NORTH SYDNEY GIRLS HIGH SCHOOL

2020

Preliminary Assessment Task 3

Preliminary Mathematics Advanced

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 15, show relevant mathematical reasoning and/or calculations

Total marks – 84

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – **74 marks** (pages 5 – 25)

- Attempt Questions 11 – 15
- Allow about 1 hour 45 minutes for this section

NAME:_____ **TEACHER:**_____

STUDENT NUMBER:_____

Question	1 – 10	11	12	13	14	15	Total
Total	/10	/14	/15	/15	/16	/14	/84

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10

---

1 Which of the following is the full factorisation of $x^2 + 3x - cx - 3c$?

A. $x(x+3-c-\frac{3c}{x})$

B. $x(x+3)+c(-x-c)$

C. $(x-c)(x-3)$

D. $(x+3)(x-c)$

2 Consider $f(x) = \begin{cases} x-7, & x \leq 1 \\ x^2+2, & x > 1 \end{cases}$

Which of the following gives the value of $f(1) - f(3)$?

A. -6

B. -9

C. -11

D. -17

3 If $f(x) = 4x - 1$ and $g(x) = x^2 + 4x$, which of the following is equivalent to $f\big(g(x)\big)$?

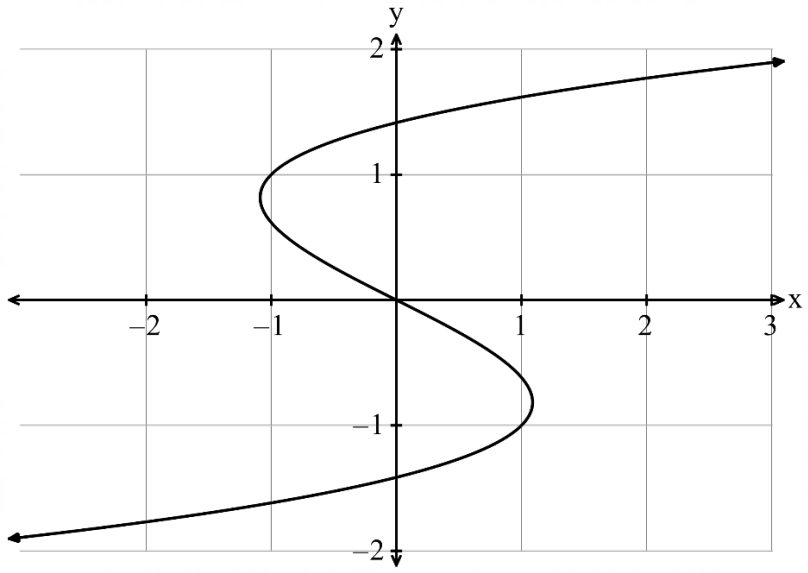
A. $16x^2 + 8x - 3$

B. $4x^2 + 16x - 1$

C. $x^2 + 8x - 1$

D. $x^2 - 1$

4 The graph of $x = y^3 - 2y$ is shown.



Which of the following describes $x = y^3 - 2y$?

A. One-to-one.

B. Many-to-one.

C. One-to-many.

D. Many-to-many.

5 Which of the following describes the function $f(x) = \cos(\sin x)$?

A. Odd.

B. Even.

C. Neither odd nor even.

D. Discontinuous.

6 What are the solutions of $2\cos x + \sqrt{3} = 0$ for $0° \leq x \leq 360°$?

A. $x = 150°,\ 210°$

B. $x = 60°,\ 300°$

C. $x = 30°,\ 150°$

D. $x = \pm 30°$

7 What is the amplitude and period of the function $f(x) = 3\cos \pi x + 1$?

A. Amplitude $= 3$, period $= 2$.

B. Amplitude $= 3$, period $= 2\pi$.

C. Amplitude $= 4$, period $= 2$.

D. Amplitude $= 4$, period $= 2\pi$.

8 For $f(x) = -3x^4 + 6x^2 - 1$, what is the value of $f'(1)$?

A. -24

B. -1

C. 0

D. 2

9 Consider the function $f(x) = |x + 2| + 1$.
Which of the following is true at the point $(-2, 1)$ of this function?

A. The function is continuous and differentiable.

B. The function is continuous and not differentiable.

C. The function is discontinuous and differentiable.

D. The function is discontinuous and not differentiable.

10 A collection of people's phone numbers is an example of what type of data?

A. Categorical nominal.

B. Categorical ordinal.

C. Quantitative discrete.

D. Quantitative continuous.

Preliminary
Mathematics Advanced

Section II Answer Booklet

74 Marks
Attempt Questions 11 – 15
Allow about 1 hour 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Additional writing space is provided at the back of this booklet on pages 26-31.
  If you use a booklet, write "see back" in the question space in the test paper.
  In the writing space, clearly indicate which question you are answering.

Question 11 (14 marks**)**

(a) Solve $x^2 - 2x - 5 = 0$ and express your answer in simplest surd form. **1**

(b) Solve this pair of simultaneous equations: **2**

$$4x - y + 6 = 0$$
$$x + 2y + 6 = 0$$

(c) Solve $|2x - 1| = 5$, for x. **2**

(d) Write down the exact value of $\operatorname{cosec}\frac{3\pi}{4}$. **1**

(e) Evaluate: **2**

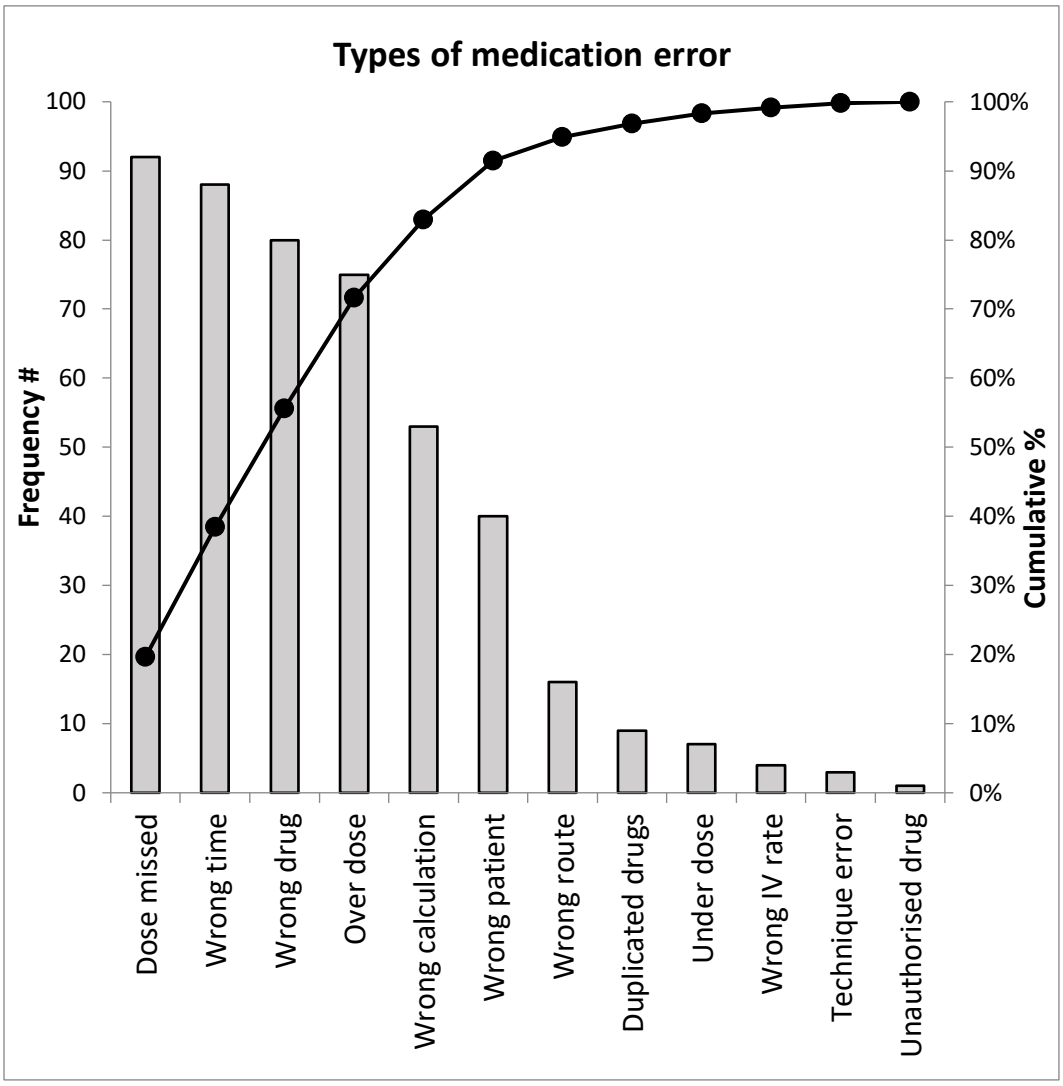
$$\lim_{x\to 2}\frac{x^2-5x+6}{x^2-4}$$

(f) Differentiate the following:

i) $y = 2x^6 - x^4 - 15$ **1**

ii) $y = (3-2x^2)^5$ **1**

(g) A hospital made the following Pareto chart to analyse medication errors and improve patient care. They decided to focus their efforts on reducing the categories that contributed the most to medication errors.



i) To cover **at least** 80% of the mistakes, how many types of medication error did they focus on? **1**

_____

_____

ii) What was the largest category they decided **not to** focus on and how many medication errors were reported for this category? **1**

