



# 2022

## YEAR 11 ASSESSMENT TASK 3

# Mathematics Advanced

### General Instructions

- Reading time – 10 minutes
- Working time – 120 minutes
- Black pen is preferred.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.
- Calculators approved by NESA may be used
- DO NOT USE LIQUID PAPER OR CORRECTION TAPE

### Total Marks – 70

#### Section I: 10 marks

- Attempt questions 1-10, using the answer sheet provided.
- Allow about 15 minutes for this section

#### Section II: 60 marks

- Attempt questions 11 - 33, answer the questions in the booklets provided.
- Allow about 105 minutes for this section

|                               | Multiple Choice   | Section 2 marks                      | TOTAL |
|-------------------------------|-------------------|--------------------------------------|-------|
| Algebraic Techniques          | 2(1)              | 13(2), 27(3), 30(3) 8                | 9     |
| Functions                     | 1(1)              | 18(2), 19(2), 25(2), 28(2) 8         | 9     |
| Linear, quadratics and cubics | 3(1)              | 16(2), 20(3), 21(4), 22(2) 11        | 12    |
| Trigonometry                  | 9(1)              | 11(1), 12(1), 14(3), 26(3), 33(2) 10 | 11    |
| Introduction to Calculus      | 7(1), 8(1), 10(1) | 15(3), 23(3), 24(3) 9                | 12    |
| Probability                   | 4(1), 5(1), 6(1)  | 17(2), 29(6), 31(3), 32(3) 14        | 17    |
| TOTAL                         | 10                | 60                                   | 70    |

NESA Number: \_\_\_\_\_ Teacher: \_\_\_\_\_

## Section I

10 marks

Attempt Questions 1 to 10

Use the multiple-choice answer sheet for Questions 1 to 10

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1. Given  $f(x) = 3x^2 + 2x$ . Calculate the value of  $f(2a)$ ?
  - A.  $6a^2 + 2a$
  - B.  $6a^2 + 4a$
  - C.  $36a^2 + 4a$
  - D.  $12a^2 + 4a$
  
2. What is 4.09784 correct to three significant figures?
  - A. 4.09
  - B. 4.10
  - C. 4.097
  - D. 4.098
  
3. What is the value of the discriminant of the quadratic equation  $x^2 + 3x - 10$ ?
  - A.  $-5$
  - B. 5
  - C. 7
  - D. 49

4. In a class of thirty Year 10 students, 15 students play piano, 14 play guitar and 7 play both. What is the probability that a randomly chosen student plays guitar, given that they play the piano?

- A.  $\frac{14}{15}$
- B.  $\frac{7}{15}$
- C.  $\frac{7}{29}$
- D.  $\frac{15}{29}$

5. Given that the random variable  $X$  has the probability distribution given in the table

| $x$        | 1    | 2    | 3    | 4    | 5    | 6    |
|------------|------|------|------|------|------|------|
| $P(X = x)$ | 0.05 | 0.23 | 0.18 | 0.33 | 0.14 | 0.10 |

Calculate  $P(X \geq 5)$ .

- A. 0.10
- B. 0.14
- C. 0.24
- D. 0.76
6. For two events  $A$  and  $B$ ,  $P(A) = \frac{3}{8}$ ,  $P(B) = \frac{4}{7}$  and  $P(A \cap B) = \frac{8}{21}$ . Calculate  $P(A|B)$ .
- A.  $\frac{2}{3}$
- B.  $\frac{21}{32}$
- C.  $\frac{3}{14}$
- D.  $\frac{3}{8}$

7. The function  $y = ax^2 - bx$  has a gradient of zero at  $x = 2$ . The  $x$ -axis intercepts of the graph of this function are:

- A.  $x = 0, 4$
- B.  $x = 0, -4$
- C.  $x = 0, \frac{1}{2}$
- D.  $x = 0, -\frac{1}{2}$

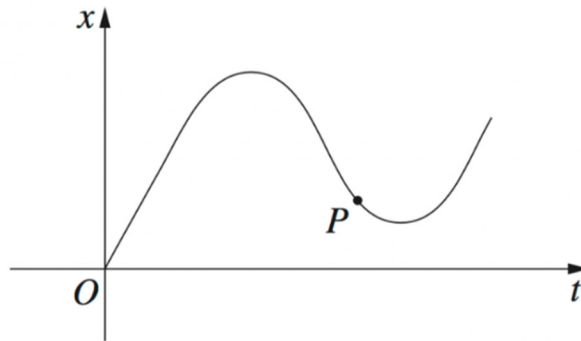
8. Given  $y = \sqrt{2f(x)}$ , calculate  $\frac{dy}{dx}$ .

