NORTH
SYDNEY
GIRLS HIGH
SCHOOL

2018	Task 3 Yearly Examination

Preliminary Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- In Questions 16 – 20, show relevant mathematical reasoning and/or calculations

Total marks – 82

Section I – 15 marks (pages 2 – 7)

- Attempt Questions 1 – 15
- Allow about 20 minutes for this section

Section II – 67 marks (pages 8 – 15)

- Attempt Questions 16 – 20
- Allow about 1 hour 40 minutes for this section

NAME:______________________________ TEACHER:________________________

STUDENT NUMBER: | | | | | | | | |

MC	16	17	18	19	20	Total
/15	/13	/14	/13	/14	/13	/82

Section I

15 marks
Attempt Questions 1-15
Allow about 20 minutes for this section

Use the multiple choice answer sheet for Questions 1-15.

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1 What is the value of $\sqrt[3]{\dfrac{2.1\times 10^{-5}}{2.7}}$ correct to three significant figures?

A. 0.020

B. 0.02

C. 0.00279

D. 0.003

2 Which of the following statements is not always true?

A. $\log_a(ab) = 1 + \log_a b$

B. $\log_a b - \log_a c = \dfrac{\log_a b}{\log_a c}$

C. $\log_a\left(b^c\right) = c\log_a b$

D. $\log_b c = \dfrac{\log_a c}{\log_a b}$

3 Which of the following is the solution to $x^2 + 7x + 6 > 0$?

A. $1 < x < 6$

B. $x < 1$ or $x > 6$

C. $-6 < x < -1$

D. $x < -6$ or $x > -1$

4 What is the domain and range of the function $f(x) = \sqrt{9 - x^2}$?

A. Domain: $-3 \le x \le 3$, Range: $0 \le y \le 3$

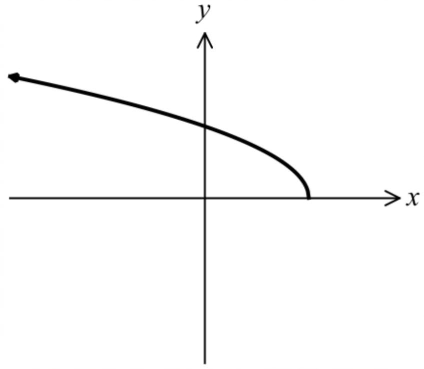
B. Domain: $-3 \le x \le 3$, Range: $-3 \le y \le 3$

C. Domain: $0 \le x \le 9$, Range: $-9 \le y \le 9$

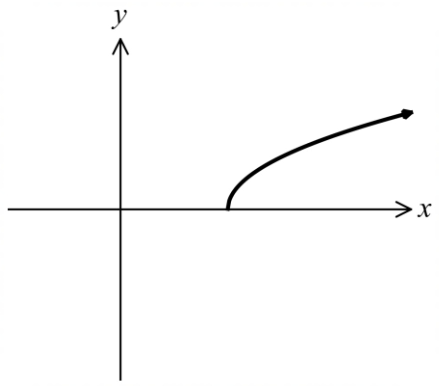
D. Domain: $0 \le x \le 9$, Range: $0 \le y \le 9$

5 Which graph best represents $y = \sqrt{6 - x}$?

A.



B.



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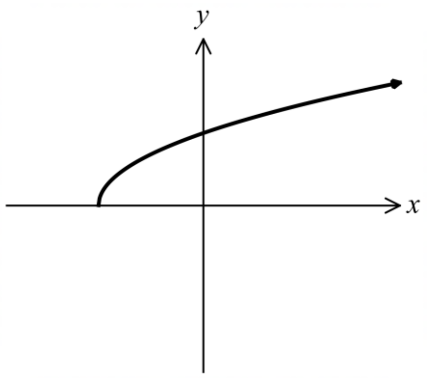
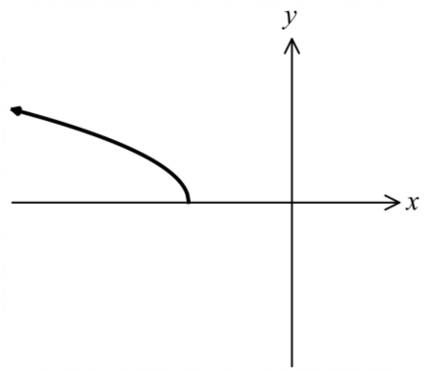


D.



6 The sum of the interior angles of a regular polygon is 2520º. What is the size of each interior angle?

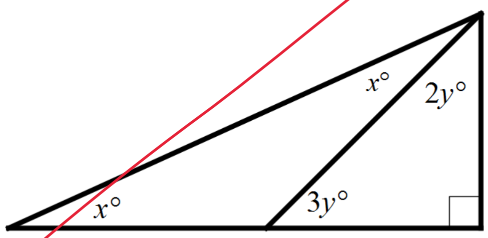
A. $140^\circ 30'$

B. 210°

C. $157^\circ 30'$

D. 315°

7 In the diagram below, what is the value of *x*?



A. 18°

B. 27°

C. 36°

D. 45°

8 Which of the following is the same as $\dfrac{\sin(\theta - 90^\circ)}{\tan(180^\circ + \theta)}$?

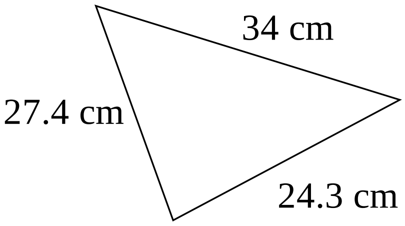
A. $\cos\theta$

B. $-\cos\theta$

C. $\dfrac{\cos^2\theta}{\sin\theta}$

D. $-\dfrac{\cos^2\theta}{\sin\theta}$

9 The smallest angle in the triangle below is θ.



What is the value of θ to the nearest degree?

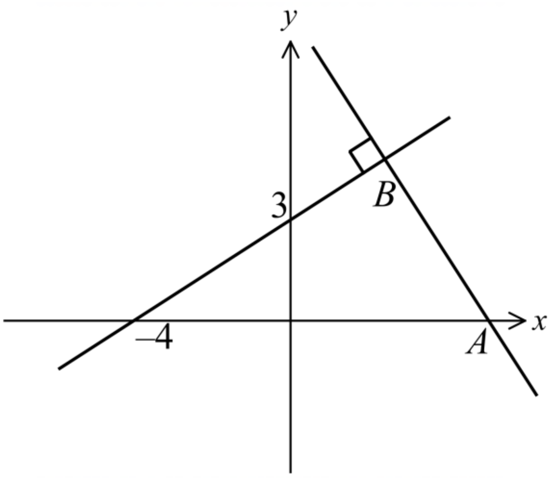
A. 30°

B. 45°

C. 53°

D. 82°

10 What is the gradient of the line *AB*?



A. $\frac{4}{3}$

B. $\frac{3}{4}$

C. $-\frac{4}{3}$

D. $-\frac{3}{4}$

11 What is the value of $f'(-3)$, if $f(x) = 3x - x^3$?

A. 24

B. 18

C. – 24

D. – 18

12 Which of the following statements is true for the equation $3x^2 - x - 3 = 0$?

A. It has no real roots

B. It has one real root

C. It has two real roots

D. It has three real roots

13 Let α and β be the roots of the equation $3x^2 - 7x + 11 = 0$.
What is the value of $\alpha^2\beta + \alpha\beta^2$?

A. $\frac{77}{9}$

B. 77

C. $-\frac{77}{9}$

D. -77

14 If $\log x : \log y = m : n$ **,** what is the value of x?

A. $\dfrac{my}{n}$

B. $m - n + y$

C. $y^{\frac{m}{n}}$

D. $\dfrac{m \log y}{n}$

15 Which of the following is an ODD function?

A. $y = |\sin x|$

B. $y = 2^{x^2}$

C. $y = x^3 + 1$

D. $y = x^2 \tan x$

Section II

67 marks
Attempt Questions 16-20
Allow about 1 hour 40 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 16-20, your responses should include relevant mathematical reasoning and/or calculations.

