Name:…………………….. Teacher……………………… Assessment No. ……………

PLC PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

Mathematics Advanced

Year 11 Yearly Examination, 2022

- Reading time – 10 minutes
- Working time – 2.5 hours
- Write using black pen
- Calculators approved by NESA may be used
- The HSC Reference Sheet will be provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations

	Marks
Multiple Choice	**/10**
Section II: Part 1	**/24**
Section II: Part 2	**/21**
Section II: Part 3	**/23**
Section II: Part 4	**/22**
Total	**/100**

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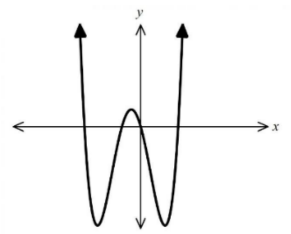
Section I: Multiple Choice (10 marks)

Answer Q1–10 on the **multiple-choice answer sheet** provided.

Allow about 15 minutes for this section.

1. Which of the graphs below represents a one to one function?

A.



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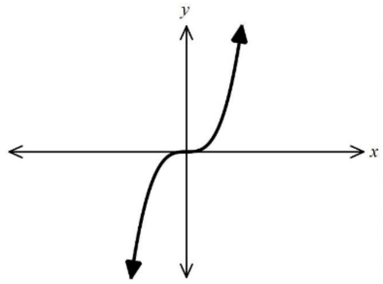

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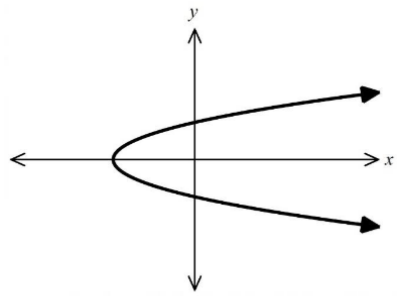

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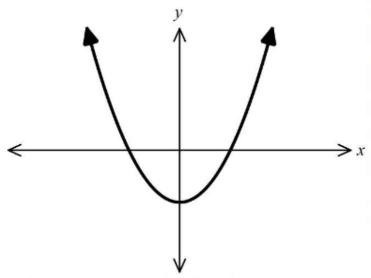


2. What is the exact value of $\cos \frac{11\pi}{6}$?

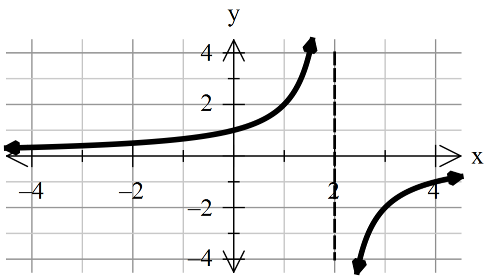
A. $-\frac{\sqrt{3}}{2}$ **B.** $-\frac{1}{2}$ **C.** $\frac{1}{2}$ **D.** $\frac{\sqrt{3}}{2}$

3. What is $\sqrt{16x^{16}}$ in simplest form?

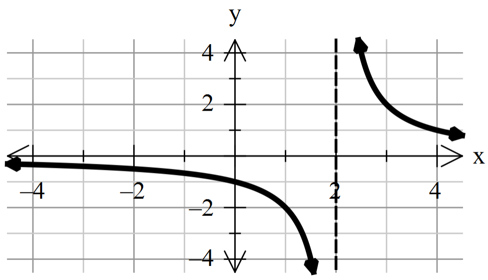
A. $8x^4$ **B.** $4x^4$ **C.** $4x^8$ **D.** $8x^8$

4. Which of the following graphs represents $f(x) = \frac{2}{2-x}$

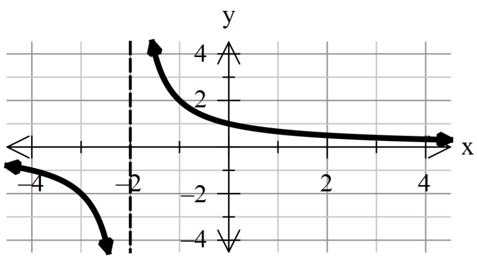
A.



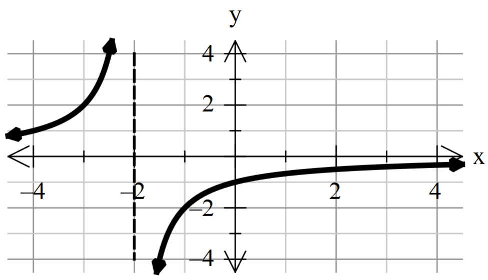
B.



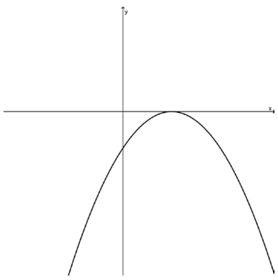
C.



D.



5. Below is the graph of $y = a(x+h)^2$, where a and h are constants.
Which statement is correct?



A. $a > 0,\ \ h > 0$

B. $a > 0,\ \ h < 0$

C. $a < 0,\ \ h > 0$

D. $a < 0,\ \ h < 0$

6. A function is defined by the rule $f(x) = \begin{cases} 1 & \text{for } x < 1 \\ x+2 & \text{for } x \geq 1 \end{cases}$

Which statement is **incorrect**?

A. The value of $f(-2)$ is 1.

B. The graph is not continuous at $x = 1$.

C. The domain is all real values for x.

D. The range is $y \geq 1$.

7. What is the derivative of $-\frac{3}{x^2}$?

A. $-\frac{3}{x}$

B. $-\frac{6}{x^3}$

C. $-\frac{3}{x^3}$

D. $\frac{6}{x^3}$

8. Let $a = e^x$. Which expression is equal to $\log_e(a^2)$?

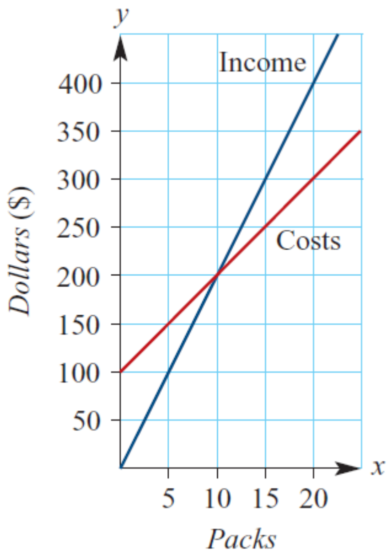
A. e^{2x}

B. e^{x^2}

C. $2x$

D. x^2

9. The graph below shows the costs of making a pack of batteries and the income received from the sale of the batteries.



How many packs of batteries are sold if the business makes a loss of $50?

A. 5 **B.** 10 **C.** 15 **D.** 20

10. There are 10 blue counters and R red counters in a bag. The probability of selecting a red counter is $\frac{4}{9}$.
How many more red counters need to be added to the bag so that the probability of selecting a red counter from the bag is $\frac{3}{5}$?

A. 7 **B.** 8 **C.** 14 **D.** 21

– End of Section I –

Section II: Part 1 of 4 (24 marks)

11. Expand and simplify $(2+\sqrt{3})(\sqrt{3}-4)$ **2**

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12. Solve $|3x+1| = 8$ **2**

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13. Solve $\tan\theta = \sqrt{3}$ for $-180° \leq \theta \leq 180°$. **2**

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14. Solve $32^x = 2^{x+1}$. **2**

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15. Differentiate $y = \sqrt{(5x^2 - 9)}$. Write the answer with a positive index. **2**

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16. Find the equation of the line parallel to $x - 3y + 18 = 0$ which passes through the point $(-1, 4)$. Answer in general form. **3**

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17. Find the centre and radius of the circle $x^2 - 6x + y^2 + 8y - 75 = 0$. **3**

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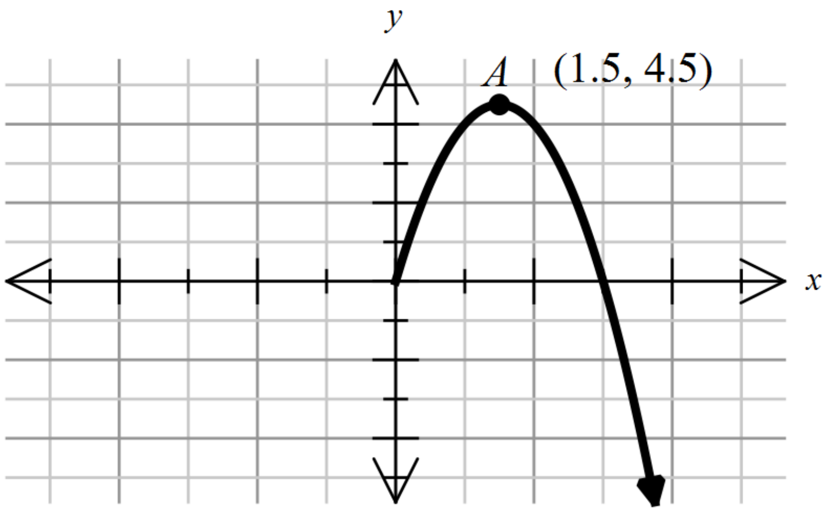
18. The graph of the function $y = f(x)$ is shown for the domain $x \geq 0$. **1**
Given that $y = f(x)$ is an odd function for all real x, draw the remainder of the function on the grid below.