- A.  $\frac{2f'(x)}{\sqrt{2f(x)}}$
- B.  $\frac{-1}{2\sqrt{2f(x)}}$
- C.  $\frac{f'(x)}{\sqrt{2f(x)}}$
- D.  $\frac{3}{2(2f'(x))}$

9. Which expression is equivalent to  $\frac{1}{\cot^2 \theta} + \sec \theta \cos \theta$  ?

- A. 1
- B.  $\sec^2 \theta$
- C.  $\tan^2 \theta$
- D.  $\operatorname{cosec}^2 \theta$

10. The graph shows the displacement  $x$  of a particle moving along a straight line as a function of time  $t$ , where  $t \geq 0$ .



Which statement describes the motion of the particle at the point  $P$ ?

- A. The velocity is negative, and acceleration is positive.
- B. The velocity is negative, and acceleration is negative.
- C. The velocity is positive, and acceleration is positive.
- D. The velocity is positive, and acceleration is negative.

**End of Section I**

Section II

60 marks  
Attempt Questions 11 to 33

Answer the questions in the spaces provided.  
Your responses should include relevant mathematical reasoning and/or calculations.  
All necessary working should be shown in every question.

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Question 11 (1 mark)

Express  $490^\circ$  in radians in terms of  $\pi$ . 1

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Question 12 (1 marks)

Calculate  $\sin\left(\frac{4\pi}{7}\right)$  to two decimal places. 1

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Question 13 (2 marks)

Simplify  $2a(b^2 - a^3) - 5a^2(b - 3a^2)$  2

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**Question 14** (3 marks)

The area of the sector of a circle formed by the centre angle of  $\frac{2\pi}{5}$  is  $\frac{33\pi}{4} \text{ cm}^2$ .

Find the radius of the circle.

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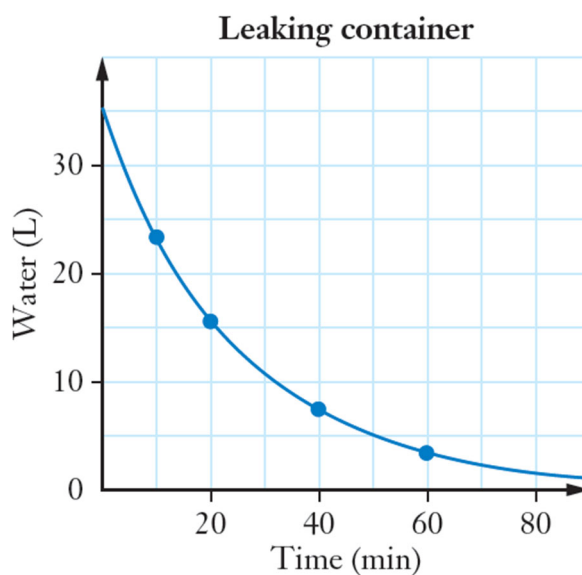
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**Question 15** (3 marks)

The amount of water in a leaking container is shown in the graph below.



- a) Calculate the average rate of loss from 20 minutes to 40 minutes.

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- b) Describe how the rate changes over time.

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**Question 16** (2 marks)

Two electrical technicians can repair five electrical lines in three hours. After a storm, 50 electrical lines need repairing. How many electrical technicians are needed to repair the electrical lines in one eight hour working day? 2

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**Question 17** (2 marks)

A survey of 950 people were asked to state their favourite team in the NRL competition.  
The results are shown in the table below.

|                         | Western Sydney team | Other teams |
|-------------------------|---------------------|-------------|
| Lives in Western Sydney | 404                 | 254         |
| Lives elsewhere         | 94                  | 198         |

Is support for a Western Sydney team independent of where a person lives? Justify your answer. 2

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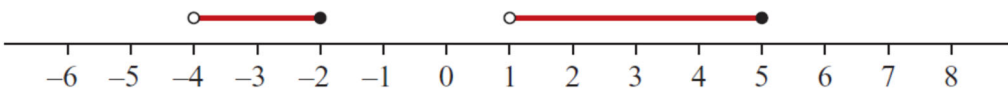
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**Question 18** (2 marks)

Describe the following solution using interval notation. 2



**Question 19 (2 marks)**

What is the domain and range for the function  $f(x) = \sqrt{4 - x}$ .