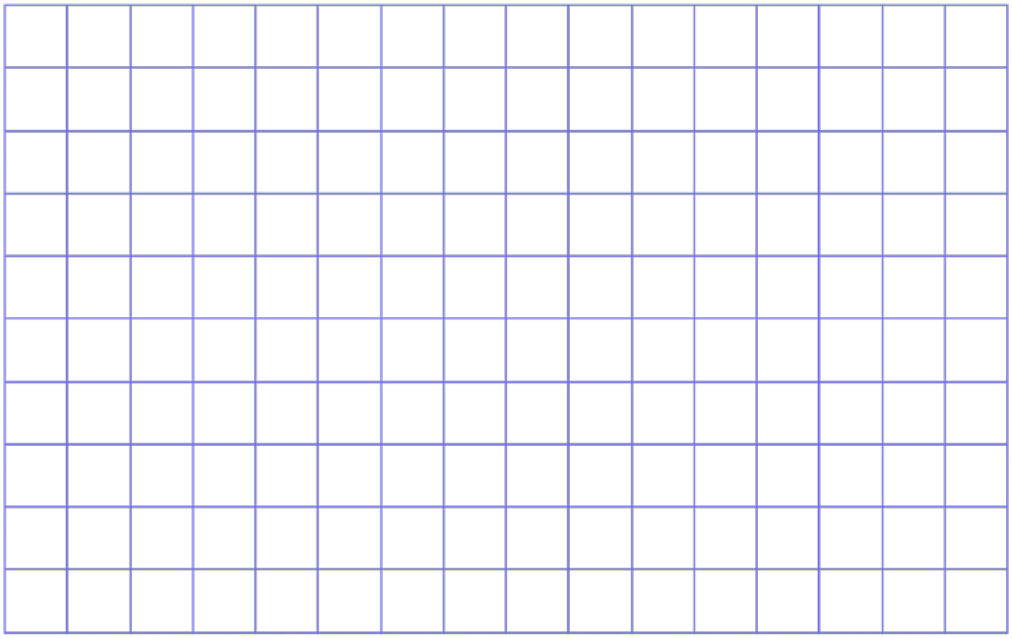
_____

_____

(h) The speed of cars travelling through an intersection were recorded. **2**

Speed (km/h)	Class Centre (km/h)	Frequency
0-10	5	18
10-20	15	15
20-30	25	3
30-40	35	8
40-50	45	19

Accurately graph this data using an appropriately chosen display.

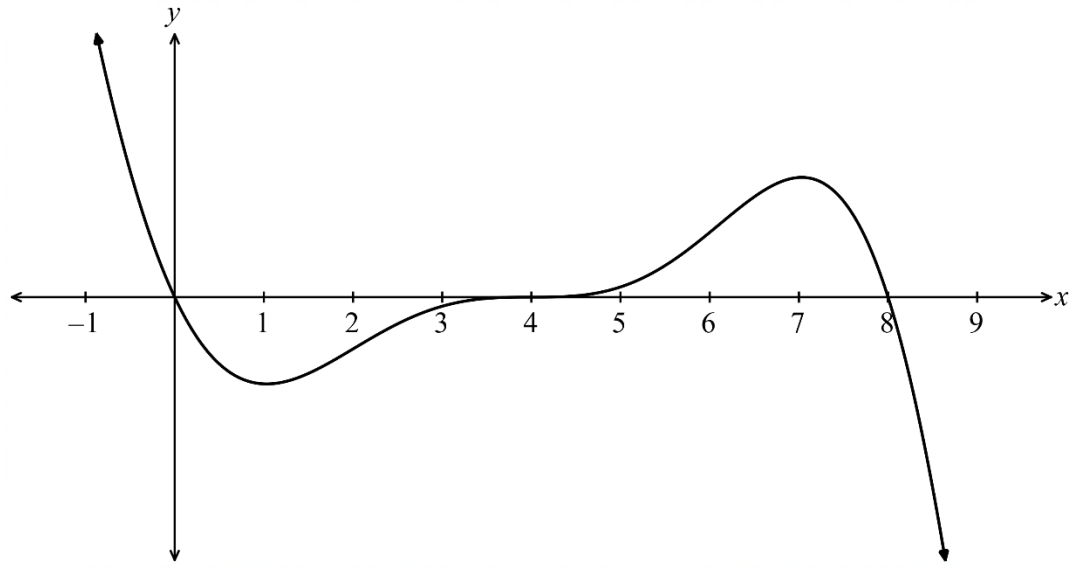


Question 12 (15 marks)

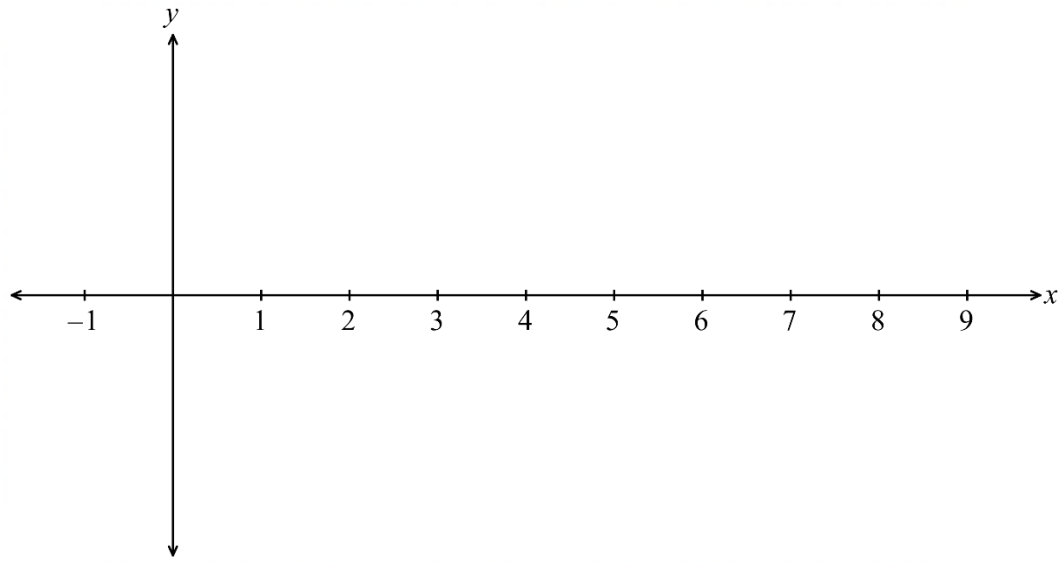
(a) Solve $2\log_2 x = \log_2(2x + 15)$, for x. **3**

(b) Find the equation of the tangent to the curve $y = \frac{1}{\sqrt{x}}$ at the point $x = 4$. **2**

(c) Consider the function $y = f(x)$ shown below.



i) Sketch the gradient function, $y = f'(x)$. **2**



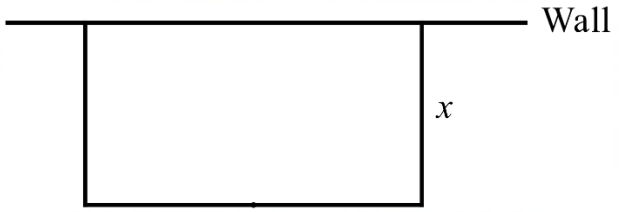
ii) Write down the domain where $f'(x) > 0$. **1**

(d) The current, I (measured in Amps), produced by a battery is inversely proportional to the resistance, R (measured in Ohms), of the circuit that the battery is connected to. When connected to a circuit with a resistance of 17 Ohms, the battery produces a current of 3 Amps.

i) Write this as an equation and find the constant of proportionality. **2**

ii) Use your equation to find the current produced when the battery is connected to a circuit with a resistance of 9 Ohms. **1**

(e) A rectangular enclosure is to be built using 80 m of temporary fencing and an existing wall.



i) Show that area of the enclosure A, in terms of the width x, can be found using $A = -2x^2 + 80x$. **1**

ii) Sketch the graph of area with respect to x. **2**

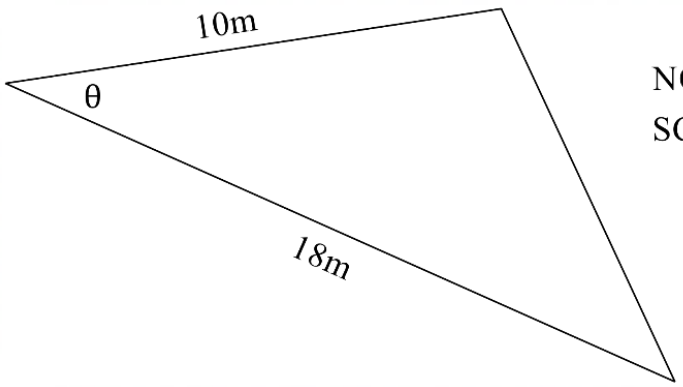
iii) Using the graph or otherwise, find the maximum area that can be enclosed. **1**

Question 13 (15 marks)

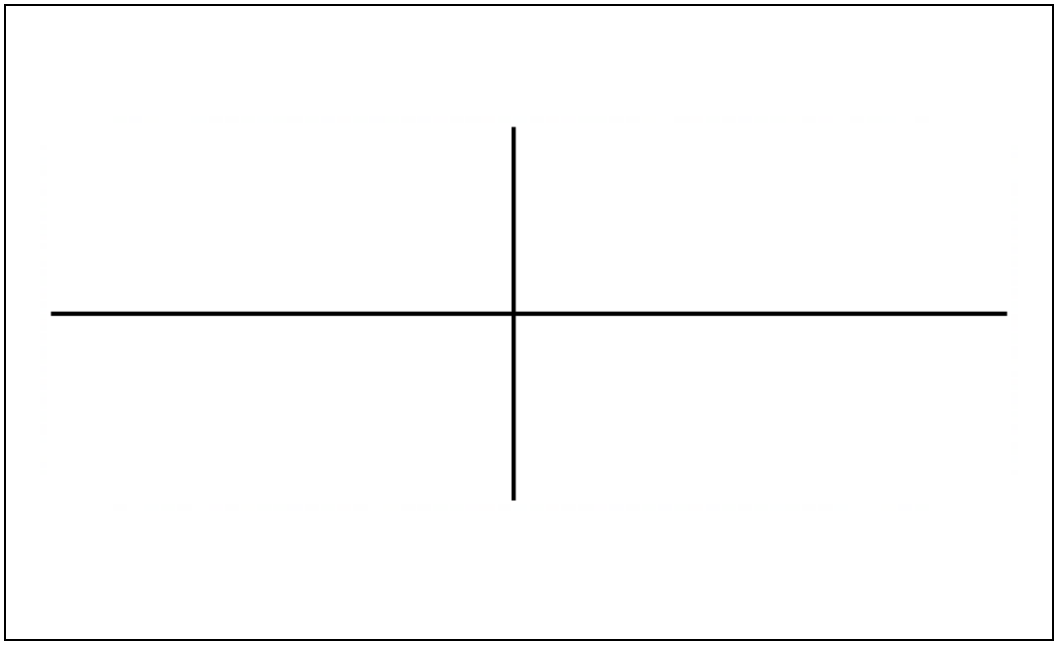
(a) Solve the following equation for $0 \leq \theta \leq 360°$: **2**

$$\tan(\theta + 25°) = 1$$

(b) If the area of this triangle is 86 m^2, find the value(s) of θ, to the nearest degree. **2**



