NORTH SYDNEY GIRLS HIGH SCHOOL

2019	**Preliminary Assessment Task 3**

Preliminary Mathematics Advanced

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 13 – 18, show relevant mathematical reasoning and/or calculations

Total marks – 80

Section I – 12 marks (pages 2 – 6)

- Attempt Questions 1 – 12
- Allow about 15 minutes for this section

Section II – 68 marks (pages 7 – 15)

- Attempt Questions 13 – 18
- Allow about 1 hour 45 minutes for this section

NAME:____________________ **TEACHER:**____________________

STUDENT NUMBER:____________________

Question	**1 – 12**	**13**	**14**	**15**	**16**	**17**	**18**	**Total**
Mark	**/12**	**/12**	**/11**	**/12**	**/11**	**/11**	**/11**	**/80**

Section I

12 marks
Attempt Questions 1-12
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-12

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1 What is the value of $\log_{31} 586$ correct to three significant figures?

A. 1.856

B. 1.86

C. 2.768

D. 2.77

2 Given $x = \log_a 2$ and $y = \log_a 3$, which expression below is equivalent to $\log_a 36$?

A. $2xy$

B. $2x + y$

C. $2x + 2y$

D. $2xy^2$

3 Which of the following is the solution to $x^2 - 5x - 6 > 0$?

A. $x \in (-1, 6)$

B. $x \in (2, 3)$

C. $x \in \{(-\infty, -1) \cup (6, \infty)\}$

D. $x \in \{(-\infty, 2) \cup (3, \infty)\}$

4 Which statement below is NOT correct for the line $5x-2y+8=0$?

A. The gradient is $\frac{5}{2}$

B. The x-intercept is $-\frac{8}{5}$

C. The y-intercept is 8

D. The line passes through the point $(2,9)$

5 What is the gradient of the curve $y=x^4-7x^2$ at the point where $x=2$?

A. 18

B. 4

C. –4

D. –20

6 Which of the following gives the solution(s) to $\cos 2\theta = -0.3746$ for $0^\circ \le \theta \le 180^\circ$?

A. 56° or 124°

B. 112°

C. 73° or 107°

D. 146°

7 Which of the following is an example of categorical, ordinal data?

A. The hair colour of the Maths staff.

B. The makes of cars in the staff carpark.

C. Survey ratings on a scale of Strongly Agree to Strongly Disagree

D. The brand of laptop used by the first 20 students to arrive at school.

8 The frequency table below shows the number of children in each household surveyed.

No. of children	Frequency
0	1
1	2
2	10
3	5
4	6
5	1

What are the mean and median for this data?

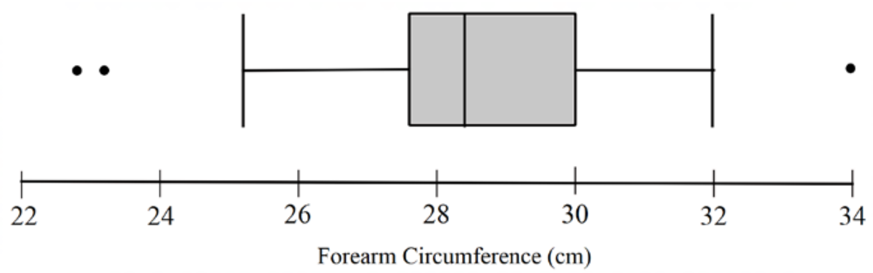
A. Mean = 2.64 Median = 2

B. Mean = 2.64 Median = 2.5

C. Mean = 4.17 Median = 2

D. Mean = 4.17 Median = 2.5

9 The boxplot below shows the distribution of the forearm circumference, in centimetres, of a group of people. The dots represent outliers.
It is known that the shaded box comprises 126 people.



How many people had a forearm circumference less than 30 cm?

A. 95

B. 189

C. 97

D. 191

10 The function $f(x)$ is a continuous odd function. Which of the following statements is necessarily true?

I. $f\left(x^2\right)$ is even

II. $f(x) + f(-x) = 0$

III. $f(0) = 0$

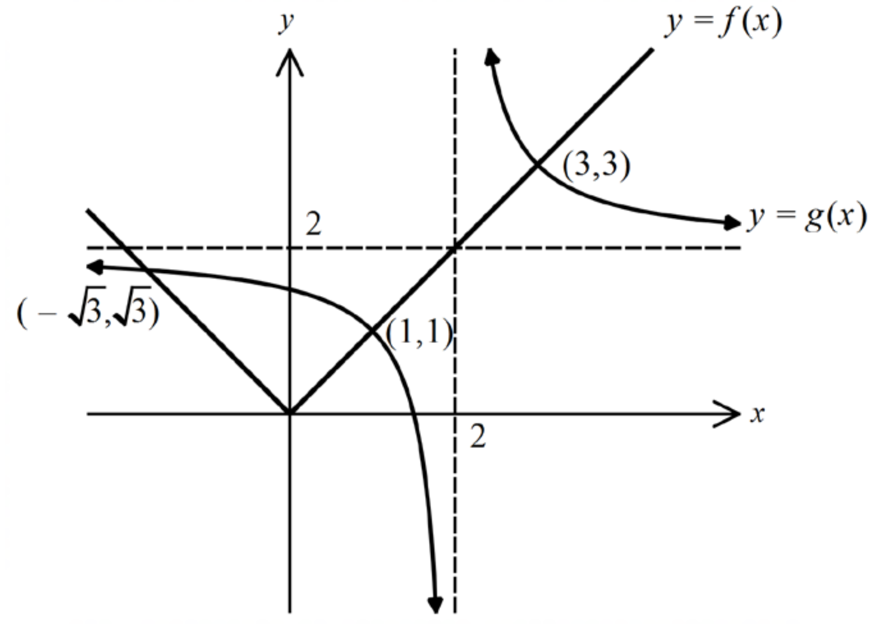
A. I only

B. II only

C. I and II only

D. I, II and III

11 The graphs of $y = f(x)$ and $y = g(x)$ intersect at $\left(-\sqrt{3}, \sqrt{3}\right)$, $(1,1)$ and $(3,3)$ as shown.