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**Question 20 (3 marks)**

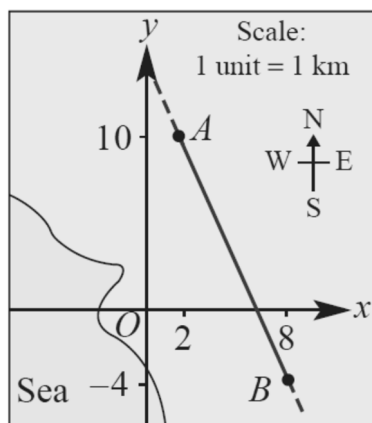
A group of friends hire a bus for \$2100 to take them to a music festival. Seven friends can no longer make the festival and withdraw from the bus hire. The remaining friends must each pay an additional \$10 to cover the cost of the bus hire. How many friends originally agreed to go on the bus?

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[illegible]

**Question 21** (4 marks)

$O(0,0)$  is the position of the air traffic control tower at Sydney Airport. A plane travelling in a straight line is identified at  $A(2, 10)$  and again at  $B(8, -4)$ .



- a) What is the equation that describes the flight of the plane?

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- b) How far south of  $O$  is the aircraft when  $x = 15$  km?

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**Question 22** (2 marks)

Determine the coordinates of the vertex of the function  $f(x) = -(x + 1)^2 + 4$ .

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**Question 23** (3 marks)

Find the gradient of the curve  $y = (x - 1)(3 - 2x)^4$  at the point where  $x = -3$ .

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**Question 24** (3 marks)

Differentiate  $f(x) = 3x^2 + 2x + 2$  using the method of first principles.

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**Question 25** (2 marks)

Given  $f(x) = \frac{1}{x}$  and  $g(x) = 3x + 2$ . Find the domain and range of the composite function  $g(f(x))$ .

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**Question 26** (3 marks)

Solve the equation  $\sin(2\theta) = -\frac{\sqrt{3}}{2}$  for the domain  $\theta \in [-\pi, \pi]$ .

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**Question 27** (3 marks)

Find the gradient of the normal for the curve  $y = \frac{x^2}{1-x}$  when  $x = -2$ .

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**Question 28** (2 marks)

Explain the difference between a one-to-one function and a many-to-one function, provide examples.

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**Question 29** (6 marks)

Claire has three packs of lollies. In bag 1 there are three mints and three sour strips, in bag 2 there are three mints and two sour strips, and in bag 3 there are two mints and one sour strip.  
Claire selects a bag at random, and then selects a lolly at random. Calculate:

a) The probability she chooses a mint from bag 1. **1**

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b) The probability she chooses a mint. **2**

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c) The probability that Claire chose bag 1, given that she selects a mint. **3**

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**Question 30** (3 marks)

A cyclist goes up a long slope at a constant speed of 15km/h. He turns around and comes down the slope at a constant speed of 40km/h. Find his average speed over a full circuit. **3**

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**Question 31** (3 marks)

James is a keen runner. He is supposed to attend running training five days per week. **3**

James finds that if he runs one day, the probability that he will run again the next day is  $\frac{4}{5}$ , and if he does not run one day, the probability that he will not run the next day is  $\frac{3}{4}$ . Suppose James runs one day, what is the probability that he runs exactly twice in the next three days?

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**Question 32** (3 marks)

Athena plays a game in which two coins are tossed. If she throws two heads, she wins \$5. If she throws two tails, she loses \$3. And for one head and one tail, she loses \$2. **3**

Calculate the financial expectation from this game. If the game is not fair, what can be done to make it a fair game?

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**Question 33** (2 marks)

Prove that

$$\frac{1 - \sin^2 x \cos^2 x}{\sin^2 x} = \cot^2 x + \sin^2 x$$
**2**

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**END OF EXAMINATION**

## Section II

60 marks

Attempt Questions 11 to 33

Answer the questions in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

All necessary working should be shown in every question.

### Question 11 (1 mark)

Express  $490^\circ$  in radians in terms of  $\pi$ .