(c) i) Sketch the graph of $y = \cos 2x$ for $-\pi \leq x \leq \pi$. **1**



ii) How many solutions are there for $\cos 2x = x + 1$ in the domain $-\pi \leq x \leq \pi$? **1**

(d) Consider $f(x) = \sqrt{x}$ and $g(x) = x^2 - 3$, state the domain of $f(g(x))$. **2**

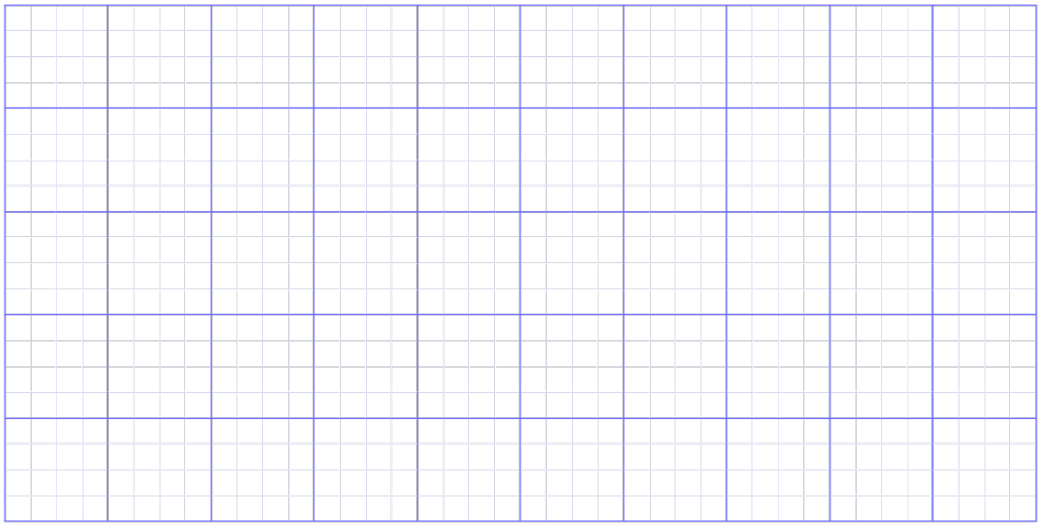
(e) Differentiate the following, leaving your answer in simplest form:

i) $f(x) = x^3(x^2 + 1)^7$ **2**

ii) $f(x) = \dfrac{2x-1}{3+x}$ **2**

(f) A small-business makes fabric hats with a selling price of $20 per unit which cost $12 per unit to produce. They have fixed operating costs of $400.

i) Graph the costs and income for this company on the same axes, from 0 to 100 units produced/sold. **1**

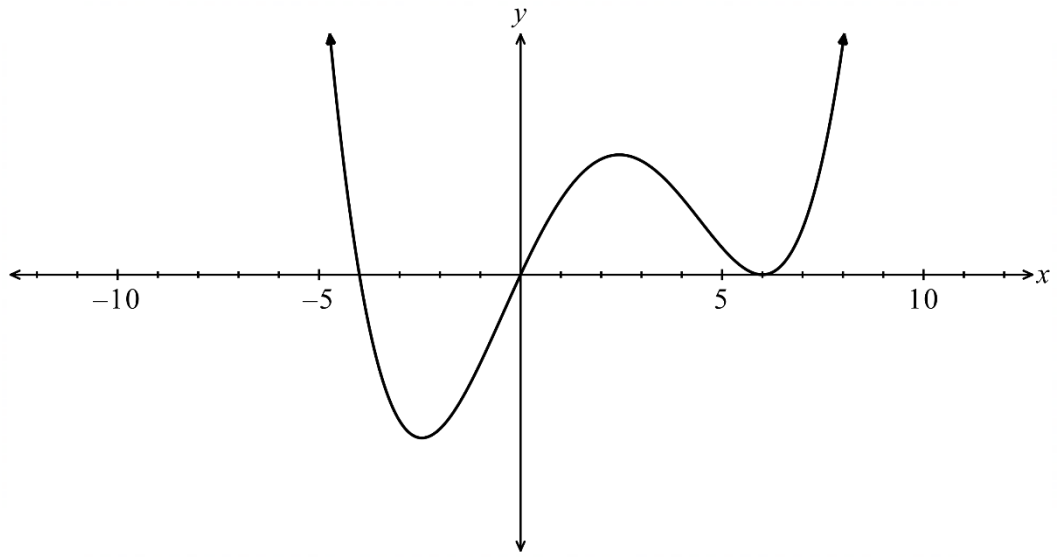


ii) Use the graph to find the break-even point, where income equals costs. **1**

iii) Shade the region on the graph that represents profit. **1**

Question 14 (16 marks)

(a) The following is a graph of a function $y = f(x)$, which is a polynomial.



i) Identify the values of the roots of the polynomial. **1**

ii) Write a possible equation for this polynomial. **1**

iii) On the graph above, sketch $y = f(2x)$. **1**

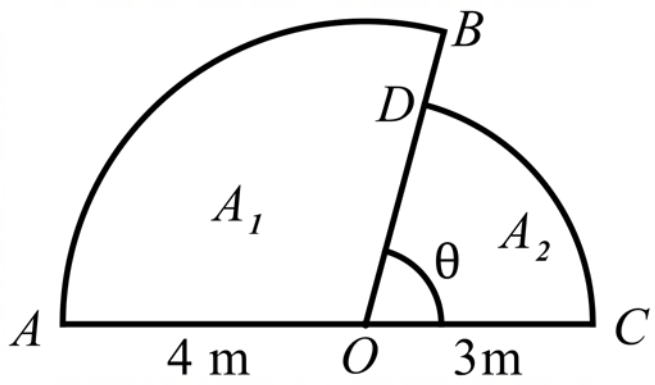
iv) Describe the transformation from part (ii). **1**

(b) Find the values of k for which the equation $x^2 + kx + 2k - 3 = 0$ has no real roots. **2**

(c) Solve $x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 8 = 0$, for x. **3**

(d) Solve $\sec^2 x = 2$, for $0 \leq x \leq \pi$. **2**

(e) Two sectors are constructed along a straight line AOC. The first sector has radius 4 m and second sector has radius 3 m, as shown below. **2**



If the area of the larger sector is twice the area of the smaller sector, find the angle at the centre of the smaller sector, θ, to two decimal places.

(f) Given that $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, differentiate $f(x) = x^2 - x$ from first principles. **3**

Question 15 (14 marks**)**

(a) The bacteria salmonella was being studied under favourable conditions in a laboratory. At time $T = 0$, the population was 10^5 bacteria and 2 hours later, it was found that the population had reached 10^6 bacteria.

i) Show that the population P, can be found using: **1**

$$P = 10^5 \times 10^{\frac{T}{2}}$$

ii) Find the time taken for the population to double, to the nearest minute. **2**

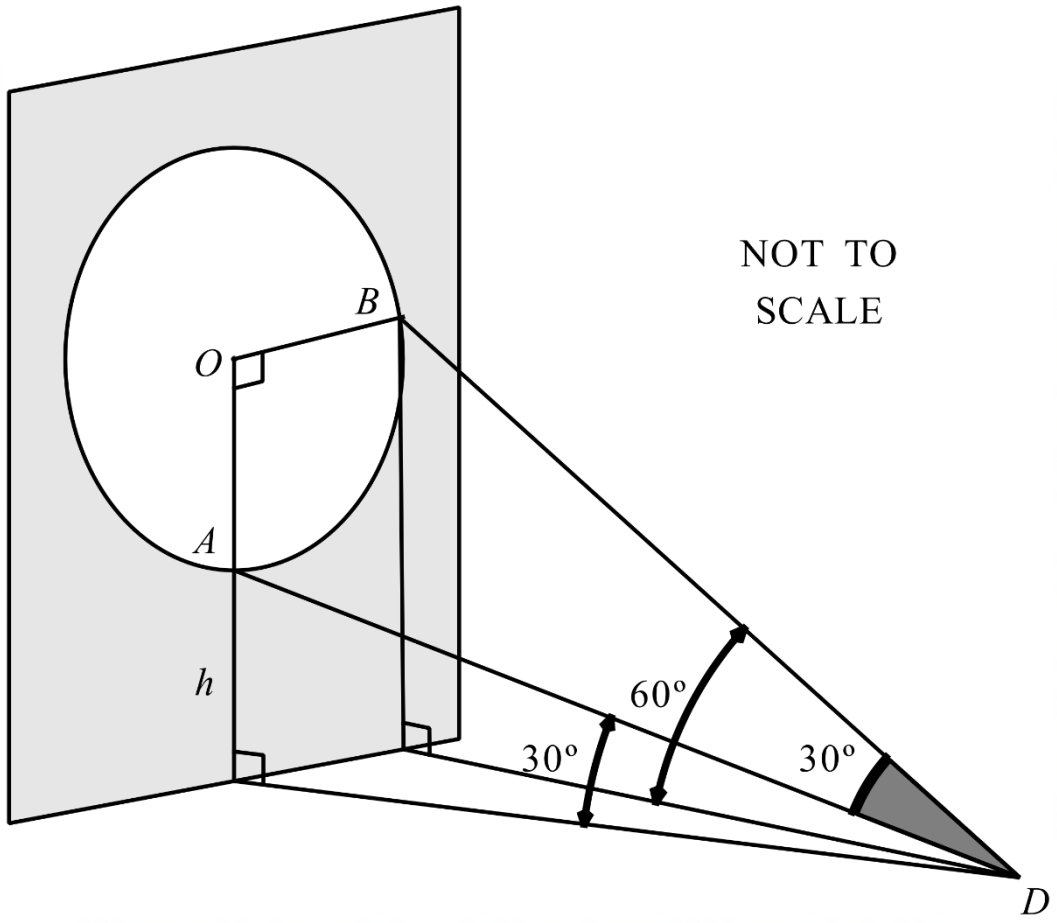
(b) If $y = 4x + c$ is tangent to $y = 2x^2 + 8x + 3$, find the value of c. **3**

(c) Prove that: **3**

$$(\text{cosec}\,\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$$

(d) A circle with centre O and radius 10 m is drawn on a vertical wall, standing on a horizontal floor. Point A is vertically below O and $\angle AOB = 90°$ as shown in the diagram.

