



PENRITH SELECTIVE HIGH SCHOOL

MATHEMATICS

2023 Year 11 Examination

2023

Mathematics Advanced

**General
Instructions**

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen.
- No correction tape/white out allowed.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.
- Write your NESA Number below and on each additional working booklet.
- Reference sheets are supplied with this examination paper.

**Total Marks:
76**

Section I – 10 marks (pages 2–5)

- Attempt questions 1–10 using the multiple-choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II – 66 marks (pages 6–23)

- Attempt questions 11–30, answer the questions in the writing booklets.
- Allow about 1 hour and 45 minutes for this section.

Topics	Functions	Probability	Trigonometry	Differential Calculus	Total
Multiple Choice	Q1 Q2 Q3 Q5	Q10	Q4 Q6 Q9	Q7 Q8	/10
Booklet 1	/14	/4	/1	/3	/22
Booklet 2		/2	/3	/16	/21
Booklet 3	/4	/3	/11	/5	/23
Total	/22	/10	/18	/26	/76

NESA Number:

Teacher:

Section I

10 marks

Attempt Questions 1 – 10

Allow 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

- 1 A relation is defined by the set of points shown below.

$$(0, -7), (-2, 1), (-2, 8), (-5, 3), (2, 0)$$

Which term best describes this relation?

- A. One-to-many
 - B. One-to-one
 - C. Many-to-one
 - D. Many-to-many
- 2 What is the domain and range of the function $y = \sqrt{2x - 3}$?

- A. D: $\left(\frac{3}{2}, \infty\right)$, R: $[0, \infty)$
- B. D: $\left[\frac{3}{2}, \infty\right)$, R: $[0, \infty)$
- C. D: $\left(-\frac{3}{2}, 0\right]$, R: $(0, \infty]$
- D. D: $\left[-\frac{3}{2}, 0\right]$, R: $(0, \infty]$

- 3 Which of the following is true for the following function $f(x) = \frac{1}{x^2} - 2x$?

- A. $f(x)$ is an even function.
- B. $f(x)$ is an odd function.
- C. $f(x)$ is neither odd nor even.
- D. $f(x)$ is a quadratic function.

- 4 Solve $\sin \theta = -\frac{1}{\sqrt{2}}$ for $-\pi \leq \theta \leq \pi$.
- A. $\theta = -\frac{5\pi}{4}$ or $\theta = \frac{\pi}{4}$
- B. $\theta = -\frac{3\pi}{4}$ or $\theta = \frac{\pi}{4}$
- C. $\theta = -\frac{5\pi}{4}$ or $\theta = \frac{5\pi}{4}$
- D. $\theta = -\frac{3\pi}{4}$ or $\theta = -\frac{\pi}{4}$
- 5 Simplifying $\frac{2^{n+1} + 2^n}{2^n - 2^{n-1}}$ gives which of the following?
- A. 6
- B. $\frac{1}{6}$
- C. $\frac{1}{2}$
- D. 2
- 6 What is the exact value of $\sin 120^\circ + \operatorname{cosec} 45^\circ$?
- A. $\frac{\sqrt{3} + \sqrt{2}}{2}$
- B. $\frac{\sqrt{5}}{2}$
- C. $\frac{\sqrt{6}}{2}$
- D. $\frac{\sqrt{3} + 2\sqrt{2}}{2}$

- 7 What is the gradient of the tangent to the curve $y = -2x^3 - 5x^2 + 3x$ when $x = -2$?
- A. -47
- B. -1
- C. 1
- D. 7
- 8 Which of the following is the derivative of $(x^2 + 12)^5$?
- A. $5(x^2 + 12)^4$
- B. $10x^2(x^2 + 12)^5$
- C. $10x(x^2 + 12)^4$
- D. $5(2x + 12)^4$
- 9 If $\cos \theta = -\frac{3}{7}$ and $180^\circ \leq \theta \leq 360^\circ$, find $\tan \theta$.
- A. $-\frac{\sqrt{10}}{3}$
- B. $\frac{2\sqrt{10}}{3}$
- C. $-\frac{10}{3}$
- D. $-\frac{2\sqrt{10}}{3}$

- 10 The table below shows a discrete probability distribution for a random variable X .

x	1	2	3	4	5
$P(X = x)$	k^2	$\frac{k}{2}$	$2k^2$	k	$\frac{k}{2}$

What is the value of k ?

- A. $k = \frac{1}{18}$
- B. $k = \frac{1}{9}$
- C. $k = \frac{1}{2}$
- D. $k = \frac{1}{3}$

End of Multiple Choice

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NESA Number:

Teacher:

Mathematics Advanced

Section II Answer Booklet 1

76 marks

Attempt Questions 11-30

Allow about 1 hour and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 17 (22 marks)

Booklet 2 – Attempt Questions 18 – 22 (21 marks)

Booklet 3 – Attempt Questions 23 – 30 (23 marks)

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- Instructions**
- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
 - Write your Teacher and NESA Number above.
-

Topics	Functions	Probability	Trigonometry	Calculus	Total
Questions	Q11 (2) Q13(7) Q14(5)	Q16 /4	Q15 /1	Q12(3)	
Total	/14	/4	/1	/3	/ 22

Question 11 (2 marks)

Simplify $\frac{2m^2 - 8}{m^2 - 2m - 8} \times \frac{3m + 6}{5m - 10}$.

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Question 12 (3 marks)

Differentiate $f(x) = 2x^2 + 3x$ using the first principles formula:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

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Question 13 (7 marks)

Let $f(x) = x + 2$ and $g(x) = -x^2 + 6x + 7$,

- (a) Show that $g(f(x)) = -x^2 + 2x + 15$.

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- (b) Sketch the function $g(f(x))$ in the space given below.
Show x and y intercepts, axis of symmetry and vertex.

3

- (c) Find the domain and range of $g(f(x))$ in interval notation.

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Question 14 (5 marks)

- (a) Expand $\left(p - \frac{1}{p}\right)^2$. **1**

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- (b) Show that $p - \frac{1}{p} = -6\sqrt{2}$, if $p = \sqrt{19} - 3\sqrt{2}$. **2**

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- (c) Hence, find the value of $p^2 + \frac{1}{p^2}$. **2**

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Question 15 (1 marks)

Express 520° in radians. Simplify fully in terms of π .

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Question 16 (4 marks)

A pair of fair 8-sided dice are rolled. The number of sevens showing is represented by a random variable X with the following probability distribution.

x	0	1	2
$P(X = x)$	$\frac{13}{64}$	$\frac{7}{32}$	$\frac{1}{64}$

- (a) Show that the expected value $E(X)$ is $\frac{1}{4}$. 1

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- (b) Hence, or otherwise, find $\text{Var}(X)$, correct to two decimal places. 2

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- (c) Calculate the standard deviation, correct to two decimal places. 1

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Section II Extra Working space

If you use this space, clearly indicate which question you are answering.