1

$$\frac{490\pi}{180} = \frac{49\pi}{18}$$

$$2.72\pi$$

### Question 12 (1 marks)

Calculate  $\sin\left(\frac{4\pi}{7}\right)$  to two decimal places.

$$= 0.9749279122$$

$$= 0.97$$

### Question 13 (2 marks)

Simplify  $2a(b^2 - a^3) - 5a^2(b - 3a^2)$

$$= 2ab^2 - 2a^4 - 5a^2b + 15a^4 = 2ab^2 + 13a^4 - 5a^2b \text{ (1)}$$

$$= a(2b^2 - 2a^3 - 5a^2b + 13a^3) = a(2b^2 + 13a^3 - 5a^2b) \text{ (1)}$$

$$= a(2b^2 - 2a^3 - 5a^2b + 13a^3)$$

**Question 14** (3 marks)

The area of the sector of a circle formed by the centre angle of  $\frac{2\pi}{5}$  is  $\frac{33\pi}{4} \text{ cm}^2$ .

Find the radius of the circle.



$$A = \frac{1}{2} r^2 \theta$$

$$\frac{33\pi}{4} = \frac{1}{2} r^2 \frac{2\pi}{5} \quad (1)$$

$$\frac{165}{4} = r^2$$

$$r = \pm \frac{\sqrt{165}}{2} \quad (1)$$

but  $r > 0$

$$\therefore r = \frac{\sqrt{165}}{2} \text{ cm} \quad (1)$$

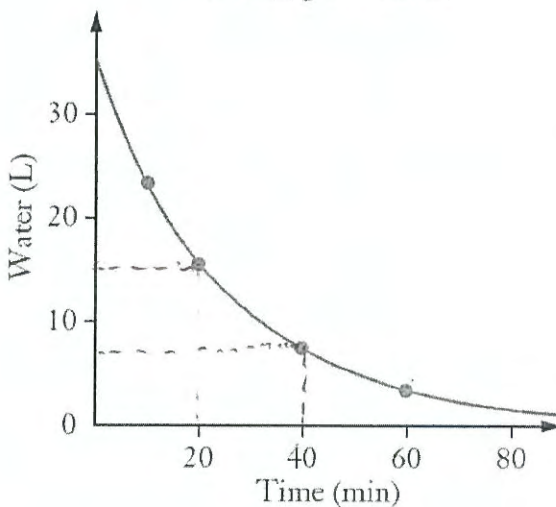
\* Students had equations in terms of degrees and radians.  $\therefore$   
 \* some students changed the area to an angle in degrees!!  $\therefore$

$$r = 6.422616289 \text{ cm}$$

**Question 15** (3 marks)

The amount of water in a leaking container is shown in the graph below.

Leaking container



a) Calculate the average rate of loss from 20 minutes to 40 minutes.

$$\text{rate} = \frac{15\text{L} - 8\text{L}}{20 - 40} = -\frac{7}{20} \text{ L/min} \quad (1)$$

decreases at  $\frac{7}{20} \text{ L/min}$

b) Describe how the rate changes over time.

As time increases  $\wedge$  the water level is going down at a decreasing rate, i.e. reducing at a decreasing rate. (1) (1)

\* you needed mention 3 things to get the two marks; most students didn't talk about time.

**Question 16 (2 marks)**

Two electrical technicians can repair five electrical lines in three hours. After a storm, 50 electrical lines need repairing. How many electrical technicians are needed to repair the electrical lines in one eight hour working day?

2 Electricians  $\rightarrow$  5 lines  $\rightarrow$  3 hours  
 $\rightarrow$  40 lines  $\rightarrow$  24 hours

8 Electricians  $\rightarrow$  160 lines  $\rightarrow$  24 hours  
 $\rightarrow$  53  $\frac{1}{3}$  lines  $\rightarrow$  8 hours.

or Electricians Lines Time

2  $\times$  5  $\times$  3  
 $x$   $\times$  50  $\times$  8

$2 \times 50 \times 3 = x \times 5 \times 8$

$300 = 40x$

$x = 7.5$

$\therefore$  8 electricians

**Question 17 (2 marks)**

A survey of 950 people were asked to state their favourite team in the NRL competition.

The results are shown in the table below.

|                         | Western Sydney team | Other teams |
|-------------------------|---------------------|-------------|
| Lives in Western Sydney | 404                 | 254         |
| Lives elsewhere         | 94                  | 198         |

Is support for a Western Sydney team independent of where a person lives? Justify your answer.