Which of the following is the solution for $f(x) > g(x)$?

A. $x < -\sqrt{3},\ x > 3$

B. $-\sqrt{3} < x < 1$

C. $x < -\sqrt{3},\ 1 < x < 2,\ x > 3$

D. $-\sqrt{3} < x < 1,\ 2 < x < 3$

12 Given that f and g are two functions such that $f(x) = 2x$ and $g(x+2) = 3x+1$, which function would be $f(g(x))$?

A. $6x - 5$

B. $6x + 1$

C. $6x - 10$

D. $6x + 2$

Section II

68 marks
Attempt Questions 13-18
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 13-18, your responses should include relevant mathematical reasoning and/or calculations.

Question 13 (12 marks**)** Use a SEPARATE writing booklet

(a) If $x\sqrt{3}+4\sqrt{y}=\sqrt{75}+\sqrt{80}+\sqrt{12}$ find x and y. **2**

(b) Solve the following equations:

(i) $x^2(x-1)+2x-2=0$ **2**

(ii) $|4m-3|=8$ **2**

(c) Convert 56° to radians leaving your answer in simplest exact form. **1**

(d) Find the exact value of $\sec\dfrac{7\pi}{6}$. **1**

(e) Simplify $\dfrac{\cos(180°-\theta)}{1+\cot^2(90°-\theta)}$. **2**

(f) Evaluate $\lim\limits_{x\to 3}\dfrac{x-3}{2x^2-x-15}$. **2**

Question 14 (11 marks) Use a SEPARATE writing booklet

(a) Consider the function $f(x) = \begin{cases} 1-x, & x \le 1 \\ (x-1)^2, & x > 1 \end{cases}$

(i) Is $f(x)$ continuous at $x = 1$? Justify your answer. **1**

(ii) Sketch the graph of $y = f(x)$. **2**

(iii) Write down the range of $f(x)$ using interval notation. **1**

(b) Differentiate the following, giving your answers with positive indices.

(i) $y = 5x^6 - 3x^4 + 2$ **1**

(ii) $y = \dfrac{1}{(2x-1)^4}$ **2**

(iii) $f(x) = x^3(x+1)^{\frac{2}{5}}$ **2**

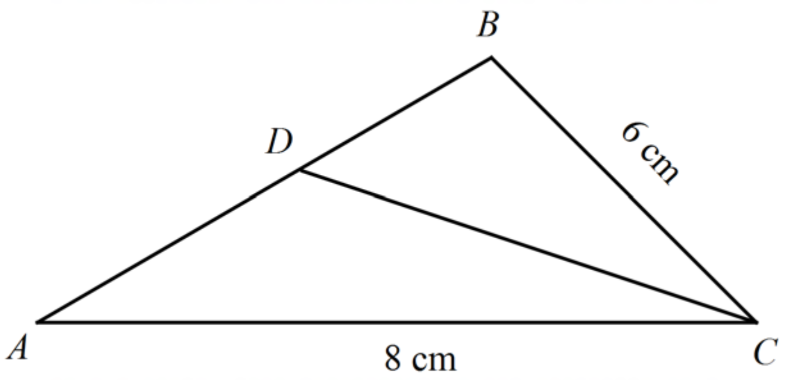
(c) Sketch $y = 2\sin x - 1$ for $0 \le x \le 2\pi$, showing the location of x and y intercepts. **2**

Question 15 (12 marks) Use a SEPARATE writing booklet

(a) Write down the domain and range of the function $y = -\sqrt{16 - x^2}$. **2**

(b) Find the values of k for which $4x^2 + (k+3)x + 1 = 0$ has no real roots. **2**

(c) In triangle ABC, CD bisects $\angle BCA$ and D lies on AB.
$\angle BCA = 60^\circ$, $AC = 8$ cm and $BC = 6$ cm.



NOT TO SCALE

(i) Calculate the length of AB correct to 1 decimal place. **2**

(ii) Find the exact area of ΔABC . **1**

(iii) Hence or otherwise calculate the exact length of CD. **2**

(d) Prove the identity $\dfrac{\cos\theta}{1-\tan\theta} + \dfrac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$. **3**

Question 16 (11 marks) Use a SEPARATE writing booklet

(a) The number of goals scored per game by Alana's netball team last season are shown below.

9 14 22 18 31 8 27 15 26 49 28 25 17 11

(i) Draw an ordered stem-and-leaf plot for the data. **2**

(ii) Comment on the shape of the distribution. **1**

(iii) Find the interquartile range. **1**

(iv) Determine whether 49 is an outlier. **1**

(b) The time x seconds, spent by each person in a random sample of 100 customers at an automatic teller machine (ATM) is recorded. The data is summarised in the table below.

Time x (in seconds)	**Number of customers**
$20 < x \leq 30$	2
$30 < x \leq 40$	7
$40 < x \leq 60$	18
$60 < x \leq 80$	27
$80 < x \leq 100$	23
$100 < x \leq 120$	13
$120 < x \leq 150$	7
$150 < x \leq 180$	3
	100

(i) Copy the table into your writing booklet, adding a class centre column. **1**

(ii) Find estimates for the mean and standard deviation of this data. **2**

Question 16 continues on Page 11

(c) (i) Sarah sets up a business running fairy parties for young children. **1**
At the start, she spends \$280 on fairy costumes and \$1500 on a jumping castle.

