NORTH
SYDNEY
GIRLS HIGH
SCHOOL

2021

Preliminary Assessment Task 3

Mathematics Advanced

General Instructions

- Downloading/ printing time – 5 minutes
- Working Time – 60 minutes
- Scanning and uploading time – 30 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided

Total marks – 41

- Attempt Questions 1-3
- Start each question on a new page.
- Show relevant mathematical reasoning and/or calculations

NAME:______________________________ **TEACHER:**____________________

STUDENT NUMBER:__________________________

Question	1	2	3	Total
Introduction to Differentiation	/6	/8	/4	**/18**
Applications of Functions	/3	/6	/5	**/14**
Probability	/5		/4	**/9**
Total	/14	/14	/13	**/41**

41 marks
Attempt Questions 1-3

Start each question on a new page.
Write your student number on the top of each page.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 1 (14 marks) **START A NEW PAGE**

(a) Differentiate $4x^5 - \dfrac{3}{x^2}$. **2**

(b) (i) Find the equation of the normal to the curve $y = (2x+1)^4$ at the point where $x = -1$. Leave your answer in general form. **3**

(ii) Find the angle the normal makes with the positive x-axis. Answer to the nearest degree. **1**

(c) At my local printing shop, it costs me \$39 for printing 600 sheets and \$51 for printing 800 sheets.

(i) Assuming that the relationship is linear, find a formula for the charge \$$C$ for printing n sheets. **2**

(ii) What is the charge for printing 1025 sheets? **1**

(d) A fruit bowl contains three apples and five oranges.
Two pieces of fruit are picked from the bowl at random without replacement.

(i) Draw a probability tree diagram to represent this situation. **1**

(ii) Find the probability that at least one apple is chosen. **1**

(iii) What is the probability that exactly one apple is chosen? **1**

(iv) Given that at least one apple was chosen, what is the probability that the second fruit chosen was an apple? **2**

Question 2 (14 marks) **START A NEW PAGE**

(a) Evaluate $\lim_{x \to 2}\left(\dfrac{x^2 - 3x + 2}{x - 2}\right)$. **2**

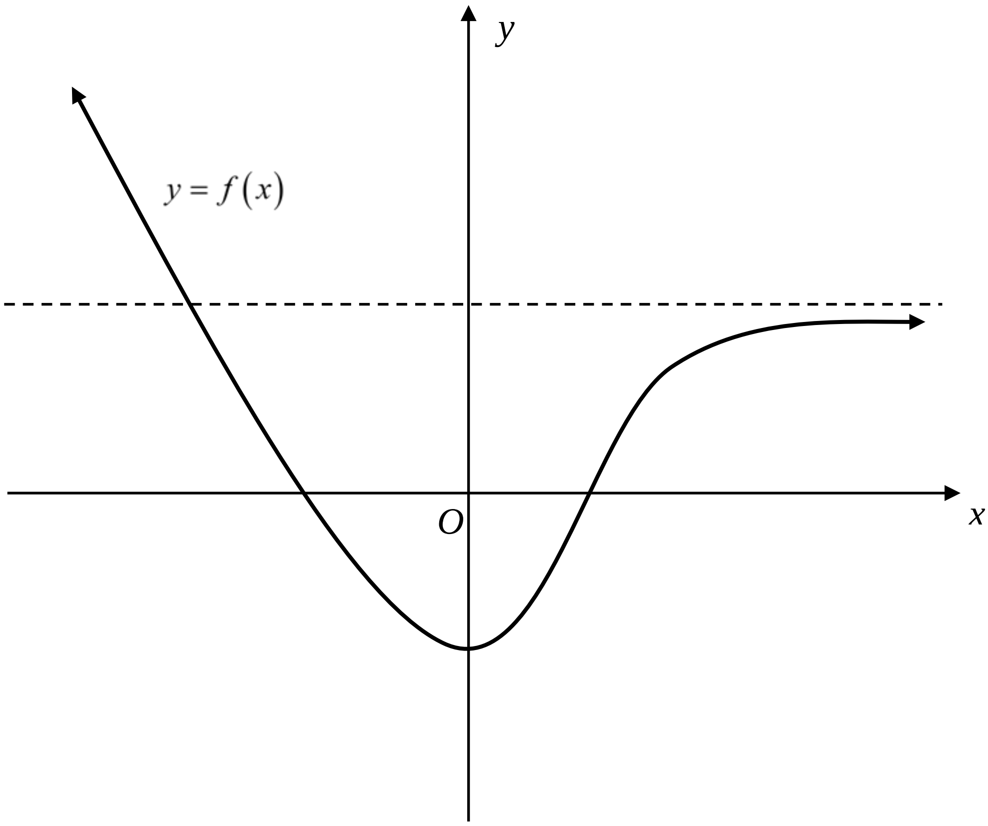
(b) Given that $f(x) = (5x+1)^3$ and $g(x) = x^2$:

(i) Find the derivative of $h(x)$, where $h(x) = f(x)g(x)$. **2**

(ii) Find the x-coordinate(s) of the point(s) where the tangent to $y = h(x)$ is horizontal. **2**

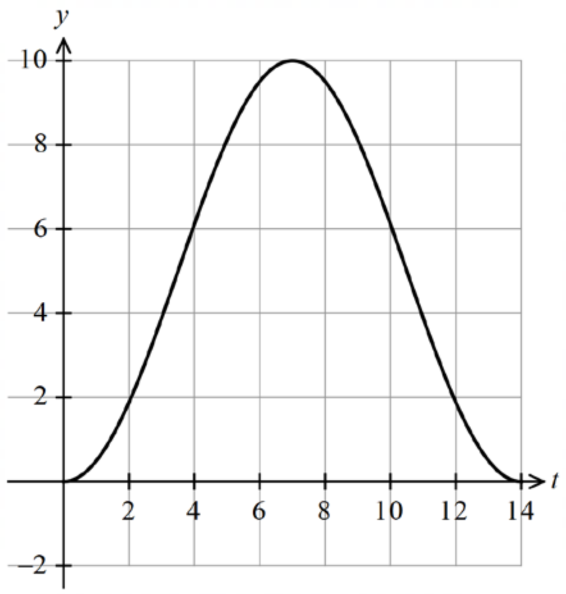
(c) The graph of $y = f(x)$ is given below. **2**

Sketch the gradient function $y = f'(x)$, clearly indicating its features at $x = 0$ and the shape of the graph as $x \to \infty$.



Question 2 continues on Page 4

(d) The UV index for a summer day in Melbourne is illustrated in the graph below, where t is the number of hours after 6am.



The UV index y, can be modelled by the equation $y = a\cos(bt) + c$.

(i) Find the values of a, b and c. **3**

(ii) A UV index greater than 7 is considered Very High. **3**
Find the times in the day when the UV index is greater than 7.
Answer to the nearest minute.

End of Question 2

Question 3 (13 marks) **START A NEW PAGE**

(a) Differentiate $f\left(x\right)=\dfrac{\sqrt{1-x}}{x+1}$. Fully simplify your answer. **2**

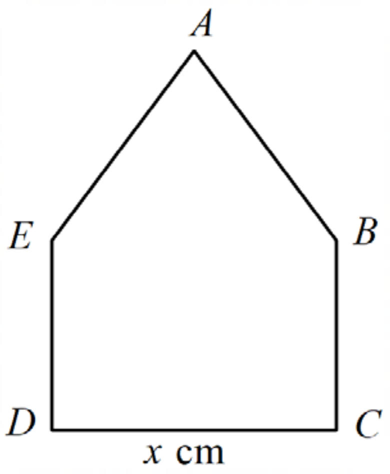
(b) Given that $f'\left(x\right)=\lim\limits_{h\to 0}\dfrac{f\left(x+h\right)-f\left(x\right)}{h}$, find the derivative of $f\left(x\right)=x^2+3x$ from first principles. **2**

(c) The probabilities of two independent events A and B are given by $P\left(A\right)=0.7$ and $P\left(B\right)=0.4$. Find $P\left(A\cup B\right)$. **2**

(d) Consider the curve $y=x^2+bx+12$. If b is an integer value chosen randomly from the interval $\left[-10,10\right]$, find the probability that $y\le 0$ for one or more values of x. **2**

Question 3 continues on Page 6

(e) A thin wire of length a cm is bent to form an irregular pentagon $ABCDE$ as shown. $BCDE$ is a rectangle and ABE is an equilateral triangle.
Let the length of CD be x cm and $A(x)$ be the area of the pentagon.