- Western Sydney reside and supporter  $\frac{404}{658}$

$= \frac{202}{329} = 0.614$

- If events A and B are independent then  $P(A \text{ and } B) = P(A) \times P(B)$

- lives elsewhere and supports Western Sydney team  $\frac{94}{292} = 0.322$

or,  $P(A|B) = P(A)$   $P(B|A) = P(B)$

LHS  $\frac{404}{658} = \frac{202}{329}$

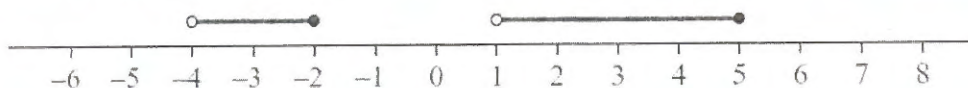
RHS  $\frac{498}{950} = \frac{249}{475}$

$\neq$  LHS  $\therefore$  not independent.

\* words only, but no justification is 1 mark only. You had to either use the table or do the math

**Question 18 (2 marks)**

Describe the following solution using interval notation.



$[-4, -2] \cup [1, 5]$

① for correct interval notation

① for "u"

**Question 19** (2 marks)

What is the domain and range for the function  $f(x) = \sqrt{4-x}$ .

2

$$4-x \geq 0$$

$$\text{domain: } x \leq 4 \quad \text{or } (-\infty, 4] \quad \checkmark$$

$f(x)$  can't be negative

$$\text{range: } y \geq 0 \quad \text{or } [0, \infty) \quad \checkmark$$

Some students thought that this is a semicircle which is not.

**Question 20** (3 marks)

A group of friends hire a bus for \$2100 to take them to a music festival. Seven friends can no longer make the festival and withdraw from the bus hire. The remaining friends must each pay an additional \$10 to cover the cost of the bus hire. How many friends originally agreed to go on the bus?

3

let  $x$  be the number of original friends

$y$  be the amount each friend contributed

$$xy = 2100$$

$$y = \frac{2100}{x}$$

$$(x-7)(y+10) = 2100 \quad \checkmark$$

$$xy + 10x - 7y - 70 = 2100$$

substituting values

$$2100 + 10x - 7\left(\frac{2100}{x}\right) - 70 = 2100$$

$$\left(10x - \frac{14700}{x} - 70 = 0\right) \times x$$

$$(10x^2 - 70x - 14700 = 0) \div 10$$

$$x^2 - 7x - 1470 = 0$$

$$(x-42)(x+35) = 0$$

$$x = 42, x = -35 \quad \text{but } x > 0 \quad \checkmark$$

$$\therefore x = 42 \text{ friends} \quad \checkmark$$

$\therefore$  There were 42 friends originally

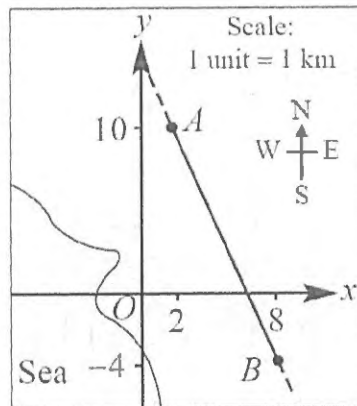
• Algebraic techniques req'd to gain full marks.

• Need to show both values of the quadratic equation and indicate that  $x > 0$ .

• 1 mark only for students who only wrote the answer using guess-and-check.

**Question 21** (4 marks)

$O(0,0)$  is the position of the air traffic control tower at Sydney Airport. A plane travelling in a straight line is identified at  $A(2, 10)$  and again at  $B(8, -4)$ .



1 mark for correct gradient.

1 mark for correct equation of the line whether in  $y=mx+c$  or  $ax+by+c=0$  form.

a) What is the equation that describes the flight of the plane?

2

$$m = \frac{-4-10}{8-2} = -\frac{7}{3} \quad \checkmark$$

$$\text{or } 7x + 3y - 44 = 0$$

$$y - 10 = -\frac{7}{3}(x - 2)$$

$$y - 10 = -\frac{7}{3}x + \frac{14}{3}$$

$$y = -\frac{7}{3}x + \frac{44}{3} \quad \checkmark$$

b) How far south of  $O$  is the aircraft when  $x = 15$  km?