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NESA Number:

Teacher:

Mathematics Advanced

Section II Answer Booklet 2

76 marks

Attempt Questions 11-30

Allow about 1 hour and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 16 (22 marks)

Booklet 2 – Attempt Questions 17 – 22 (21 marks)

Booklet 3 – Attempt Questions 23 – 30 (23 marks)

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- Instructions**
- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
 - Write your Teacher and NESA Number above.
-

Topics	Functions	Probability	Trigonometry	Calculus	Total
Questions		Q20 /2	Q17 /3	Q18 / 3 Q19 /2 Q21 /4 Q22 /7	
Total		/2	/3	/16	/21

Question 17 (3 marks)

Solve $4\cos^2 \theta = 3$ for $0 \leq \theta \leq 2\pi$.

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Question 18 (3 marks)

Show that $\frac{d}{dx} \left(\frac{3x}{5x^2 + 1} \right) = \frac{3(1 - 5x^2)}{(5x^2 + 1)^2}$.

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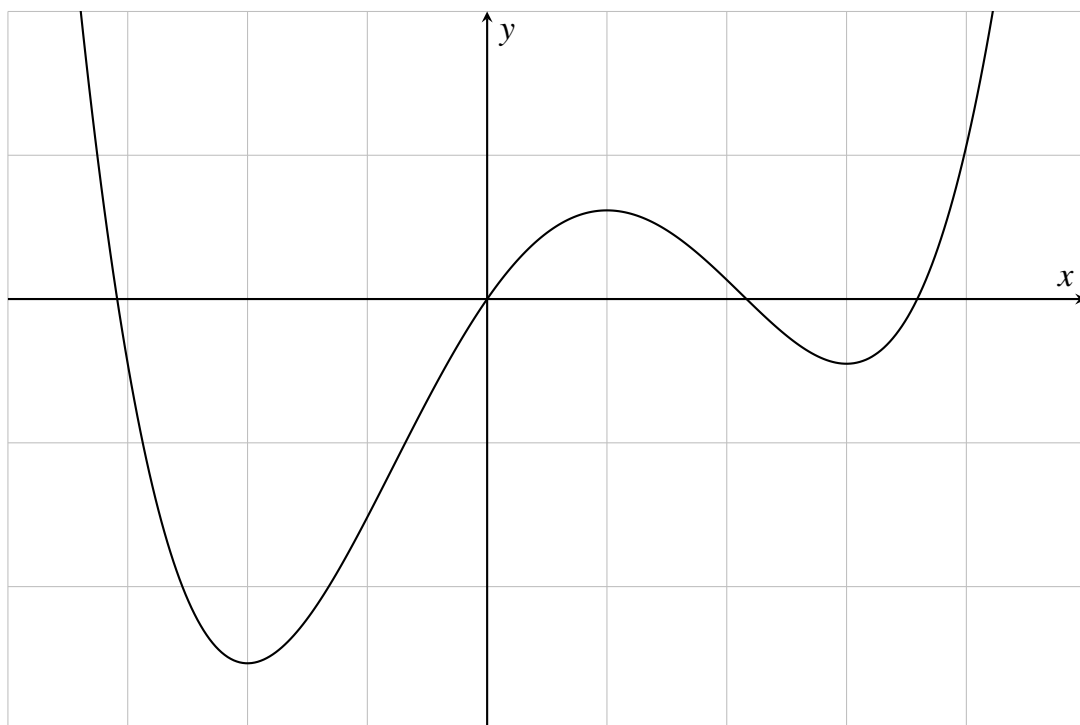
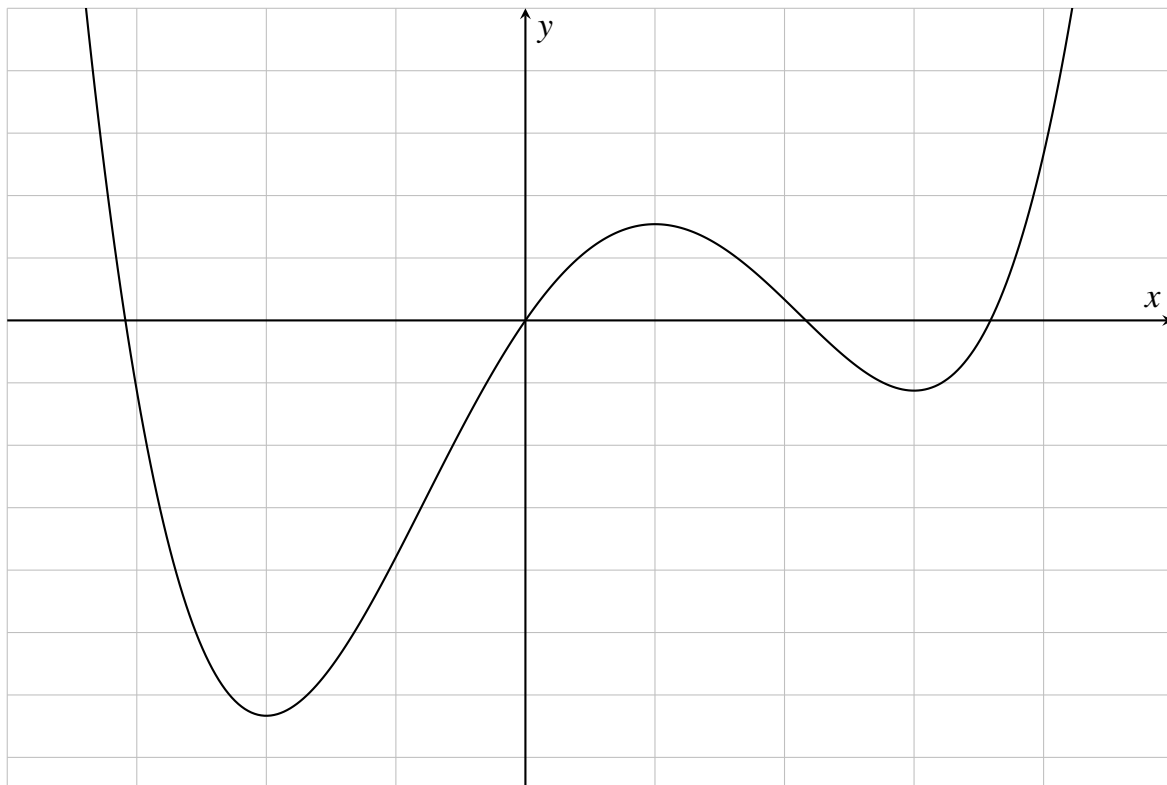
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Question 19 (2 marks)

Sketch the derivative function of the function $y = f(x)$ given below. You may sketch the $y = f'(x)$ on the same Cartesian plane as the graph shown below.