A laser pointer is aimed from a point D on the ground. The angle of elevation from D to A and B are 30° and 60° respectively. The laser beam starts at A and sweeps through and angle of 30° to reach B.



Let A be h m above the ground.

i) Write expressions for AD and BD in terms of h. **2**

Continued on next page.

ii) Find the exact value of h. **3**

End of questions.

Additional writing space.

Clearly indicate which question you are answering.

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Question	1 – 10	11	12	13	14	15	Total
Algebra	1	5	3				9
Functions	4		3	3	4	3	17
Linear and quadratic functions			4		5	3	12
Trigonometry	2	1		6	4	8	21
Differentiation	2	4	5	6	3		20
Statistics	1	4					5
Total	**/10**	**/14**	**/15**	**/15**	**/16**	**/14**	**/84**

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Total marks – 84

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10 marks
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Use the multiple choice answer sheet for Questions 1-10

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B. $x(x+3)+c(-x-c)$

C. $(x-c)(x-3)$

D. $(x+3)(x-c)$

2 Consider $f(x) = \begin{cases} x-7, & x \leq 1 \\ x^2+2, & x > 1 \end{cases}$

Which of the following gives the value of $f(1) - f(3)$?

$1 - 7 - (3^2 + 2)$

A. -6

B. -9

C. -11

D. -17

3 If $f(x) = 4x - 1$ and $g(x) = x^2 + 4x$, which of the following is equivalent to $f(g(x))$?

$4(x^2+4x) - 1$

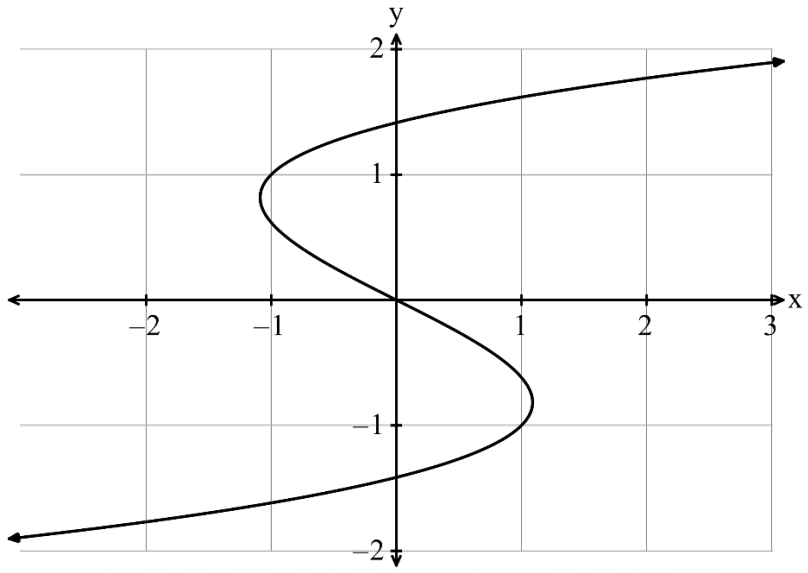
A. $16x^2 + 8x - 3$

B. $4x^2 + 16x - 1$

C. $x^2 + 8x - 1$

D. $x^2 - 1$

4 The graph of $x = y^3 - 2y$ is shown.



Which of the following describes $x = y^3 - 2y$?

A. One-to-one.

B. Many-to-one.

C. One-to-many.

D. Many-to-many.

5 Which of the following describes the function $f(x) = \cos(\sin x)$?

A. Odd.

B. Even.

C. Neither odd nor even.

D. Discontinuous.

$\cos(\sin(-x))$
$= \cos(-\sin x)$
$= \cos(\sin x)$

6 What are the solutions of $2\cos x + \sqrt{3} = 0$ for $0° \leq x \leq 360°$?

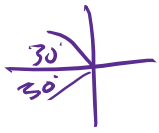
A. $x = 150°,\ 210°$

B. $x = 60°,\ 300°$

C. $x = 30°,\ 150°$

D. $x = \pm 30°$

$\cos x = -\frac{\sqrt{3}}{2}$



7 What is the amplitude and period of the function $f(x) = 3\cos \pi x + 1$?

A. Amplitude $= 3$, period $= 2$.

B. Amplitude $= 3$, period $= 2\pi$.

C. Amplitude $= 4$, period $= 2$.

D. Amplitude $= 4$, period $= 2\pi$.

8 For $f(x) = -3x^4 + 6x^2 - 1$, what is the value of $f'(1)$?

A. -24

B. -1

C. 0

D. 2

$f'(x) = -12x^3 + 12x$

$f'(1) = -12 + 12 = 0$

9 Consider the function $f(x) = |x + 2| + 1$.

Which of the following is true at the point $(-2, 1)$ of this function?

A. The function is continuous and differentiable.

B. The function is continuous and not differentiable.

C. The function is discontinuous and differentiable.

D. The function is discontinuous and not differentiable.

10 A collection of people's phone numbers is an example of what type of data?

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Preliminary
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Question 11 (14 marks)

(a) Solve $x^2 - 2x - 5 = 0$ and express your answer in simplest surd form. **1**

$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-5)}}{2} = \frac{2 \pm \sqrt{24}}{2}$

$= 1 \pm \sqrt{6}$

(b) Solve this pair of simultaneous equations: **2**

$4x - y + 6 = 0$ ①

$x + 2y + 6 = 0$ ②

$2 \times ① + ②$

$2(4x - y + 6) + x + 2y + 6 = 0$

$8x - 2y + 12 + x + 2y + 6 = 0$

$9x + 18 = 0$

$x = \frac{-18}{9} = -2$

Sub into ②

$-2 + 2y + 6 = 0$

$y = \frac{-4}{2} = -2$

(c) Solve $|2x - 1| = 5$, for x. **2**

$2x - 1 = 5$ or $2x - 1 = -5$

$2x = 6$ $2x = -4$

$x = 3$ $x = -2$

(d) Write down the exact value of $\operatorname{cosec}\frac{3\pi}{4}$. **1**

$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$

$\operatorname{cosec}\frac{3\pi}{4} = \frac{1}{\sin\frac{3\pi}{4}} = \sqrt{2}$

(e) Evaluate: **2**

$$\lim_{x\to 2}\frac{x^2-5x+6}{x^2-4}$$

$= \lim_{x\to 2}\frac{(x-2)(x-3)}{(x-2)(x+2)}$

$= \lim_{x\to 2}\frac{x-3}{x+2}$

$= \frac{2-3}{2+2} = \frac{-1}{4}$

(f) Differentiate the following:

i) $y = 2x^6 - x^4 - 15$ **1**

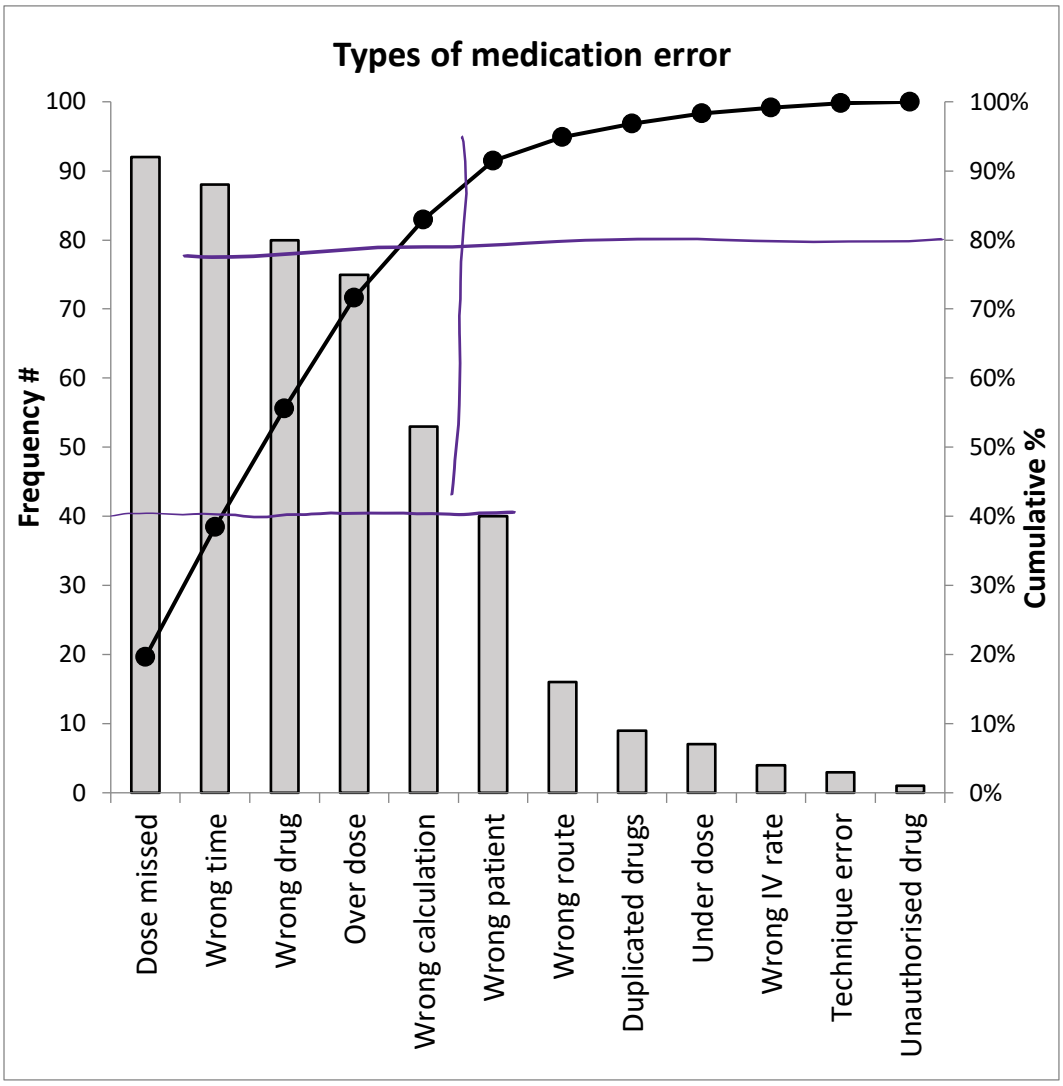
$\frac{dy}{dx} = 12x^5 - 4x^3$

ii) $y = (3-2x^2)^5$ **1**

$\frac{dy}{dx} = 5(3-2x^2)^4 \times (-4x)$

$= -20x(3-2x^2)^4$

(g) A hospital made the following Pareto chart to analyse medication errors and improve patient care. They decided to focus their efforts on reducing the categories that contributed the most to medication errors.