Question 16 (13 marks**)** Use a SEPARATE writing booklet

(a) Simplify $\dfrac{36-8x}{28}$. **1**

(b) Express $\dfrac{15}{2\sqrt{3}-3}$ in simplest form with a rational denominator. **2**

(c) Solve $7-2(x-1)<13$. **2**

(d) Solve $|2x-3|>5$. **2**

(e) Solve $5x^2-3x=1$. **2**

(f) Make z the subject of $v=2-\dfrac{1}{z^2}$. **2**

(g) Consider the function $f(x)=\dfrac{mx-1}{x+1}$, where m is a positive real constant. **2**

Simplify $f(2)-f(1)$.

Question 17 (14 marks**)** Use a SEPARATE writing booklet

(a) Differentiate, writing your answer with positive indices where applicable:

(i) $\dfrac{5}{x^3} - 3\sqrt{x} + 1$ **2**

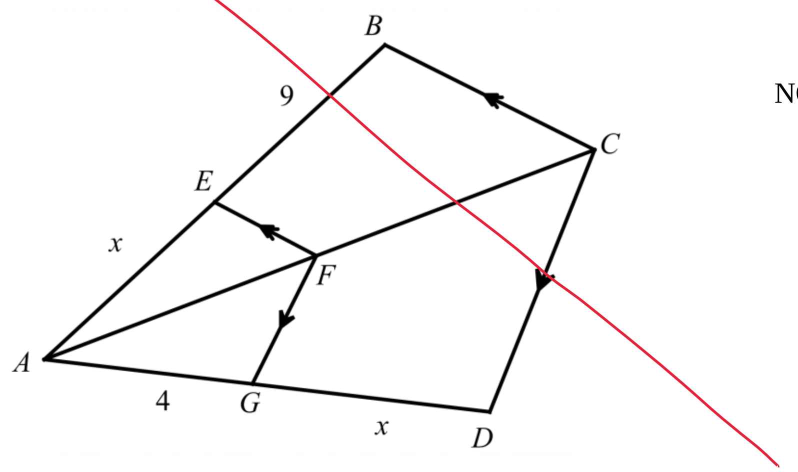
(ii) $\left(x^2 + 3x\right)^6$ **1**

(b) Evaluate $\lim\limits_{x \to 2} \left(\dfrac{x^3 - 8}{x^2 - 4} \right)$. **2**

(c) (i) Simplify $\operatorname{cosec}\theta \tan\theta$ as a single trigonometric ratio. **1**

(ii) Hence, or otherwise, solve $\operatorname{cosec}\theta \tan\theta + 8\cos^2\theta = 0$ for $0° \le \theta \le 360°$. **2**

(d) In the diagram below, $FG \parallel CD$ and $EF \parallel BC$. **2**
Deduce the value of x giving reasons.



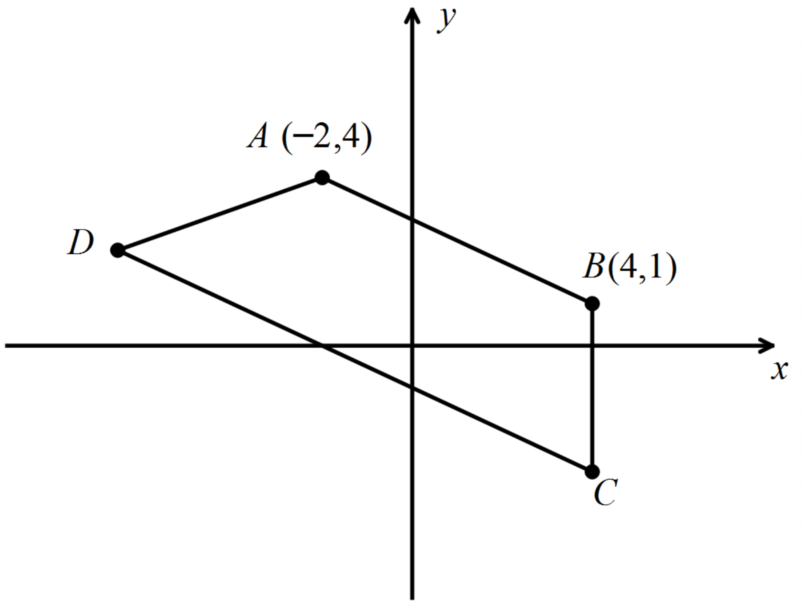
NOT TO SCALE

(e) (i) Sketch the graph of the function $f\left(x\right) = x^2 + 4x + 3$ showing all intercepts and the coordinates of the vertex. **2**

(ii) Hence, or otherwise, find the value(s) of k for which the equation $\left|x^2 + 4x + 3\right| = k$ has exactly two solutions. **2**

Question 18 (13 marks) Use a SEPARATE writing booklet

(a) In the trapezium *ABCD*, the coordinates of *A* and *B* are $(-2,4)$ and $(4,1)$ respectively. The equation of *CD* is $x+2y+2=0$.



(i) Find the length of *AB* in exact form. **1**

(ii) The line *BC* is parallel to the *y*-axis. Find the coordinates of *C*. **1**

(iii) Find the perpendicular distance from *A* to *CD*. **1**

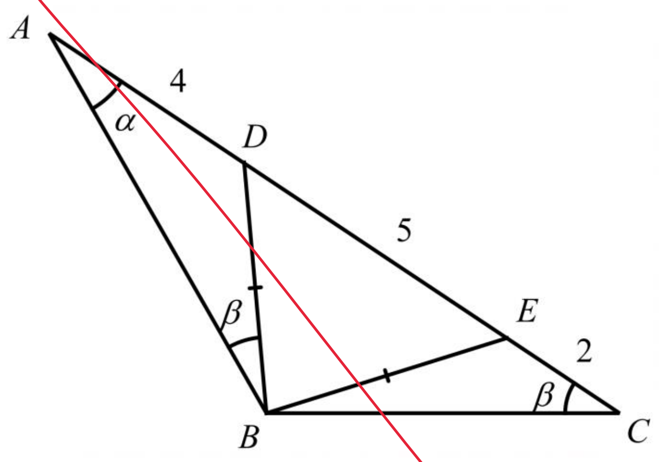
(iv) If the area of the trapezium is 32 square units, find the exact length of *CD*. **2**

(b) Find the equation of the normal to the curve $y=\sqrt{2x-5}$ at the point $(3,1)$. **3**

Question 18 continues on Page 11

Question 18 (continued)

(c) In ΔABC, $\angle BAC = \alpha$ and $\angle BCA = \beta$. BD is constructed such that $\angle ABD = \beta$. Also $BD = BE$ as shown.



