Name ……………………… Teacher ………………………… Assessment No. ………………….

Mathematics Advanced

Year 11 Yearly Examination, 2023

- Reading time – 10 minutes
- Working time – 2.5 hours
- Write using black pen
- Calculators approved by NESA may be used
- The HSC Reference Sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

	Marks
Multiple Choice	**/10**
Section II: Part 1	**/23**
Section II: Part 2	**/22**
Section II: Part 3	**/22**
Section II: Part 4	**/23**
Total	**/100**

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Section I: 10 Marks

Allow about 15 minutes for this section.

Answer Q1-10 on the **multiple-choice answer sheet** provided.

1. If $f(x) = x^2 + x + 1$, what is the value of $f(-2)$?

A. 2

B. 3

C. 5

D. 7

2. If $3^x = 7$, what is the value of x?

A. $\frac{\log 7}{\log 3}$

B. $\log\left(\frac{7}{3}\right)$

C. $\log 4$

D. $\log\left(\frac{3}{7}\right)$

3. Which of the following *correctly* describes the sketch of $y = -f(-x)$?

A. It is a reflection of $y = f(-x)$ in the x-axis

B. It is a reflection of $y = f(x)$ in the x-axis

C. It is a reflection of $y = f(-x)$ in the y-axis

D. It is a reflection of $y = f(x)$ in the y-axis

4. If $R\sin\theta = 2$ and $R\cos\theta = 3$, what is the value of R^2 ?

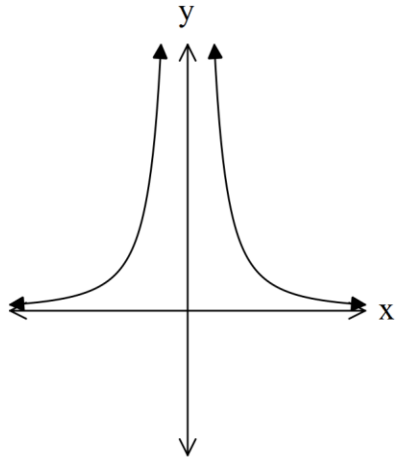
A. 1

B. 5

C. 13

D. 25

5. The diagram below shows $y = \frac{1}{x^2}$



Which of the following statements is correct?

A. $y = \frac{1}{x^2}$ is a one-to-one function

B. $y = \frac{1}{x^2}$ is a many-to-one function

C. $y = \frac{1}{x^2}$ is a one-to-many relation

D. $y = \frac{1}{x^2}$ is a many-to-many relation

6. What is the derivative of $\frac{e^{2x}}{2x}$?

A. $\frac{4x-2}{e^{4x}}$

B. $\frac{2-4x}{e^{4x}}$

C. $\frac{e^{2x}(1-2x)}{2x^2}$

D. $\frac{e^{2x}(2x-1)}{2x^2}$

7. If $\log_a 2 = 0.36$ and $\log_a 5 = 0.83$, what is the value of $\log_a 20$?

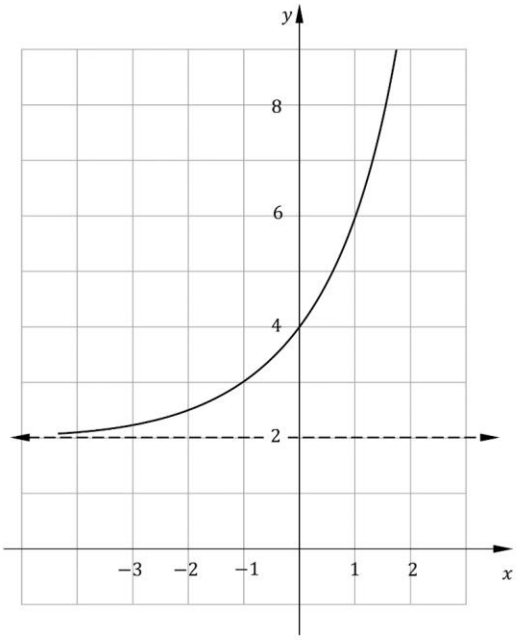
A. 0.72

B. 1.19

C. 1.55

D. 2.27

8.