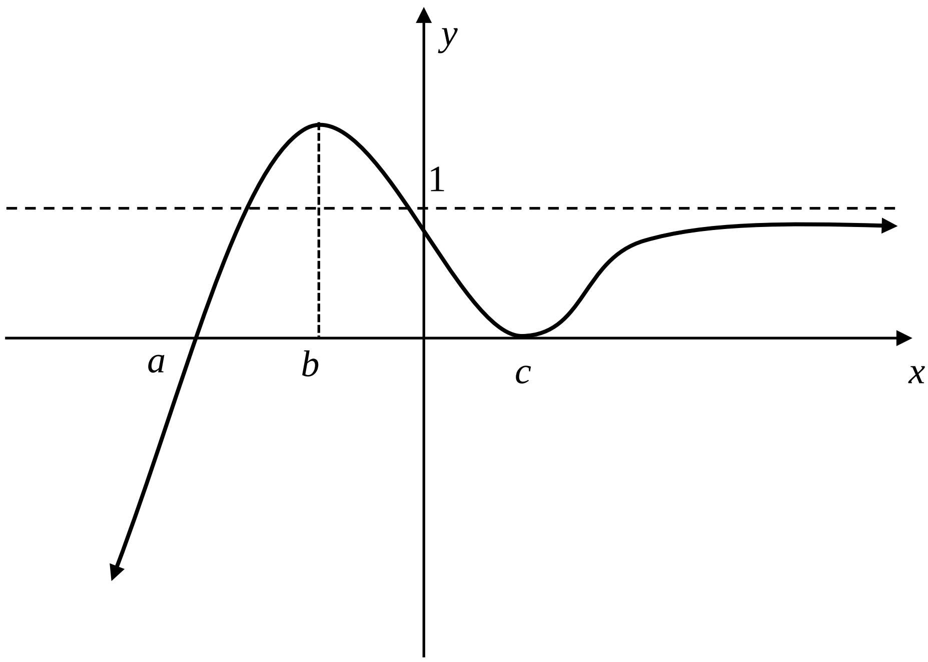
She also pays \$12.80 after each party to have her fairy outfit cleaned.
Write a formula for the costs, $\$C$ incurred by Sarah for n parties.

(ii) Sarah charges \$95 per party. By first finding an expression for her earnings, find the minimum number of parties that Sarah needs to run before she starts making a profit. **2**

End of Question 16

Question 17 (11 marks) Use a SEPARATE writing booklet

(a) The graph of $y = f(x)$ is shown below.



(i) For what values of x is $f'(x) < 0$? **1**

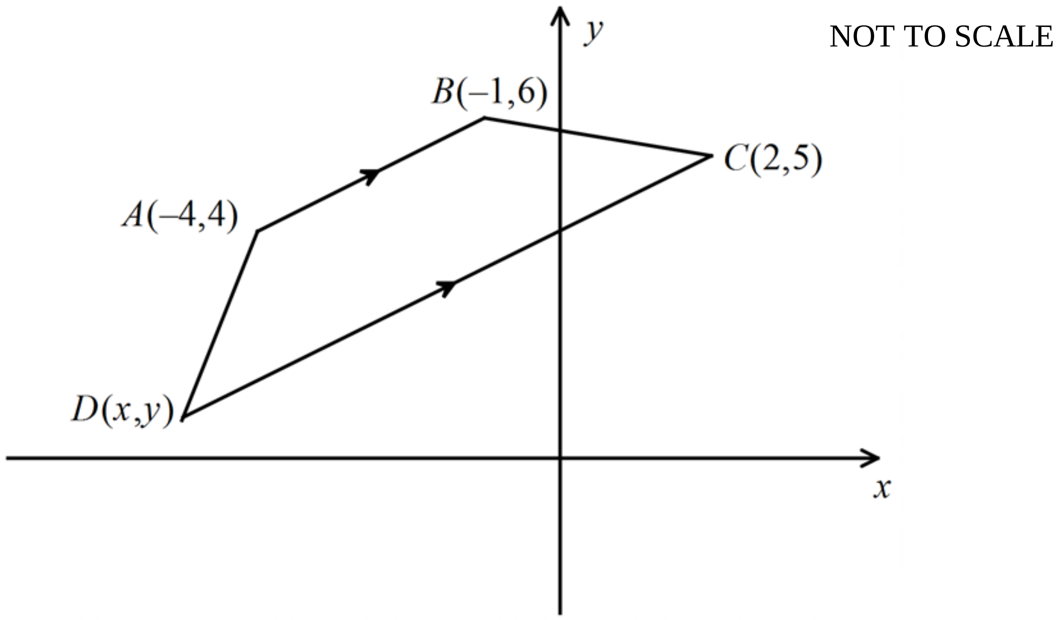
(ii) Sketch the graph of the gradient function $y = f'(x)$ clearly showing the location of key features. **2**

Your sketch should be about one-third of a page.

(b) Solve $2 - \sec^2\theta = 2\tan\theta$ in the domain $[0, \pi]$. **3**

Question 17 continues on Page 13

(c) In the diagram below, $ABCD$ is a trapezium with AB parallel to CD.
D is the point (x, y). The length of CD is $\sqrt{117}$ units.