(i) Explain why the length of BC is $\dfrac{a-3x}{2}$. **1**

(ii) Show that $A(x)=\dfrac{x}{4}\left(2a-\left(6-\sqrt{3}\right)x\right)$. **2**

(iii) Find the maximum possible area of the pentagon as an exact value. **2**

End of paper

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Question 1

a) $y = 4x^5 - \frac{3}{x^2}$

$= 4x^5 - 3x^{-2}$

$\frac{dy}{dx} = 20x^4 - (-6)\, x^{-3}$

$= 20x^4 + \frac{6}{x^3}$ 1 mark for each term

b) i) $y = (2x+1)^4$

$\frac{dy}{dx} = 4(2x+1)^3 \times 2$

$= 8(2x+1)^3$ ✓

At $x = -1$ $\frac{dy}{dx} = 8(-2+1)^3$

$= -8$

$\therefore$ gradient of normal is $\frac{1}{8}$ ✓

when $x = -1$, $y = 1$

$\therefore$ eqn of normal is

$y - 1 = \frac{1}{8}(x+1)$

$8y - 8 = x + 1$

$x - 8y + 9 = 0$ ✓

ii) Let the angle made with the +ve x-axis be θ.

$\tan\theta = \frac{1}{8}$

$\theta = 7°7'30''$

$\theta = 7°$ (nearest degree) ✓

c) i) $n = 600 \quad C = 39$

$n = 800 \quad C = 51$

linear relationship $\therefore C = mn + b$

$\therefore \quad m = \dfrac{51 - 39}{800 - 600} = \dfrac{3}{50}$ or 0.06 ✓

$39 = \dfrac{3}{\cancel{50}} \times \overset{12}{\cancel{600}} + b$

$b = 39 - 36$

$= 3$

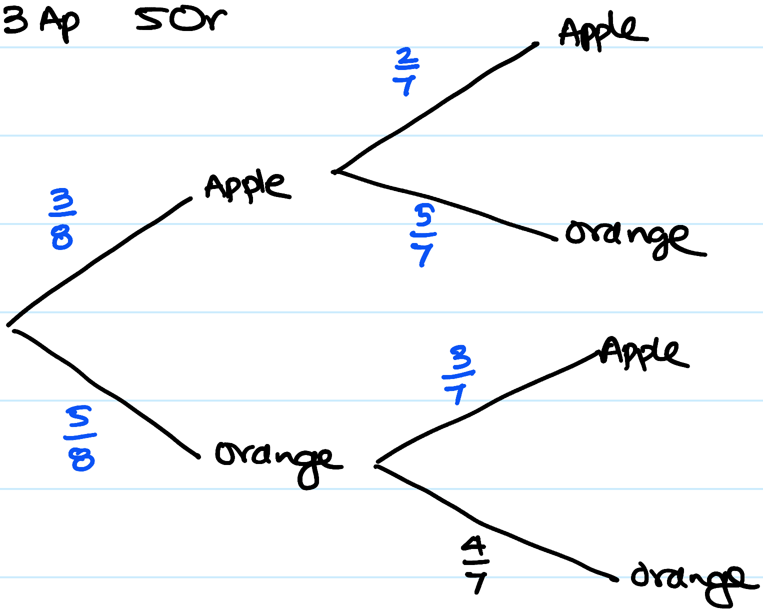
$\therefore \quad C = 0.06n + 3$ ✓

(ii) $n = 1025$ then $C = 0.06 \times 1025 + 3$

$= 64.5$

It costs \$64.50 to print 1025 sheets. ✓

d) i) 3 Ap 5 Or



ii) Prob (at least 1 apple) $= 1 - P(\text{no apple})$

$= 1 - P(OO)$

$= 1 - \frac{5}{8} \times \frac{4}{7}$

$= \frac{9}{14}$

iii) Prob (exactly 1 apple) $= P(AO) + P(OA)$

$= \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7}$

$= \frac{15}{28}$

iv) Prob (second fruit apple | at least 1 apple)

$$= \frac{P(OA) + P(AA)}{P(\text{at least 1 apple})}$$

$$= \frac{\frac{3}{8} \times \frac{2}{7} + \frac{5}{8} \times \frac{3}{7}}{\frac{9}{14}}$$

$$= \frac{7}{12}$$

Question 1

Part a

Very well done.