The graph of $y = ka^x + c$ is shown above.

What are the values of k and c ?

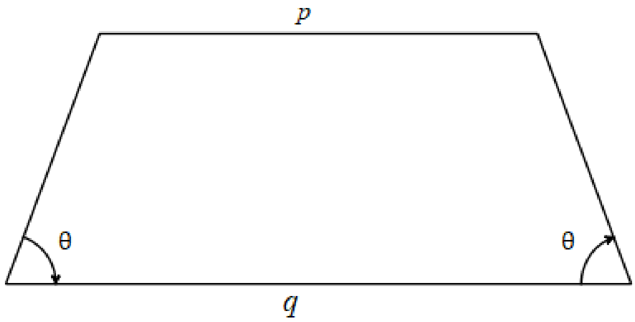
A. $k = 1,\ c = 0.$

B. $k = 1,\ c = 2.$

C. $k = 2,\ c = 2.$

D. $k = 2\ ,\ c = 0.$

9. Find an expression for the area of the trapezium below.



A. $\frac{1}{4}\left(q^2 - p^2\right)\tan\theta$

B. $\frac{1}{4}\left(q^2 - p^2\right)\cot\theta$

C. $\frac{1}{2}\left(q^2 - p^2\right)\tan\theta$

D. $\frac{1}{2}\left(q^2 + p^2\right)\tan\theta$

10. If $P(B) = 0.5$ and $P(A \cup B) = 0.8$, what is the value of $P(\bar{A}|\bar{B})$?

A. 0.2

B. 0.3

C. 0.4

D. 0.5

- End of Section I –

Section II: 90 Marks

Allow about 2 hours and 15 minutes for this section.

- Answer Q11-44 in the space provided
- Extra writing paper is provided at the end of each part
- If you use this space, clearly indicate which question you are answering
- Show relevant mathematical reasoning and/or calculations

Section II: Part 1 of 4 (23 Marks)

11. Show that $(2\sqrt{3}+1)(2-\sqrt{3}) = 3\sqrt{3}-4$. **1**

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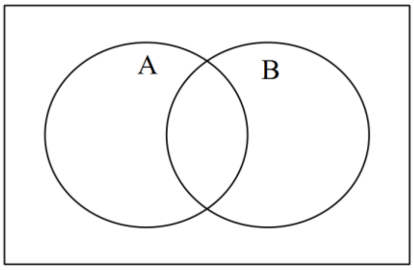
12. Shade $\bar{A} \cap B$ on the Venn diagram below. **1**



13. Express $\frac{3}{x^2-2x} + \frac{2}{x^2-7x+10}$ as a single fraction in simplest form. **2**

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14. Find the centre and radius of the circle represented by the equation. **2**

$$x^2 - 4x + y^2 + 6y + 4 = 0$$

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15. Find the exact value of $\sec A$ if $\tan A > 0$ and $\sin A = -\frac{3}{5}$. **2**

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16. Show that the line $y = 5x - 2$ is a tangent to the parabola $y = x^2 + 3x - 1$. **2**

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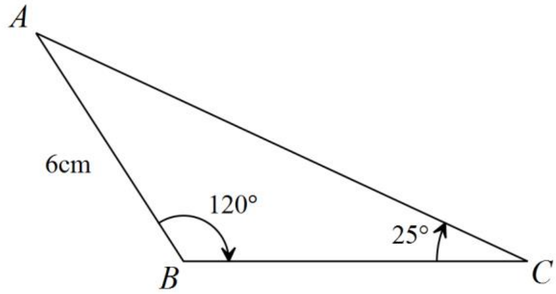
17. In triangle ABC below, $AB = 6\text{cm}$, $\angle ABC = 120^\circ$, $\angle ACB = 25^\circ$.



a) Find the length BC to the nearest cm. **2**

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b) Hence, or otherwise, calculate the area of triangle ABC. **1**
Give your answer correct to nearest cm^2.

