PENRITH HIGH SCHOOL

2015
PRELIMINARY YEARLY EXAMINATION

Mathematics

General Instructions:

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Answer all Questions in the booklets provided

Total marks–70

SECTION I **Pages 3–5**

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

SECTION II **Pages 6–9**

60 marks

- Attempt Questions 11–14
- Allow about 1 hours 45 minutes for this section

Student Name:_____ Teacher Name:_____

This paper MUST NOT be removed from the examination room

Assessor: R Young

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the provided multiple–choice answer sheet for Questions 1–10

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1. What is the value of $\frac{\pi^2}{6}$, correct to 3 significant figures?

   (A) 1.64

   (B) 1.65

   (C) 1.644

   (D) 1.645

2. What are the solutions of $2x^2 - 8x - 3 = 0$?

   (A) $x = \frac{-8 \pm \sqrt{40}}{4}$

   (B) $x = \frac{8 \pm \sqrt{40}}{4}$

   (C) $x = \frac{7 \pm \sqrt{88}}{4}$

   (D) $x = \frac{-7 \pm \sqrt{88}}{4}$

3. $x^3 - 27$ in factorised form is

   (A) $(x + 3)(x^2 + 3x + 9)$

   (B) $(x - 3)(x^2 + 3x + 9)$

   (C) $(x - 3)(x^2 + 6x + 9)$

   (D) $(x + 3)(x^2 + 6x + 9)$

4. Which of the following is true?

(A) An even function has a point of symmetry at the origin.

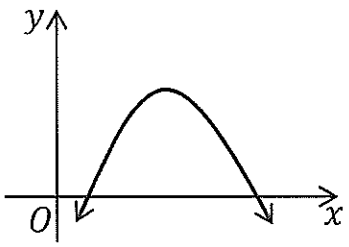
(B) An even function is symmetrical about the x – axis.

(C) An odd function has point symmetry about the origin.

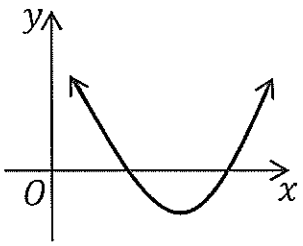
(D) An odd function is symmetrical about the y – axis.

5. Which graph represents a positive definite function.

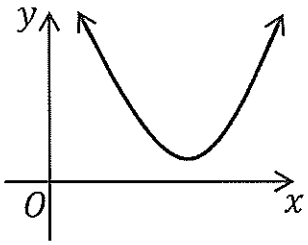
(A)



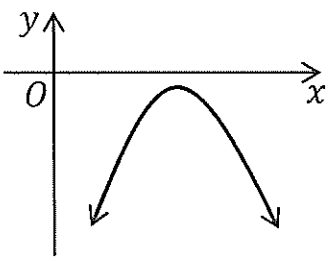
(B)



(C)



(D)



6. Which statement defines the domain of the function $f(x) = \sqrt{4 - x^2}$?

(A) $x < 4$

(B) $-4 \leq x \leq 4$

(C) $-2 \leq x \leq 2$

(D) $-2 < x < 2$

7. The exact value of $\cos(-135°)$ is:

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{-1}{\sqrt{2}}$

(C) $-0 \cdot 707$

(D) $0 \cdot 707$

8. If $sec\theta = \frac{-5}{4}$ and $tan\theta > 0$ then,

(A) $sin\theta = \frac{3}{5}$

(B) $sin\theta = -\frac{3}{5}$

(C) $cos\theta = \frac{4}{5}$

(D) $tan\theta = -\frac{3}{5}$

9. The equation of the normal to the curve $y = x^2 + 2x$ at the point where $x = 2$

(A) is parallel to the x – axis.

(B) is perpendicular to the x – axis.

(C) cuts the x - axis at a positive value of x.

(D) cuts the x - axis at a negative value of x.