A second copy of the Cartesian plane is provided below in case you make a mistake on the first.



Question 20 (2 marks)

Mr Ferrarin has a new hobby of kite flying with his son, Henry, but is weary of the weather on certain days. On a given day, the probability that it is cloudy is 0.35, the probability that it is windy is 0.6, and the probability that it is both windy and cloudy is 0.25.

What is the probability that it is cloudy or windy (or both)?

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Question 21 (4 marks)

Find the equation of the normal to the curve at $y = 4x^2(2x - 9)$ at the point $(1, -28)$.

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Question 22 (7 marks)

A particle is moving in a straight line. Its displacement, x m, from a fixed point, O , after t seconds is given by $x = 3t^2 - 6t$ where $t \geq 0$.

- (a) Find the expressions for the velocity and acceleration functions in terms of t . **2**

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- (b) Find when the particle is at rest. **1**

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- (c) Find the velocity and acceleration of the particle at 0.5 seconds. **2**

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- (d) Is the particle speeding up or slowing down when $t = 0.5$ seconds? Explain how you determined this. **2**

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Section II Extra Working space

If you use this space, clearly indicate which question you are answering.

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NESA Number:

Teacher:

Mathematics Advanced

Section II Answer Booklet 3

76 marks

Attempt Questions 11-30

Allow about 1 hour and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 17 (22 marks)

Booklet 2 – Attempt Questions 18 – 22 (21 marks)

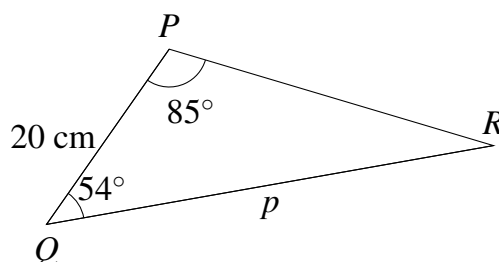
Booklet 3 – Attempt Questions 23 – 30 (23 marks)

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- Instructions**
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Topics	Functions	Probability	Trigonometry	Calculus	Total
Questions	Q26 /2 Q28 /2	Q25 /3	Q23 /3 Q29 /3 Q30 /5	24 /2 27 /3	
Total	/4	/3	/11	/5	/23

Question 23 (3 marks)

In $\triangle PQR$, $\angle P = 85^\circ$, $\angle Q = 54^\circ$ and $PQ = 20$ cm as shown in the diagram below.



- (a) Show that $p = 30.37$ cm, correct to two decimal places.

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- (b) Calculate the area of $\triangle PQR$, answer to two decimal places.

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Question 24 (2 marks)

Find the values of x for which the graph of the function $f(x) = \frac{1}{x^2 - 9}$ is discontinuous.

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Question 25 (3 marks)

A music magazine surveyed 70 people and asked them whether they liked Ice Spice or Taylor Swift. The survey found that 43 people liked Ice Spice and 32 people liked Taylor Swift and 7 people did not like Taylor Swift nor Ice Spice.

(a) Represent this data in a Venn Diagram in the space below. 2

(b) A person is selected at random. What is the probability that the person likes Taylor Swift given that they person likes Ice Spice? 1

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Question 26 (2 marks)

Solve $|4x - 5| = 15$

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Question 27 (3 marks)

Differentiate $y = (x\sqrt{x} - 2)^4$.

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Question 28 (2 marks)

Give reasons why $x^2 + 4x + y^2 - 6y + 15 = 0$ is not an equation of a circle.

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Question 29 (3 marks)

Prove that $\frac{1 + \cos \phi}{1 - \cos \phi} - \frac{1 - \cos \phi}{1 + \cos \phi} = 4 \cot \phi \operatorname{cosec} \phi$.

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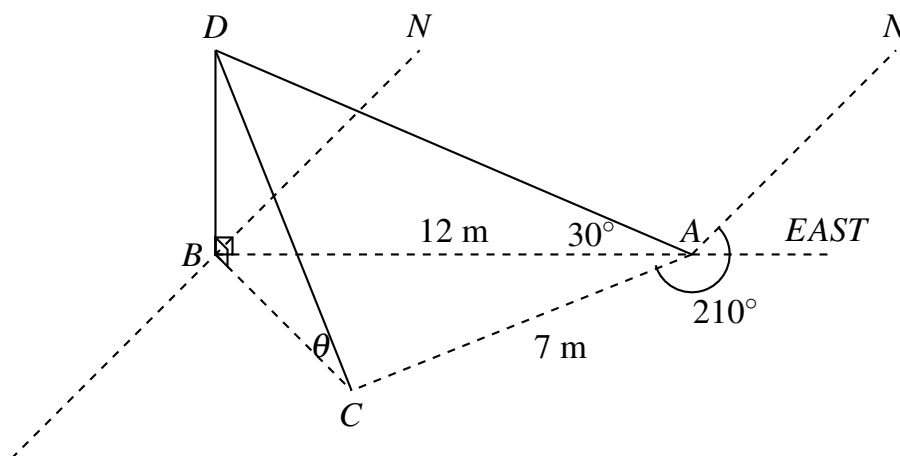
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Question 30 (5 marks)

A flagpole, BD , sits on flat level ground. It is supported by two cables, AD and CD , as shown below. The cable AD is inclined at 30° to the horizontal and is tied to the floor 12 metres from the foot of the flagpole.

The second cable is tied to the floor at point C , 7 metres from A and on a bearing of 210° from A .



- (a) Show that $\angle BAC = 60^\circ$. **1**

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- (b) Find the exact height of the flagpole, BD . **1**

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- (c) Find the exact distance B to C and hence, find θ , correct to the nearest degree **3**

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End of paper

Section II Extra Working space

If you use this space, clearly indicate which question you are answering.