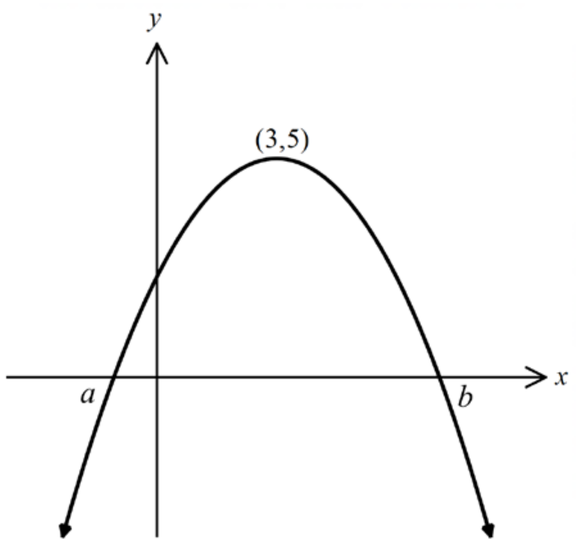
(i) Show that $(x-2)^2 + \left(\dfrac{2x-4}{3}\right)^2 = 117$. **2**

(ii) Find the coordinates of D given that it lies to the left of the y-axis. **3**

End of Question 17

Question 18 (11 marks) Use a SEPARATE writing booklet

(a) The curve $y = f(x)$ has a vertex at $(3,5)$ and x-intercepts at a and b as shown.



(i) Write down the coordinates of the vertex of the curve $y = f(x+k)$ **1**

(ii) The function $y = f(x)$ is transformed so that its x intercepts are now at $3a$ and $3b$ respectively and the y-coordinate of the vertex is 10. Write down the equation of the transformed graph in terms of $f(x)$. **1**

(b) Find the equation of the tangent to the curve $y = \dfrac{x^2 - 1}{2x+1}$ at $x = 1$. **3**

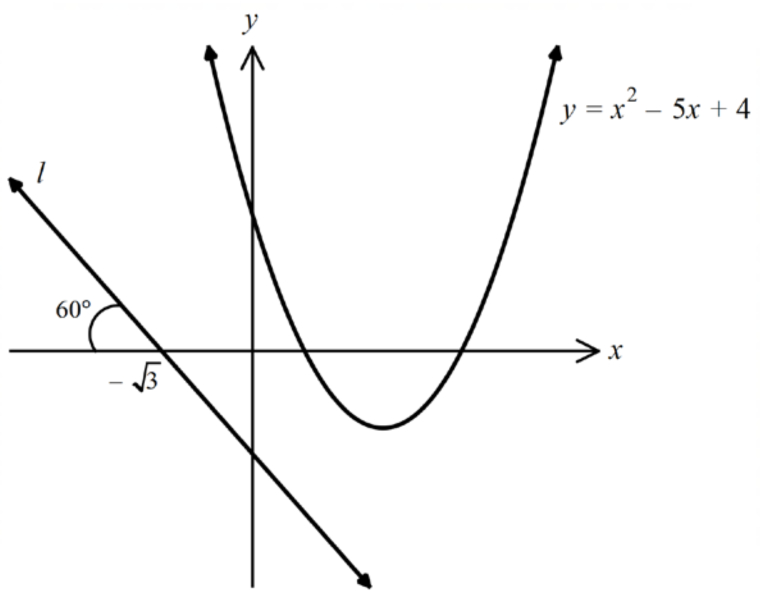
Give your answer in general form.

Question 18 continues on Page 15

(c) The standard deviation of a sample of 10 scores is 3.75. **2**
Another score is now added to the sample. Phoebe notices that the mean of the sample is unchanged. Calculate the standard deviation of the sample after the addition of the new score. Answer correct to two decimal places and show all working.

Note: The formula for standard deviation is $\sigma = \sqrt{\dfrac{\sum\left(x_i - \overline{x}\right)^2}{n}}$

(d) The diagram below shows a line l and the parabola $y = x^2 - 5x + 4$ **4**



The points A and B lie on the parabola and the line respectively such that A lies directly above B. Find the minimum length of AB as an exact value.

End of paper

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Prelim Maths T3 2019.

Section 1 MC B C C C B A C A B D C C

① $\log_{31} 586 = \frac{\log_{10} 586}{\log_{10} 31} = 1.85595...$

$= 1.86.$

B

② $x = \log_a 2 \quad y = \log_a 3 \qquad \log_a 36 = \log_a (2^2 \times 3^2)$

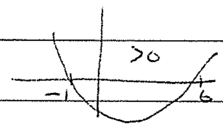
$= 2\log_a 2 + 2\log_a 3$

$= 2x + 2y.$

C

③ $x^2 - 5x - 6 > 0.$

$(x-6)(x+1) > 0$



$x < -1 \;+\; x > 6.$

C

④ $5x - 2y + 8 = 0$

$2y = 5x + 8$

$y = \frac{5}{2}x + 4.$

C

A ✓

B. ✓

C. ×

D ✓

⑤. $y = x^4 - 7x^2$

$\frac{dy}{dx} = 4x^3 - 14x$

$x = 2. \quad m = 4\times 8 - 14\times 2$

$= 32 - 28$

$= 4.$

B

⑥. $\cos 2\theta = -0.3746 \qquad 0° \le \theta \le 180°$

$2\theta = 111.9995926 \qquad 0° \le 2\theta \le 360 \qquad$ 2nd Q + 3rd Q.

or 248.0004074

$\theta = 124°$ or $56°$

A

7. a) categ. nom

b). categ. nom

c). survey ratings - categ. ord. ✓

d). categ. nom

C

8.

x no.	f	fx
0	1	0
1	2	2
2	10	20
3	5	15
4	6	24
5	1	5

$\Sigma f = 25$ | $\Sigma fx = 6\cancel{6}$

mean $= \frac{6\cancel{6}}{25} = 2.6\cancel{4}$.

$\bar{x} = 2.64$.

median $= \frac{12^{th} + 13^{th}}{2}$

$= \frac{2+2}{2}$

$= 2$.

A

9. $126 = 50\%$.

$63 = 25\%$

less than $30 = 75\%$

$= 126 + 63$

$= \underline{189}$.