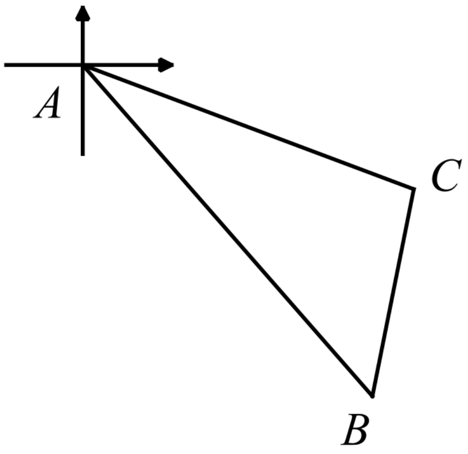
(i) Prove that ΔABC and ΔBEC are similar. **3**

(ii) Hence, or otherwise, find the exact length of BC. **2**

End of Question 18

Question 19 (14 marks) Use a SEPARATE writing booklet

(a) A party of hikers leaves the camp, A, at 6:00 am and walks in the direction 150° T until they reach point B at the base of a cliff. They then travel on a bearing of 015° T and set up camp for the night when they reach point C at 2:00 pm. The bearing and distance of C from A are 120° T and 10 km respectively.



(i) Copy the diagram into your writing booklet and add in the given information. **1**

(ii) Explain why $\angle ABC = 45°$. **1**

(iii) Calculate the average speed of the hike from A to C, correct to one decimal place. **3**

(b) Prove that $\dfrac{1+\sin t}{1-\sin t} = (\sec t + \tan t)^2$. **2**

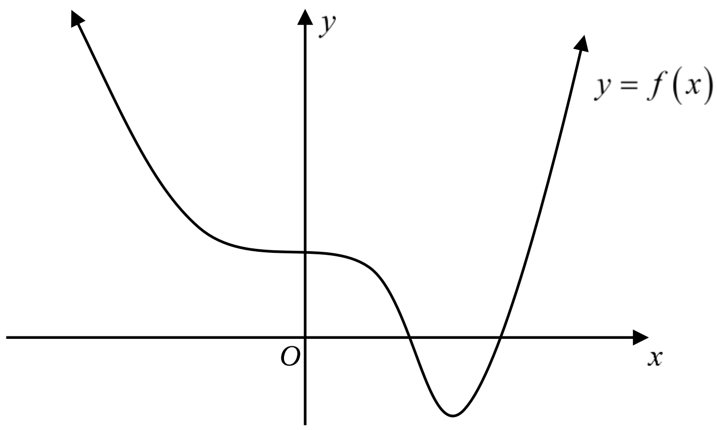
(c) Use the substitution $u = m^{0.2}$ to solve $m^{0.4} - 2m^{0.2} - 3 = 0$. **3**

(d) Determine whether the line $x + y + 3 = 0$ is a tangent to the curve $y = 2x^2 + 3x - 1$. **2**

Question 19 continues on Page 13

Question 19 (continued)

(e) The graph of $y = f(x)$ is shown below. **2**



Copy the graph into your writing booklet.

Underneath your graph, sketch the graph of $y = f'(x)$, lining up all the important features.

End of Question 19

Question 20 (13 marks) Use a SEPARATE writing booklet

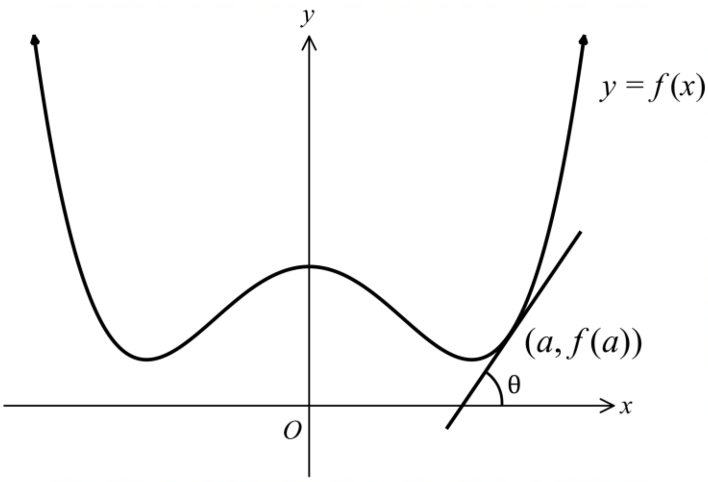
(a) (i) On the same number plane, sketch the curves $y = \sin x$ and $y = \sqrt{3}\cos x$ for $0° \le x \le 360°$. **2**

(ii) Hence solve $\sin x > \sqrt{3}\cos x$ in the domain $0° \le x \le 360°$. **2**

(b) Given that $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$, differentiate $f(x) = \frac{2}{x}$ from first principles. **3**

(c) The diagram below shows an even differentiable function $y = f(x)$. **2**

The tangent at $(a, f(a))$ makes an angle of θ with the x-axis as shown.

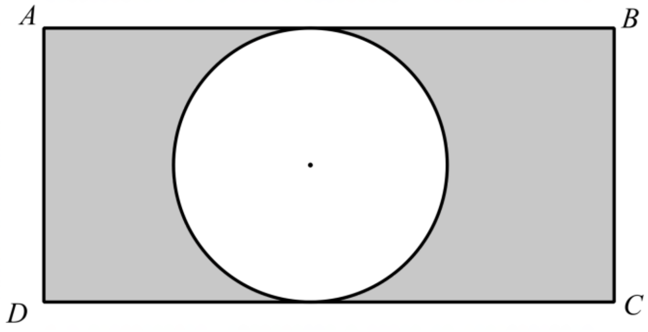


Justify geometrically, or otherwise, why the derivative of an even function is an odd function.

Question 20 continues on page 15

Question 20 (continued)

(d) A circle is inscribed within a rectangle $ABCD$ such that it is always in contact with two opposite sides of the rectangle, as shown. The perimeter of rectangle $ABCD$ is 40 cm. Let $AB = x$ and $BC = y$.



(i) Show that the area of the shaded region, R cm^2 is given by **2**

$$R = 20y - \left(\frac{4+\pi}{4}\right)y^2 \ .$$

(ii) Find the largest value of R, giving your answer as an exact value. **2**

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End of paper

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2018	Task 3 Yearly Examination

Preliminary Mathematics Solutions

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- In Questions 16 – 20, show relevant mathematical reasoning and/or calculations

Total marks – 80

Section I – 15 marks (pages 2 – 6)

- Attempt Questions 1 – 15
- Allow about 20 minutes for this section

Section II – 65 marks (pages 7 – 12)

- Attempt Questions 16 – 20
- Allow about 1 hour 40 minutes for this section

NAME:______________________________ TEACHER:________________________

STUDENT NUMBER:

MC	16	17	18	19	20	Total
/15	/13	/14	/13	/14	/13	/82

Section I

15 marks
Attempt Questions 1-15
Allow about 20 minutes for this section

Use the multiple choice answer sheet for Questions 1-15.