10. The derivative of $f(x) = \frac{1}{x}$ at $x = 0$,

(A) is positive

(B) is negative

(C) $\pm\infty$

(D) does not exist

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 11–14, your responses should include relevant mathematical reasoning and/or calculations

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Question 11 (15 marks) Use a new writing booklet.

a) Rationalise the denominator of $\frac{1}{\sqrt{5}-3}$ 2

b) Factorise $2x^2 + 5x - 3$ 2

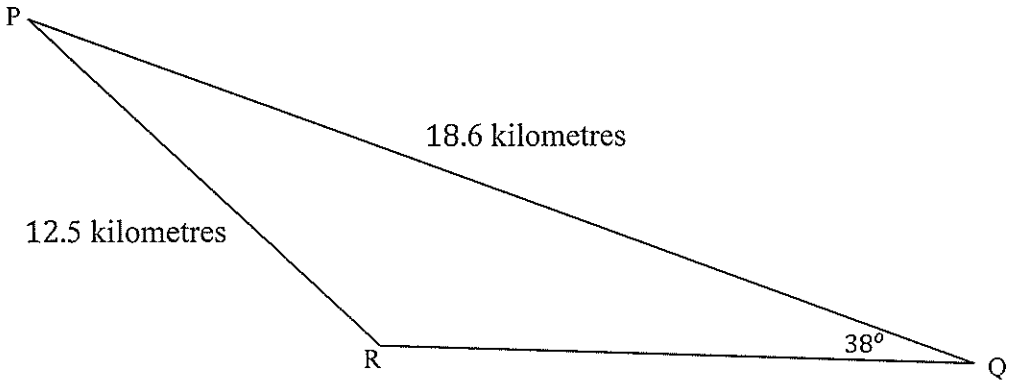
c) Solve for x, $|3x + 2| \leq 3$ 2

d) Express $\frac{2x}{x^2+2x-3} + \frac{2}{x-1}$ as a single fraction in simplest form. 2

e) Shade the region that satisfies $y > 3x - 1$ and $y \leq 5$ simultaneously. 3

f) Prove that $tan\theta + cot\theta = cosec\theta sec\theta$. 2

g)

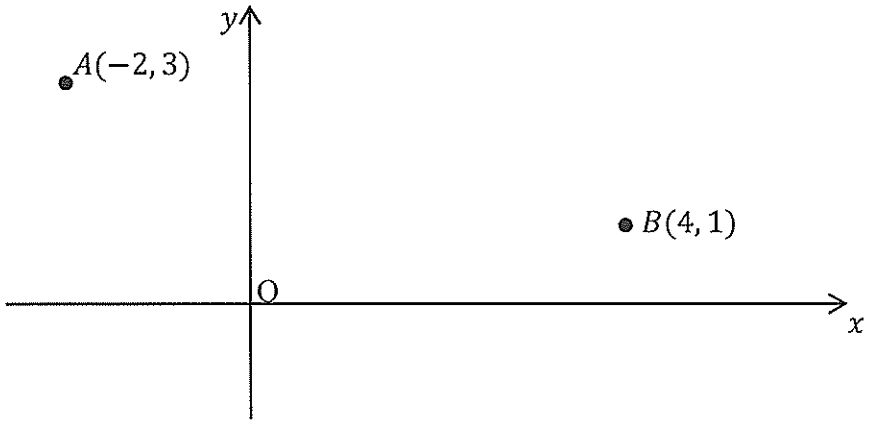


In the diagram above, $PQ = 18.6$ kilometres, $PR = 12.5$ kilometres and $\angle PQR = 38°$. $\angle PRQ$ is obtuse. Find the size of $\angle PRQ$ correct to the nearest minute. 2

Question 12 (15 marks) Use a new writing booklet.