Part b

i) Well done. A few students found the equation of the tangent instead of the normal and are advised to read the question carefully. Many students did not leave their answer in general form as required in the question.

ii) Well done. Some students incorrectly had $\tan\theta = -\frac{1}{8}$ and then had $\theta = -7°$. They had trouble converting this to an angle between the normal and the positive x-axis.

Part c

i) Quite a few students used the linear equation $C = nx$ rather than $C = nx + b$. Other students incorrectly worked out the cost of printing one sheet by dividing $39 by 600 instead of finding the gradient of (600, 39) and (800, 51). They ignored the fact that there was a fixed cost as well as a per sheet cost.

ii) Very well done.

Part d

i) Well done

ii) Most students correctly used the complement (1-P(O,O)) to find this.

iii) Well done.

iv) Some students found this difficult. Some either calculated the numerator or denominator correctly but not both. Conditional probability was not well understood by many students.

Question 2

a) $\lim_{x \to 2} \left(\frac{x^2-3x+2}{x-2}\right) = \lim_{x \to 2} \frac{\cancel{(x-2)}(x-1)}{\cancel{x-2}}$ ✓

$= \lim_{x \to 2} (x-1)$

$= 2-1$

$= 1$ ✓

b) i) $f(x) = (5x+1)^3 \qquad g(x) = x^2$

$h(x) = f(x)\, g(x)$

$f(x) = (5x+1)^3 \qquad g(x) = x^2$

$f'(x) = 15(5x+1)^2 \qquad g'(x) = 2x$ ✓

$h'(x) = f'g + fg'$ Product Rule

$= 15(5x+1)^2 \times x^2 + (5x+1)^3 \times 2x$

$= 15x^2(5x+1)^2 + 2x(5x+1)^3$ ✓

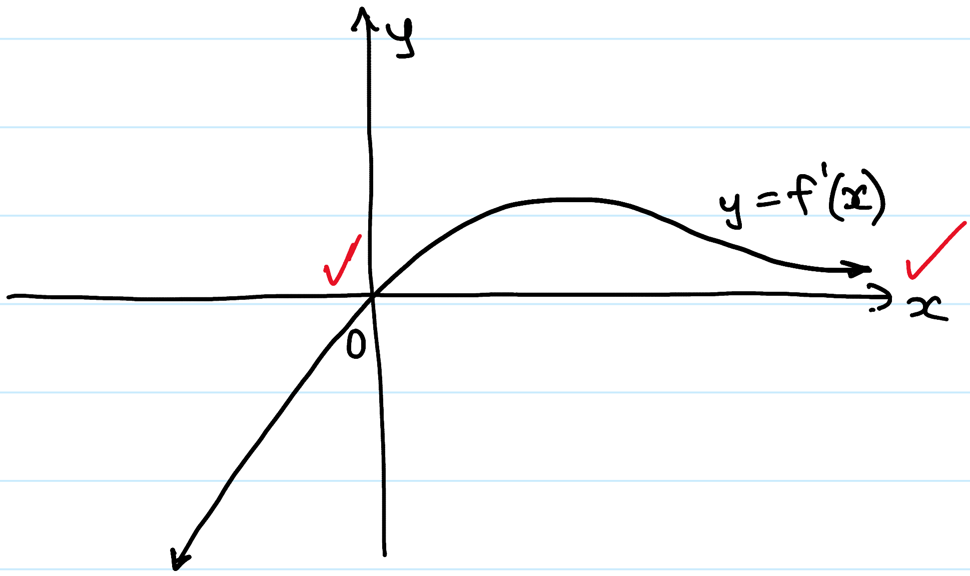
ii) $h'(x) = x(5x+1)^2 [15x + 2(5x+1)]$

$= x(5x+1)^2 (25x+2)$ ✓

$h'(x) = 0$ where the tangent is horizontal

$h'(x) = 0 \Rightarrow x = 0, -\frac{1}{5}, -\frac{2}{25}$ ✓

c)



d) i) $a = -5$ vertical dilation

$b = \frac{\pi}{7}$ horizontal dilation

$c = \frac{10+0}{2} = 5$ vertical shift

ii) solving $y=7$ $\quad -5\cos\left(\frac{\pi t}{7}\right) + 5 = 7$

$$\cos\frac{\pi t}{7} = -\frac{2}{5}$$

$$\frac{\pi t}{7} = 1.9823,\ 4.3009\ (4dp)$$

$$t = 4.4169,\ 9.5831\ (4dp)$$

$$= 4h\ 25m,\ 9h\ 35m$$

$\therefore$ the UV index is greater than 7 between 10:25 am and 3:35 pm

Question 2

Part a

Well done. A few incorrect factorisations but nearly all found a limit correctly.