i) To cover **at least** 80% of the mistakes, how many types of medication error did they focus on? **1**

5

ii) What was the largest category they decided **not to** focus on and how many medication errors were reported for this category? **1**

Wrong patient, 40

(h) The speed of cars travelling through an intersection were recorded. **2**

Speed (km/h)	Class Centre (km/h)	Frequency
0-10	5	18
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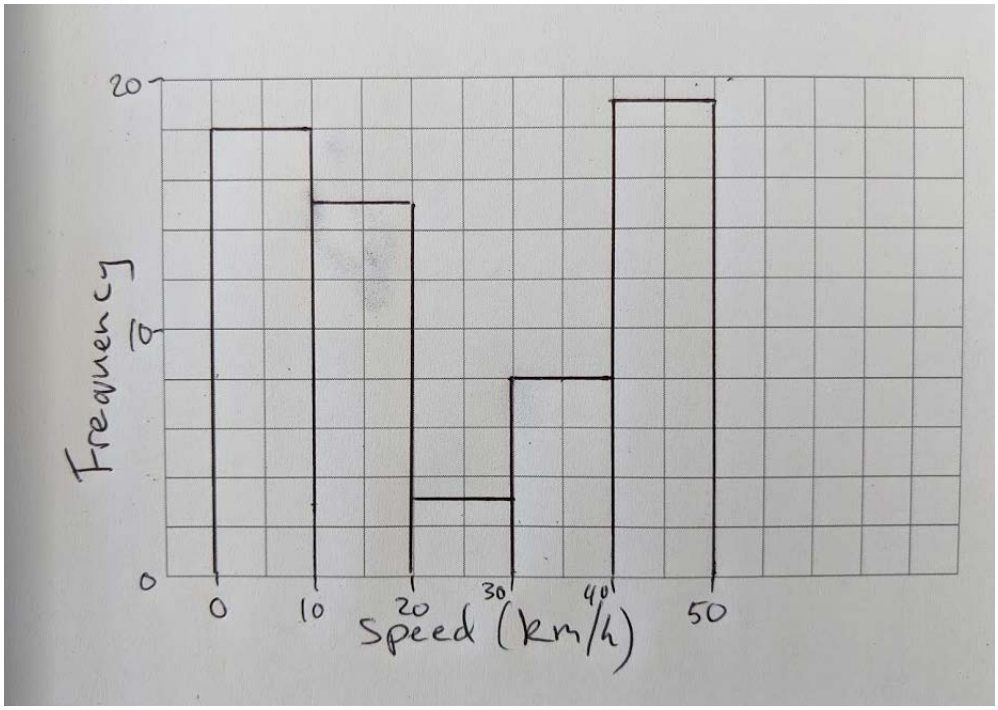


Question 12 (15 marks)

(a) Solve $2\log_2 x = \log_2(2x + 15)$, for x. **3**

$\log_2 x^2 = \log_2(2x+15)$

$x^2 = 2x + 15$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5, -3 \quad (x > 0)$

(b) Find the equation of the tangent to the curve $y = \frac{1}{\sqrt{x}}$ at the point $x = 4$. **2**

$y = x^{-\frac{1}{2}}$

$y' = -\frac{1}{2}x^{-\frac{3}{2}}$

$= -\frac{1}{2\sqrt{x^3}}$

$y'(4) = \frac{-1}{2\sqrt{4^3}} = \frac{-1}{16}$

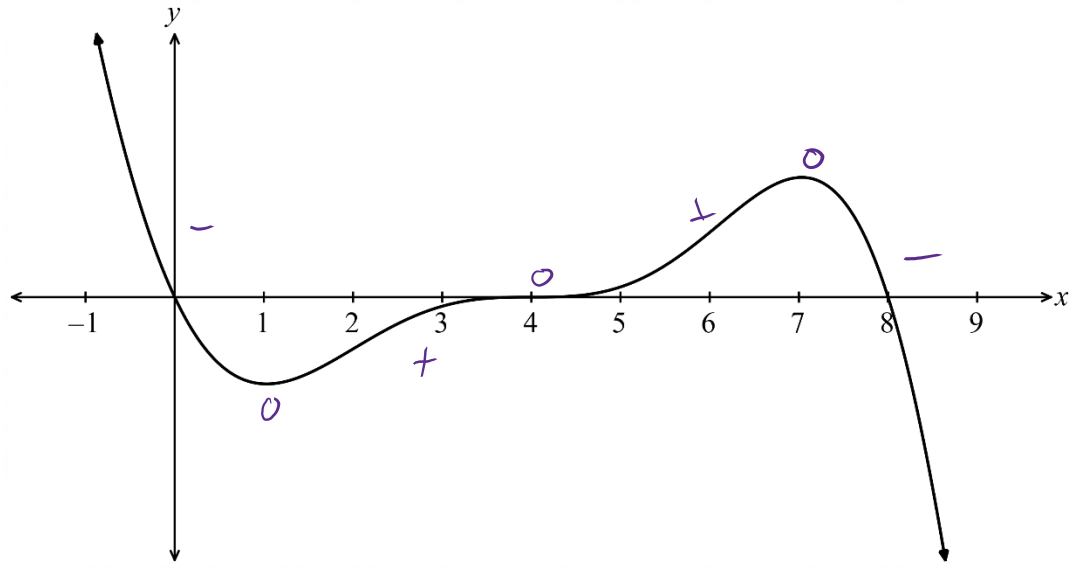
at $x = 4$, $y = \frac{1}{\sqrt{4}} = \frac{1}{2}$

$y - \frac{1}{2} = -\frac{1}{16}(x-4)$

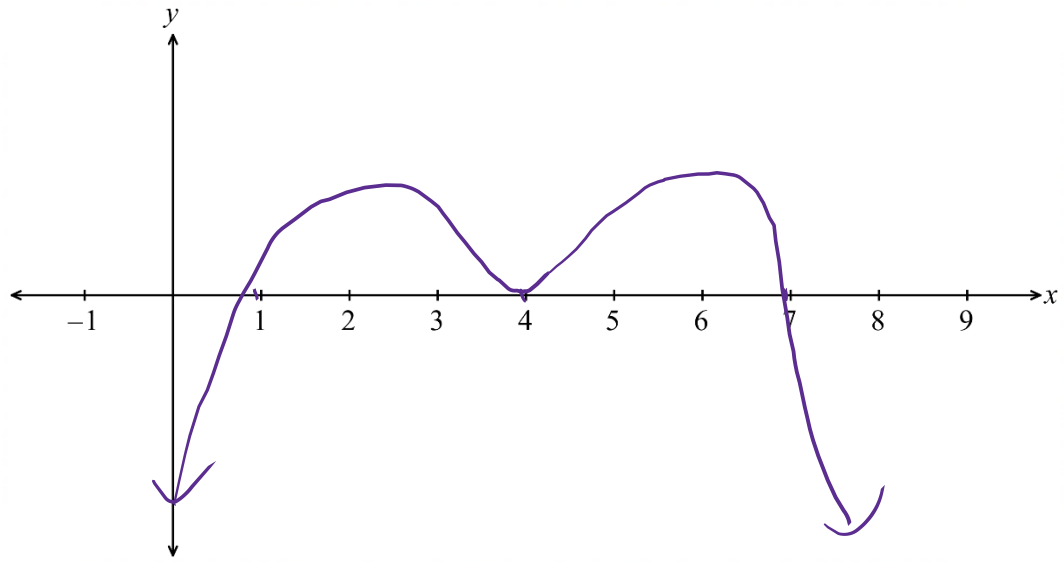
$= -\frac{1}{16}x + \frac{1}{4}$

$x + 16y - 12 = 0$

(c) Consider the function $y = f(x)$ shown below.



i) Sketch the gradient function, $y = f'(x)$. **2**



ii) Write down the domain where $f'(x) > 0$. **1**

$1 < x < 4, \; 4 < x < 7 \quad (x \neq 4)$

(d) The current, I (measured in Amps), produced by a battery is inversely proportional to the resistance, R (measured in Ohms), of the circuit that the battery is connected to. When connected to a circuit with a resistance of 17 Ohms, the battery produces a current of 3 Amps.

i) Write this as an equation and find the constant of proportionality. **2**

$I = \frac{k}{R}$

$3 = \frac{k}{17}$

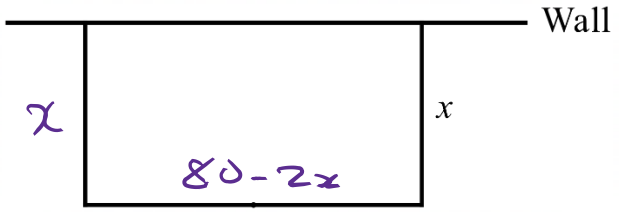
$k = 3 \times 17 = 51$

$I = \frac{51}{R}$

ii) Use your equation to find the current produced when the battery is connected to a circuit with a resistance of 9 Ohms. **1**

$I = \frac{51}{9} = 5\frac{2}{3}\ \Omega$

(e) A rectangular enclosure is to be built using 80 m of temporary fencing and an existing wall.