19. Evaluate $\log_b 36$ if $\log_b 3 = 0.68$ and $\log_b 4 = 0.86$ **2**

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20. Find the exact value of $\tan\theta$ if $\operatorname{cosec}\theta = 4$, for $90° \leq \theta \leq 180°$. **2**

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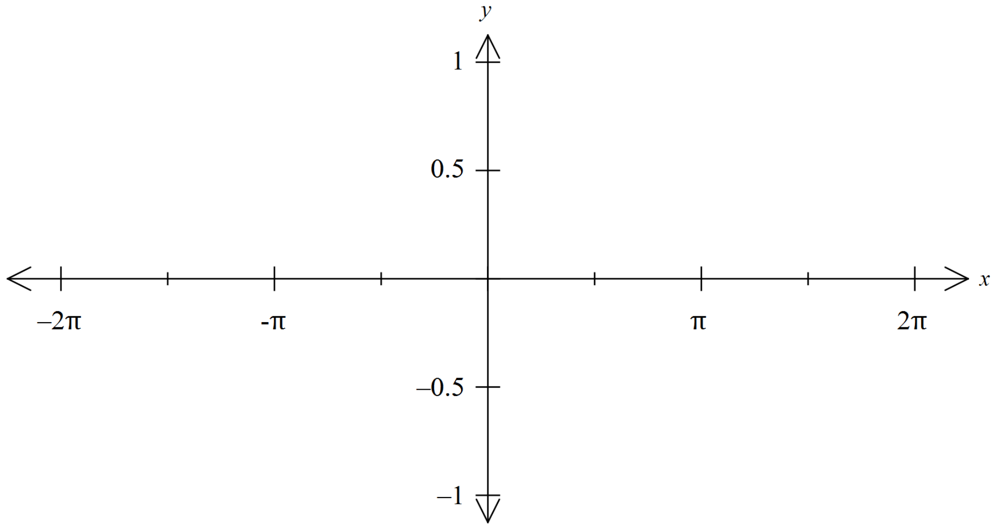
21. Sketch $y = \cos x$ for $-2\pi \leq x \leq 2\pi$ on the axes below. **1**



22. Simplify $\frac{(x^{-3}y^2)^{-3}}{x^{-2}y^7} \times \frac{x^2y^{-3}}{(xy^3)^2}$ expressing your answer with positive indices. **2**

End of Section II: Part 1

Section II: Part 2 of 4 (21 marks)

23. Differentiate $f(x) = x^2 + 3x$ from first principles, **2**

using the formula $f\,'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

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24. Solve $\log_2 x - \log_2 7 = \log_2 \left(\frac{x-3}{2}\right)$ **2**

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25. Find the exact gradient of the tangent of the function $f(x) = 3x^2e^{5x}$ at $x = 1$. **2**

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26. Differentiate $y = \frac{x^2+1}{4x-7}$, giving your answer in simplest form. **3**

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27. The discrete random variable X has the following probability distribution.

X	1	2	3
$P(X = x)$	a	b	0.2

Given that E(X) = 1.9

a. Find the values of a and b. **2**

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b. Use one of the formulae on the reference sheet to calculate Var (X). Show working. **2**

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28. Let A and B be independent events, where $P(A) = 0.3$ and $P(B) = 0.6$

a. Find $P(A \cap B)$ **1**

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b. Find $P(A \cup B)$ **1**

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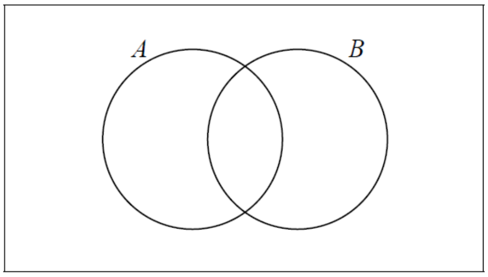
c. On the Venn diagram below, shade the region $A \cap B'$ **1**



d. Find $P(A \cap B')$ **1**

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29. A semicircle has domain $[-1,1]$and range $[-1,0]$. **1**
What is the equation of the semicircle?

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30. Differentiate $y = x^2 + bx + c$ and hence find b and c, given that the line $y = -3x + 8$ is a normal to the curve at the point $(3, -1)$. **3**

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End of Section II: Part 2

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Section II: Part 3 of 4 (23 marks)

31. The population of an ant colony is modelled by the equation $P = 1000e^{0.34t}$ where t is measured in weeks.

a. Find the population after 10 weeks (answer to the nearest 10) **1**

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b. When will the population exceed 80 000 ants? **2**

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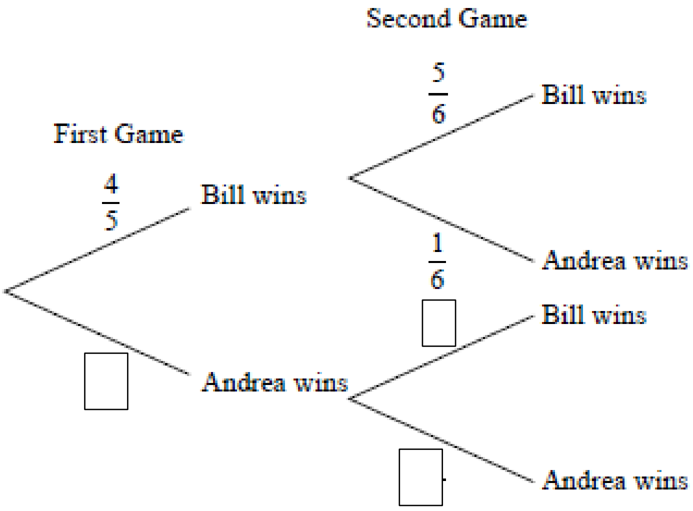
32. Bill and Andrea play two games of tennis. The probability that Bill wins the first game is $\frac{4}{5}$.

If Bill wins the first game, the probability he wins the second game is $\frac{5}{6}$.

If Bill loses the first game, the probability that he wins the second game is $\frac{2}{3}$.

a. Complete the tree diagram below. **1**



b. Find the probability that Bill wins at least one game. **1**

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c. Given that Bill wins at least 1 game, find the probability he wins both games. **1**

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33.

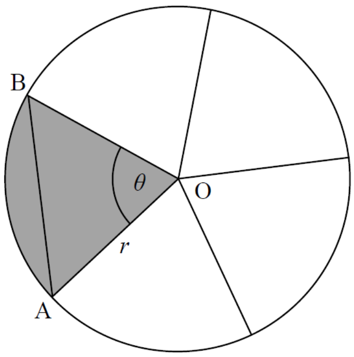

diagram not to scale

The diagram above shows a circle, centre O and radius r mm.
The circle is divided into five equal sectors.
One sector is OAB and $\angle AOB = \theta$ radians.
The area of the sector AOB is 20π mm^2.

a. Find the value of r. **2**

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b. Find the length of the chord AB. Give the answer to 1 decimal place. **2**

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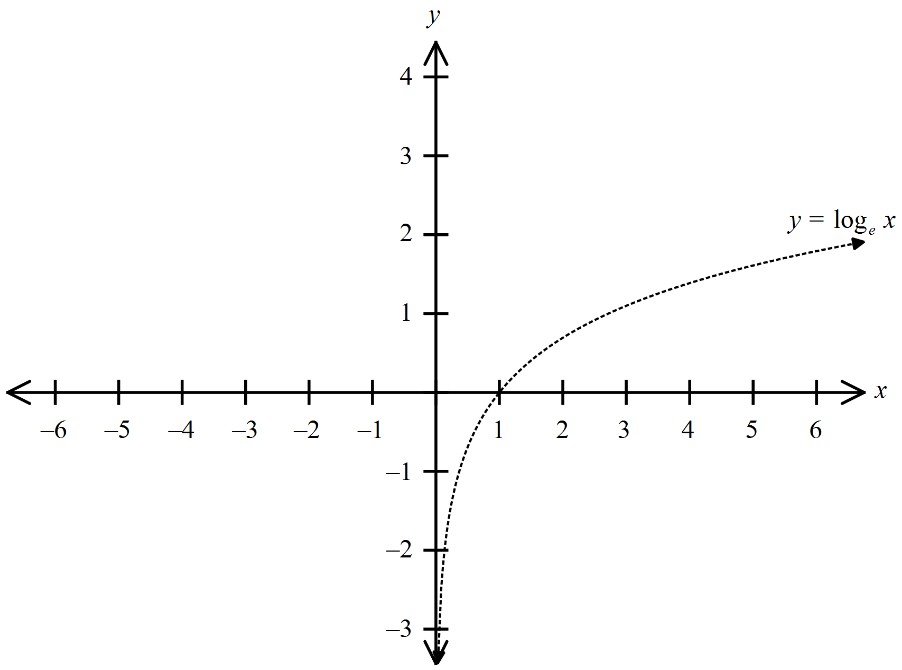
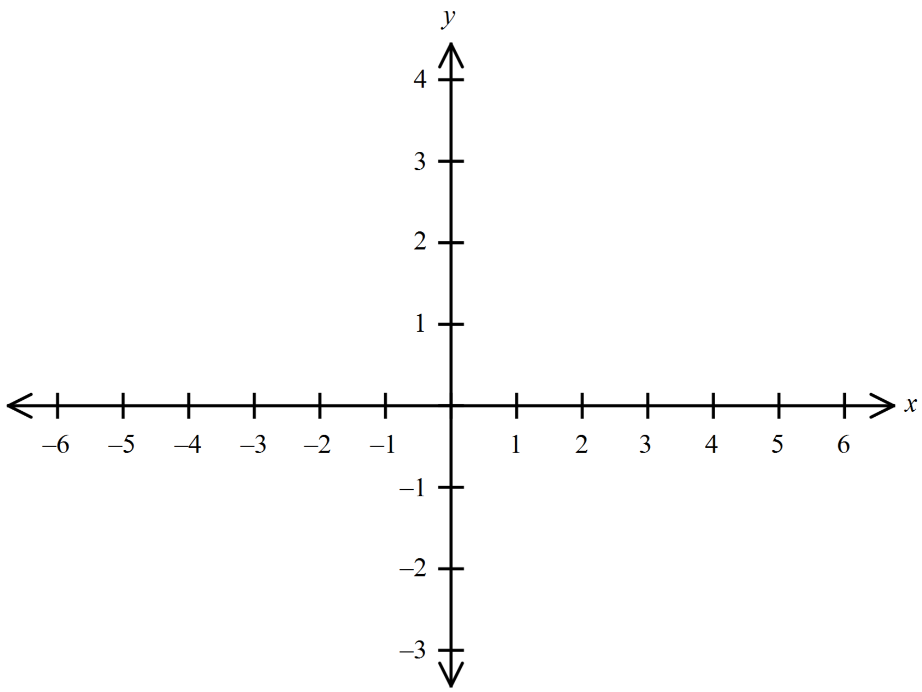
34. Consider the function $y = \log_e x$. **2**