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MATHEMATICS

2023 Year 11 Examination

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Mathematics Advanced

**General
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Total Marks:
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Section I – 10 marks (pages 2–5)

- Attempt questions 1–10 using the multiple-choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II – 66 marks (pages 6–24)

- Attempt questions 11–30, answer the questions in the writing booklets.
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Topics	Functions	Probability	Trigonometry	Differential Calculus	Total
Multiple Choice	Q1 Q2 Q3 Q5	Q10	Q4 Q6 Q9	Q7 Q8	/10
Booklet 1	/14	/4	/1	/3	/22
Booklet 2		/2	/3	/16	/21
Booklet 3	/4	/3	/11	/5	/23
Total	/22	/10	/18	/26	/76

Solutions

NESA Number:

Teacher:

Section I

10 marks

Attempt Questions 1 – 10

Allow 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

- 1 A relation is defined by the set of points shown below.

$$(0, -7), (-2, 1), (-2, 8), (-5, 3), (2, 0)$$

Which term best describes this relation?

- ☒ A. One-to-many
B. One-to-one
C. Many-to-one
D. Many-to-many

- 2 What is the domain and range of the function $y = \sqrt{2x-3}$?

A. D: $\left(\frac{3}{2}, \infty\right)$, R: $[0, \infty)$

☒ B. D: $\left[\frac{3}{2}, \infty\right)$, R: $[0, \infty)$

C. D: $\left(-\frac{3}{2}, 0\right]$, R: $(0, \infty]$

D. D: $\left[-\frac{3}{2}, 0\right]$, R: $(0, \infty]$

$$2x - 3 \geq 0$$

$$2x \geq 3$$

$$x \geq \frac{3}{2}$$

$$D: \left[\frac{3}{2}, \infty\right), \quad R: [0, \infty)$$

- 3 Which of the following is true for the following function $f(x) = \frac{1}{x^2} - 2x$?

A. $f(x)$ is an even function.

B. $f(x)$ is an odd function.

☒ C. $f(x)$ is neither odd nor even.

D. $f(x)$ is a quadratic function.

$$f(-x) = \frac{1}{(-x)^2} - 2(-x)$$

$$= \frac{1}{x^2} + 2x$$

$$\neq f(x)$$

4 Solve $\sin \theta = -\frac{1}{\sqrt{2}}$ for $-\pi \leq \theta \leq \pi$.

A. $\theta = -\frac{5\pi}{4}$ or $\theta = \frac{\pi}{4}$

B. $\theta = -\frac{3\pi}{4}$ or $\theta = -\frac{\pi}{4}$

C. $\theta = -\frac{5\pi}{4}$ or $\theta = \frac{5\pi}{4}$

☒ D. $\theta = -\frac{3\pi}{4}$ or $\theta = -\frac{\pi}{4}$



$$\theta = \frac{\pi}{4}$$

$$\theta = -\frac{\pi}{4}, -\pi + \frac{\pi}{4}$$

$$= -\frac{\pi}{4}, -\frac{3\pi}{4}$$

5 Simplifying $\frac{2^{n+1} + 2^n}{2^n - 2^{n-1}}$ gives which of the following?

☒ A. 6

B. $\frac{1}{6}$

C. $\frac{1}{2}$

D. 2

$$\frac{2^n(2+1)}{2^{n-1}(2-1)}$$

$$= \frac{2^n}{2^{n-1}} \times 3 = 6$$

6 What is the exact value of $\sin 120^\circ + \operatorname{cosec} 45^\circ$?

A. $\frac{\sqrt{3} + \sqrt{2}}{2}$

B. $\frac{\sqrt{5}}{2}$

C. $\frac{\sqrt{6}}{2}$

☒ D. $\frac{\sqrt{3} + 2\sqrt{2}}{2}$

$$\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{1}$$

$$= \frac{\sqrt{3} + 2\sqrt{2}}{2}$$

7 What is the gradient of the tangent to the curve $y = -2x^3 - 5x^2 + 3x$ when $x = -2$?

A. -47

☒ B. -1

C. 1

D. 7

$$\begin{aligned}y' &= -6x^2 - 10x + 3 \\y'(-2) &= -6(-2)^2 - 10(-2) + 3 \\&= -6 \times 4 + 20 + 3 \\&= -24 + 20 + 3 \\&= -1\end{aligned}$$

8 Which of the following is the derivative of $(x^2 + 12)^5$?

A. $5(x^2 + 12)^4$

B. $10x^2(x^2 + 12)^5$

☒ C. $10x(x^2 + 12)^4$

D. $5(2x + 12)^4$

$$\begin{aligned}&5(x^2 + 12)^4 \times 2x \\&= 10x(x^2 + 12)^4\end{aligned}$$

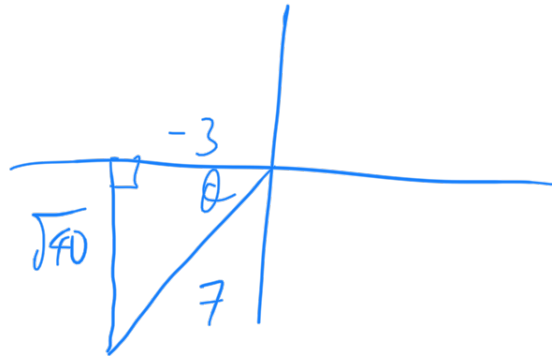
9 If $\cos \theta = -\frac{3}{7}$ and $180^\circ \leq \theta \leq 360^\circ$, find $\tan \theta$.

A. $-\frac{\sqrt{10}}{3}$

☒ B. $\frac{2\sqrt{10}}{3}$

C. $-\frac{10}{3}$

D. $-\frac{2\sqrt{10}}{3}$



$$\begin{aligned}\tan \theta &= \frac{-\sqrt{40}}{-3} \\&= \frac{2\sqrt{10}}{3}\end{aligned}$$
$$\begin{aligned}49 - (-3)^2 \\&= 49 - 9 \\&= 40\end{aligned}$$

- 10 The table below shows a discrete probability distribution for a random variable X .

x	1	2	3	4	5
$P(X = x)$	k^2	$\frac{k}{2}$	$2k^2$	k	$\frac{k}{2}$

What is the value of k ?

- A. $k = \frac{1}{18}$
B. $k = \frac{1}{9}$
C. $k = \frac{1}{2}$
☒ D. $k = \frac{1}{3}$

$$3k^2 + 2k = 1$$

$$3k^2 + 2k - 1 = 0$$

$$\begin{array}{l} 3k \\ \times \\ k+1 \end{array}$$

$$(3k-1)(k+1) = 0$$
$$k = \frac{1}{3} \text{ or } k = -1$$

but $k > 0$

End of Multiple Choice

Question 11 (2 marks)

Simplify $\frac{2m^2-8}{m^2-2m-8} \times \frac{3m+6}{5m-10}$

common mistakes include

- expanding the brackets

$$= \frac{2(m^2-4) \times 3(m+2)}{(m-4)(m+2) \times 5(m-2)}$$

- $3m+6 = 3(m+3)$

$$= \frac{2(m+2)(m-2) \times 3(m+2)}{5(m-4)(m+2)(m-2)}$$

- incorrect factorising of m^2-3m-8

$$= \frac{6(m+2)}{5(m-4)}$$

Overall it was okay.