2

$$y = -\frac{7}{3}(15) + \frac{44}{3} \quad \checkmark$$

1 mark for the line of substitution.

$$y = -\frac{61}{3} \text{ or } -20\frac{1}{3}$$

1 mark for the answer.

$\therefore$  the aircraft is  $20\frac{1}{3}$  km south of  $O$ .

**Question 22** (2 marks)

Determine the coordinates of the vertex of the function  $f(x) = -(x + 1)^2 + 4$ .

2

$$f(x) = a(x-h)^2 + k \quad \checkmark$$

the vertex is at  $(-1, 4)$   $\checkmark$

1 mark for x-coordinate

1 mark for y-coordinate

**Question 23** (3 marks)

Find the gradient of the curve  $y = (x - 1)(3 - 2x)^4$  at the point where  $x = -3$ .

3

$$\begin{array}{l}
 u = (x-1) \quad \left| \quad v = (3-2x)^4 \right. \\
 u' = 1 \quad \left| \quad v' = 4(3-2x)^3(-2) \right. \checkmark \\
 \quad \quad \quad \left| \quad v' = -8(3-2x)^3 \right. \checkmark
 \end{array}$$

not required to show.

$$y' = u'v + v'u$$

$$y' = (3-2x)^4 - 8(3-2x)^3(x-1) \checkmark \quad \text{or} \quad (3-2x)^3(11-10x)$$

at  $x = -3$

$$y' = (3-2(-3))^4 - 8(3-2(-3))^3(-3-1)$$

$$y' = 29889$$

∴ the gradient is 29889  $\checkmark$

2 marks for correct derivative using product rule.  
1 mark for gradient

**Question 24** (3 marks)

Differentiate  $f(x) = 3x^2 + 2x + 2$  using the method of first principles.

3

**Question 23** (3 marks)

Find the gradient of the curve  $y = (x - 1)(3 - 2x)^4$  at the point where  $x = -3$ .

3

**Question 24** (3 marks)

Differentiate  $f(x) = 3x^2 + 2x + 2$  using the method of first principles.

3

Use the reference sheet

$$\left. \begin{aligned} f(x+h) &= 3(x+h)^2 + 2(x+h) + 2 \\ &= 3(x^2 + 2xh + h^2) + 2x + 2h + 2 \\ &= 3x^2 + 6xh + 3h^2 + 2x + 2h + 2 \end{aligned} \right\} \text{1 mark}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \left. \right\} \text{1 mark}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h + 2 - (3x^2 + 2x + 2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{2x} + 2h + \cancel{2} - \cancel{3x^2} - \cancel{2x} - \cancel{2}}{h} \quad \left. \right\} \text{1 mark}$$

$$= \lim_{h \rightarrow 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(6x + 3h + 2)}{h}$$

$$= \lim_{h \rightarrow 0} 6x + 3h + 2$$

$$= 6x + 3(0) + 2$$

$$= 6x + 2$$

11

} 1 mark.

lost 1 mark  
for  $3h + 6x + 2$

Question 25 (2 marks)

Given  $f(x) = \frac{1}{x}$  and  $g(x) = 3x + 2$ . Find the domain and range of the composite function  $g(f(x))$ .

2

$$g(f(x)) = g\left(\frac{1}{x}\right) = 3\left(\frac{1}{x}\right) + 2$$

$$g(f(x)) = \frac{3}{x} + 2$$

(1 mark)

Domain: all real,  $x \neq 0$  or  $(-\infty, 0) \cup (0, \infty)$  (1 mark for both)

Range: all real,  $y \neq 2$  or  $(-\infty, 2) \cup (2, \infty)$  (1 mark for both)

Question 26 (3 marks)

Solve the equation  $\sin(2\theta) = -\frac{\sqrt{3}}{2}$  for the domain  $\theta \in [-\pi, \pi]$ .