B

10. D.

11. C

12. $f(x) = 2x$ $\quad g(x+2) = 3x+1$.

$f(g(x)) = f(3x-5)$ $\quad = 3(x+2) - 5$.

$= 2(3x-5)$ $\quad g(x) = 3x - 5$

$= 6x - 10$.

C.

Section 2

Q13.

a) $x\sqrt{3} + 4\sqrt{y} = \sqrt{75} + \sqrt{80} + \sqrt{12}$

$= 5\sqrt{3} + 4\sqrt{5} + 2\sqrt{3}$ (1) - for progress

$= 7\sqrt{3} + 4\sqrt{5}$

$\therefore\ x = 7$ and $y = 5$. (1) for answers

b) Solve

i) $x^2(x-1) + 2(x-1) = 0$

$(x-1)(x^2+2) = 0$ (1) - for progress

$\therefore\ \underline{\underline{x = 1}}$ or $x^2 = -2$ (1) for 1 answer

no solution — other answer must be discounted.

ii) $|4m-3| = 8$

$4m - 3 = 8$ or $4m - 3 = -8$

$4m = 11$ $\qquad$ $4m = -5$ (1) for progress

$m = 11/4$ $\qquad$ $m = -5/4$ (1) for answers

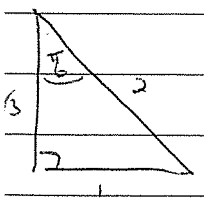
check $\left|4 \times \frac{11}{4} - 3\right| = |8|$ ✓ $\qquad$ $\left|4 \times \frac{-5}{4} - 3\right| = |-8|$ $= 8.$ ✓ — checking not req'd.

c). $180° = \pi$ radians

$\therefore\ 56° = \frac{\pi}{180} \times 56$ (1)

$= \frac{14\pi}{45}$ rads.

d). $\sec\frac{7\pi}{6} = \frac{1}{\cos\frac{7\pi}{6}}$



$= \frac{1}{\cos(\pi + \frac{\pi}{6})}$ ← 3rd quad $\therefore$ cos is neg.

$= \frac{1}{-\frac{\sqrt{3}}{2}}$ (1)

$= -\frac{2}{\sqrt{3}}$

e) $\dfrac{\cos(180^\circ-\theta)}{1+\cot^2(90-\theta)} = \dfrac{-\cos\theta}{1+\tan^2\theta}$ $\qquad \cot(90-\theta) = \tan\theta.$

$= \dfrac{-\cos\theta}{\sec^2\theta}$

$= -\cos\theta \div \dfrac{1}{\cos^2\theta}$

1 - for progress

$= -\cos\theta \times \cos^2\theta$

$= -\cos^3\theta.$ $\qquad$ 1 - for answer.

f). $\lim\limits_{x\to 3} \dfrac{x-3}{2x^2-x-15} = \lim\limits_{x\to 3} \dfrac{\cancel{x-3}}{(2x+5)(\cancel{x-3})}$

$= \lim\limits_{x\to 3} \dfrac{1}{2x+5}$

1 - for progress

$= \dfrac{1}{11}.$ $\qquad$ 1 - for answer.

Q 14.

a). $f(x) = \begin{cases} 1-x & x \leq 1 \\ (x-1)^2 & x > 1 \end{cases}$

i). $f(1) = 1 - 1$ using first part

$= 0.$

$f(x \to 1^+) = (\overset{\text{say}}{1.1} - 1)^2$ using 2nd part

$= 0.01$ which is close to 0.

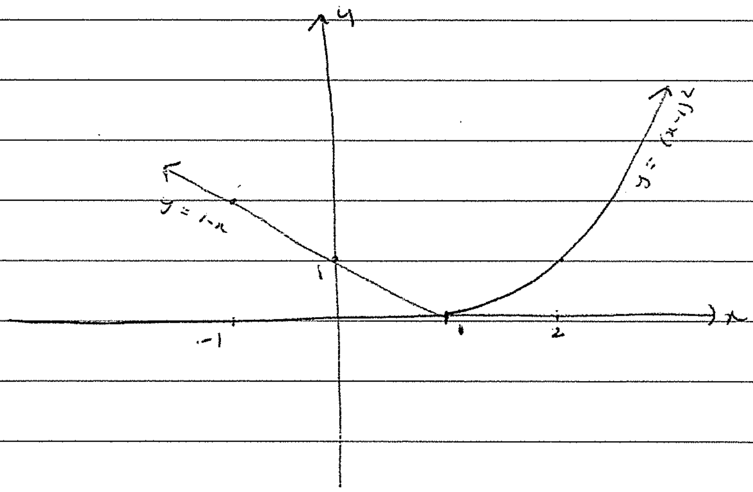
$f(x=1) = (1-1)^2$ in 2nd pt.

$= 0.$

$\therefore$ as $x \to 1^-$ + $x \to 1^+$ both results give $f(x) = 0$

$\therefore$ the fnc is continuous. at $x = 1$

ii).



iii). Range $y \geq 0$ ie $[0, \infty)$

b). i) $y = 5x^6 - 3x^4 + 2$

$\frac{dy}{dx} = 30x^5 - 12x^3$

ii). $y = \frac{1}{(2x-1)^4} = (2x-1)^{-4}$

$y' = -4(2x-1)^{-5} \times 2$

$= \frac{-8}{(2x-1)^5}$

iii). $f(x) = x^3 (x+1)^{2/5}$

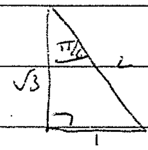
$f'(x) = x^3 \times \frac{2}{5}(x+1)^{-3/5} \times 1 + (x+1)^{2/5} \times 3x^2$