Part b

(i) Students received 2/2 if they showed the correct application of the product rule. There was no deduction of marks if they went on to simplify and made an error. However, errors made in their factorisation impacted on their ability to solve part (ii).

(ii) Students could not receive full marks here if they had an incorrect, simpler equation to solve due to errors from part (i). Too many students were expanding their derivative from part (i). The link of factorising fully then solving a polynomial equal to zero was not understood by a significant number of students.

Part c

Reasonably well done except with how f'(x) approaches 0+. Greater care must be taken that it is not drawn parallel to the x axis as it will imply a constant gradient (Which it isn't). Also have the curve of f'(x) **approaching** the asymptote needs some greater attention to detail. Be aware that gradient function had only **one** stationary point, so could only cut or touch the x axis **once** in your drawing.

Part d

(i) This was not done very well overall. Students should have been able to quickly find the values of a, b and c by just looking at the graph drawn and doing the appropriate transformations as well as applying Period = 2pi/b. Most used the method of solving 3 equations simultaneously using 3 distinct points of substitution. This often led to errors and misinterpretation of answers. Also very time consuming.

(ii) Many CFPE (carried over previous errors) and students could not gain full marks if there incorrect values of a, b, and c simplified the equation to solve from what it was meant to be. Students should have stayed in radians as this just complicated the question by converting to degrees and then became confused trying to understand what this is as a time. Too many didn't finish the question by stating the times of the day when the UV index was above 7. Most just solved the trig equation and therefore couldn't get the extra mark for the actual times of day. Students need to be reminded that if they can't see a degrees symbol then it is implied as radians.

Question 3

a) $y = \dfrac{\sqrt{1-x}}{x+1}$

$u = (1-x)^{\frac{1}{2}}$ $\quad$ $v = x+1$

$u' = -\frac{1}{2}(1-x)^{-\frac{1}{2}}$ $\quad$ $v' = 1$

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2}$$

$$= \frac{-\frac{1}{2}(1-x)^{-\frac{1}{2}}(x+1) - (1-x)^{\frac{1}{2}} \times 1}{(x+1)^2} \checkmark$$

$$= \frac{-\frac{1}{2}(1-x)^{-\frac{1}{2}}\left[(x+1) + 2(1-x)\right]}{(x+1)^2}$$

$$= \frac{-(3-x)}{2(1-x)^{\frac{1}{2}}(x+1)^2}$$

$$= \frac{(x-3)}{2(x+1)^2\sqrt{1-x}} \checkmark$$

b) $f(x) = x^2 + 3x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - (x^2 + 3x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h - \cancel{x^2} - \cancel{3x}}{h}$$ ✓

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 3)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x + h + 3)$$

$$= 2x + 3$$ ✓

c) $P(A) = 0.7$ $\quad P(B) = 0.4$

$P(A \cap B) = 0.7 \times 0.4$ $\quad$ (A & B are independent) ✓

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.4 - 0.7 \times 0.4$

$= 0.82$ ✓

d) If $y \leq 0$ then the curve $y = x^2 + bx + 12$ will touch or cross the axis ie. $\Delta \geq 0$

$\Delta = b^2 - 48$

$\Delta \geq 0$ $\quad\quad b^2 \geq 48$

$\quad\quad b < -\sqrt{48}$ or $b > \sqrt{48}$ ✓

$\sqrt{48} \doteq 6.92$

As b is an integer betn -10 and 10

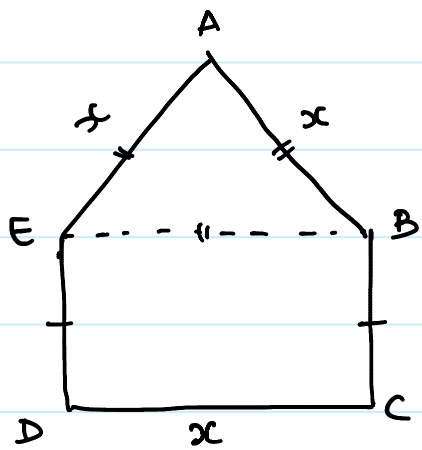
possible values of b such that $\Delta \geq 0$

are $-10, -9, -8, -7, 7, 8, 9, 10$

Total # values of b $= 21$ $(-10$ to 10 incl $0)$

$\therefore$ required probability $= \dfrac{8}{21}$ ✓

e)