Use the sketch below of $y = \log_e x$ to sketch $y = \log_e(x+2) - 1$ on the same axes. Show **any asymptotes and the *y* intercept** on your graph.



Use your sketch from part (a) to draw a sketch of $y = -\log_e(x+2) + 1$. **1**

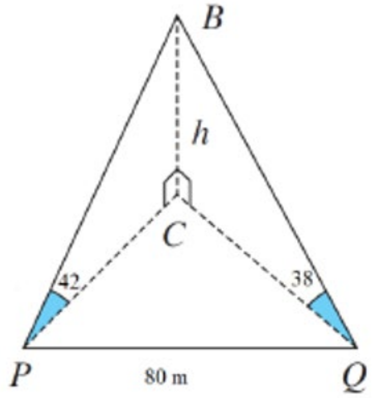
Show **any asymptotes and the *y* intercept** on your graph.



35. The top of a tower is due north of P and its angle of elevation is $42°$. Q is due west of the tower and is 80 metres from P. The angle of elevation from Q to the top of the tower is $38°$. **4**

Let the height of the tower be h metres and let C be the base of the tower on the ground.

Calculate the height of the tower correct to 1 decimal place.



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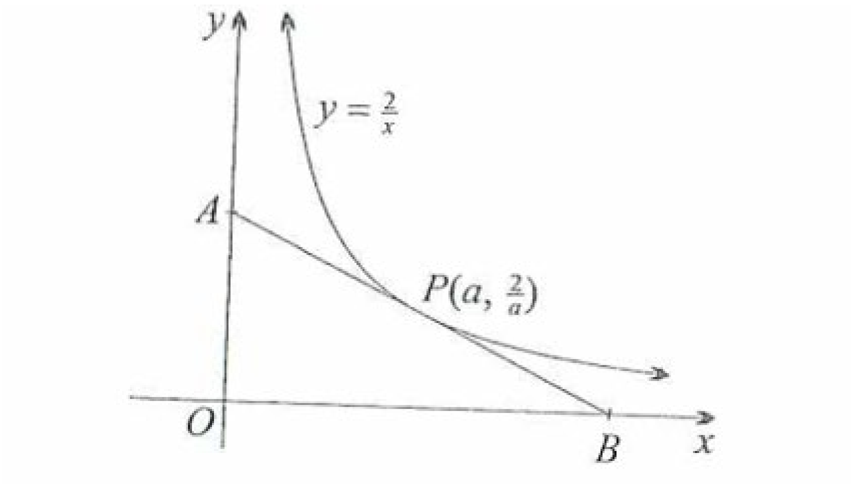

The diagram above shows part of the hyperbola $y = \frac{2}{x}$ and the tangent to the hyperbola drawn at point $P\ \left(a, \frac{2}{a}\right)$.
The tangent intersects the y-axis and x-axis at A and B respectively.

a. Find the gradient of the curve at the point $P\ \left(a, \frac{2}{a}\right)$ in terms of a. **2**

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b. Hence, show the equation of the tangent to the curve $y = \frac{2}{x}$ at the point $P\ \left(a, \frac{2}{a}\right)$ is $2x + a^2y - 4a = 0$. **2**

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Question 36 continues over the page

c. Show that the area of $\Delta\, AOB$ is a constant. **2**

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End of Section II: Part 3

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Section II: Part 4 of 4 (22 marks)

37. Consider $f(x) = \frac{1}{\sqrt{x}}$.

a. State the domain and range of $f(x)$. **1**

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b. For what value of x does $f(x) = 2$? **1**

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c. Find $f'(x)$ and hence calculate $f'(3)$, leaving your answer in simplified surd form with a rational denominator. **2**

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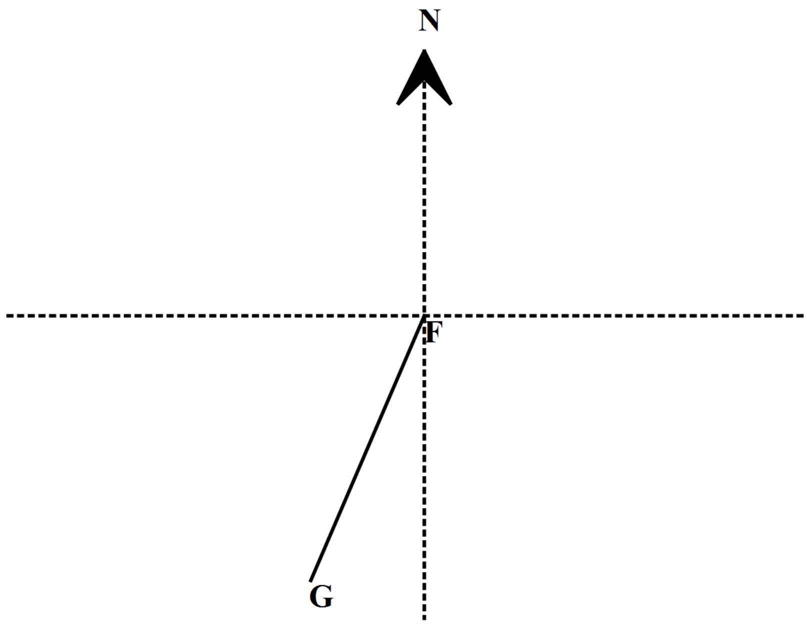
38. The distance between the towns of Ford (F) and Galen (G) is 160 km. The bearing of Galen from Ford is 214T°. The distance from Galen to Holden (H) is 220 km. Holden is due west of Ford.



a. Mark the information on the diagram above. **1**

b. Find the size of $\angle FHG$ correct to the nearest degree. **2**

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c. What is the bearing of H from G? **1**

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39. Prove $\sec x + \tan x + \cot x = \frac{1+\sin x}{\sin x \cos x}$ **2**

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40. Use the discriminant to show that $x^2 - 2x + 4 > 0$ **2**

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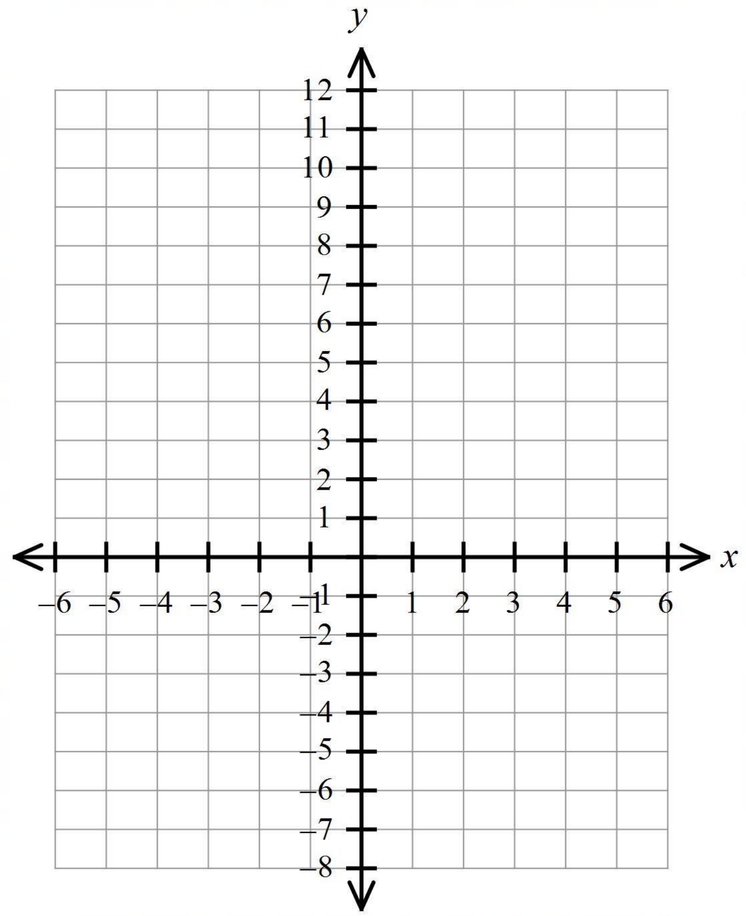
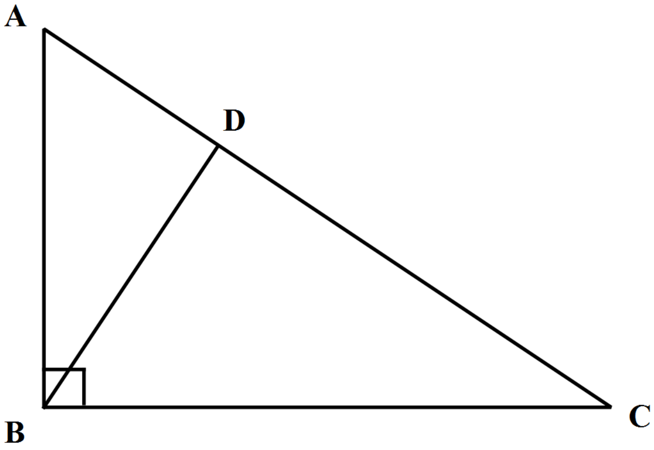


a. On the grid above, draw the graph of the function $y = |3 - 2x|$. **2**
Clearly show coordinates of x and y intercept(s).