---

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A. 0.020

B. 0.02

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2 Which of the following statements is not always true?

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B. $\log_a b-\log_a c=\dfrac{\log_a b}{\log_a c}$

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D. $\log_b c=\dfrac{\log_a c}{\log_a b}$

3 Which of the following is the solution to $x^2+7x+6>0$?

A. $1<x<6$

B. $x<1$ or $x>6$

C. $-6<x<-1$

D. $x<-6$ or $x>-1$

4 What is the domain and range of the function $f(x)=\sqrt{9-x^2}$?

A. Domain: $-3 \le x \le 3$, Range: $0 \le y \le 3$

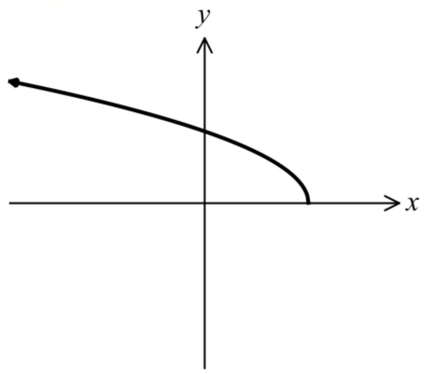
B. Domain: $-3 \le x \le 3$, Range: $-3 \le y \le 3$

C. Domain: $0 \le x \le 9$, Range: $-9 \le y \le 9$

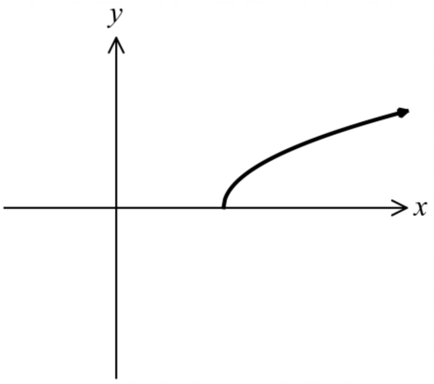
D. Domain: $0 \le x \le 9$, Range: $0 \le y \le 9$

5 Which graph best represents $y=\sqrt{6-x}$?

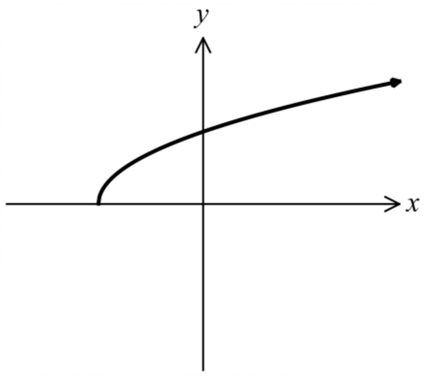
A.



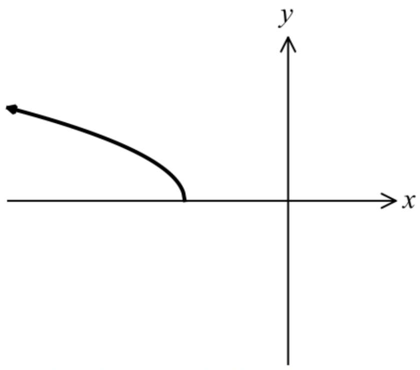
B.



C.



D.



6 The sum of the interior angles of a regular polygon is 2520°. What is the size of each interior angle?

A. $140^{\circ}30'$

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C. $157^{\circ}30'$

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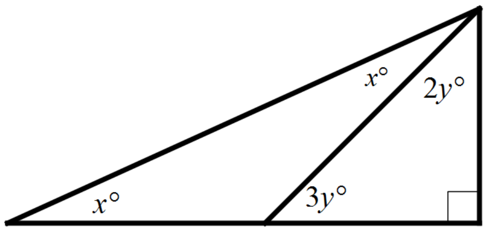
7 In the diagram below, what is the value of *x*?



A. 18°

B. 27°

C. 36°

D. 45°

8 Which of the following is the same as $\dfrac{\sin(\theta - 90°)}{\tan(180° + \theta)}$?

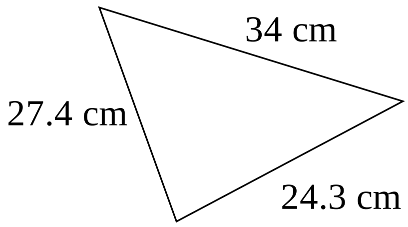
A. $\cos\theta$

B. $-\cos\theta$

C. $\dfrac{\cos^2\theta}{\sin\theta}$

D. $-\dfrac{\cos^2\theta}{\sin\theta}$

9 The smallest angle in the triangle below is θ.



What is the value of θ to the nearest degree?

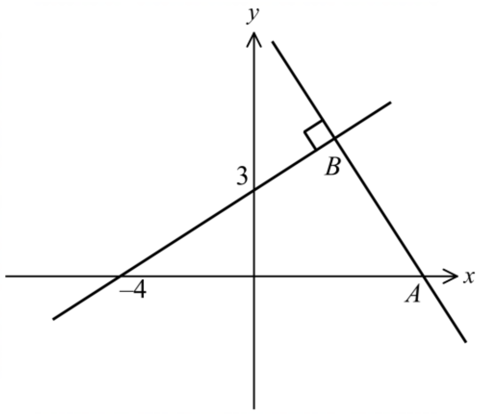
A. 30°

B. 45°

C. 53°

D. 82°

10 What is the gradient of the line *AB*?



A. $\frac{4}{3}$

B. $\frac{3}{4}$

C. $-\frac{4}{3}$

D. $-\frac{3}{4}$

11 What is the value of $f'(-3)$, if $f(x) = 3x - x^3$?

A. 24

B. 18

C. – 24

D. – 18

12 Which of the following statements is true for the equation $3x^2 - x - 3 = 0$?

A. It has no real roots

B. It has one real root

C. It has two real roots

D. It has three real roots

13 Let α and β be the roots of the equation $3x^2 - 7x + 11 = 0$.
What is the value of $\alpha^2\beta + \alpha\beta^2$?

A. $\frac{77}{9}$

B. 77

C. $-\frac{77}{9}$

D. -77

14 If $\log x : \log y = m : n$, what is the value of x?

A. $\frac{my}{n}$

B. $m - n + y$

C. $y^{\frac{m}{n}}$

D. $\frac{m \log y}{n}$

15 Which of the following is an ODD function?

A. $y = |\sin x|$

B. $y = 2^{x^2}$

C. $y = x^3 + 1$

D. $y = x^2 \tan x$

Section II

65 marks
Attempt Questions 16-20
Allow about 1 hour 40 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 16-20, your responses should include relevant mathematical reasoning and/or calculations.