Question 12 (3 marks)

Differentiate $f(x) = 2x^2 + 3x$ using the first principles formula:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2(x+h)^2 + 3(x+h) - (2x^2 + 3x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2(x^2 + 2xh + h^2) + 3x + 3h - 2x^2 - 3x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3x} + 3h - \cancel{2x^2} - \cancel{3x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(4x + 2h + 3)}{h}$$

$$f'(x) = 4x + 3 \quad (h=0)$$

well done.

- Very few made algebraic mistakes

Question 13 (7 marks)

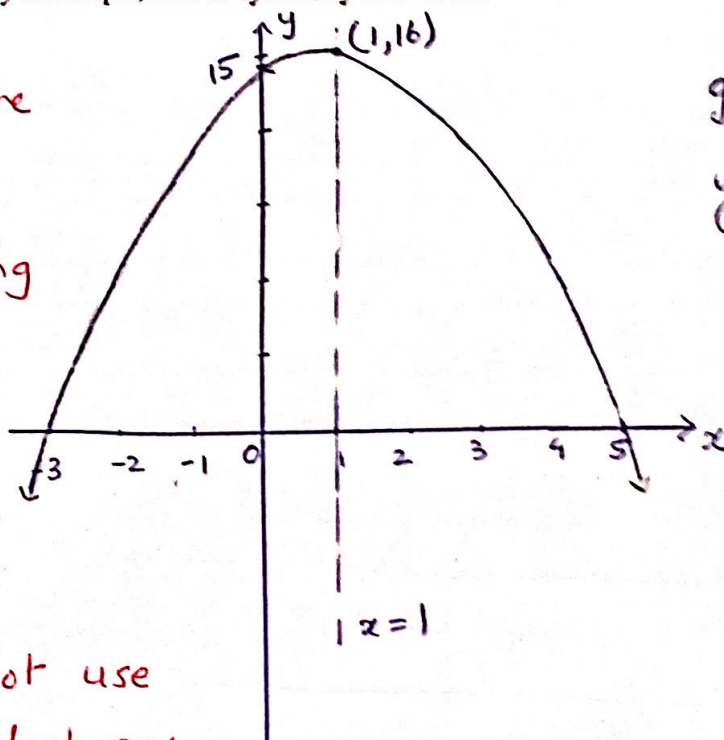
Let $f(x) = x+2$ and $g(x) = -x^2 + 6x + 7$,

- (a) Show that $g(f(x)) = -x^2 + 2x + 15$.

$$\begin{aligned} g(x+2) &= -(x+2)^2 + 6(x+2) + 7 \\ &= -(x^2 + 4x + 4) + 6x + 12 + 7 \\ &= -x^2 - 4x - 4 + 6x + 19 \\ &= -x^2 + 2x + 15 \end{aligned}$$

- (b) Sketch the function $g(f(x))$ in the space given below. Show x and y intercepts, axis of symmetry and vertex.

students have done a good work by finding x-intercept, y-intercept, axis of sym and vertex.



Very few did not use a ruler and lost one mark.

well done

-very few students did algebraic errors.

$$g(f(x)) = -x^2 + 2x + 15$$

y-intercept: 15

x-intercept: $y=0$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$$x = -3, 5$$

axis of sym

$$x = -\frac{b}{2a}$$

$$x = 1$$

vertex: at $x=1$,

$$y = 16$$

- (c) Find the domain and range of $g(f(x))$ in interval notation.

$$D_{g(f(x))} : x \in (-\infty, \infty)$$

$$R_{g(f(x))} : y \in (-\infty, 16]$$

-very few students forgot to write in interval notation

-few students wrote $[16, -\infty)$ instead

Question 14 (5 marks)

(a) Expand $\left(p - \frac{1}{p}\right)^2$.

$$= p^2 - 2p \cdot \frac{1}{p} + \frac{1}{p^2}$$

$$= p^2 - 2 + \frac{1}{p^2}$$

- very few wrote
 $p^2 - \frac{2p}{p} + \frac{1}{p^2}$, $p^2 - \frac{2}{p} + \frac{1}{p^2}$,
 $p^2 - 2p + \frac{1}{p^2}$, hence lost
the mark.

(b) Show that $p - \frac{1}{p} = -6\sqrt{2}$, if $p = \sqrt{19} - 3\sqrt{2}$.

2

$$p - \frac{1}{p} = (\sqrt{19} - 3\sqrt{2}) - \frac{1}{(\sqrt{19} - 3\sqrt{2})}$$

$$= (\sqrt{19} - 3\sqrt{2}) - \frac{1}{\sqrt{19} - 3\sqrt{2}} \times \frac{\sqrt{19} + 3\sqrt{2}}{\sqrt{19} + 3\sqrt{2}} = \frac{(\sqrt{19})^2 - (3\sqrt{2})^2}{\sqrt{19} + 3\sqrt{2}}$$

$$= \sqrt{19} - 3\sqrt{2} - \frac{\sqrt{19} + 3\sqrt{2}}{(\sqrt{19})^2 - (3\sqrt{2})^2}$$

$$= \sqrt{19} - 3\sqrt{2} - \frac{\sqrt{19} + 3\sqrt{2}}{19 - 18} = -6\sqrt{2} - (\sqrt{19} + 3\sqrt{2})$$

(c) Hence, find the value of $p^2 + \frac{1}{p^2}$.

$$p^2 + \frac{1}{p^2} = \left(p - \frac{1}{p}\right)^2 + 2 \quad (\text{using a})$$

$$= (-6\sqrt{2})^2 + 2$$

$$= 72 + 2$$

$$= 74$$

- poor attempt
- As it is a "show
that", many stu
did not show
 $(\sqrt{19})^2 - (3\sqrt{2})^2 = 1$,
hence lost 1 m

- many students were
losing 1 mark for
not using "Hence"
- overall poor
attempt

Question 15 (1 marks)

Express 520° in radians. Simplify fully in terms of π .

$$= \frac{\pi}{180^\circ} \times 520^\circ$$

$$= \frac{26\pi}{9} \text{ radians}$$

- well done.

Question 16 (4 marks)

A pair of fair 8-sided dice are rolled. The number of sevens showing is represented by a random variable X with the following probability distribution.

x	0	1	2
$P(X = x)$	$\frac{13}{64}$	$\frac{7}{32}$	$\frac{1}{64}$

- (a) Show that the expected value $E(X)$ is $\frac{1}{4}$.