b. Hence solve the equation $|3 - 2x| = 7$ graphically. **2**

42. A triangle ABC is right angled at B.
D is the point on AC such that BD is perpendicular to AC.
Let $\angle BAC = \theta$.



a. You are given that $6AD + BC = 5AC$ **2**
Show that $6\cos\theta + \tan\theta = 5\sec\theta$.
Hint: Consider ΔABC and ΔABD

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Question 42 continues over the page

b. Deduce that $6\sin^2\theta - \sin\theta - 1 = 0$ **2**

c. Find the value of θ. **2**

End of Section II
END OF EXAM

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Extra Writing Space

If you use this space, clearly indicate which question you are answering.

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Extra Writing Space

If you use this space, clearly indicate which question you are answering.

Assessment Number…………………………

Name:…………………….. Teacher……………………… Assessment No. ……………

PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

Mathematics Advanced

Year 11 Yearly Examination, 2022

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- Working time – 2.5 hours
- Write using black pen
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- The HSC Reference Sheet will be provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations

	Marks
Multiple Choice	/10
Section II: Part 1	/24
Section II: Part 2	/21
Section II: Part 3	/23
Section II: Part 4	/22
Total	/100

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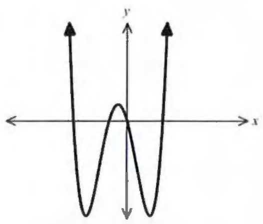
Section I: Multiple Choice (10 marks)

Answer Q1–10 on the **multiple-choice answer sheet** provided.

Allow about 15 minutes for this section.

1. Which of the graphs below represents a one to one function? Pass vertical and horizontal line test

A.



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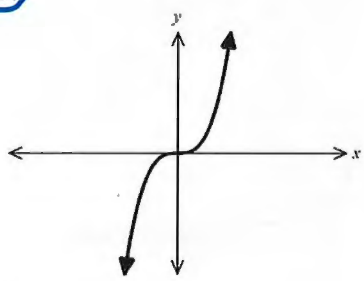
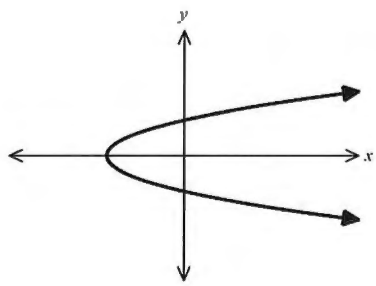


C.



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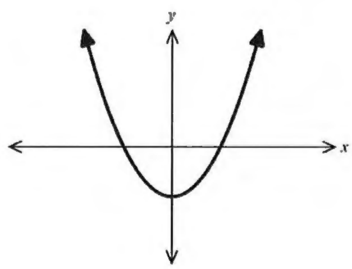


2. What is the exact value of $\cos \frac{11\pi}{6}$? $= \cos 330^\circ = \cos 30^\circ$

A. $-\frac{\sqrt{3}}{2}$ B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. $\frac{\sqrt{3}}{2}$

3. What is $\sqrt{16x^{16}}$ in simplest form? $\sqrt{16} = 4$ $(x^{16})^{1/2} = x^8$

A. $8x^4$ B. $4x^4$ C. $4x^8$ D. $8x^8$

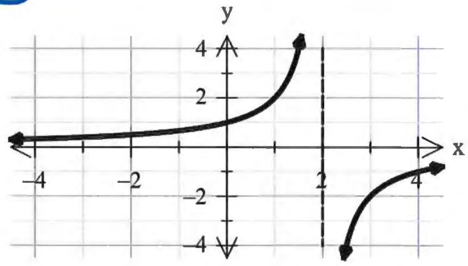
4. Which of the following graphs represents $f(x) = \frac{2}{2-x}$

when $x = 1$ $y = 2$

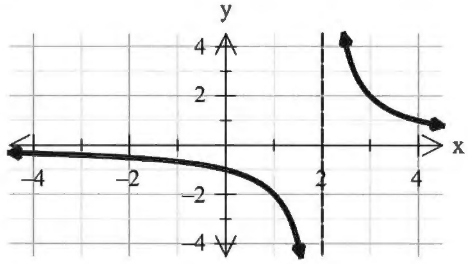
$2 - x \neq 0$

$x \neq 2$

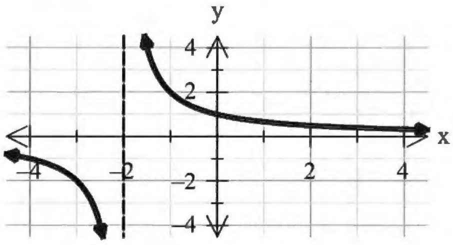
A.



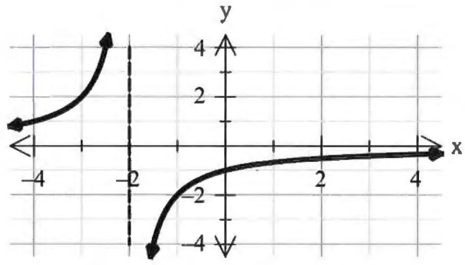
B.



C.



D.

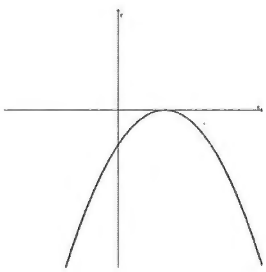


5. Below is the graph of $y = a(x + h)^2$, where a and h are constants.
Which statement is correct?

concave down $a < 0$

x intercept > 0

$\therefore$ $h < 0$



A. $a > 0,\ h > 0$

B. $a > 0,\ h < 0$

C. $a < 0,\ h > 0$

D. $a < 0,\ h < 0$

6. A function is defined by the rule $f(x) = \begin{cases} 1 & \text{for } x < 1 \\ x+2 & \text{for } x \geq 1 \end{cases}$ $f(1)=3$

Which statement is **incorrect?**

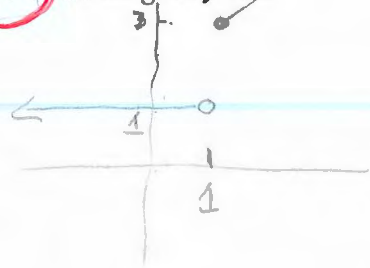
D is correct answer!

A. The value of $f(-2)$ is 1. T

B. The graph is not continuous at $x = 1$.

C. The domain is all real values for x. T

D. The range is $y \geq 1$.



7. What is the derivative of $-\frac{3}{x^2}$? $y = -3x^{-2}$ $\frac{dy}{dx} = 6x^{-3} = \frac{6}{x^3}$

A. $-\frac{3}{x}$

B. $-\frac{6}{x^3}$

C. $-\frac{3}{x^3}$

D. $\frac{6}{x^3}$

8. Let $a = e^x$. Which expression is equal to $\log_e(a^2)$? $\log_e e^{2x} = 2x \log_e e$

$a^2 = (e^x)^2 = e^{2x}$ $= 2x \times 1$

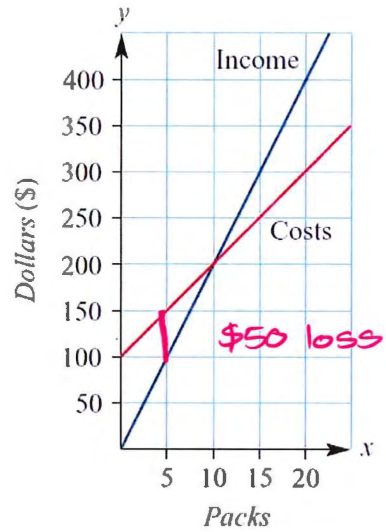
A. e^{2x}

B. e^{x^2}

C. $2x$

D. x^2 $= 2x$

9. The graph below shows the costs of making a pack of batteries and the income received from the sale of the batteries.



How many packs of batteries are sold if the business makes a loss of $50?