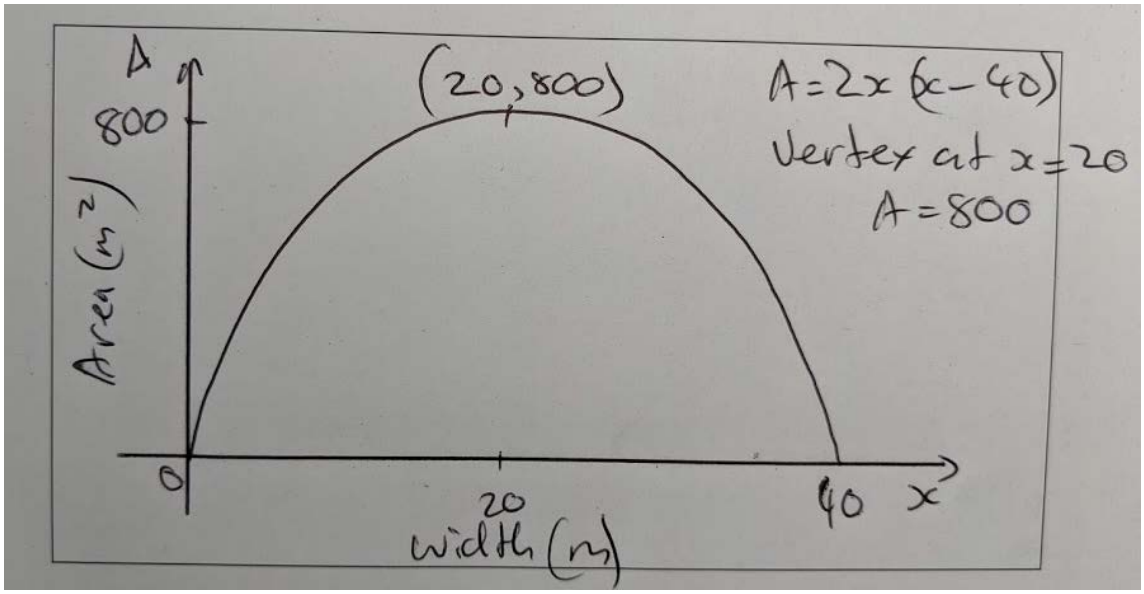
i) Show that area of the enclosure A, in terms of the width x, can be found using $A = -2x^2 + 80x$. **1**

length = 80-2x

$A = x(80-2x)$

$= -2x^2 + 80x$

ii) Sketch the graph of area with respect to x. **2**



iii) Using the graph or otherwise, find the maximum area that can be enclosed. **1**

800 m²

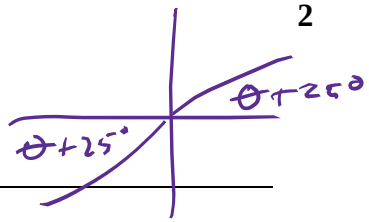
Question 13 (15 marks)

(a) Solve the following equation for $0 \leq \theta \leq 360°$: **2**

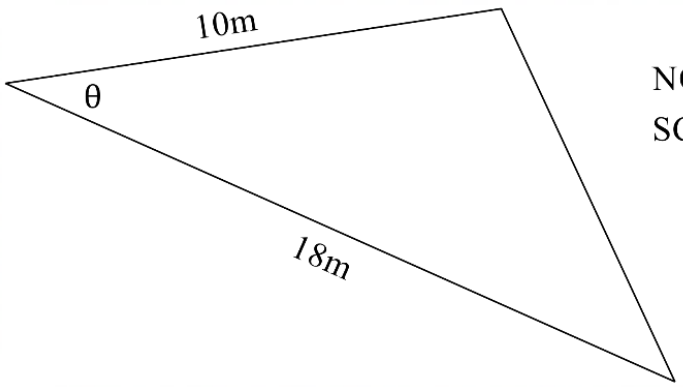
$$\tan(\theta + 25°) = 1$$

$\theta + 25° = 45°, 125°$

$= 20°, 105°$



(b) If the area of this triangle is 86 m^2, find the value(s) of θ, to the nearest degree. **2**



NOT TO SCALE

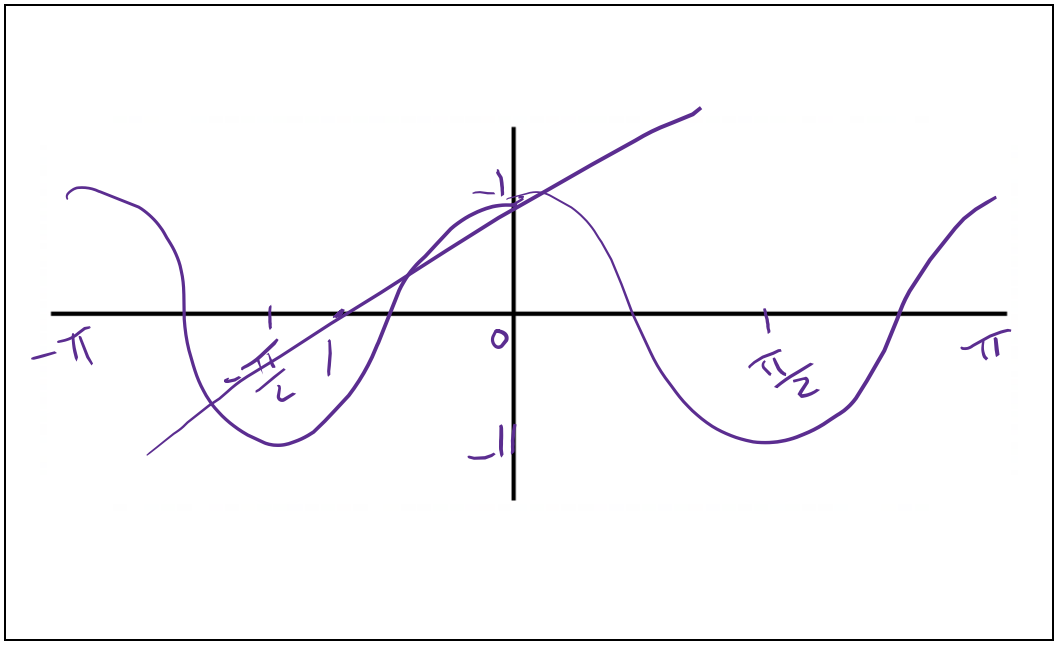
$A = \frac{1}{2} ab \sin C$

$86 = \frac{1}{2} \times 10 \times 18 \sin C$

$\sin C = 86/90$

$C = 73°$ and $180 - 73 = 107°$

(c) i) Sketch the graph of $y = \cos 2x$ for $-\pi \leq x \leq \pi$. **1**



ii) How many solutions are there for $\cos 2x = x + 1$ in the domain $-\pi \leq x \leq \pi$? **1**

$\frac{\pi}{4} \approx 0.8$, $\frac{\pi}{2} \approx 1.6$

3 solutions

(d) Consider $f(x) = \sqrt{x}$ and $g(x) = x^2 - 3$, state the domain of $f(g(x))$. **2**

$f(g(x)) = \sqrt{x^2-3}$

$\therefore \quad x^2 - 3 \geq 0$

$x^2 \geq 3$

$x \leq -\sqrt{3}, \sqrt{3} \leq x$

(e) Differentiate the following, leaving your answer in simplest form:

i) $f(x) = x^3(x^2+1)^7$ **2**

$u = x^3 \qquad v = (x^2+1)^7$

$u' = 3x^2 \qquad v' = 14x(x^2+1)^6$

$y' = uv' + vu'$

$= x^3 14x(x^2+1)^6 + (x^2+1)^7 3x^2$

$= 14x^4(x^2+1)^6 + 3x^2(x^2+1)(x^2+1)^6$

$= (17x^4 + 3x^2)(x^2+1)^6$

$= x^2(17x^2+3)(x^2+1)^6$

ii) $f(x) = \dfrac{2x-1}{3+x}$ **2**

$u = 2x-1 \qquad v = 3+x$

$u' = 2 \qquad v' = 1$

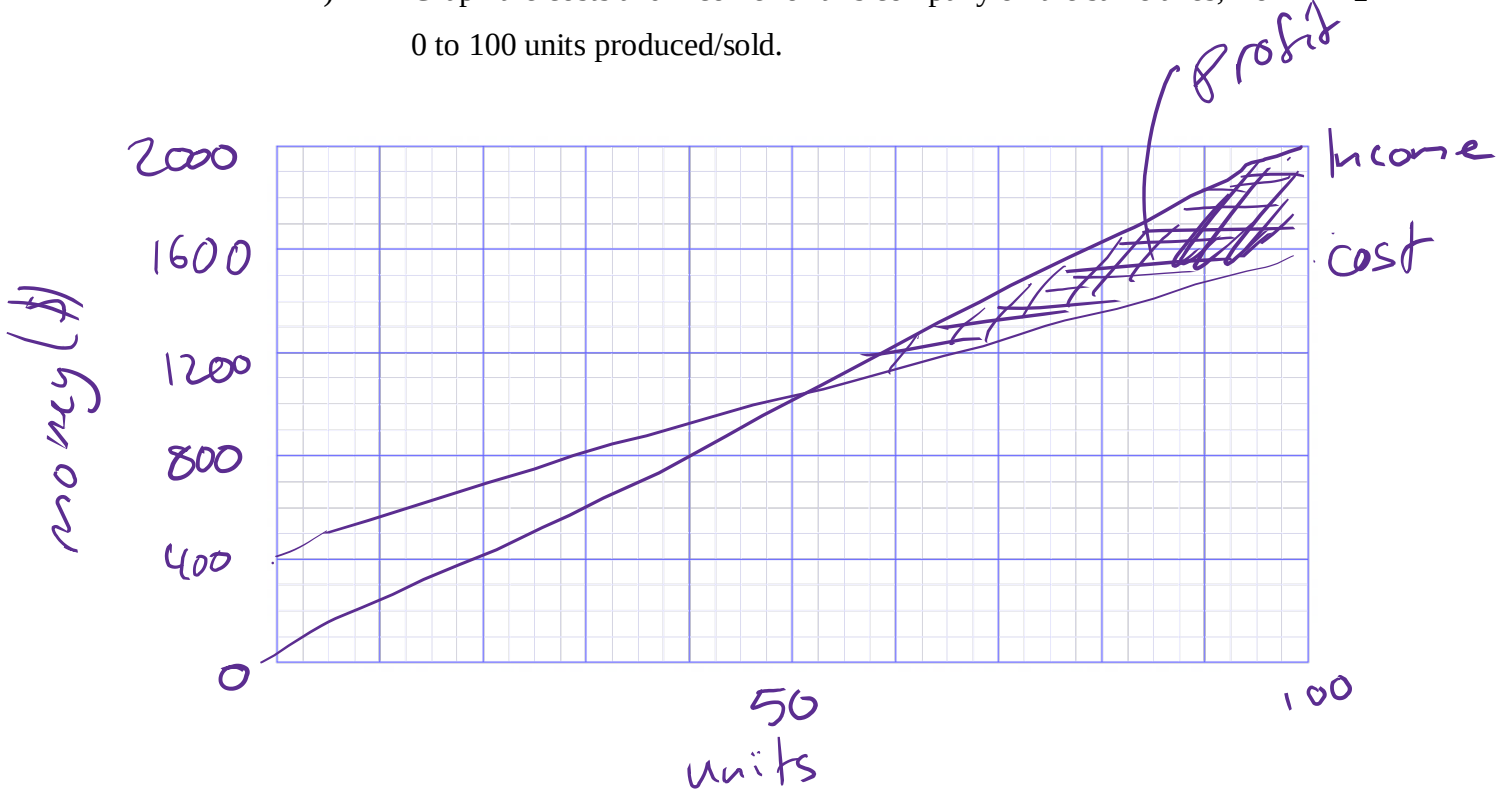
$y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{2(3+x) - (2x-1)}{(3+x)^2}$

$= \dfrac{6 + 2x - 2x + 1}{(3+x)^2} = \dfrac{7}{(3+x)^2}$

(f) A small-business makes fabric hats with a selling price of $20 per unit which cost $12 per unit to produce. They have fixed operating costs of $400.

i) Graph the costs and income for this company on the same axes, from 0 to 100 units produced/sold. **1**



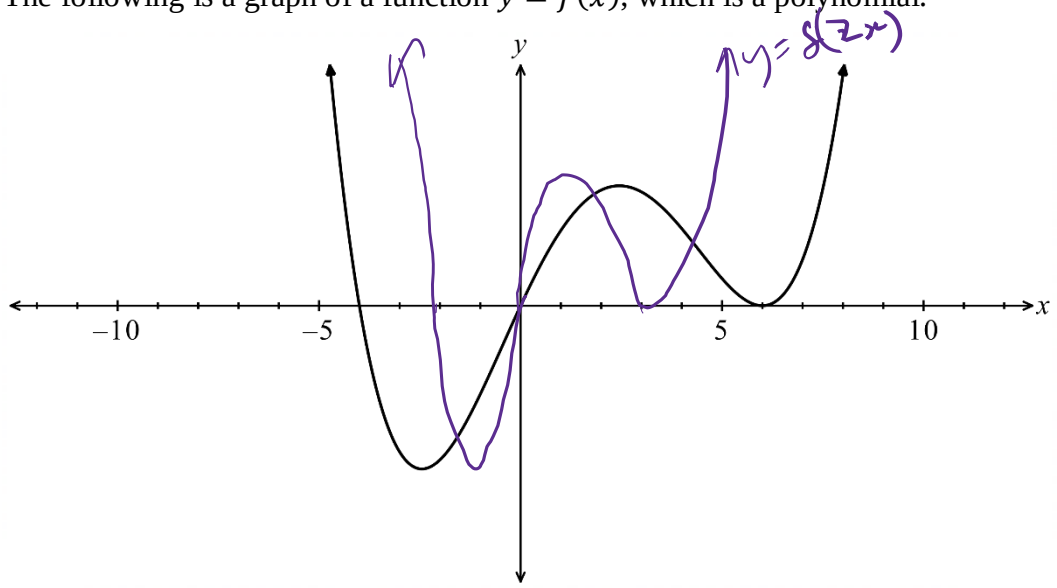
ii) Use the graph to find the break-even point, where income equals costs. **1**

(50, 1000)

iii) Shade the region on the graph that represents profit. ✓ **1**

Question 14 (16 marks)

(a) The following is a graph of a function $y = f(x)$, which is a polynomial.



i) Identify the values of the roots of the polynomial. **1**

$-4, 0, 6$

ii) Write a possible equation for this polynomial. **1**

$y = (x+4)x(x-6)^2$

iii) On the graph above, sketch $y = f(2x)$. **1**

iv) Describe the transformation from part (ii). **1**

horizontal dilation by a factor of $\frac{1}{2}$

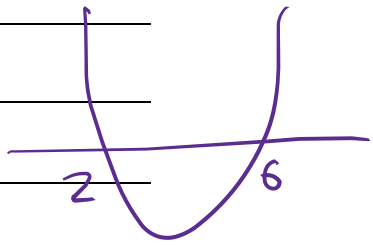
(b) Find the values of k for which the equation $x^2 + kx + 2k - 3 = 0$ has no real roots. **2**

$\Delta = b^2 - 4ac < 0$

$k^2 - 4 \times 1 \times (2k-3) < 0$

$k^2 - 8k + 12 < 0$

$(k-6)(k-2) < 0$



$2 < k < 6$

(c) Solve $x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 8 = 0$, for x. **3**

Let $u = x^{\frac{1}{3}}$

$u^2 + 2u - 8 = 0$

$(u+4)(u-2) = 0$

$u = -4$ or $u = 2$

$x^{\frac{1}{3}} = -4$ or $x^{\frac{1}{3}} = 2$

$x = -64$ or $x = 8$

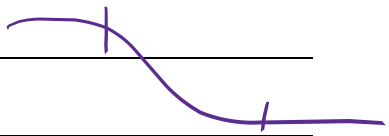
(d) Solve $\sec^2 x = 2$, for $0 \leq x \leq \pi$. **2**

$\sec x = \pm\sqrt{2}$

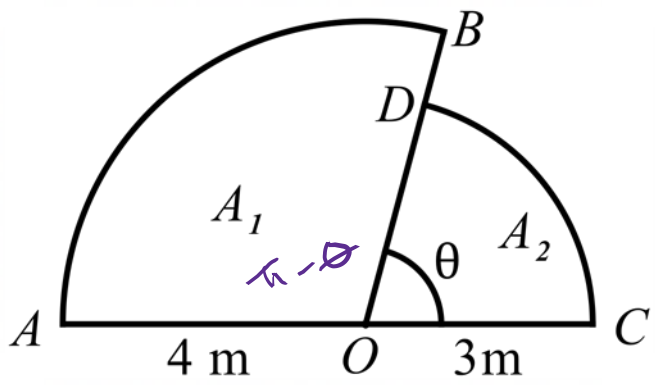
$\frac{1}{\cos x} = \pm\sqrt{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$



(e) Two sectors are constructed along a straight line AOC. The first sector has radius 4 m and second sector has radius 3 m, as shown below. **2**



If the area of the larger sector is twice the area of the smaller sector, find the angle at the centre of the smaller sector, θ, to two decimal places.

$A = \frac{1}{2} r^2 \theta$

$A_L = 2A_S$

$\frac{1}{2} \times 4^2 (\pi - \theta) = 2 \times \frac{1}{2} \times 3^2 \theta$

$8\pi - 8\theta = 9\theta$

$8\pi = 17\theta$

$\theta = \frac{8\pi}{17}$

$\theta \approx 1.48$ (2 d.p.)

(f) Given that $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, differentiate $f(x) = x^2 - x$ from first principles. **3**

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - (x+h) - (x^2 - x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x - 1 + h)}{h}$$

$$= 2x - 1$$

Question 15 (14 marks)

(a) The bacteria salmonella was being studied under favourable conditions in a laboratory. At time $T = 0$, the population was 10^5 bacteria and 2 hours later, it was found that the population had reached 10^6 bacteria.

i) Show that the population P, can be found using: **1**

$$P = 10^5 \times 10^{\frac{T}{2}}$$

at $t=2$ $\quad 10^6 = 10^5 \times 10^{\frac{2}{2}}$

$= 10^6$

$=$ LHS

ii) Find the time taken for the population to double, to the nearest minute. **2**

$2 \times 10^5 = 10^5 \times 10^{\frac{T}{2}}$

$2 = 10^{\frac{T}{2}}$

$\log_{10} 2 = \frac{T}{2}$

$2 \log_{10} 2 = T$

$T = 36$ minutes

(b) If $y = 4x + c$ is tangent to $y = 2x^2 + 8x + 3$, find the value of c. 2 methods **3**