Q14

b) iii) cont'd.

$$f'(x) = \frac{2x^3}{(x+1)^{3/5}} + 3x^2(x+1)^{2/5}.$$

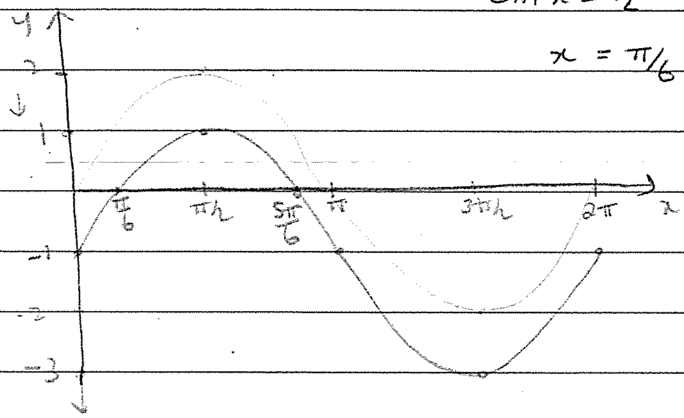
c). $y = 2\sin x - 1 \qquad 0 \le x \le 2\pi.$



$2\sin x = 1$

$\sin x = 1/2$

$x = \pi/6,\ 5\pi/6.$



Question 14

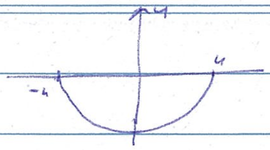
a) ii) In addition to the x – intercept, an additional point needed to be shown on the parabola.

This additional point helps to make the parabola you have drawn distinct from any other parabola.

c) 1 mark was awarded for correct intercept and 1 mark was awarded for correct shape. The domain was specified in radians but a number of students drew a graph using degrees.

Q 15.

a) $y = -\sqrt{16 - x^2}$

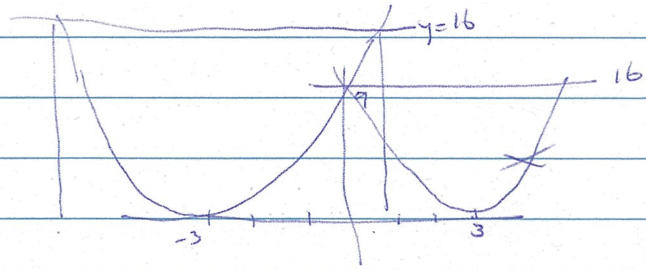


D: $-4 \leq x \leq 4$

R: $-4 \leq y \leq 0$. (2).

b). $4x^2 + (k+3)x + 1 = 0$

no real roots $\therefore \Delta < 0$.



$\therefore b^2 - 4ac < 0$

$(k+3)^2 - 4 \times 4 \times 1 < 0$

$(k+3)^2 - 16 < 0$

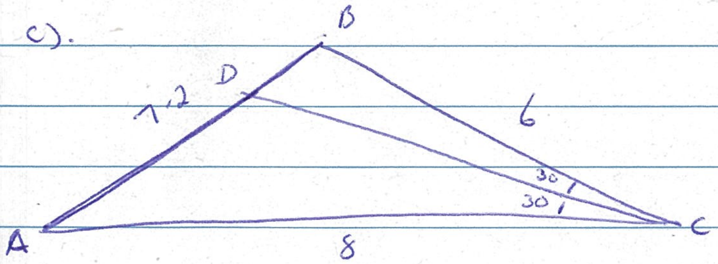
$(k+3)^2 < 16$.

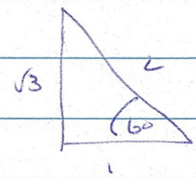
$k+3 < 4$ or $k+3 > -4$

$k < 1$ $\quad$ $k > -7$.

ie $-7 < k < 1$.

c).





i). by cosine rule $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 60$

$= \overset{100}{36 + 64} - 96 \times 1/2$

$= 52$

$c = 7.2111..$

$c \doteqdot 7.2$ cm (1dp). (2)

ii) $A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 6 \times 8 \times \sin 60$

$= 24 \times \frac{\sqrt{3}}{2}$ (1)

$= \underline{12\sqrt{3} \text{ cm}^2}$

Q15.

c) ii). ΔADC $A = \frac{1}{2} \times 8 \times CD \times \sin 30$

ΔBDC $A = \frac{1}{2} \times 6 \times CD * \sin 30$

$\therefore 12\sqrt{3} = 3CD \times \frac{1}{2} + 4CD \times \frac{1}{2}$

$= 1.5CD + 2CD$

$= \frac{7}{2} CD$

$24\sqrt{3} = 7CD$

$CD = \frac{24\sqrt{3}}{7}$ cm (2)

d). $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$

LHS $= \frac{\cos\theta}{1 - \frac{\sin\theta}{\cos\theta}} \times \frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{1 - \frac{\cos\theta}{\sin\theta}} \times \frac{\sin\theta}{\sin\theta}$

$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta}$

$-(\cos\theta - \sin\theta)$

$= \frac{\cos^2\theta - \sin^2\theta}{(\cos\theta - \sin\theta)}$

$= \frac{\cancel{(\cos\theta - \sin\theta)}(\cos\theta + \sin\theta)}{\cancel{(\cos\theta - \sin\theta)}}$

$= \cos\theta + \sin\theta$

$=$ RHS.

3

Year 11 Advanced:

Marker's comments

Q15 (a) Many students could not recognise this semi-circle.

(b) Few students did not realise that $\Delta = 0$ and many students could not solve their quadratic inequation.

(c) (i) and (ii) Well done

(iii) Many students did not use the composite area of two triangles.

(d) Many students could not decide which identities should be used.