1

$$E(X) = 0 \times \frac{13}{64} + 1 \times \frac{7}{32} + 2 \times \frac{1}{64} \quad \text{--- well done.}$$

$$= \frac{1}{4}$$

- (b) Hence, or otherwise, find $\text{Var}(X)$, correct to two decimal places.

2

$$\text{Var}(X) = \sum x^2 p(x) - (E(X))^2$$

$$= 0^2 \times \frac{13}{64} + 1^2 \times \frac{7}{32} + 2^2 \times \frac{1}{64} - \left(\frac{1}{4}\right)^2$$

$$= \frac{7}{32}$$

$$= 0.22 \text{ (2 dp)}$$

- (c) Calculate the standard deviation, correct to two decimal places.

1

$$\text{standard Deviation} = \sqrt{\text{Var}(X)}$$

$$= 0.47 \text{ (2 dp)}$$

Comments (b) :

- Some students used $\sum (x - \mu)^2 p(x)$ instead and got the correct answer
- very few used $\frac{1}{4}$ instead of $\left(\frac{1}{4}\right)^2$
- very few forgot to write correct to 2 dp.

Comments (c) :

- incorrect use of formula
- forgot to correct to 2 dp.

Question 17 (3 marks)

Solve $4\cos^2 \theta = 3$ for $0 \leq \theta \leq 2\pi$.

$$\cos^2 \theta = \frac{3}{4}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\text{related angle } \theta = \frac{\pi}{6} \text{ (1) since } \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6} \text{ (1)}$$

$$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \text{ (1)}$$

-	+
S	A
T	C
-	+

cos is positive in 1st and 4th
cos is negative in 2nd and 3rd
total 4 solutions

- penalised 1 mark if solution was in degrees

- penalised 1 mark for 2 out of 4 solutions.

common mistakes

• Not working in π

• incorrect θ (ie used $\frac{\pi}{3}$ instead)

• Not finding all 4 solutions.

Question 18 (3 marks)

Show that $\frac{d}{dx} \left(\frac{3x}{5x^2+1} \right) = \frac{3(1-5x^2)}{(5x^2+1)^2}$.

$$u = 3x \quad v = 5x^2+1 \quad \text{use quotient rule}$$

$$u' = 3 \quad v' = 10x \quad y' = \frac{vu' - uv'}{v^2} \quad (\text{formula sheet})$$

$$\frac{d}{dx} \left(\frac{3x}{5x^2+1} \right) = \frac{(5x^2+1)(3) - (3x)(10x)}{(5x^2+1)^2} \text{ (1)}$$

$$= \frac{15x^2+3-30x^2}{(5x^2+1)^2} \text{ (1)}$$

$$= \frac{3-15x^2}{(5x^2+1)^2}$$

$$= \frac{3(1-5x^2)}{(5x^2+1)^2} \text{ (1)}$$

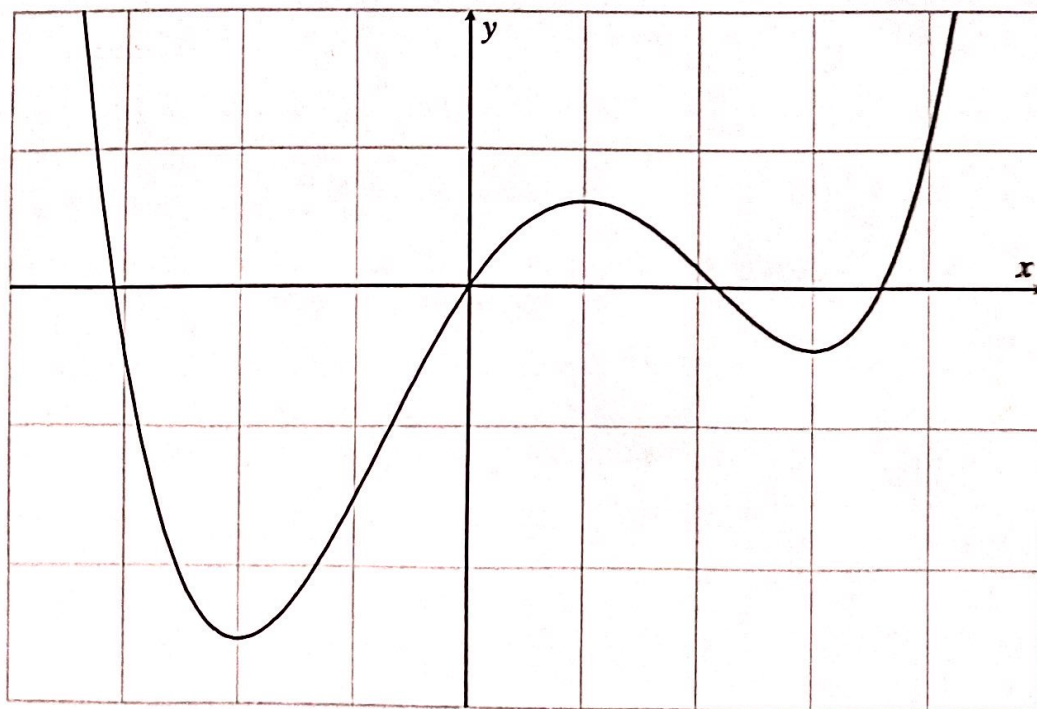
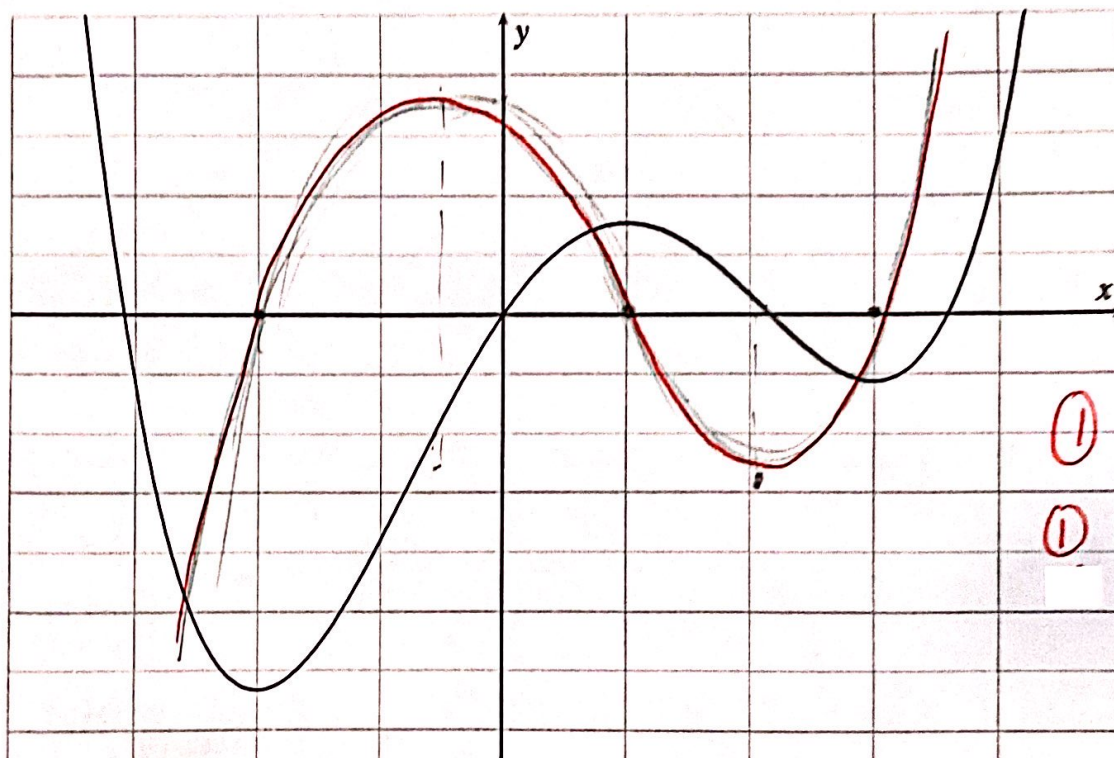
common mistakes