Question 16 (13 marks**)** Use a SEPARATE writing booklet

(a)	Simplify $\dfrac{36-8x}{28}$.	**1**

$$\frac{36-8x}{28}=\frac{\cancel{4}(9-2x)}{\cancel{28}\,7}=\frac{9-2x}{7}$$

(b)	Express $\dfrac{15}{2\sqrt{3}-3}$ in simplest form with a rational denominator.	**2**

$$\frac{15}{2\sqrt{3}-3}\times\frac{2\sqrt{3}+3}{2\sqrt{3}+3}=\frac{15\left(2\sqrt{3}+3\right)}{12-9}=\frac{5\,\cancel{15}\left(2\sqrt{3}+3\right)}{\cancel{3}}=5\left(2\sqrt{3}+3\right)$$

(c)	Solve $7-2(x-1)<13$.	**2**

$$\begin{aligned}7-2(x-1)&<13\\ 7-2x+2&<13\\ -2x&<4\\ x&>\frac{4}{-2}\\ x&>-2\end{aligned}$$

(d)	Solve $\lvert 2x-3\rvert>5$.	**2**

$|2x-3|>5$

$2x-3<-5$ or $2x-3>5$

$2x<-2$ or $2x>8$

$x<-1$ or $x>4$

(e)	Solve $5x^2 - 3x = 1$.	**2**

$$5x^2 - 3x - 1 = 0$$

$$x = \frac{3 \pm \sqrt{9 - 4 \times 5 \times -1}}{10}$$

$$= \frac{3 \pm \sqrt{29}}{10}$$

(f)	Make z the subject of $v = 2 - \frac{1}{z^2}$.	**2**

$$v = 2 - \frac{1}{z^2}$$

$$\frac{1}{z^2} = 2 - v$$

$$z^2 = \frac{1}{2 - v}$$

$$z = \pm\sqrt{\frac{1}{2 - v}}$$

(g)	Consider the function $f(x) = \frac{mx - 1}{x + 1}$, where m is a positive real constant. Simplify $f(2) - f(1)$.	**2**

$$f(2) - f(1) = \frac{2m - 1}{3} - \frac{m - 1}{2}$$

$$= \frac{2(2m - 1) - 3(m - 1)}{6}$$

$$= \frac{4m - 2 - 3m + 3}{6}$$

$$= \frac{m + 1}{6}$$

Markers Comment
Most students did really well in all parts of question 16. Common mistakes include forgetting to change the inequality sign when dividing by a negative sign in part (c) and omitting the +/– sign in part (f).

Question 17 (14 marks**)** Use a SEPARATE writing booklet1

(a) Differentiate, writing your answer with positive indices where applicable:

(i) $\dfrac{5}{x^3}-3\sqrt{x}+1$ **2**

Let $y=\dfrac{5}{x^3}-3\sqrt{x}+1=5x^{-3}-3x^{\frac{1}{2}}+1$

$$\frac{dy}{dx}=15x^{-4}-\frac{3}{2}x^{-\frac{1}{2}}$$

$$=-\frac{15}{x^4}-\frac{3}{2\sqrt{x}}$$

(ii) $\left(x^2+3x\right)^6$ **1**

Let $y=\left(x^2+3x\right)^6$

$$\frac{dy}{dx}=6(2x+3)\left(x^2+3x\right)^5$$

(b) Evaluate $\lim\limits_{x\to 2}\left(\dfrac{x^3-8}{x^2-4}\right)$. **2**

$$\lim_{x\to 2}\left(\frac{x^3-8}{x^2-4}\right)=\lim_{x\to 2}\frac{\cancel{(x-2)}\left(x^2+2x+4\right)}{\cancel{(x-2)}(x+2)}$$

$$=\frac{2^2+2(2)+4}{2+2}=3$$

(c) (i) Simplify $\operatorname{cosec}\theta\tan\theta$ as a single trigonometric ratio. **1**

$$\operatorname{cosec}\theta\tan\theta=\frac{1}{\sin\theta}\times\frac{\sin\theta}{\cos\theta}=\frac{1}{\cos\theta}\text{ or }\sec\theta$$

(ii) Hence, or otherwise, solve $\operatorname{cosec}\theta\tan\theta+8\cos^2\theta=0$ for $0°\le\theta\le 360°$. **2**

$\operatorname{cosec}\theta\tan\theta+8\cos^2\theta=0$ use result from (i)

$\dfrac{1}{\cos\theta}+8\cos^2\theta=0 \quad \times\cos\theta$

$1+8\cos^3\theta=0$

$\cos^3\theta=-\dfrac{1}{8}$

$\cos\theta=-\dfrac{1}{2}$

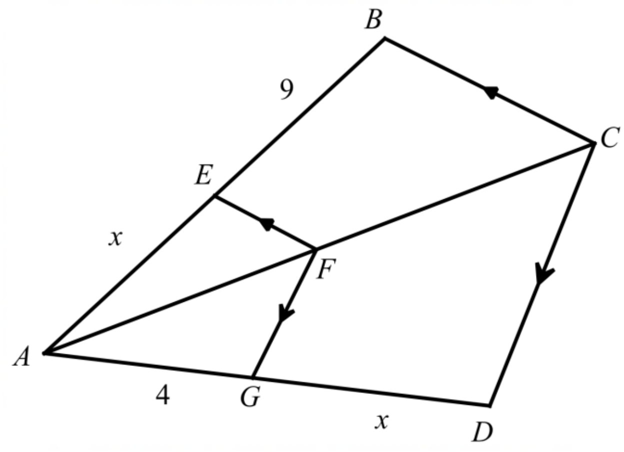
$\theta=120°,240°$

Markers Comment

When students got to $1+8\cos^3\theta = 0$ in their solution, many decided to factorise using sum of two cubes rather than continuing to $\cos^3\theta = -\frac{1}{8}$ as described in the solutions. This was a less efficient method because students also needed to explain that the quadratic factor had no real roots.

(d) In the diagram below, $FG \parallel CD$ and $EF \parallel BC$. **2**

Deduce the value of x giving reasons.



NOT TO SCALE

$\frac{AG}{GD} = \frac{AF}{FC}$ (parallel lines preserve ratios of intercepts; $FG \parallel CD$)

$\frac{AE}{EB} = \frac{AF}{FC}$ (parallel lines preserve ratios of intercepts; $EF \parallel BC$)

$\therefore \frac{AG}{GD} = \frac{AE}{EB}$ or $\frac{x}{9} = \frac{4}{x}$

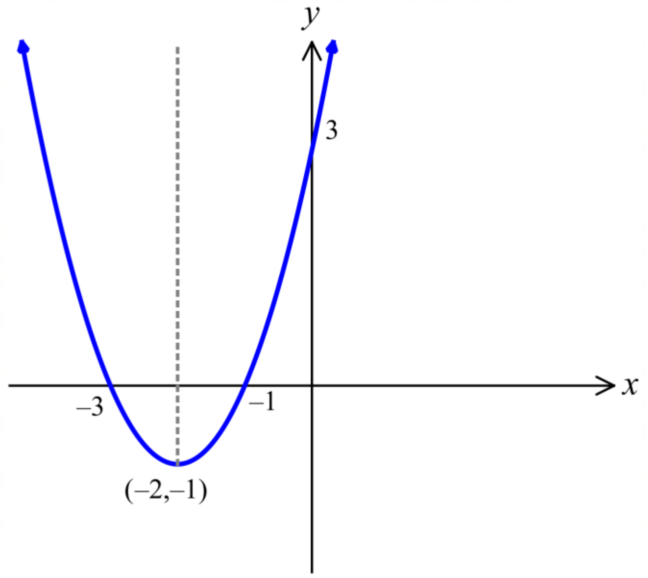
$x^2 = 36$

$x = 6$

Markers Comment

This was poorly set out. There are two different sets of parallel lines and students should have considered the ratios on each of these separately and then equated the ratios.