A. 5 **B.** 10 **C.** 15 **D.** 20

10. There are 10 blue counters and R red counters in a bag. The probability of selecting a red counter is $\frac{4}{9}$. $P(\text{Blue}) = \frac{5}{9} = \frac{10}{18}$ $\therefore$ 18 counters in total

How many more red counters need to be added to the bag so that the probability of selecting a red counter from the bag is $\frac{3}{5}$? $\therefore$ 8 red

A. 7 **B.** 8 **C.** 14 **D.** 21

Changes to $P(\text{red}) = \frac{3}{5}$ $P(\text{blue}) = \frac{2}{5}$

$= \frac{10}{25}$

$\therefore$ 10 Blue 15 red

$15 - 8 = 7$ more red.

– End of Section I –

Assessment Number.................

Section II: Part 1 of 4 (24 marks)

11. Expand and simplify $(2+\sqrt{3})(\sqrt{3}-4)$ **2**

$= 2\sqrt{3} - 8 + 3 - 4\sqrt{3}$ Well done

$= -2\sqrt{3} - 5$

12. Solve $|3x+1| = 8$ **2**

Absolute value equations require solving 2 cases.

$3x+1=8$

$3x=7$

$x=\frac{7}{3}$

$-(3x+1)=8$

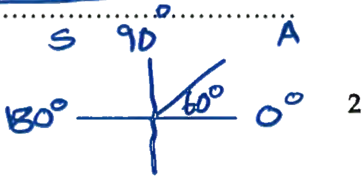
$-3x-1=8$

$-3x=9$

$x=-3$

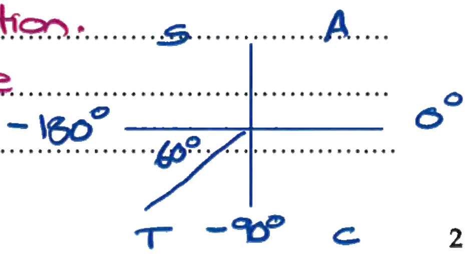
13. Solve $\tan\theta = \sqrt{3}$ for $-180° \le \theta \le 180°$. **2**

✓ degrees so answer in degrees.



$\theta = 60°, -120°$

Always check domain in question.
Do not assume negative angle solution is the same number as positive angle.



14. Solve $32^x = 2^{x+1}$. **2**

$(2^5)^x = 2^{x+1}$

make base same on both sides of equation.

$2^{5x} = 2^{x+1}$

equate indices

$5x = x+1$

$4x = 1$

$x = \frac{1}{4}$

Assessment Number.................

15. Differentiate $y = \sqrt{(5x^2 - 9)}$. Write the answer with a positive index. **2**

$y = (5x^2-9)^{\frac{1}{2}}$ chain rule not product rule

$y' = \frac{1}{2}(5x^2-9)^{-\frac{1}{2}} \times 10x$

$= \frac{5x}{\sqrt{5x^2-9}}$ or $\frac{5x}{(5x^2-9)^{\frac{1}{2}}}$

16. Find the equation of the line parallel to $x - 3y + 18 = 0$ which passes through the point $(-1, 4)$. Answer in general form. **3**

$3y = x + 18$

$y = \frac{x+18}{3}$

Grad $= \frac{1}{3}$

$y - 4 = \frac{1}{3}(x+1)$

$3y - 12 = x + 1$

$x - 3y + 13 = 0$

General form
- no fractions
- coefficient of x is positive

17. Find the centre and radius of the circle $x^2 - 6x + y^2 + 8y - 75 = 0$. **3**

$x^2 - 6x + 9 + y^2 + 8y + 16 = 75 + 9 + 16$

$(x-3)^2 + (y+4)^2 = 100$

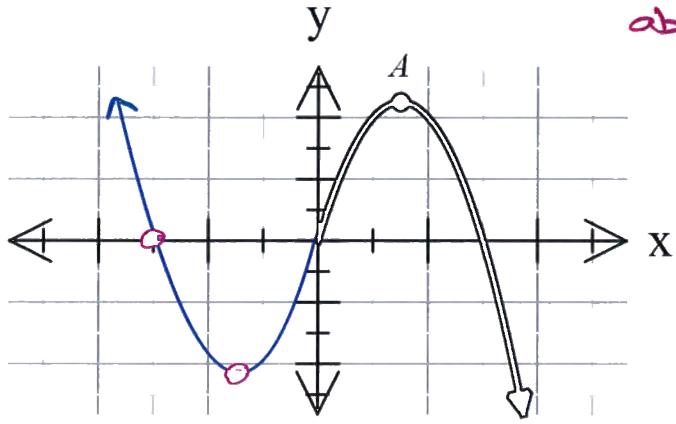
Centre $= (3, -4)$

Radius $= 10$

learn process of completing square.

18. The graph of the function $y = f(x)$ is shown for the domain $x \geq 0$. **1**
Given that $y = f(x)$ is an odd function for all real x, draw the remainder of the function on the grid below. ~~Mark the point A' that corresponds to $A = (1.5, 4.5)$ and write its coordinates on the graph.~~

→ rotational symmetry about origin



be accurate with obvious points such as x, y intercepts, max, min points.

19. Evaluate $log_b 36$ if $log_b 3 = 0.68$ and $log_b 4 = 0.86$ **2**

$\log_b 36 = \log_b (4 \times 9)$

$= \log_b 4 \; -2\log_b 3$

$= 0.86 \; + 2 \times 0.68$

$= 2.22$

$\log_b 3^2$ is not $(0.68)^2$. Use log laws $\log_b 3^2 = 2\log_b 3$

20. Find the exact value of $\tan\theta$ if $\operatorname{cosec}\theta = 4$, for $90° \leq \theta \leq 180°$. **2**

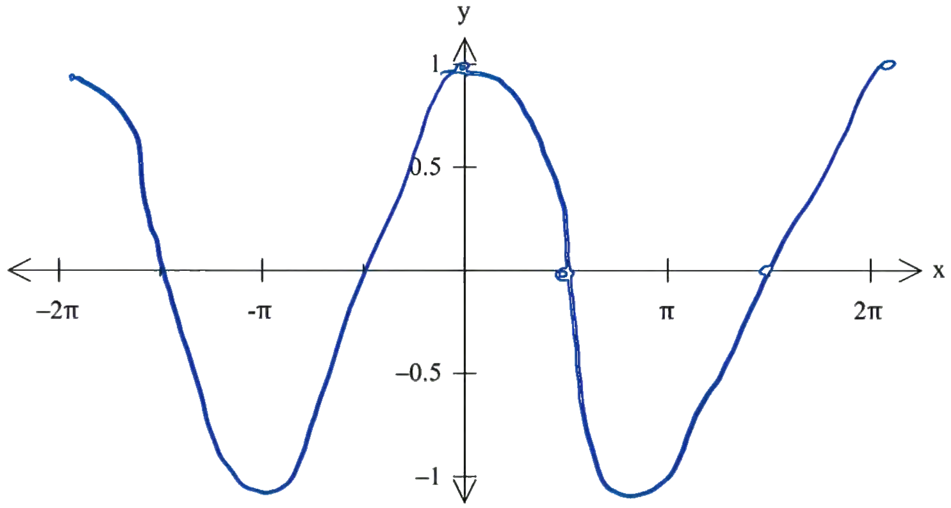
$\operatorname{cosec} \theta = 4$ 2nd quadrant

reciprocal ratios are on reference sheet → $\sin \theta = \frac{1}{4}$

4, 1, θ, $\sqrt{15}$ (Pythagoras)

$\tan \theta = -\frac{1}{\sqrt{15}}$

21. Sketch $y = \cos x$ for $-2\pi \leq x \leq 2\pi$ on the axes below. **1**



learn basic trig graphs.

22. Simplify $\frac{(x^{-3}y^2)^{-3}}{x^{-2}y^7} \times \frac{x^2y^{-3}}{(xy^3)^2}$ expressing your answer with positive indices. **2**

$\frac{x^{+9}y^{-6}}{x^{-2}y^7} \times \frac{x^2y^{-3}}{x^2y^6}$

$= \frac{x^{11}y^{-9}}{x^0y^{13}}$

$= x^{11}y^{-22}$

$= \frac{x^{11}}{y^{22}}$

OR

$\frac{x^{+9}y^{-6}}{x^{-2}y^7} \times \frac{x^2y^{-3}}{x^2y^6}$

$= x^{11}y^{-13} \times x^0y^{-9}$

$= x^{11}y^{-22}$

$= \frac{x^{11}}{y^{22}}$

It is sensible to write all working to avoid errors.

Space out work so it is easy to read.

End of Section II: Part 1

Section II: Part 2 of 4 (21 marks)

23. Differentiate $f(x) = x^2 + 3x$ from first principles, using the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ **x 2**

$= \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - (x^2+3x)}{h}$

$= \lim_{h \to 0} \frac{\boxed{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}}{h}$

Lots of mistakes here.