In the diagram above, A, B and O are the points $(-2, 3)$, $(4, 1)$ and $(0, 0)$ respectively.

a)



i. Show that the equation of AB is $x + 3y - 7 = 0$ 1

ii. Find the distance AB 1

iii. Find M, the midpoint of AB 1

iv. Show that the equation of the line through M, perpendicular to AB is $3x - y - 1 = 0$. 1

v. Hence find D the point of intersection with the y axis of the line in part iv. 1

vi. Hence find the area of the triangle BAD. 2

b) If α and β are the roots of the equation $2x^2 - 3x + 7 = 0$, find:

i. $\alpha + \beta$ and $\alpha\beta$ 1

ii. $\alpha\beta - \alpha - \beta + 1$ 1

c) For what values of k does the equation $4x^2 - 4x + k = 0$ have real roots? 2

d) Solve $3^{2x} - 4.3^x + 3 = 0$ 2

e) If one root of $3x^2 - 8x + k = 0$ is three times the other root, find k. 2

Question 13 (15 marks) Use a new writing booklet.

a) A parabola is defined by $x^2 = 12y$.

i. What is the coordinates of the focus? 1

ii. What is the equation of the directrix? 1

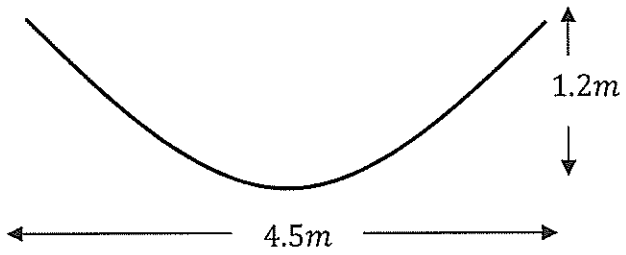
b) Find the centre and radius of the circle $x^2 - 4x + y^2 - 2y - 20 = 0$. 2

c) A point $P(x, y)$ moves so that AP is perpendicular to PB. Given $A = (2, 1)$ and $B = (-3, 6)$, find the of the locus of P. 3

d) Find the equation of the parabola with vertex $(2, 4)$ and focus $(-3, 4)$. 2

e) Find the equation of the locus of a point that is always 3 units from the line $3x + 4y - 1 = 0$. 2

f) A satellite dish is to be 4.5 m wide and 1.2 m deep. How far from the vertex is the focus? 2



g. Find the equation of the locus of the midpoints of all chords of length 4 units of the circle with equation $x^2 + y^2 - 4x + 2y = 4$. 2

Question 14 (15 marks) Use a new writing booklet.

a) Differentiate the following with respect to x.

i. $2x^2 - 7x + 1$ 1

ii. $\dfrac{x^2 + 3x + 6}{x}$ 2

iii. $\dfrac{5}{\sqrt[3]{x}}$ 2

b) Differentiate $f(x) = x^2 + 4x$ from first principles. 2

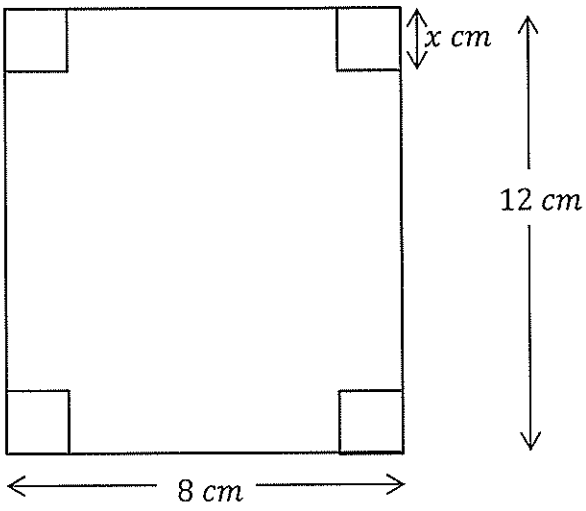
c) Find the value of $\lim\limits_{x\to\infty} \dfrac{3x^2}{7x^2+1}$ 2