$2x^2+8x+3-4x-c=0$

tangent, so 1 solution

$\Delta = 0 = 4^2 - 4\times2\times(3-c)$

$= 16-24+8c$

$= -8+8c$

$\therefore c=1$

gradient $= 4 = 4x+8$

$-4 = 4x$, $\therefore x = -1$

at $x=-1$, $y = 2(-1)^2-8+3 = -3$

$-3 = 4(-1)+c$

$\therefore c = 1$

(c) Prove that: **3**

$$(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

$LHS = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{\cos^2\theta - 1}$

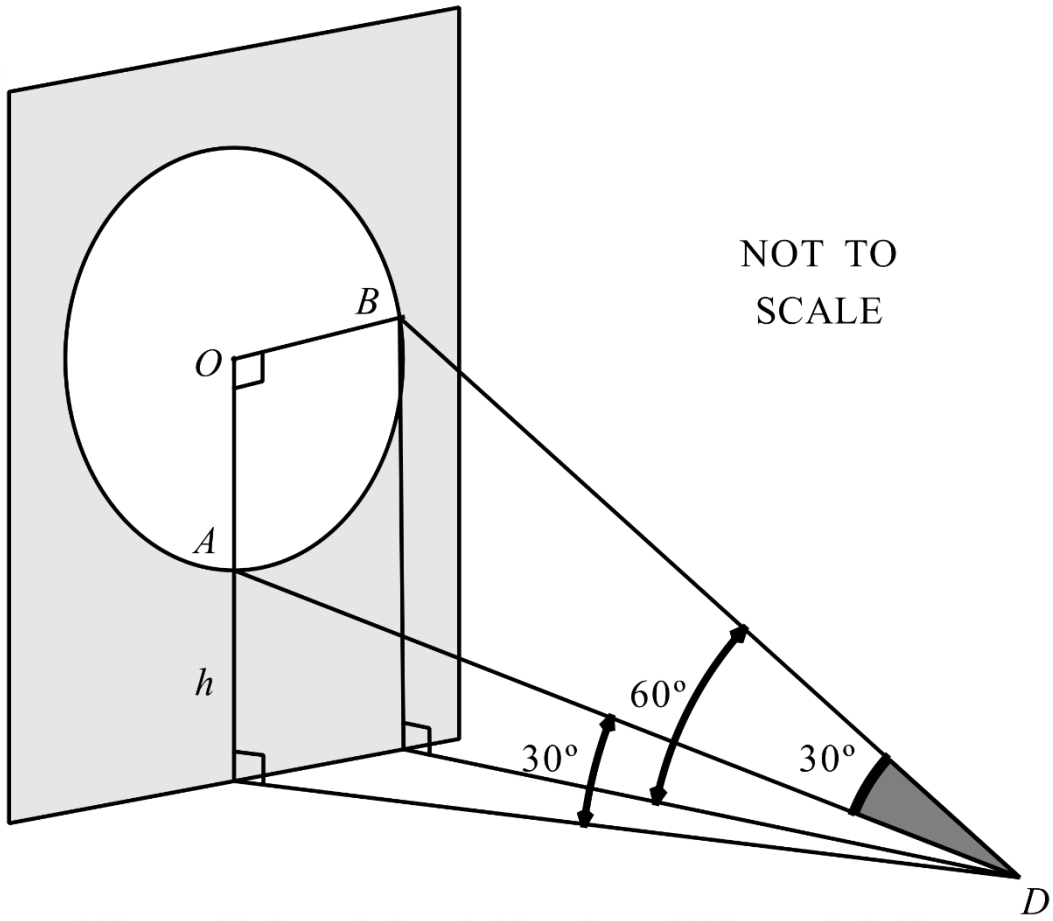
$= \frac{(1-\cos\theta)^2}{(\cos\theta-1)(\cos\theta+1)}$

$= \frac{1-\cos\theta}{1+\cos\theta}$

$= RHS$

(d) A circle with centre O and radius 10 m is drawn on a vertical wall, standing on a horizontal floor. Point A is vertically below O and $\angle AOB = 90°$ as shown in the diagram.

A laser pointer is aimed from a point D on the ground. The angle of elevation from D to A and B are 30° and 60° respectively. The laser beam starts at A and sweeps through and angle of 30° to reach B.



Let A be h m above the ground.

i) Write expressions for AD and BD in terms of h. **2**

$\sin 30° = \frac{h}{AD}$

$AD = \frac{h}{\sin 30°} = 2h$

$\sin 60° = \frac{h+10}{BD}$

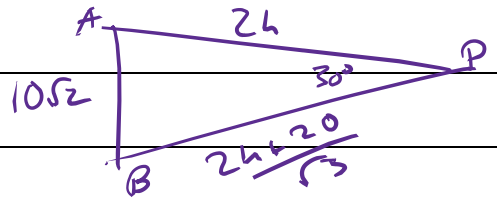
$BD = \frac{h+10}{\sin 60°} = \frac{2h+20}{\sqrt{3}}$

Continued on next page.

ii) Find the exact value of h. **3**

In $\triangle AOB$, $AB = \sqrt{10^2+10^2} = 10\sqrt{2}$

In $\triangle ADB$



$c^2 = a^2 + b^2 - 2ab\cos C$

$(10\sqrt{2})^2 = (2h)^2 + \left(\frac{2h+20}{\sqrt{3}}\right)^2 - 2\times 2h \times \left(\frac{2h+20}{\sqrt{3}}\right)\frac{\sqrt{3}}{2}$

$200 = 4h^2 + \frac{4h^2}{3} + \frac{80h}{3} + \frac{400}{3} - 4h^2 - 40h$

$600 = 4h^2 + 80h - 120h + 400$

$200 = 4h^2 - 40h$

$4h^2 - 40h - 200 = 0$

$h^2 - 10h - 50 = 0$

$h = \frac{10 \pm \sqrt{10^2 - 4\times 50}}{2}$

$h = \frac{10 \pm \sqrt{300}}{2}$

$= \frac{10 \pm 10\sqrt{3}}{2}$

$\therefore h = 5 \pm 5\sqrt{3}$

End of questions.

$h > 0$

$\therefore h = 5 + 5\sqrt{3}$

2020 Year 11 Mathematics Advanced – Markers comments

Question 11

a->f) Apart from a couple of students, only silly errors were made.

g) i) Some students listed the types of medication errors, rather than answering how many ie 5.

ii) Quite a few students answered "dose missed" as it was the largest category, rather than the largest category they did not focus on.
Also, some students who got i) incorrect could not give correct follow on answer from their part i).
This question was not well done by many students.

h) A common error was leaving a gap between the columns with 10 ,', no gap.
Also, quite a few students had 5, 15, 25 on the horizontal axis, but didn't acknowledge that these were class centres.

Question 12

a) Generally well done.

Many students ignored final answers and rejected any answers that did not satisfy the equation.

b) Generally well done.

c) i) Generally well done.

ii) for f'(x) > 0, many students did not exclude x = 4 i.e. x is not equal to 4.

d) i) Many students did not understand "I is inversely proportional to R".

ii) Well done.

e) i) Well done.

ii) Most students ignore that A can't be negative.

iii) Well done.

Question 13

a) Generally well done. Most common mistake was forgetting the second solution. Some students gave a third solution that was not in the stated limits.

b) Mostly well done. To get both marks you had to either give the second angle, or specify that you had excluded it. A few students used $A = \frac{1}{2}AB\cos C$. You can check the formula on your reference sheet.

c) i) Mostly well done. Common mistakes were drawing: $y = 2\cos x$ or another variation other than $\cos 2x$. Many students did not label their axes!

ii) Students had difficulty if their graph in part i) was inaccurate.

d) Many mistakes and generally poorly done. Most common mistakes were missing the "equal to" part of $\geq, \leq$ or the square root signs. Also, many students treated this as a semicircle.

|

e) i) Generally well done. Mostly common mistake was not differentiating $(x^2 + 1)^7$ correctly using the chain rule. Many students didn't simplify fully.

ii) Mostly well done. Most common mistake was not substituting into the quotient rule correctly. Some students attempted to use the product rule, but used it incorrectly.

f) Not well done. Some students did not even attempt this question, which involved drawing two straight lines.

i) Plan your graph before you draw it. Many students choose scales that did not give them enough room to fit the whole graph. Some students did not draw their lines accurately or label their axes.

ii) Mostly well done. This had to match your graph above.

iii) Mostly well done. Students had to graph the section between the lines where income was above costs.

Question 14

a) i) Well done most students got the mark

ii) Generally well done, the two main errors were:

- Not squaring the $(x-6)$ factor,
- Writing (x^2-6) instead of $(x-6)^2$
- Not writing this as an equation, just writing down the factors.

iii) Again generally well done but the main errors were

- Horizontally dilating the function by a factor of 2 instead of a half.
- When dilating horizontally by a half, also dilating vertically, or not keeping the y-values un-dilated.

iv) Main errors were:

- Not using the language associated with the translation, but instead describing the process to arrive at the correct function. Such as "***Divide all the x-values by 2 and keep the y-values the same***".
- Using a factor of 2 instead of a factor of a half.

b) Most students completed this efficiently and accurately but the main errors were:

- The most common error was getting to the correct quadratic inequation for *k* but not solving that correctly (Mostly not using a graph)
- Getting the co-efficients of the *x*-term and constant term wrong,
- Simple algebraic errors.

c) Generally very well done with correct substitutions being made. And solving for both values.

- Some students didn't use a substitution and didn't reduce the equation to a quadratic so received no marks.

  Most students who made a correct substitution went on to gain full marks, however there were quite a few who made the following errors:

- When solving $\sqrt[3]{x} = -4$ stated that there were no solutions. They came to this conclusion a number of ways but the most common was to apply a log to both sides and then conclude that $\log(-4)$ has no value.

d) A lot of students lost some or all of the marks in this question. Most students were able to turn the equation into $\tan^2 x = 1$ or $\cos^2 x = \frac{1}{2}$. Students who could not get to this point did not earn any marks.

- The first errors was not taking the positive and negative square root, thus reducing the amount of solution

- Not using radians
- Not considering the domain
- Deciding which quadrant the solutions were in.

e) A lot of students used the wrong formula when using degrees for θ. They were able to arrive at the correct answer but through incorrect working.

f) Very well done, most students were able to achieve full marks for this.

Question 15

a) (i) Students needed to verify that the conditions at t=0 and t=2 were both satisfied by the equation given in the question. This required substitution into the equation to be shown.

(ii) Generally well done. Some students incorrectly confused 2×10^5 with $\left(2\times10\right)^5$.

b) As long as students were confident in using a method to answer this question, they generally were successful apart from some minor algebraic errors. Unfortunately, if students weren't sure of a method then they found it difficult to obtain any marks because they could not make a start.

c) Generally well done, although many students produced quite a convoluted proof. See the solution for the most efficient method.

d) (i) Question asked for expressions for AD and BD so these needed to be made the subject of any answer.

(ii) Many students incorrectly assumed that ABD was a right-angled triangle. Very few students were awarded 3 marks because, even if they correctly applied the cosine rule, they then made algebraic errors.