$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$

$= \lim_{h \to 0} \frac{h(2x+h+3)}{h}$ $\Rightarrow \frac{h}{h} \neq 0$

$= \lim_{h \to 0} 2x + h + 3$

$= 2x + 3$

24. Solve $\log_2 x - \log_2 7 = \log_2\left(\frac{x-3}{2}\right)$ **2**

$\log_2\left(\frac{x}{7}\right) = \log_2\left(\frac{x-3}{2}\right)$

$\frac{x}{7} = \frac{x-3}{2}$ $\Rightarrow \log_2 x - \log_2 7 \neq \log_2 (x-7)$

$2x = 7x - 21$

$21 = 5x$ $\Rightarrow$ Generally well done

$x = \frac{21}{5}$

25. Find the exact gradient of the tangent of the function $f(x) = 3x^2e^{5x}$ at $x = 1$. **2**

$f'(x) = 3x^2 \cdot 5e^{5x} + e^{5x} \cdot 6x$

$= 15x^2e^{5x} + 6xe^{5x}$

$u = 3x^2$ $v = e^{5x}$

$\frac{du}{dx} = 6x$ $\frac{dv}{dx} = 5e^{5x}$

$f'(1) = 15e^5 + 6e^5$

$= 21e^5$

⇒ Many students did not recognise the need to use Product Rule

26. Differentiate $y = \frac{x^2+1}{4x-7}$, giving your answer in simplest form. **3**

$u = x^2+1$ $v = 4x-7$

$\frac{du}{dx} = 2x$ $\frac{dv}{dx} = 4$

$\frac{dy}{dx} = \frac{(4x-7)2x - 4(x^2+1)}{(4x-7)^2}$

$= \frac{8x^2 - 14x - 4x^2 - 4}{(4x-7)^2}$ ← Lots of algebra mistakes made with bracket expansion.

$= \frac{4x^2 - 14x - 4}{(4x-7)^2}$

⇒ Generally Well Done

$= \frac{2(2x^2 - 7x - 2)}{(4x-7)^2}$

27. The discrete random variable X has the following probability distribution.

X	1	2	3
$P(X=x)$	a	b	0.2

Given that E(X) = 1.9

a. Find the values of a and b. **2**

sum of probabilities

$a+b+0.2=1$

$a+b=0.8$ (1)

E(x)

$a+2b+0.6=1.9$

$a+2b=1.3$ (2)

(2) − (1) $b=0.5$

$a=0.3$

⇒ Generally Well Done.

b. Use one of the formulae on the reference sheet to calculate Var (X). Show working. **2**

$\text{Var}(x)=\left[1^2\times0.3+2^2\times0.5+3^2\times0.2\right]-1.9^2$

$=4.1-1.9^2$

$=0.49$

$\text{Var}(x)=(1-1.9)^2\times0.3+(2-1.9)^2\times0.5+(3-1.9)^2\times0.2$

$=0.49$

⇒ Poorly Done.

⇒ Check the reference sheet for the correct formula to use.

28. Let A and B be independent events, where $P(A) = 0.3$ and $P(B) = 0.6$

a. Find $P(A \cap B)$ **1**

$P(A \cap B) = P(A)P(B)$

$= 0.3 \times 0.6$

$= 0.18$

⇒ Generally Well Done

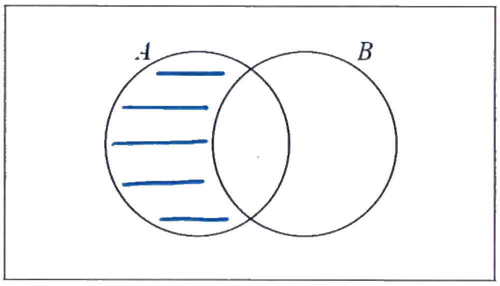
b. Find $P(A \cup B)$ **1**

$= 0.3 + 0.6 - 0.18$

$= 0.72$

⇒ Generally Well Done

c. On the Venn diagram below, shade the region $A \cap B'$ **1**



d. Find $P(A \cap B')$ **1**

$0.3 - 0.18 = 0.12$

⇒ Generally Well Done.

29. A semicircle has domain $[-1,1]$ and range $[-1,0]$. **1**
What is the equation of the semicircle?

$y = -\sqrt{1 - x^2}$

⇒ Poorly Done.

⇒ Lots of students wrote $\sqrt{x^2 + y^2}$ and did not realise the need to rearrange the formula.

⇒ Lots of students missed the negative.

30. Differentiate $y = x^2 + bx + c$ and hence find b and c, given that the line $y = -3x + 8$ is a normal to the curve at the point $(3, -1)$. **3**

$y' = 2x + b$

At $x = 3$ Grad $= \frac{1}{3}$

$\frac{1}{3} = 2 \times 3 + b$

$y = -3x + 8$

Grad $= -3$

ie Grad normal $= -3$

Grad tangent $= \frac{1}{3}$

$b = -5\frac{2}{3}$

$y = x^2 - \frac{17}{3}x + c$

sub $(3, -1)$

$-1 = 9 - 17 + c$

$-1 = -8 + c$

$c = 7$

⇒ Lots of students directly substituted the point into the equation of normal. They need to realise that differentiation finds the gradient function, and so on from there.

End of Section II: Part 2

Section II: Part 3 of 4 (23 marks)

31. The population of an ant colony is modelled by the equation $P = 1000e^{0.34t}$ where t is measured in weeks.

a. Find the population after 10 weeks (answer to the nearest 10) **1**

$P = 1000e^{0.34\times 10}$

$= 29960$ (nearest 10)

Round to the nearest 10

b. When will the population exceed 80 000 ants? **2**

$80\,000 = 1000e^{0.34t}$

$80 = e^{0.34t}$

$\log_e 80 = 0.34t$

Most students got to converting to a logarithm

$t = \frac{1}{0.34} \ln 80$

$= 12.8...$

Many were unable to solve the equation. Please review.

After 12.8 weeks

allowed 12.8 or rounded to 13.

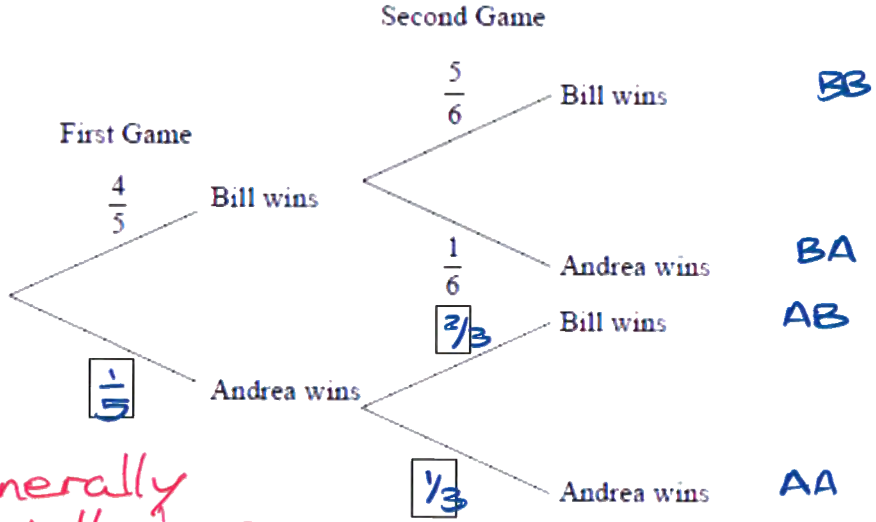
If students used guess and check they received credit for correct answer only.

32. Bill and Andrea play two games of tennis. The probability that Bill wins the first game is $\frac{4}{5}$.

If Bill wins the first game, the probability he wins the second game is $\frac{5}{6}$.

If Bill loses the first game, the probability that he wins the second game is $\frac{2}{3}$.

a. Complete the tree diagram below. **1**



Generally well done.

b. Find the probability that Bill wins at least one game. **1**

$P = 1 - P(AA)$ this is most efficient method

$= 1 - \frac{1}{5} \times \frac{1}{3} = \frac{14}{15}$

c. Given that Bill wins at least 1 game, find the probability he wins both games. **1**

$P(BB \mid \text{at least } 1) = \dfrac{\frac{4}{5} \times \frac{5}{6}}{\frac{14}{15}}$ — P(wins both) — P(wins at least 1)

$= \frac{20}{30} \times \frac{15}{14}$

$= \frac{5}{7}$

33.

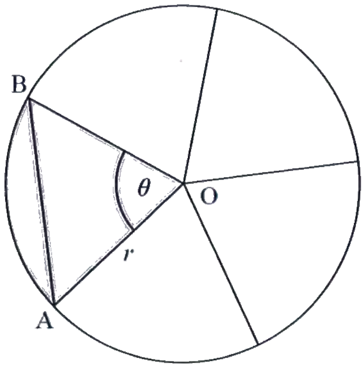


diagram not to scale

The diagram above shows a circle, centre O and radius r mm.
The circle is divided into five equal sectors.
One sector is OAB and $\angle AOB = \theta$ radians.
The area of the sector AOB is 20π mm^2.

a. Find the value of r. **2**

$\theta = \frac{2\pi}{5}$

$A = \frac{1}{2}r^2\theta$ this formula requires the angle in radians

$20\pi = \frac{1}{2} \times r^2 \times \frac{2\pi}{5}$

$20\pi = \frac{\pi r^2}{5}$

$100 = r^2$

$r = 10$

b. Find the length of the chord AB. Give the answer to 1 decimal place. **2**

$AB^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \cos \frac{2\pi}{5}$

$= 138.1966\ldots$ Many found arc length.