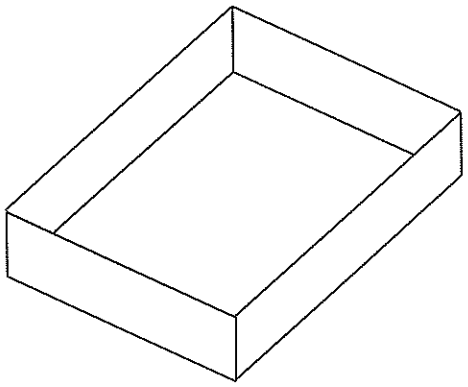
d) A function is defined by $f(x) = x^3 - 3x^2 - 9x + 22$.

i. Find the coordinates of the stationary points and determine their nature. 2

ii. Sketch the graph of $y = f(x)$ showing the turning points and where the curve meets the y axis. 1

e) An open rectangular prism is to be made from a flat sheet of metal $12cm \times 8cm$. Squares of side $x$cm are cut from each corner then the metal is bent to form the prism.





i. Show that the volume of the prism is given by $V = 4x^3 - 40x^2 + 96x$. 1

ii. Find the value of x that maximises the volume of the prism. Leave your answer in exact form. 2

End of paper

YEAR 11 YEARLY MATHEMATICS SOLUTIONS

Multiple Choice

1.A 2.B 3.B 4.C 5.C 6.C 7.B 8.B 9.C 10.D

Question 11

a. $\frac{1}{\sqrt{5}-3} \times \frac{\sqrt{5}+3}{\sqrt{5}+3}$

$= \frac{\sqrt{5}+3}{5-9}$

$= \frac{\sqrt{5}+3}{-4}$ or $\frac{-5-3}{4}$

b. $(2x-1)(x+3)$

c. $3x+2 \leq 3$ $\quad 3x+2 \geq -3$

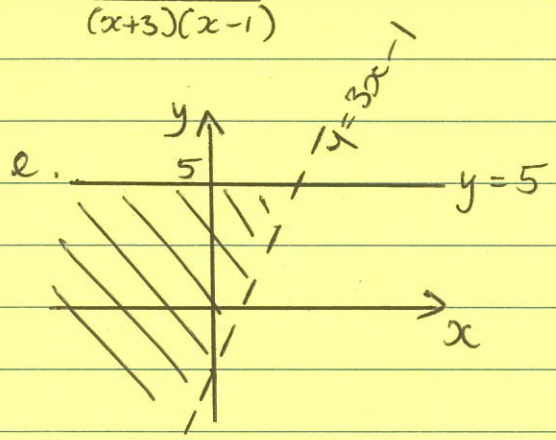
$3x \leq 1$ $\quad 3x \geq -5$

$x \leq \frac{1}{3}$ $\quad x \geq -\frac{5}{3}$

d. $\frac{2x}{(x+3)(x-1)} + \frac{2(x+3)}{(x-1)(x+3)}$

$= \frac{4x+6}{(x+3)(x-1)}$

e.



f. $LHS = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta}$ $= \operatorname{cosec}\theta\sec\theta$