Q16.

a)

i)

Stem	leaf
0	8 9
1	1 4 5 7 8
2	2 5 6 7 8
3	1
4	9

Stem	leaf
0	8 9
1	1 4 5 7 8
2	2 5 6 7 8
3	1
4	9

(2)

ii) There is a slight positive skew (1)

iii) IQR. median $= \frac{18+22}{2}$
$= 20$

$Q_1 = 14$ $Q_3 = 27$

$IQR = 27 - 14$
$= 13$ (1)

iv). $1.5 \times IQR = 19.5$

$Q_3 + 19.5 = 27 + 19.5$
$= 46.5$.

Therefore 49 is more than 1½ times IQR above the upper quartile
So it is an outlier. (1)

b) i)

time (x)	c.c (1)	f. no. of customers	
$20 < x \leq 30$	25	2	50
$30 < x \leq 40$	35	7	245
$40 < x \leq 60$	50	18	900
$60 < x \leq 80$	70	27	1890
$80 < x \leq 100$	90	23	2070
$100 < x \leq 120$	110	13	1430
$120 < x \leq 150$	135	7	945
$150 < x \leq 180$	165	3	495
		Σf 100.	8025 $\Sigma f.cc.$

$\Sigma f \times cc$

ii) $\bar{x} = 80.25$ from calculator.

$\sigma_n = 30.9788\ldots$

$\doteq 31.$ (lead) /2

$\left(\frac{sx}{\text{sample}}\right) = 31.13$

OR.

$\mu = \frac{\Sigma f cc}{\Sigma f}$

$= \frac{8025}{100}$

$= 80.25$

Q16 cont'd.

c) i). costs $= 280 + 1500 + 12.80 \times n$.

$\boxed{C = 1780 + 12.80n}$. (1)

ii). $E = 95n$.

Break even $C = E$.

$95n = 1780 + 12.80n$

$82.2n = 1780$

$n = \frac{1780}{82.2}$

$= 21.6545\ldots$

So Sarah must run 22 parties before she starts making a profit.

2

Q16 2Unit Yr11 feedback

a) (i) mostly done well. Students should space the “leaves” equally as that helps to determine the symmetry / skew.

(ii)

(iii)

(iv) Many students wrote yes or no without justification. “Determine” indicates you are expected to show some work. Some students struggled using the formula provided on the reference sheet.

b) (i) many students made errors in finding the class centres.

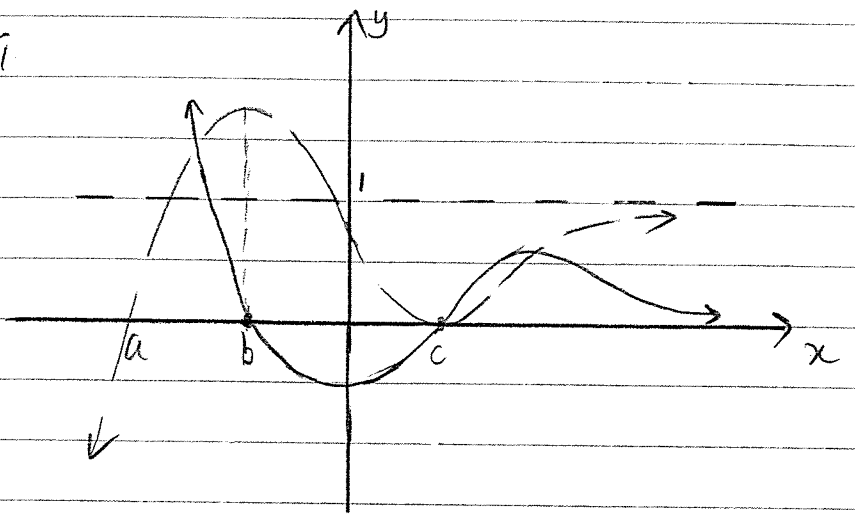
(ii) Most students could calculates both the mean and standard deviation. Use of the formula for either was not necessary.

c (ii) Students should write “let E = earnings” then writing the expression. The number of parties should have been rounded up to allow for a profit.

Question 17

a) i. $f'(x) < 0$ for $b < x < c$

ii



b). $2 - \sec^2\theta = 2\tan\theta$ $\quad [0, \pi]$

$2 - (1 + \tan^2\theta) = 2\tan\theta$

$2 - 1 - \tan^2\theta = 2\tan\theta$

$\tan^2\theta + 2\tan\theta - 1 = 0$

$\tan\theta = \frac{-2 \pm \sqrt{2^2 - 4\times1\times(-1)}}{2}$

$= \frac{-2 \pm 2\sqrt{2}}{2}$

$= -1 \pm \sqrt{2}$

$\tan\theta = -1 + \sqrt{2}$ or $\tan\theta = -1 - \sqrt{2}$

$\theta = 0.3927\ldots$ $\qquad \theta = 1.963$

$= \frac{\pi}{8}$ $\qquad = \frac{5\pi}{8}$

c) i) $CD = \sqrt{(x-2)^2 + (y-5)^2}$

$\sqrt{117} = \sqrt{(x-2)^2 + (y-5)^2}$

$117 = (x-2)^2 + (y-5)^2$

$m_{AB} = \frac{6-4}{1--4}$

$= \frac{2}{3}$

$AB \parallel CD \quad \therefore m_{AB} = m_{CD}$

$\therefore m_{CD} = \frac{2}{3}$

equation of CD

$y - 5 = \frac{2}{3}(x-2)$

$\therefore 117 = (x-2)^2 + \left[\frac{2}{3}(x-2)\right]^2$

$= (x-2) + \left(\frac{2x-4}{3}\right)^2$

ii) $(x-2)^2 + \left(\frac{2x-4}{3}\right)^2 = 117$

$(x-2)^2 + \left(\frac{2(x-2)}{3}\right)^2 = 117$

$(x-2)^2 + \frac{4}{9}(x-2)^2 = 117$

$(1+\frac{4}{9})(x-2)^2 = 117$

$\frac{13}{9}(x-2)^2 = 117$

$(x-2)^2 = 81$

$x - 2 = \pm 9$

$x = \pm 9 + 2$

$= 11, -7$

The point lies to the left of the x-axis

$\therefore x < 0$

Sub $x = -7$ in

$y - 5 = \frac{2}{3}(x-2)$

$y = \frac{2}{3}(-7-2) + 5$

$= -1$

$\therefore$ D is $(-7, -1)$

Q18.