3

$$\sin 2\theta = -\frac{\sqrt{3}}{2}$$

$$-\pi \leq \theta \leq \pi$$

$$-2\pi \leq 2\theta \leq 2\pi$$

related angle is  $\frac{\pi}{3}$

$$2\theta = \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, -\frac{\pi}{3}, -\pi + \frac{\pi}{3}$$

$$2\theta = \frac{4\pi}{3}, \frac{5\pi}{3}, -\frac{\pi}{3}, -\frac{2\pi}{3}$$

$$\theta = \frac{4\pi}{6}, \frac{5\pi}{6}, -\frac{\pi}{6}, -\frac{2\pi}{6}$$

(1 mark for related angle)

$$= \frac{2\pi}{3}, \frac{5\pi}{6}, -\frac{\pi}{6}, -\frac{\pi}{3}$$

$$\therefore \theta = \left\{ -\frac{\pi}{6}, -\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6} \right\} \text{ have two marks}$$

1 mark

12

1 mark

Question 27 (3 marks)

Find the gradient of the normal for the curve  $y = \frac{x^2}{1-x}$  when  $x = -2$ .

3

Lots  
not  
differentiating  
using  
m.

$$y' = \frac{(1-x)(2x) - x^2(-1)}{(1-x)^2}$$

$$= \frac{2x - 2x^2 + x^2}{(1-x)^2}$$

Using the  
quotient rule  
 $y' = \frac{vu' - uv'}{v^2}$

$$u = x^2 \quad v = 1-x$$

$$u' = 2x \quad v' = -1$$

$$y' = \frac{2x - x^2}{(1-x)^2} = \frac{x(2-x)}{(1-x)^2}$$

Not  
done  
well.

$$y'(-2) = \frac{-2(2 - (-2))}{(1 - (-2))^2}$$

1 mark

$$= \frac{-2(4)}{3^2}$$

$$= \frac{-8}{9}$$

$$\therefore m = -\frac{8}{9}$$

1 mark

gradient of normal is  $\frac{9}{8}$

Since  $m_1 \times m_2 = -1$ .

-1 missing  
example

Question 28 (2 marks)

Explain the difference between a one-to-one function and a many-to-one function, provide examples.

2

One-to-one function means each  $x$  value has a one  $y$ -value

Eg.  $y = x$



many to one function means each  $x$  value has ~~just~~ a  $y$ -value. Many  $x$  values have the same  $y$ -value.

Eg. parabola



Done well.  
Well explained

**Question 29 (6 marks)**

Claire has three packs of lollies. In bag 1 there are three mints and three sour strips, in bag 2 there are three mints and two sour strips, and in bag 3 there are two mints and one sour strip.

Claire selects a bag at random, and then selects a lolly at random. Calculate:

- a) The probability she chooses a mint from bag 1.

1

$$P(\text{mint from bag 1}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \quad \text{poorly done.}$$

- b) The probability she chooses a mint.

2

$$\begin{aligned} P(\text{mint}) &= \overset{\text{Bag 1}}{\left(\frac{1}{3} \times \frac{1}{2}\right)} + \overset{\text{Bag 2}}{\left(\frac{1}{3} \times \frac{3}{5}\right)} + \overset{\text{Bag 3}}{\left(\frac{1}{3} \times \frac{2}{3}\right)} \\ &= \frac{1}{6} + \frac{1}{5} + \frac{2}{9} \\ &= \frac{53}{90} \end{aligned}$$

- c) The probability that Claire chose bag 1, given that she selects a mint.

3

$$\begin{aligned} P(\text{Bag 1} / \text{mint}) &= \frac{P(\text{mint} \cap \text{Bag 1})}{P(\text{M})} \\ &= \frac{\left(\frac{1}{6}\right)}{\left(\frac{53}{90}\right)} \quad \begin{array}{l} \text{from part a} \\ \text{from part b} \end{array} \\ &= \frac{15}{53} \end{aligned}$$

**Question 30** (3 marks)

A cyclist goes up a long slope at a constant speed of 15km/h. He turns around and comes down the slope at a constant speed of 40km/h. Find his average speed over a full circuit. 3

let  $x$  be length of slope in km.