$= RHS$

g. $\frac{\sin\angle PRQ}{18.6} = \frac{\sin 38^\circ}{12.5}$

$\sin\angle PRQ = \frac{18.6\sin 38}{12.5}$

$\sin\angle PRQ = 0.9161\ldots$

$\angle PRQ = 66.36^\circ$ or 113.63°

obtuse $\angle PRQ = 113^\circ 38'$

Question 12

ai. $m = \frac{3-1}{-2-4} = \frac{-1}{3}$

$y-1 = -\frac{1}{3}(x-4)$

$3y-3 = -x+4$

$x+3y-7=0$

ii. $AB = \sqrt{(1-3)^2+(4+2)^2}$

$= \sqrt{4+36}$

$= \sqrt{40} = 2\sqrt{10}$

iii. $M: \quad x = \frac{-2+4}{2} = 1$

$y = \frac{3+1}{2} = 2 \quad M(1,2)$

iv $m_\perp = 3 \quad y-2 = 3(x-1)$

$y-2 = 3x-3$

$3x-y-1=0$

Question 12 continued

av. $x=0 \quad 0-y-1=0$

$y=-1$

$\therefore D(0,-1)$

vi. distance $= \sqrt{(2+1)^2+(1-0)^2}$

$= \sqrt{10}$

Area $= \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10}$

$= 10 \text{ units}^2$

bi. $\alpha+\beta = \frac{3}{2} \quad \alpha\beta = \frac{7}{2}$

ii. $\alpha\beta - (\alpha+\beta) + 1$

$= \frac{7}{2} - \frac{3}{2} + 1$

$= 3$

c. $\Delta \geq 0$

$\Delta = b^2 - 4ac$

$4^2 - 4.4.k \geq 0$

$16(1-k) \geq 0$

$1 \geq k$

$k \leq 1$

d. $(3^x)^2 - 4(3^x) + 3 = 0$

let $m = 3^x$

$m^2 - 4m + 3 = 0$

$(m-3)(m-1) = 0$

$m = 3, 1$

$3^x = 3 \quad 3^x = 1$

$x = 1, 0$

e. roots α and 3α

$4\alpha = \frac{8}{3} \quad 3\alpha^2 = \frac{k}{3}$

$\alpha = \frac{2}{3} \quad 3\left(\frac{2}{3}\right)^2 = \frac{k}{3}$

$k = 4$

Question 13

ai. $x^2 = 12y \quad x^2 = 4ay$

$= 4 \times 3y$

focus $(0,3)$

ii. $y = -3.$

b. $x^2 - 4x + 4 + y^2 - 2y + 1 = 25$

$(x-2)^2 + (y-1)^2 = 25$

centre $(2,1)$ radius $= 5$

c. $m_{PA} = \frac{y-1}{x-2} \quad m_{PB} = \frac{y-5}{x+3}$

$m_{PA} \times m_{PB} = -1$

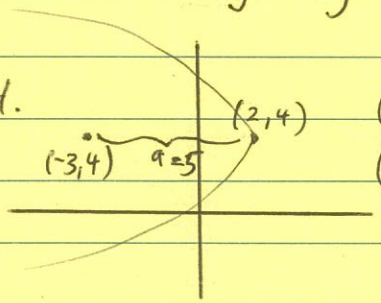
$\frac{y-1}{x-2} \times \frac{y-5}{x+3} = -1$

$y^2 - 6y + 5 = -(x^2 + x - 6)$

$y^2 - 6y + 5 = -x^2 - x + 6$

$x^2 + x + y^2 - 6y - 1 = 0$

d.



$(y-4)^2 = -4.5(x-2)$

$(y-4)^2 = -20(x-2)$

e. $PA = 2PB$

$\sqrt{(x+2)^2+y^2} = 2\sqrt{(x-1)^2+y^2}$

$x^2 + 4x + 4 + y^2 = 4[x^2 - 2x + 1 + y^2]$

$0 = 3x^2 - 12x + 3y^2$

$0 = x^2 - 4x + y^2$

Question 13 continued

f. $x^2 = 4ay \quad (2,4)$

$(2)^2 = 4a(2)$

$4 = 8a$

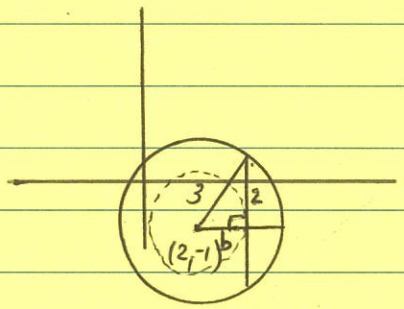
$a = \frac{1}{2}$

g. $x^2 - 4x + y^2 + 2y = 4$

$x^2 - 4x + 4 + y^2 + 2y + 1 = 9$

$(x-2)^2 + (y+1)^2 = 9$

centre $(2,-1)$ radius $= 3$.