- incorrect formula for quotient rule
- used product rule instead which made the question harder.
- algebraic errors
- poor setting out

Question 19 (2 marks)

Sketch the derivative function of the function $y = f(x)$ given below. You may sketch the $y = f'(x)$ on the same Cartesian plane as the graph shown below.

A second copy of the Cartesian plane is provided below in case you make a mistake on the first.



Question 20 (2 marks)

Mr Ferrarin has a new hobby of kite flying with his son, Henry, but is weary of the weather on certain days. On a given day, the probability that it is cloudy is 0.35, the probability that it is windy is 0.6, and the probability that it is both windy and cloudy is 0.25.

What is the probability that it is cloudy or windy (or both)?

$$P(W \cup C) = 0.6 + 0.35 - 0.25 \quad (1)$$

$$= 0.7 \quad (1)$$

Question 21 (4 marks)

Find the equation of the normal to the curve at $y = 4x^2(2x - 9)$ at the point $(1, -28)$.

$$u = 4x^2 \quad v = 2x - 9 \quad \text{product rule } y' = vu' + uv'$$

$$u' = 8x \quad v' = 2$$

$$y' = 8x(2x - 9) + 2(4x^2)$$

$$= 16x^2 - 72x + 8x^2$$

$$= 24x^2 - 72x \quad (1) \text{ for correct differentiation}$$

$$\text{at } x=1 \quad y' = 24(1)^2 - 72(1)$$

$$= -48$$

$$\therefore m_{\text{Tangent}} = -48 \quad (1)$$

$$\text{hence } m_{\text{normal}} = \frac{1}{48} \quad (1) \quad (m_T \times m_N = -1)$$

$$y - y_1 = m(x - x_1)$$

$$y - (-28) = \frac{1}{48}(x - 1)$$

$$y + 28 = \frac{1}{48}x - \frac{1}{48}$$

$$y = \frac{1}{48}x - \frac{1345}{48} \quad \left. \begin{array}{l} (y = mx + c \text{ form}) \\ (1) \text{ for correct equation} \end{array} \right\}$$

$$\text{or } x - 48y - 1345 = 0 \quad \left. \begin{array}{l} (general \text{ form}) \end{array} \right\}$$

Common mistakes

- basic algebra mistakes
- cannot differentiate using product rule
- don't know how to find the gradient after differentiation.

Question 22 (7 marks)

A particle is moving in a straight line. Its displacement, x m, from a fixed point, O , after t seconds is given by $x = 3t^2 - 6t$ where $t \geq 0$.

- (a) Find the expressions for the velocity and acceleration functions in terms of t . 2

..... $\dot{x} = 6t - 6$ (1)

..... $\ddot{x} = 6$ (1)

- (b) Find when the particle is at rest.

..... particle at rest means $\dot{x} = 0$

..... $6t - 6 = 0$

..... $6t = 6$
..... $t = 1$ after 1 second (1)

common mistake

- a lot of students¹ thought 'at rest' means $t = 0$. (initially)
- 'at rest' means velocity is zero.

- (c) Find the velocity and acceleration of the particle at 0.5 seconds. 2

..... $\dot{x} = 6(0.5) - 6$

..... $= -3 \text{ ms}^{-1}$ (1)

..... $\ddot{x} = 6 \text{ ms}^{-2}$ (1)

..... velocity is -3 ms^{-1} and acceleration 6 ms^{-2}

common mistake

- not writing down units ms^{-1} and ms^{-2}

- (d) Is the particle speeding up or slowing down when $t = 0.5$ seconds? Explain how you determined this. 2

..... Particle is slowing down (1) Since velocity is -3 ms^{-1}

..... which is negative, and acceleration is 6 ms^{-2}

..... positive, opposite sign indicate the particle is

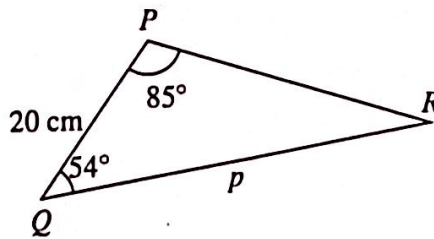
..... slowing down (1) for explanation

common mistake

- lack of understanding, a lot students think velocity negative means its ~~slow~~ slowing down.

Question 23 (3 marks)

In $\triangle PQR$, $\angle P = 85^\circ$, $\angle Q = 54^\circ$ and $PQ = 20$ cm as shown in the diagram below.



- (a) Show that $p = 30.37$ cm, correct to two decimal places.

2

Many students didn't show this. They lost a mark!

$\angle PRQ + 85 + 54 = 180$
 $\angle PRQ = 41$

$\frac{p}{\sin 85} = \frac{20}{\sin 41}$
 $p = \frac{20 \sin 85}{\sin 41} = 30.37$

This line is also needed. Otherwise you have not shown.

- (b) Calculate the area of $\triangle PQR$, answer to two decimal places.