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18. Solve $\log_2(x+2) - \log_2(x-5) = 3$. **2**

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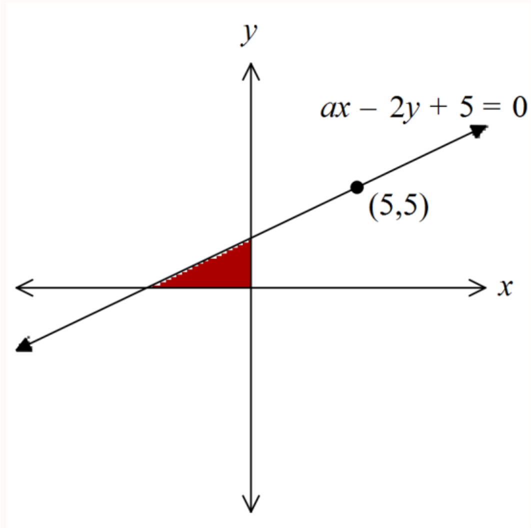
19. The point $(5,5)$ lies on the line $ax - 2y + 5 = 0$ as shown in the diagram below. **3**

Find the area of the shaded region in the diagram.



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20. State the domain and range of $f(x) = \frac{1}{2x+1}$. **2**

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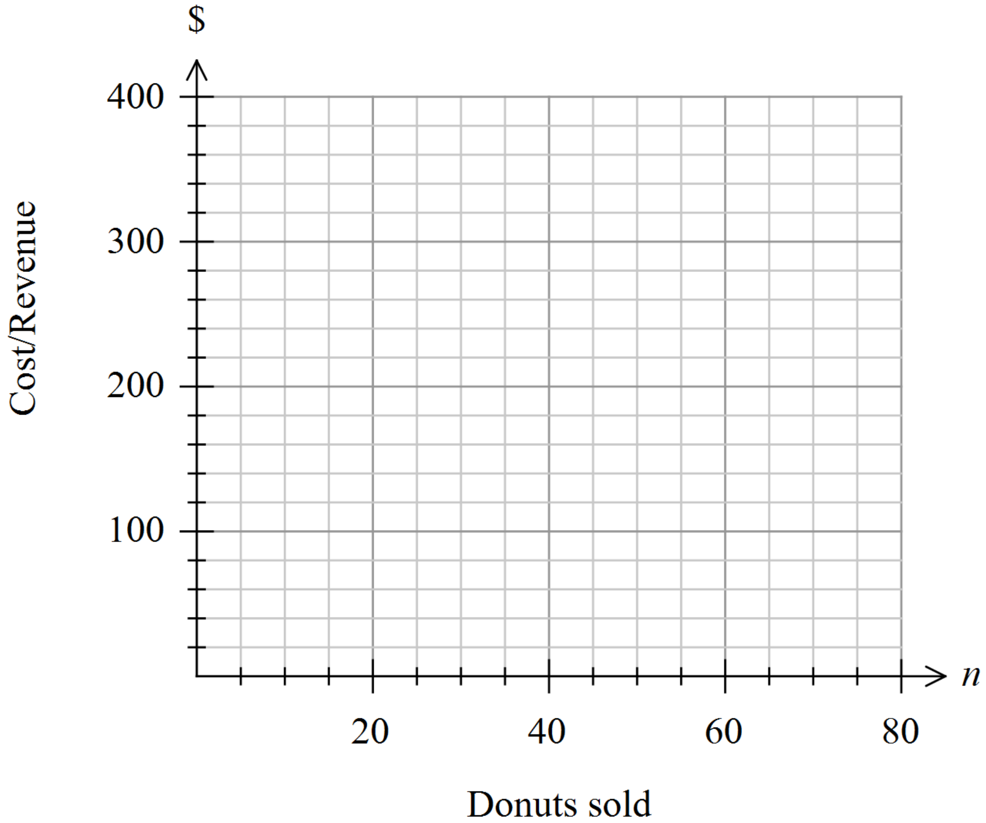
21. Jenny decides to go into the donut business. She buys some start up equipment to the value of \$140. She has worked out that it will cost her \$1.50 to make each donut, and she intends to sell them for \$5.00 each.

a) Graph her costs and revenue on the grid below. **2**



b) Use your graphs to find the number of donuts Jenny needs to sell to break even. **1**

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- End of Section II Part 1 –

Extra Writing Space

If you use this space, clearly indicate which question you are answering.