$b^2 = 3^2 - 2^2$

$= 9 - 4$

$b = \sqrt{5}$

∴ locus is circle with centre $(2,-1)$ and radius $\sqrt{5}$

$(x-2)^2 + (y+1)^2 = 5$

Question 14

a.i. $4x - 7$

ii. $y = x + 3 + 6x^{-1}$

$y' = 1 - 6x^{-2}$

iii. $y = 5x^{-\frac{1}{3}}$

$y' = -\frac{5}{3}x^{-\frac{4}{3}}$

b. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 4(x+h) - (x^2 + 4x)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 4x + 4h - x^2 - 4x}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 + 4h}{h}$

$= \lim_{h \to 0} 2x + h + 4$

$= 2x + 4$

c. $\lim_{x \to \infty} \frac{\frac{3x^2}{x^2}}{\frac{7x^2}{x^2} + \frac{1}{x^2}}$

$= \lim_{x \to \infty} \frac{3}{7 + \frac{1}{x^2}}$

$= \frac{3}{7 + \frac{1}{\infty^2}} = \frac{3}{7}$

di. $f(x) = x^3 - 3x^2 - 9x + 22$

$f'(x) = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$0 = (x-3)(x+1)$

$x = 3, -1$

$f(3) = -5 \quad f(-1) = 27$

$(3,-5) \quad (-1,27)$

x	-2	-1	0
$f'(x)$	15	0	-9

↗ ↔ ↘

$(-1,27)$ max

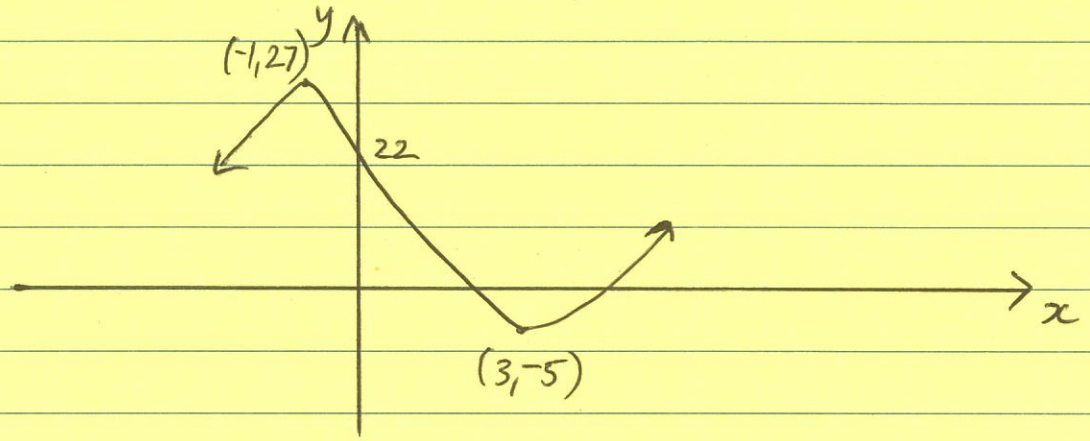
x	2	3	4
$f'(x)$	-9	0	15

↘ ↔ ↗

$(3,-5)$ min

Question 14 continued

dii. $f(x) = x^3 - 3x^2 - 9x + 22$



ei. $V = x(8-2x)(12-2x)$

$= (8x - 2x^2)(12-2x)$

$= 96x - 40x^2 + 4x^3$

ii. $\frac{dV}{dx}$ $= 12x^2 - 80x + 96$

$4(3x^2 - 20x + 24) = 0$

$x = \frac{20 \pm \sqrt{20^2 - 4.3.24}}{6}$

$= \frac{20 \pm \sqrt{112}}{6}$

$= \frac{10 \pm 2\sqrt{7}}{3}$

test x	5	$\frac{10+2\sqrt{7}}{3}$	6
$\frac{dV}{dx}$	-1 ↘	0 ↔	12 ↗

x	1	$\frac{10-2\sqrt{7}}{3}$	2
$\frac{dV}{dx}$	7 ↗	0 ↔	-4 ↘

∴ max volume when $x = \frac{10-2\sqrt{7}}{3}$