$$\text{time up} = \frac{x}{15}$$

$$\text{time down} = \frac{x}{40}$$

$$\begin{aligned} \text{total time} &= \frac{x}{15} + \frac{x}{40} \\ &= \frac{11x}{120} \end{aligned}$$

$$\text{Average speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{2x}{\left(\frac{11x}{120}\right)}$$

$$= 2x \times \frac{120}{11x}$$

$$= \frac{240}{11}$$

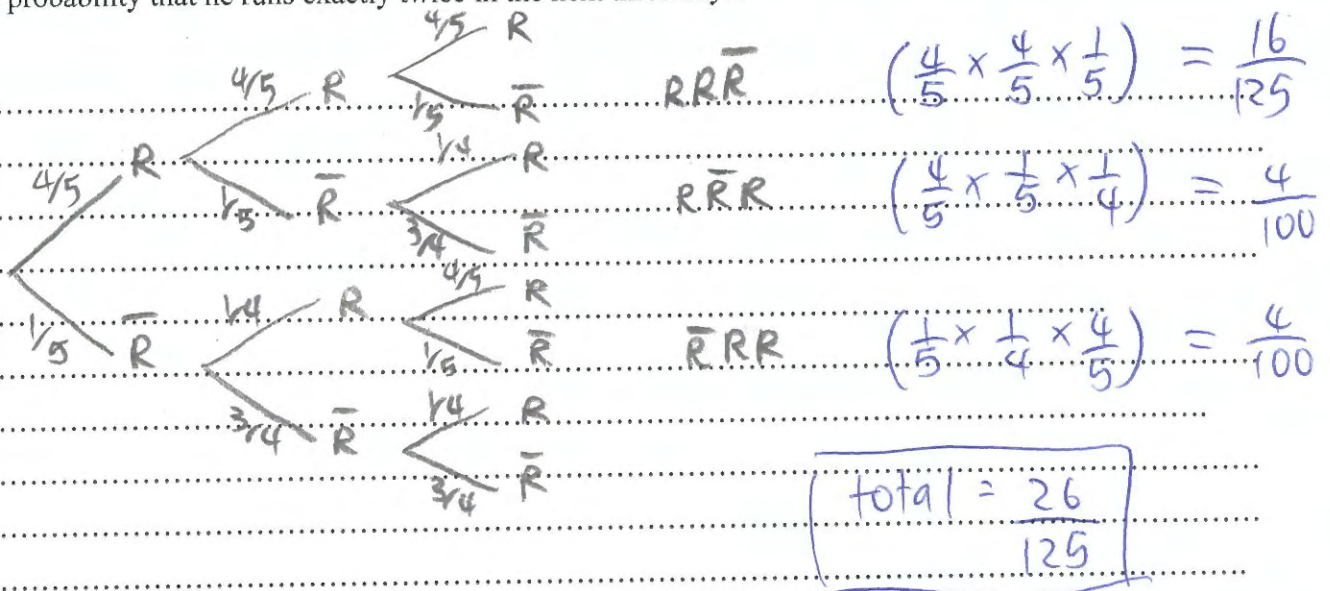
$$= 21.82 \text{ km/hr.}$$

very poorly done!

**Question 31** (3 marks)

James is a keen runner. He is supposed to attend running training five days per week. 3

James finds that if he runs one day, the probability that he will run again the next day is  $\frac{4}{5}$ , and if he does not run one day, the probability that he will not run the next day is  $\frac{3}{4}$ . Suppose James runs one day, what is the probability that he runs exactly twice in the next three days?



**Question 32 (3 marks)**

Athena plays a game in which two coins are tossed. If she throws two heads, she wins \$5. If she throws two tails, she loses \$3. And for one head and one tail, she loses \$2.

**3**

Calculate the financial expectation from this game. If the game is not fair, what can be done to make it a fair game?

$$E(X) = \frac{1}{4}(5) + \frac{1}{4}(-3) + \frac{1}{2}(-2)$$

$$= -0.5$$

$\therefore$  not fair

If Athena wins \$7, the game will be fair.

Feedback: Students commented on the probability being not fair and awarded no marks.

**Question 33 (2 marks)**

Prove that

$$\frac{1 - \sin^2 x \cos^2 x}{\sin^2 x} = \cot^2 x + \sin^2 x$$

**2**

$$\text{LHS} = \frac{1 - \sin^2 x \cos^2 x}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x} - \frac{\sin^2 x \cos^2 x}{\sin^2 x}$$

$$= \csc^2 x - \cos^2 x$$

$$= (1 + \cot^2 x) - (1 - \sin^2 x)$$

$$= \cot^2 x + \sin^2 x$$

$$= \text{RHS}$$

Feedback:

→ Many students did this!

$$\frac{1 - \sin^2 x \cos^2 x}{\sin^2 x}$$

$$= \frac{\cos^2 x \cos^2 x}{\sin^2 x}$$

→ RHS to LHS was a little easier.

END OF EXAMINATION