$CD = BE = x$ (rectangle)

$\therefore$ $AE = AB = BE = x$ (equilateral triangle)

$BC + DE + 3x = a$ (perimeter)

$2BC + 3x = a$ (as $BC = DE$)

$\therefore$ $BC = DE = \dfrac{a - 3x}{2}$ ✓

continued.

e) $A(x) = \text{Area } BCDE + \text{Area } ABE$

$$= x\left(\frac{a-3x}{2}\right) + \frac{1}{2}x^2 \sin 60^\circ \quad ✓$$

$$= \frac{x(a-3x)}{2} + \frac{x^2\sqrt{3}}{4}$$

$$= \frac{x}{4}\left(2(a-3x) + x\sqrt{3}\right)$$

$$= \frac{x}{4}\left(2a - 6x + x\sqrt{3}\right)$$

$$= \frac{x}{4}\left(2a - (6-\sqrt{3})x\right) \quad ✓ \text{ as reqd}$$

(iii) $A(x)$ is a concave down parabola

$\therefore$ maximum value occurs at the vertex.

vertex is half-way between the intercepts

intercepts at $x=0$ and $x = \dfrac{2a}{6-\sqrt{3}}$

$\therefore$ max at $x = \dfrac{a}{6-\sqrt{3}}$ ✓

$$A_{max} = \frac{a}{4(6-\sqrt{3})}\left(2a - \cancel{(6-\sqrt{3})}\frac{a}{\cancel{6-\sqrt{3}}}\right)$$

$$= \frac{a^2}{4(6-\sqrt{3})} \quad ✓$$

$$A_{max} = \frac{a^2(6+\sqrt{3})}{132} \text{ cm}^2$$

Question 3

Part a

Generally, not well done. First mark for setting up quotient rule or product rule. Second mark for a properly simplified solution.

- Many students did not end up with a negative when differentiating $\sqrt{1-x}$.
- Only some students were able to simplify the resulting algebraic fraction. Those that used the product rule instead of the quotient rule had more difficulty, although it was possible.

Part b

Generally, well done. First mark for correctly expanding and replacing function notation. Second mark for correctly cancelling to obtain the final answer.

- You should always expect the f(x) term to cancel completely and you also know from normal differentiation techniques what the final answer should be: So, if you end up with a different result at the end it should be easy to identify whether you made errors.

Part c

Many students got full marks. First mark for finding the intersection, correctly labelled, $P(A\cap B)$. Second for applying the formula and calculating 0.82.

- Many students simply multiplied the probabilities together and they should review the difference between $P(A\cup B)$ and $P(A\cap B)$.
- Also, many students added the probabilities together, giving a total probability greater than one! This should have notified you that you had done something wrong.

Part d

Generally, not well done. First mark for correctly working with the discriminant. Second mark for the correct final expression. There was also an alternate solution some students tried using the vertex, which resulted in the same inequality.

- Many people saw the $y \leq 0$, assumed that meant negative definite and looked for $\Delta < 0$. However, the parabola is concave up and the question was instead asking when is it <u>not</u> positive-definite.
- Several people formed the discriminant and then did the subsequent calculations wrong.
- Many people did not handle the inequality properly, especially when taking the square root.
- $4\times 12 = 36$ was surprisingly common, if you are prone to this sort of error, you should make a habit of double checking your answers.

Part e

(i) Poorly done. This was a relatively straightforward communication mark. The answer is given and you needed to explain how to arrive at it without missing any reasons.

(ii) Generally, well done. One mark for the correct expressions for the area of the rectangle and triangle. Subsequent mark for correctly working towards the answer.

(iii) Many students lost marks here. First mark for finding the x-coordinate of the vertex, second for subbing that into the area and correctly working towards the solution. Easiest method involved using the axis of symmetry of the parabola, however finding the derivative also worked. Many students lost marks for incorrect algebra and are advised to focus on laying out working in a logical manner so it is easier to work with.