1

$A = \frac{1}{2} ab \sin C$
 $= \frac{1}{2} (20)(p) \sin 54$
 $= 245.69 \text{ cm}^2$

Many students rounded wrong. If using the exact value of p (which you should do), then the answer is 245.69. If using $p = 30.37$, then the answer is 245.698 = 245.70

Question 24 (2 marks)

Find the values of x for which the graph of the function $f(x) = \frac{1}{x^2 - 9}$ is discontinuous.

$x^2 - 9 \neq 0$
 $x^2 \neq 9$
 $x \neq \pm 3$

$\therefore$ The graph is discontinuous when $x = 3$ and $x = -3$

NOTE: The question asks when is the graph discontinuous. The answer to this is $x = 3$ and $x = -3$.

NOT $x \neq 3$ and $x \neq -3$.

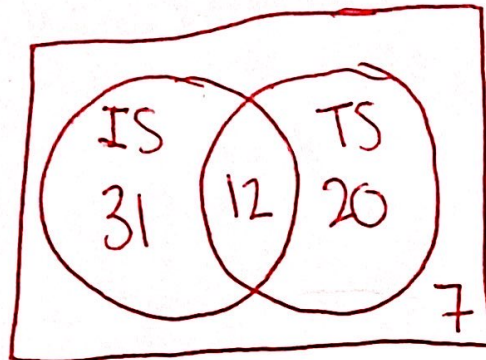
-19- No mark was deducted though.

Question 25 (3 marks)

A music magazine surveyed 70 people and asked them whether they liked Ice Spice or Taylor Swift. The survey found that 43 people liked Ice Spice and 32 people liked Taylor Swift and 7 people did not like Taylor Swift nor Ice Spice.

- (a) Represent this data in a Venn Diagram in the space below.

2



- (b) A person is selected at random. What is the probability that the person likes Taylor Swift given that they person likes Ice Spice?

1

$$P(TS|IS) = \frac{P(TS \cap IS)}{P(TS)} = \frac{\frac{12}{70}}{\frac{43}{70}} = \frac{12}{43}$$

Question 26 (2 marks)

Solve $|4x - 5| = 15$

$$\begin{aligned} 4x - 5 &= 15 & 4x - 5 &= -15 \\ 4x &= 20 & 4x &= -10 \\ x &= 5 & x &= -10/4 \end{aligned}$$

$$x = 5 \text{ or } -5/2$$

$x\sqrt{x} = x \times x^{\frac{1}{2}} = x^{\frac{3}{2}}$ } Too many students didn't know this.

Question 27 (3 marks)

Differentiate $y = (x\sqrt{x} - 2)^4$.

$$y = (x^{\frac{3}{2}} - 2)^4$$

$$\frac{dy}{dx} = 4(x^{\frac{3}{2}} - 2)^3 \times \frac{3}{2}x^{\frac{1}{2}}$$

$$= 6x^{\frac{1}{2}}(x^{\frac{3}{2}} - 2)^3$$

Question 28 (2 marks) NOTE: It is not acceptable to say the radius can't be negative. In this case r^2 is negative, which means $r = \sqrt{-2}$. This is not a negative number. It is not real.

Give reasons why $x^2 + 4x + y^2 - 6y + 15 = 0$ is not an equation of a circle.

$$x^2 + 4x + 4 + y^2 - 6y + 9 = -15 + 4 + 9$$

$$(x+2)^2 + (y-3)^2 = -2$$

$$r^2 = -2$$

$$\therefore r = \sqrt{-2}$$

However you can't square root a negative number. Therefore the radius and hence circle doesn't exist.

Question 29 (3 marks)

Prove that $\frac{1 + \cos \phi}{1 - \cos \phi} - \frac{1 - \cos \phi}{1 + \cos \phi} = 4 \cot \phi \operatorname{cosec} \phi$.

$$\text{LHS} = \frac{(1 + \cos \phi)^2 - (1 - \cos \phi)^2}{(1 - \cos \phi)(1 + \cos \phi)}$$

$$= \frac{(1 + 2\cos \phi + \cos^2 \phi) - (1 - 2\cos \phi + \cos^2 \phi)}{1 - \cos^2 \phi}$$

$$= \frac{4\cos \phi}{\sin^2 \phi}$$

$$= 4 \times \frac{\cos \phi}{\sin \phi} \times \frac{1}{\sin \phi}$$

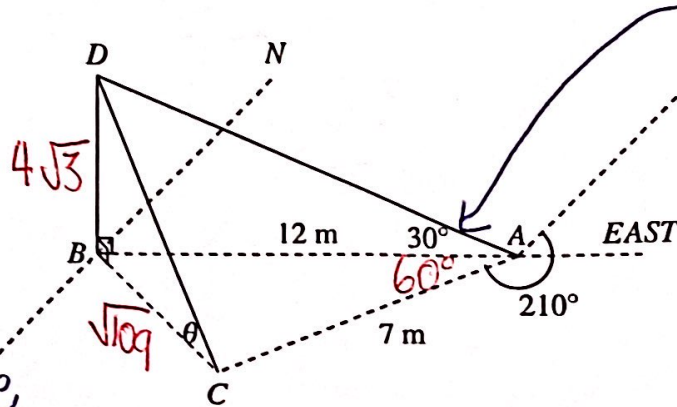
$$= 4 \cot \phi \operatorname{cosec} \phi$$

$$= \text{RHS}$$

Question 30 (5 marks)

A flagpole, BD , sits on flat level ground. It is supported by two cables, AD and CD , as shown below. The cable AD is inclined at 30° to the horizontal and is tied to the floor 12 metres from the foot of the flagpole.

The second cable is tied to the floor at point C , 7 metres from A and on a bearing of 210° from A .



NOTE: This 30° is in the vertical plane. It is not part of the revolution on the ground.

A very good idea is to add things to the diagram as you find them, as I have done here. This will help save time in the next parts.

(a) Show that $\angle BAC = 60^\circ$.

$$\angle NAB = 90^\circ \quad | \quad 360 = 210 + 90 + \angle BAC \quad | \quad \angle BAC = 60^\circ$$

(b) Find the exact height of the flagpole, BD .

$$\tan 30 = \frac{BD}{12} \quad | \quad BD = 12 \tan 30 = 12 \times \frac{1}{\sqrt{3}} = 4\sqrt{3}$$

(c) Find the exact distance B to C and hence, find θ , correct to the nearest degree

$$BC^2 = 12^2 + 7^2 - 2(12)(7)\cos 60$$

$$= 109$$

$$BC = \sqrt{109}$$

$$\tan \theta = \frac{4\sqrt{3}}{\sqrt{109}}$$

$$\theta = \tan^{-1}\left(\frac{4\sqrt{3}}{\sqrt{109}}\right) = 34^\circ$$

End of paper