(e)	(i)	Sketch the graph of the function $f(x) = x^2 + 4x + 3$ showing and all intercepts and the coordinates of the vertex.	**2**

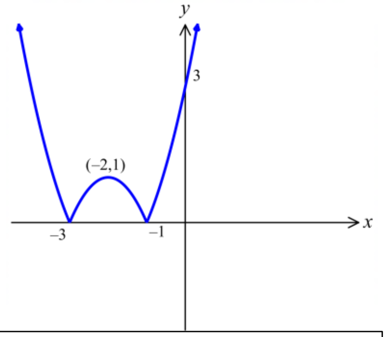


Markers Comment
When asked to sketch a function, your graph should be approx. a third of a page. Use a ruler to draw the axes and label the axes. The scale does not have to be the same on both axes, but should be consistent within the same axis. A poor scale usually leads to a graph that is not symmetrical.

(ii)	Hence, or otherwise, find the value(s) of k for which the equation $\left|x^2 + 4x + 3\right| = k$ has exactly two solutions.	**2**

Alongside is the graph of $y = \left|x^2 + 4x + 3\right|$

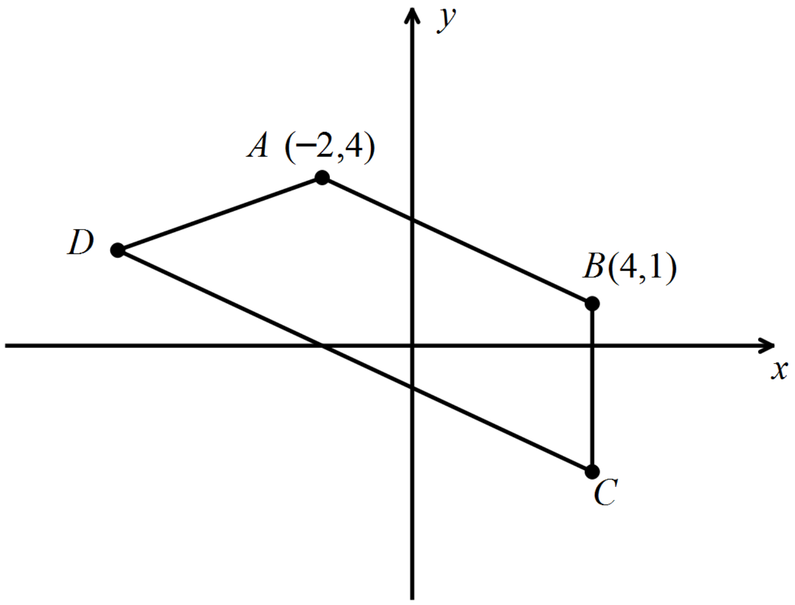
From the graph, there will be exactly two solutions for $k = 0$ or $k > 1$.



Markers Comment
Students really needed to use the 'hence' and consider the graph in part (i) in order to be successful at this question. Those who just ploughed into algebra and used the discriminant did not successfully interpret their solutions.

Question 18 (13 marks) Use a SEPARATE writing booklet

(a) In the trapezium *ABCD*, the coordinates of *A* and *B* are $(-2,4)$ and $(4,1)$ respectively. The equation of *CD* is $x+2y+2=0$.



(i) Find the length of *AB* in exact form. **1**

$$AB^2 = (4+2)^2 + (1-4)^2$$
$$= 36 + 9 = 45$$
$$AB = \sqrt{45} = 3\sqrt{5}$$

(ii) The line *BC* is parallel to the *y*-axis. Find the coordinates of *C*. **1**

Sub $x = 4$ into equation of *CD*

$4 + 2y + 2 = 0$

$2y = -6$

$y = -3$

$C = (4, -3)$

(iii) Find the perpendicular distance from *A* to *CD*. **1**

$$d = \frac{|-2+2(4)+2|}{\sqrt{1^2+2^2}}$$
$$= \frac{8}{\sqrt{5}}$$

(iv) If the area of the trapezium is 32 square units, find the exact length of CD. **2**

$$A=\frac{h}{2}(a+b)$$

$$32=\frac{1}{2}\times\frac{8}{\sqrt{5}}\left(3\sqrt{5}+CD\right)$$

$$3\sqrt{5}+CD=32\times2\times\frac{\sqrt{5}}{8}=8\sqrt{5}$$

$$CD=8\sqrt{5}-3\sqrt{5}=5\sqrt{5}$$

Markers Comment
Parts (i) & (ii) were done well. For (iii) some students did not substitute correctly into the denominator of the perpendicular distance formula. (iv) was done well by most, however some students didn't know the formula for Area of a Trapezium.

(b) Find the equation of the normal to the curve $y=\sqrt{2x-5}$ at the point $(3,1)$. **3**

$$\frac{dy}{dx}=\frac{1}{2\sqrt{2x-5}}\times2=\frac{1}{\sqrt{2x-5}}$$

At $x=3$, $\dfrac{dy}{dx}=\dfrac{1}{\sqrt{2(3)-5}}=1$

$m_N=-1$

Equation of normal at $(3,1)$ is:

$$y-1=-1(x-3)$$

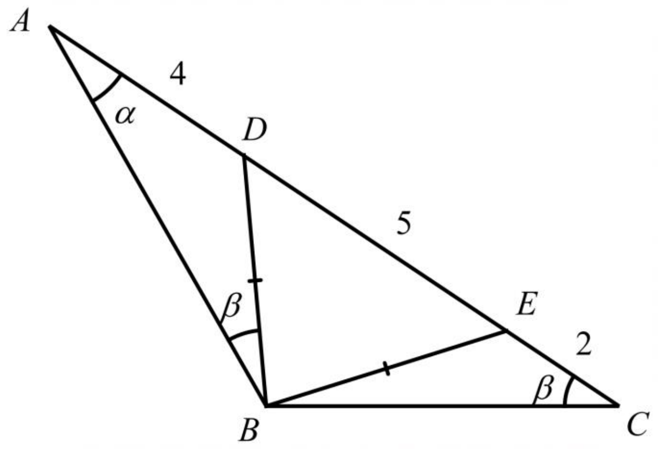
$$x+y-4=0$$

Markers Comment
Mostly done well. The most common error was in finding the derivative of the function; many students forgot to multiply by 2 (the derivative of $2x-5$). Those students are encouraged to revise the chain rule.

Question 18 continues on Page 11

Question 18 (continued)

(c) In ΔABC, $\angle BAC = \alpha$ and $\angle BCA = \beta$. BD is constructed such that $\angle ABD = \beta$. Also $BD = BE$ as shown.



