $f(x)$ with range $(0, \infty)$ is such that $f(a) \times f(b) = f(a+b)$ for any real numbers a and b. ~~3~~ 2 **MA11-1**

Show that $f\left(\frac{a}{2}\right) = \sqrt{f(a)}$ for any real number a.

$$f\left(\frac{a}{2}\right) \times f\left(\frac{a}{2}\right) = f\left(\frac{a}{2} + \frac{a}{2}\right)$$

$$\left(f\left(\frac{a}{2}\right)\right)^2 = f(a)$$

$$f\left(\frac{a}{2}\right) = \sqrt{f(a)}$$

25. Nick constructs a swimming pool in the shape below.
Nick's pool design is made up of a sector and a semi-circle,
the radius(r) of the sector is equal to the diameter of the semi-circle.

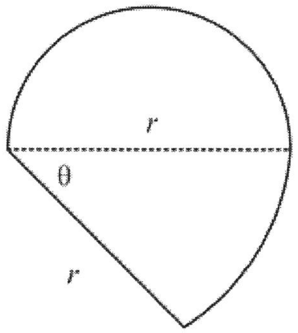

Figure not to scale

a. Find the perimeter of the pool in terms of r and θ. **2** **MA11-3**

$$P = \frac{1}{2} \times \pi \times r + r\theta + r$$

$$P = \frac{\pi r}{2} + r\theta + r$$

3 MA11-3

Nick wants to be able to use this design in several different properties over the next year, so he decides to let the area of the pool equal 1 square unit for future ease in scaling the pool sizes.

b. Show that, if this is the case, the perimeter can be given by:

$$P = \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right).$$

$$A = \frac{1}{2} \times \pi \times \left(\frac{r}{2}\right)^2 + \frac{1}{2}r^2\theta$$

$$1 = \frac{\pi r^2}{8} + \frac{4 \times r^2\theta}{2 \times 4}$$

$$1 = \frac{\pi r^2}{8} + \frac{4r^2\theta}{8}$$

$$8 = \pi r^2 + 4r^2\theta$$

$$\frac{8 - \pi r^2}{4r^2} = \theta \qquad \theta = \frac{2}{r^2} - \frac{\pi}{4}$$

$$P = \frac{\pi r}{2} + r + r\left(\frac{2}{r^2} - \frac{\pi}{4}\right)$$

$$= \frac{2\cancel{\times}\pi r}{2\cancel{4}2} + r + \frac{2}{r} - \frac{\pi r}{4}$$

$$= \frac{\pi r}{4} + r + \frac{2}{r}$$

$$= \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right)$$

END OF EXAM