(a) (i) $y = f(x+k)$ is a translation to the left by k units

$\therefore$ vertex is at $(3-k, 5)$

(ii) intercepts are at 3a and 3b. So a dilation horizontally

vertex is at $(3, 10)$ so a vertical dilation

New equation is $y = 2f\left(\frac{x}{3}\right)$

(b) $y = \dfrac{x^2-1}{2x+1}$ $\quad u = x^2 - 1 \quad v = 2x+1$

$u' = 2x \quad v' = 2$

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2}$$

$$= \frac{2x(2x+1) - 2(x^2-1)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2 + 2}{(2x+1)^2}$$

$$= \frac{2x^2 + 2x + 2}{(2x+1)^2}$$

At $x = 1$, $\dfrac{dy}{dx} = \dfrac{6}{9} = \dfrac{2}{3}$

$m_T = \dfrac{2}{3}$

At $x = 1$, $y = 0$ $\quad \therefore$ point is $(1, 0)$

Eqn of tangent is $\quad y - 0 = \dfrac{2}{3}(x-1)$

$3y = 2x - 2$

$2x - 3y - 2 = 0$ in general form

(c) $\sigma_{10} = 3.75$

If the mean is unchanged after addition of the 11^{th} score, the new score is the same as the mean

ie. $x_{11} = \bar{x}$.

Now $\sqrt{\dfrac{\sum (x_i - \bar{x})^2}{10}} = 3.75$

$$\frac{\sum (x_i - \bar{x})^2}{10} = (3.75)^2 = 14.0625$$

$$\sum (x_i - \bar{x})^2 = 140.625$$

$$\sigma_{11} = \sqrt{\frac{\sum_{i=1}^{10} (x_i - \bar{x})^2 + (x_{11} - \bar{x})^2}{11}}$$

$$= \sqrt{\frac{140.625}{11}} \quad \text{as} \quad (x_{11} - \bar{x})^2 = 0 \quad \because x_{11} = \bar{x}$$

$$= 3.57548...$$

$$= 3.58 \text{ (2dp)}.$$

(d) gradient of line $= \tan 120^\circ$ (angle made with positive x-axis)

$= -\sqrt{3}$

Passes through $(-\sqrt{3}, 0)$

Eqn of line: $y - 0 = -\sqrt{3}(x + \sqrt{3})$

$y = -\sqrt{3}x - 3$

As point B lies directly beneath A, they have the same x-coordinate & length AB is the difference in their y-coordinates

A is $(x, x^2 - 5x + 4)$

B is $(x, -\sqrt{3}x - 3)$

$$AB = (x^2 - 5x + 4) - (-\sqrt{3}x - 3)$$
$$= x^2 - (5 - \sqrt{3})x + 7$$

This is a quadratic expression representing a concave up parabola. Its minimum value is achieved at the vertex

vertex is at $x = \dfrac{-b}{2a}$

$$= \frac{5 - \sqrt{3}}{2}.$$

When $x = \dfrac{5 - \sqrt{3}}{2}$, the length AB is given by

$$AB = \left(\frac{5-\sqrt{3}}{2}\right)^2 - (5-\sqrt{3})\left(\frac{5-\sqrt{3}}{2}\right) + 7$$

$$= \frac{25 + 3 - 10\sqrt{3}}{4} - \frac{25 + 3 - 10\sqrt{3}}{2} + 7$$

$$= \frac{\cancel{28} - 10\sqrt{3} \cancel{- 2(28)} + 20\sqrt{3} \cancel{+ 28}}{4}$$

$$= \frac{10\sqrt{3}}{4}$$

$$= \frac{5\sqrt{3}}{2} \text{ units}$$

$\therefore$ the minimum length of AB is $\dfrac{5\sqrt{3}}{2}$ units.

Q18 comments

a) Many students are still developing their understanding of function transformations.

b) Generally well done. However, some students did not correctly quote/use the quotient rule. Students are reminded that this result is in the Reference Sheet provided and are advised to copy the result down before using it. Read the question carefully and note what form the answer is required. Some students did not leave their final answer in general form.

c) To get full credit, students needed to explain that if the mean was unchanged, the new score added must be the same as the mean of the 10 scores. Many students could get the correct value for the new standard deviation but could not earn full credit as they made no reference to the new score and how that impacts $\sum(x_i - \bar{x})^2$.

d) Not very well done. Most students found or attempted to find the equation of the line l. A common error was to assume that the gradient of the line was $\tan 60° = \sqrt{3}$. The gradient is the tan ratio of the angle formed with the positive direction of the x-axis which is 120°in this instance. Follow on credit was awarded.

- Many students solved the parabola and the line simultaneously – this makes no sense as it is equivalent to finding the points of intersection – from the diagram it should be apparent there are no intersections.
- Others assumed that the minimum occurs at the vertex of the parabola. This is where the y-values on the parabola are minimum not where AB is minimum.
- Several students stated that the distance AB is minimised when the line is parallel to the tangent to the parabola. This happens to be true but is not a standard result that can be quoted.