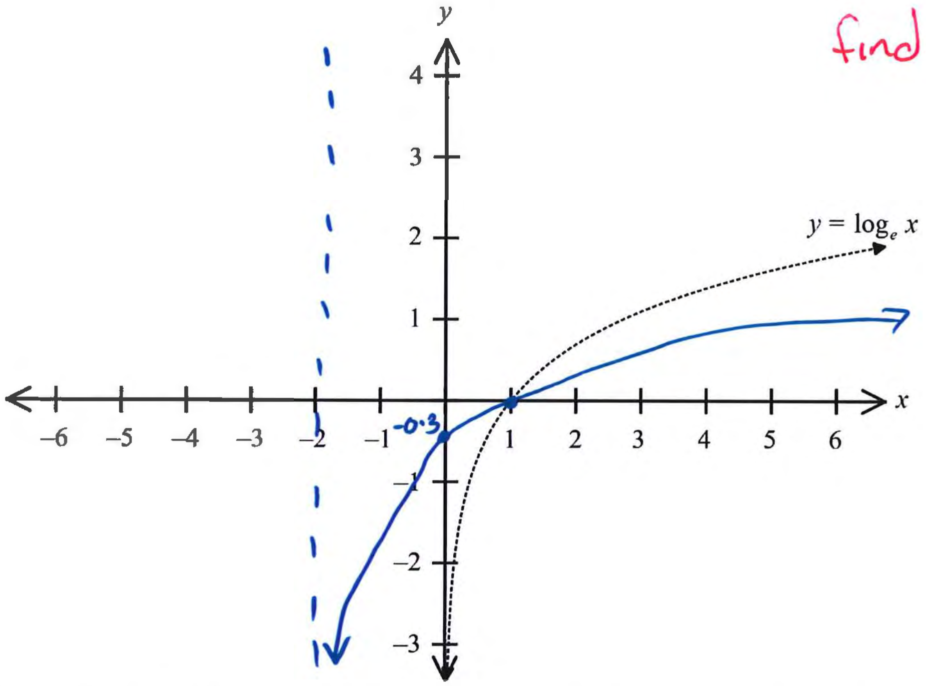
$AB = 11.8$ mm Read the question!

34. Consider the function $y = \log_e x$. **2**

Use the sketch below of $y = \log_e x$ to sketch $y = \log_e(x+2) - 1$ on the same axes. Show **any asymptotes and the y intercept** on your graph.

$y = \ln 2 - 1$
$= -0.3$

find y-intercept

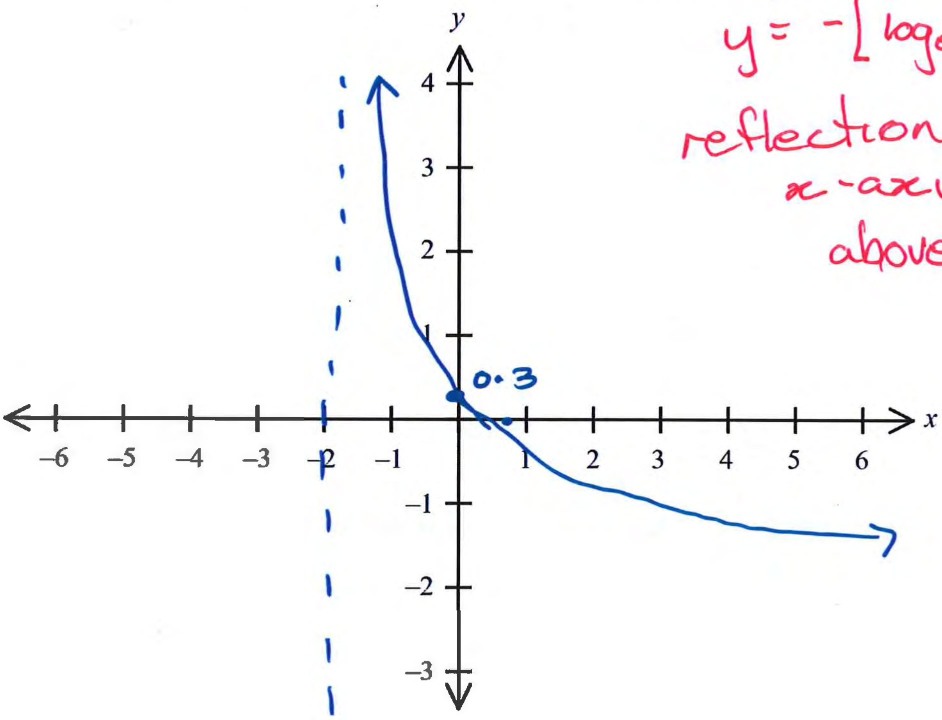


Use your sketch from part (a) to draw a sketch of $y = -\log_e(x+2) + 1$. **1**

Show **any asymptotes and the y intercept** on your graph.

$y = -[\log_e(x+2) - 1]$

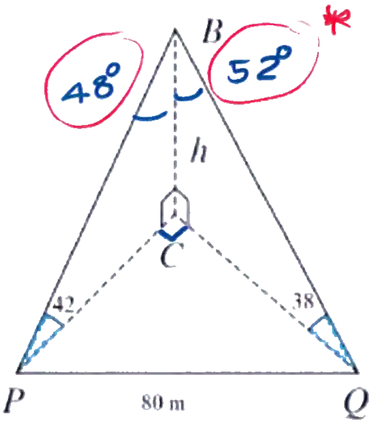
reflection in the x-axis of above.



35. The top of a tower is due north of P and its angle of elevation is 42°. Q is due west of the tower and is 80 metres from P. The angle of elevation from Q to the top of the tower is 38°. **4**

Let the height of the tower be h metres and let C be the base of the tower on the ground.

Calculate the height of the tower correct to 1 decimal place.



This is a standard 3D Trig question. Please review.

* Using the complementary angle avoids working with fractions

$\frac{PC}{h} = \tan 48^\circ$ $\qquad$ $\frac{QC}{h} = \tan 52^\circ$

$PC = h\tan 48^\circ$ $\qquad$ $QC = h\tan 52^\circ$

$PC^2 + QC^2 = 80^2$ Pythagoras because $\angle PCQ = 90^\circ$, otherwise use cosine rule.

$(h\tan 48^\circ)^2 + (h\tan 52^\circ)^2 = 80^2$

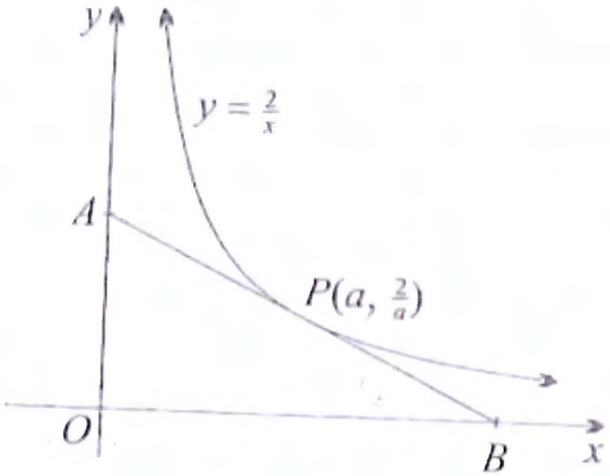
Factorise

$h^2 = \frac{80^2}{\tan^2 48^\circ + \tan^2 52^\circ}$

solve.

$h = 47.2$ m

36.



The diagram above shows part of the hyperbola $y = \frac{2}{x}$ and the tangent to the hyperbola drawn at point $P\left(a, \frac{2}{a}\right)$.

if numerator is constant, express with a negative index....

The tangent intersects the y-axis and x-axis at A and B respectively.

a. Find the gradient of the curve at the point $P\left(a, \frac{2}{a}\right)$ in terms of a. **2**

$y = 2x^{-1}$ ... then basic rule

$y' = -2x^{-2}$

$= \frac{-2}{x^2}$ convert back

At $x = a$ Grad $= \frac{-2}{a^2}$ or $-2a^{-2}$

b. Hence, show the equation of the tangent to the curve $y = \frac{2}{x}$ at the point $P\left(a, \frac{2}{a}\right)$ is $2x + a^2y - 4a = 0$. **2**

$y - \frac{2}{a} = \frac{-2}{a^2}(x - a)$ Use point gradient formula

$a^2y - 2a = -2x + 2a$ removing fractions immediately by $(\times a^2)$ reduces risk of errors.

$2x + a^2y - 4a = 0$

Also accepted $y = mx + c$ method.

Question continued over the page

c. Show that the area of $\Delta\, AOB$ is a constant. **2**

Find x and y-intercepts

when $x = 0$ $\quad a^2 y - 4a = 0$

$$y = \frac{4a}{a^2}$$

$$= \frac{4}{a}$$ height

$y = 0$ $\quad 2x - 4a = 0$

$$x = \frac{4a}{2}$$

$$= 2a$$ base

$$A = \frac{1}{2} \times 2a \times \frac{4}{a}$$

$= 4\ u^2$ which is a constant

...

End of Section II: Part 3

Section II: Part 4 of 4 (22 marks)

37. Consider $f(x) = \frac{1}{\sqrt{x}}$.

a. State the domain and range of $f(x)$. **1**

Some students did not find the range. Common error was including zero.

D: $x > 0$

R: $y > 0$

b. For what value of x does $f(x) = 2$? **1**

Some students subbed $x=2$, while others did the square root of $\frac{1}{2}$.

$\frac{1}{\sqrt{x}} = 2$

$\frac{1}{2} = \sqrt{x}$

$x = \frac{1}{4}$

c. Find $f'(x)$ and hence calculate $f'(3)$, leaving your answer in simplified surd form with a rational denominator. **2**

Students incorrectly differentiated.