(i) Prove that ΔABC and ΔEBC are similar. **3**

$\angle EDB = \angle DAB + \angle DBA$ (exterior angle ΔADB)

$= \alpha + \beta$

$\angle EDB = \angle DEB$ (angles opposite equal sides BD and BE)

$= \alpha + \beta$

$\angle EBD = \angle DEB - \angle ECB$ (exterior angle ΔEBC)

$= \alpha + \beta - \beta$

$= \alpha$

In ΔABC and ΔBEC

$\angle BAC = \angle EBC = \alpha$

$\angle ACB = \angle BCE = \beta$

$\Delta ABC \mathbin{|||} \Delta BEC$ (matching angles are equal OR equiangular)

Markers Comment

Students who used the exterior angle of a triangle found the proof easier to do. It was disappointing to see some students using congruence here.

(ii)	Hence, or otherwise, find the exact length of BC.	**2**

$$\frac{BC}{AC} = \frac{EC}{BC} \text{ (matching sides of similar triangles are in the same proportion)}$$

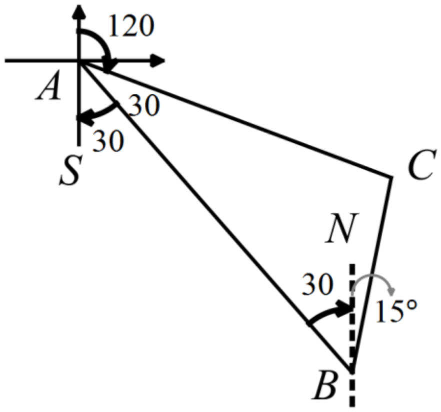
$$\begin{aligned} BC^2 &= AC \times EC \\ &= (4+5+2) \times 2 \\ &= 22 \\ BC &= \sqrt{22} \end{aligned}$$

Markers Comment
Some students wrote an equation of the ratios of 2 pairs of sides and solved but didn't write "matching sides in similar triangles". This incurred a 0.5 mark penalty.

End of Question 18

Question 19 (14 marks) Use a SEPARATE writing booklet

(a) A party of hikers leaves the camp, A, at 6:00 am and walks in the direction 150° T until they reach point B at the base of a cliff. They then travel on a bearing of 015° T and set up camp for the night when they reach point C at 2:00 pm. The bearing and distance of C from A are 120° T and 10 km respectively.



(i) Copy the diagram into your writing booklet and add in the given information. **1**

See above.

(ii) Explain why $\angle ABC = 45°$. **1**

$\angle SAB = \angle ABN = 30°$ (alternate angles on parallel lines)

$\angle NBC = 15°$

$\angle ABC = 30° + 15° = 45°$

(iii) Calculate the average speed of the hike from A to C, correct to one decimal place. **3**

$\angle CAB = 150° - 120° = 30°$

$\angle ACB = 180° - 30° - 45° = 105°$ (angle sum of triangle)

$$\frac{AC}{\sin 45°} = \frac{AB}{\sin 105°} \text{ using the Sine Rule}$$

$$AB = \frac{AC \sin 105°}{\sin 45°} = \frac{10 \sin 105°}{\sin 45°} = 13.660..$$

$$\frac{BC}{\sin 30°} = \frac{AC}{\sin 45°} \text{ using Sine Rule}$$

$$BC = \frac{AC \sin 30°}{\sin 45°} = \frac{10 \sin 30°}{\sin 45°} = 7.071...$$

$$\text{Average speed } = \frac{AB + BC}{8} = \frac{20.731..}{8} = 2.591.. \text{ or } 2.6 \text{ km per hour}$$

(b) Prove that $\dfrac{1+\sin t}{1-\sin t}=(\sec t+\tan t)^2$. **2**

$$LHS=\frac{1+\sin t}{1-\sin t}\times\frac{1+\sin t}{1+\sin t}=\frac{(1+\sin t)^2}{1-\sin^2 t}$$
$$=\frac{(1+\sin t)^2}{\cos^2 t}$$
$$=\left(\frac{1+\sin t}{\cos t}\right)^2$$
$$=\left(\frac{1}{\cos t}+\frac{\sin t}{\cos t}\right)^2$$
$$=(\sec t+\tan t)^2$$
$$=RHS$$

(c) Use the substitution $u=m^{0.2}$ to solve $m^{0.4}-2m^{0.2}-3=0$. **3**

$m^{0.4}-2m^{0.2}-3=0$

$u^2-2u-3=0$

$(u-3)(u+1)=0$

$u=3$ or $u=-1$

$m^{0.2}=3$ or $m^{0.2}=-1$

$m=243$ or $m=-1$

Markers Comment

Most student could solve the equation using the substitution and hence $m^{0.2}=3$ or $m^{0.2}=-1$. However, most student could not solve $m^{0.2}=-1$ correctly as they used log, instead of index law. i.e $\left(m^{0.2}\right)^5=(-1)^5$. The correct answer is -1 but most student answered “no solution”.

(d) Determine whether the line $x+y+3=0$ is a tangent to the curve $y=2x^2+3x-1$. **2**

Solving simultaneously for points of intersection

$x+\left(2x^2+3x-1\right)+3=0$

$2x^2+4x+2=0$

$x^2+2x+1=0$

$(x+1)^2=0$ or $x=-1$

As there is only one point of intersection, the line is tangent to the curve at $(-1,-2)$

Markers Comment
Poorly done.
Most common incorrect answer:

$2x^2+3x-1=4x+3$

$\therefore x=-1$

The gradient of $x+y+3=0$ is $m_1=-1$

$y=2x^2+3x-1$

$\therefore y'=4x+3$

Substitute $x=-1$, then $y'=-1$

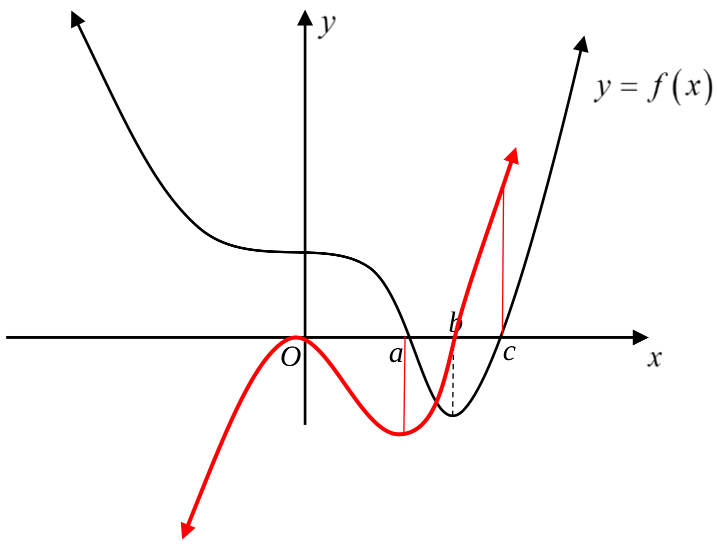
Therefore tangent as $y'=m_1$

But this only shows that they are parallel. To get full mark, student must evaluate the *y*-value then show that it is an equation of the tangent to the curve.

(e) The graph of $y=f(x)$ is shown below. **2**

Sketch the graph of $y=f'(x)$ showing all important features.

Mark the positions *a*, *b*, and *c* on your diagram.

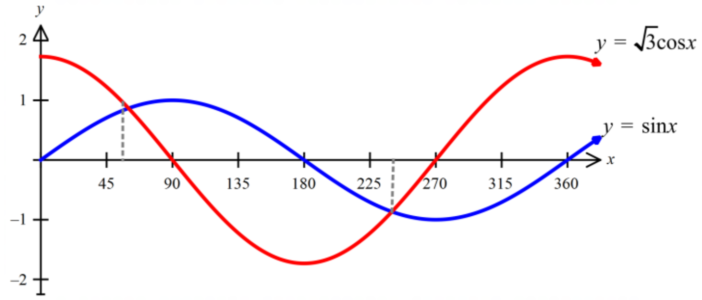


Markers Comment
This question was marked generously.

End of Question 19

Question 20 (13 marks) Use a SEPARATE writing booklet

(a)	(i)	On the same number plane, sketch the curves $y = \sin x$ and $y = \sqrt{3}\cos x$ for $0° \le x \le 360°$.	**2**



Markers Comment
Generally well done. When graphing two curves on the same number plane, you must label which graph is which. Intercepts always need to be marked unless advised not to.

	(ii)	Hence solve $\sin x > \sqrt{3}\cos x$ in the domain $0° \le x \le 360°$.	**2**

$\sin x = \sqrt{3}\cos x$

$\tan x = \sqrt{3}$

$x = 60°, 240°$

From the graph, $\sin x > \sqrt{3}\cos x$ for $60° < x < 240°$.

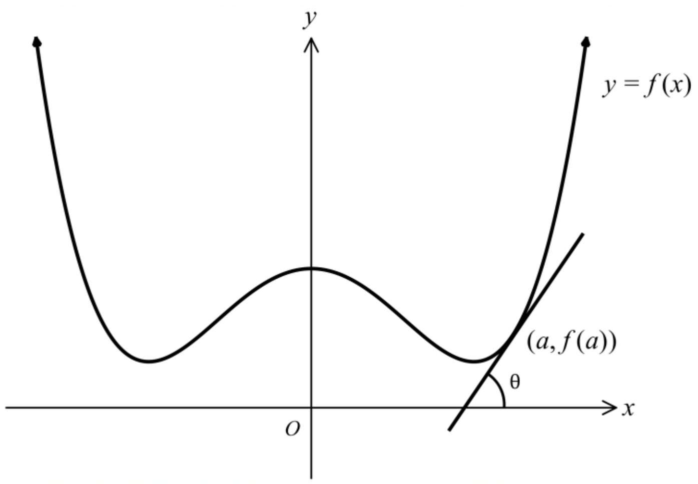
Markers Comment
For a two mark question, you cannot just write down the final answer – working must be shown. If the domain in the question is in degrees you should answer in degrees.

(b) Given that $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$, differentiate $f(x) = \dfrac{2}{x}$ from first principles. **3**

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} \times \frac{x(x+h)}{x(x+h)} \\ &= \lim_{h \to 0} \frac{2x - 2(x+h)}{xh(x+h)} \\ &= \lim_{h \to 0} \frac{\cancel{2x} - \cancel{2x} - 2h}{xh(x+h)} \\ &= \lim_{h \to 0} \frac{-2\cancel{h}}{x\cancel{h}(x+h)} \\ &= -\frac{2}{x^2} \end{aligned}$$

Markers Comment
Generally well done. Students are advised to show all steps of working for such questions. They are akin to a proof or a "show that" question.

(c) The diagram below shows an even differentiable function $y = f(x)$. **2**
The tangent at $(a, f(a))$ makes an angle of θ with the x-axis as shown.



Justify geometrically, or otherwise, why the derivative of an even function is an odd function.

$f'(a) = \tan\theta$

$f'(-a) = \tan(180-\theta)$ by symmetry in the y-axis as $f(x)$ is even

$= -\tan\theta$

$= -f'(a)$

$\therefore f'(x)$ is odd

Alternately, $f(-x) = f(x)$ as $f(x)$ is even

$f'(-x) = f'(x) \times -1$ Chain Rule

$= -f'(x)$

$\therefore f'(x)$ is odd

Markers Comment

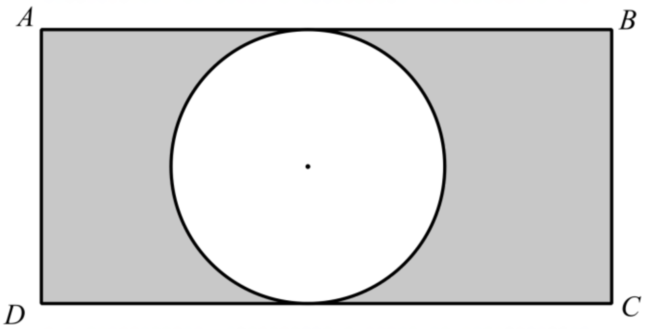
This part was poorly done. Most students failed to engage with the information given in the question and proceeded to answer the question as if it started at "Justify geometrically, or otherwise, why the derivative of an even function is an odd function". Several long descriptions followed with little attention to rigour or use of appropriate mathematical language.

Many responses assumed the function was x^4 and stated the derivative was $4x^3$ which is a very specific case of the question. Also these responses failed to engage with the idea of an even /odd function simply stating that they were even or odd.

Question 20 continues on page 15

Question 20 (continued)

(d) A circle is inscribed within a rectangle $ABCD$ such that it is always in contact with two opposite sides of the rectangle, as shown. The perimeter of rectangle $ABCD$ is 40cm. Let $AB = x$ and $BC = y$.



(i) Show that the area of the shaded region, R cm^2 is given by **2**

$$R = 20y - \left(\frac{4+\pi}{4}\right)y^2 \ .$$

Perimeter

$2x + 2y = 40$

$x = 20 - y$

$R = \text{rectangle} - \text{circle}$

$$= xy - \pi\left(\frac{y}{2}\right)^2$$

$$= (20 - y)y - \frac{\pi}{4}y^2$$

$$= 20y - y^2 - \frac{\pi}{4}y^2$$

$$= 20y - \left(1 + \frac{\pi}{4}\right)y^2$$

$$= 20y - \left(\frac{4+\pi}{4}\right)y^2$$

Markers Comment
This part was generally well done.

	(ii)	Find the largest value of R, giving your answer as an exact value.	**2**

This is a concave down parabola. So a maximum occurs at the vertex.

Axis of symmetry is $y = \dfrac{-b}{2a} = \dfrac{-20}{-2\left(\dfrac{4+\pi}{4}\right)} = \dfrac{40}{4+\pi}$

Maximum value of R is given by

$$\begin{aligned} R_{\max} &= 20\left(\frac{40}{4+\pi}\right) - \left(\frac{4+\pi}{4}\right)\left(\frac{40}{4+\pi}\right)^2 \\ &= \frac{800}{4+\pi} - \frac{400}{4+\pi} \\ &= \frac{400}{4+\pi} \end{aligned}$$

Markers Comment
Many students used calculus to find the stationary point. However, there was a penalty for not justifying why the stationary point was a maximum.
Some students correctly found the y value but did not find the value of R, either forgetting to finish the question or thinking that the y-value itself was the maximum value. The confusion perhaps stemmed from the fact that in this question y is the independent variable being graphed on the x-axis.

End of paper