$f(x) = x^{-1/2}$

$f'(x) = -\frac{1}{2}x^{-3/2}$

$= -\frac{1}{2\sqrt{x^3}}$

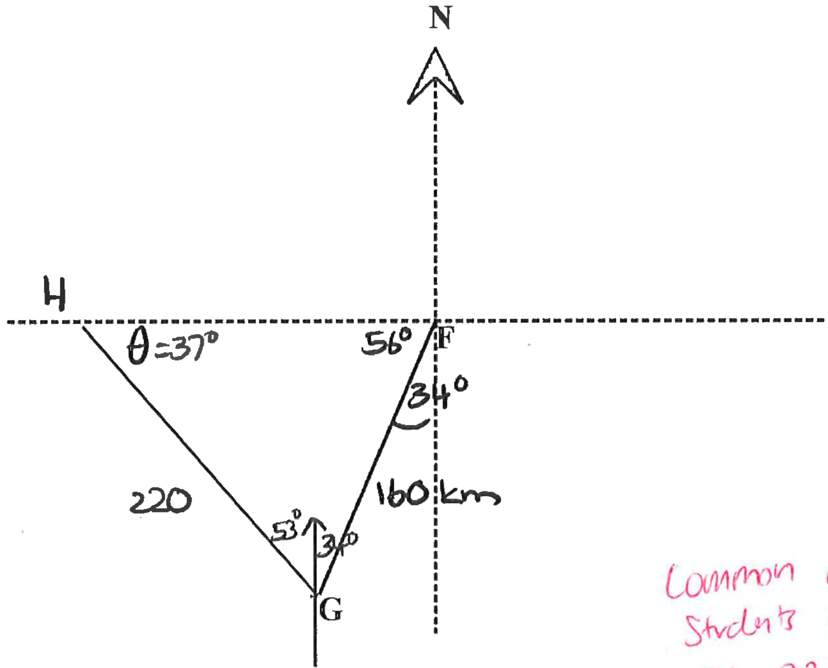
$f'(3) = -\frac{1}{2\sqrt{27}}$ or $-\frac{1}{2}(3)^{-3/2}$

$= -\frac{1}{6\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

Some students did not fully simplify or did not rationalize correctly.

$= \frac{-\sqrt{3}}{18}$

38. The distance between the towns of Ford (F) and Galen (G) is 160 km. The bearing of Galen from Ford is 214T°. The distance from Galen to Holden (H) is 220 km. Holden is due west of Ford.



Common error: Students labelled HF as 220 km.

a. Mark the information on the diagram above. **1**

b. Find the size of $\angle FHG$ correct to the nearest degree. **2**

$$\frac{\sin\theta}{160} = \frac{\sin 56^\circ}{220}$$

$$\sin\theta = \frac{160\sin 56^\circ}{220}$$

$$\theta = 37^\circ$$

Completed well.

c. What is the bearing of H from G? **1**

$180 - 37 - 56 - 34 = 53^\circ$

Bearing $= 307^\circ$ T

Students need to review bearing. Completed poorly.

38. Prove $\sec x + \tan x + \cot x = \frac{1+\sin x}{\sin x \cos x}$ **2**

$LHS = \frac{1}{\cos x} + \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$= \frac{\sin x + \sin^2 x + \cos^2 x}{\sin x \cos x}$

$= \frac{1 + \sin x}{\sin x \cos x}$

Completed well.

Make sure to clearly show all steps.

39. Use the discriminant to show that $x^2 - 2x + 4 > 0$ **2**

$\Delta < 0$ no real roots.

$a > 0$

$\Delta = b^2 - 4ac$

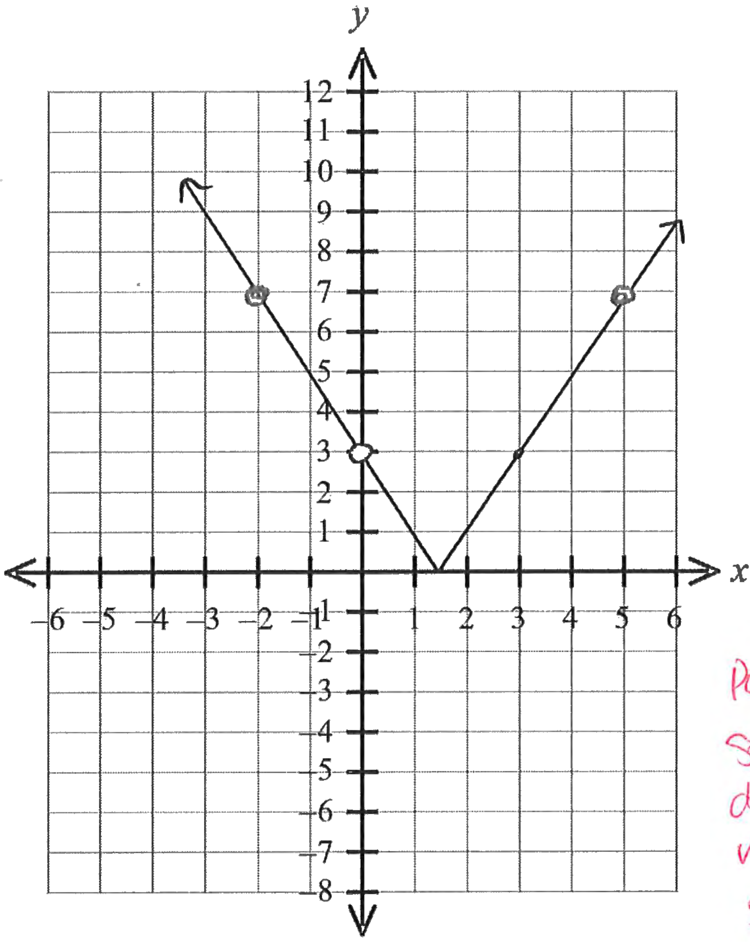
$= (-2)^2 - 4(1)(4)$

$= 4 - 16$

< 0

Students need to show discriminate is less than zero and $a > 0$.

40.



Poorly completed. Students are to draw an absolute value showing x and y intercepts.

a. On the grid above, draw the graph of the function $y = |3 - 2x|$. Clearly show coordinates of x and y intercept(s). $x = 0$ $y = 3$ **2**

b. Hence solve the equation $|3 - 2x| = 7$ graphically. $0 = 3 - 2x$ **2**

$2x = 3$

$x = 1.5$

$-3 + 2x = 7$

$2x = 10$

$x = 5$

$3 - 2x = 7$

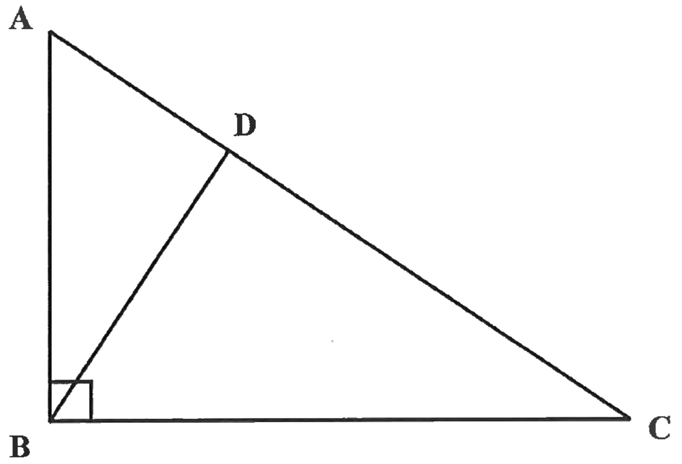
$-2x = 4$

$x = -2$

Solution $x = -2, 5$

Students are to use the graph to solve part (b).

41. A triangle ABC is right angled at B.
D is the point on AC such that BD is perpendicular to AC.
Let $\angle BAC = \theta$.



a. You are given that $6AD + BC = 5AC$ **2**
Show that $6\cos\theta + \tan\theta = 5\sec\theta$.
Hint: Use ΔABC and ΔABD

$\cos\theta = \frac{AD}{AB}$ In ΔABD

$AD = AB\cos\theta$

$\tan\theta = \frac{BC}{AB}$ In ΔABC

$BC = AB\tan\theta$

$\sec\theta = \frac{AC}{AB}$ In ΔABC

$AC = AB\sec\theta$

$6AD + BC = 5AC$ Given

$6AB\cos\theta + AB\tan\theta = 5AB\sec\theta$

$6\cos\theta + \tan\theta = 5\sec\theta$

Students are to use the instructions provided in the question to form three expressions.

Question 43 continues over the page

b. Deduce that $6\sin^2\theta - \sin\theta - 1 = 0$ **2**

From part a. $6\cos\theta + \tan\theta = 5\sec\theta$

$$6\cos\theta + \frac{\sin\theta}{\cos\theta} = \frac{5}{\cos\theta}$$

$$6\cos^2\theta + \sin\theta = 5$$

$$6(1-\sin^2\theta) + \sin\theta = 5$$

$$6 - 6\sin^2\theta + \sin\theta = 5$$

$$6\sin^2\theta - \sin\theta - 1 = 0$$

Deduce means to 'determine by reasoning'. Many students did not use part (a) to complete this question.

c. Find the value of θ. **2**

$$(2\sin\theta - 1)(3\sin\theta + 1) = 0$$

$\sin\theta = \frac{1}{2}$ $\sin\theta = -\frac{1}{3}$

$\theta = 30^\circ$ $\theta > 90^\circ$

θ can only be positive and acute.

End of Section II

END OF EXAM