Section II: Part 2 of 4 (22 Marks)

22. Determine algebraically whether the function $f(x) = \frac{1+x^2}{x^3}$ is even, odd or neither. Show all working. **2**

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23. Differentiate the following with respect to x.

a) $y = \sqrt[3]{x^2}$ **2**

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b) $y = \frac{x^3-x^2-1}{x}$ **2**

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c) $f(x) = 4x(1-2x)^5$ **2**

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24. Show that $\frac{d}{dx}\left(\frac{x}{\sqrt{x-1}}\right) = \frac{x-2}{2\sqrt{(x-1)^3}}$. **3**

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25. Use the result $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative of $3x^2 + 1$. **2**

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26. Find the value(s) of θ if $\tan^2 \theta = 3$ for $0 \leq \theta \leq 2\pi$. **2**

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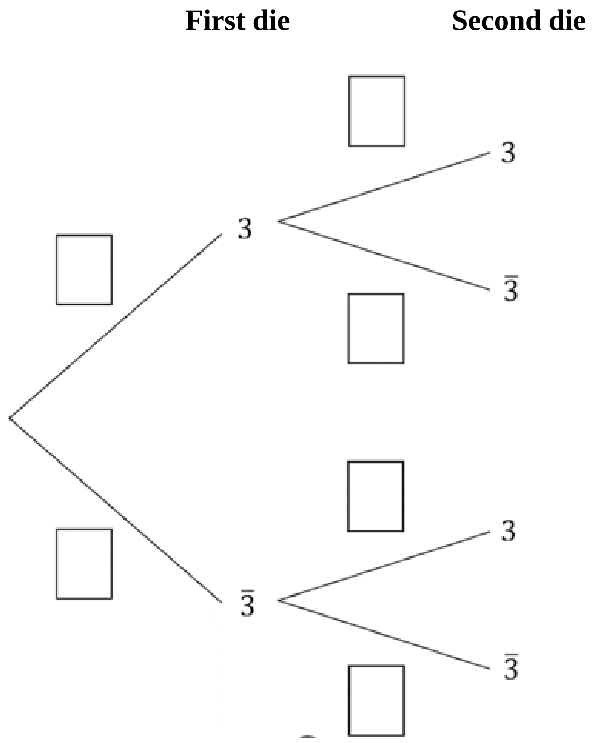
27. Two dice are rolled. The first die is a standard die with numbers 1, 2, 3, 4, 5, 6 while the second die is non-standard and has the numbers 3, 3, 3, 4, 4, 4.

The number of threes is then recorded.

a) Insert the correct probabilities into the six boxes on the probability tree below, to show the probability of whether or not a 3 is rolled. **2**



b) Calculate the probability of rolling at least one 3. **1**

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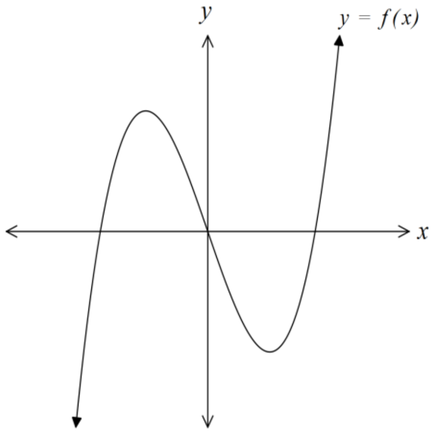
28. Consider the graph of $y = f(x)$ shown on the axes below.



Sketch the derivative function, $y = f'(x)$, on the same axes. **1**

29. Find the equation of the tangent of $y = 2e^{3x}$ when $x = 0$, leaving your answer in general form. **3**

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- End of Section II Part 2 –

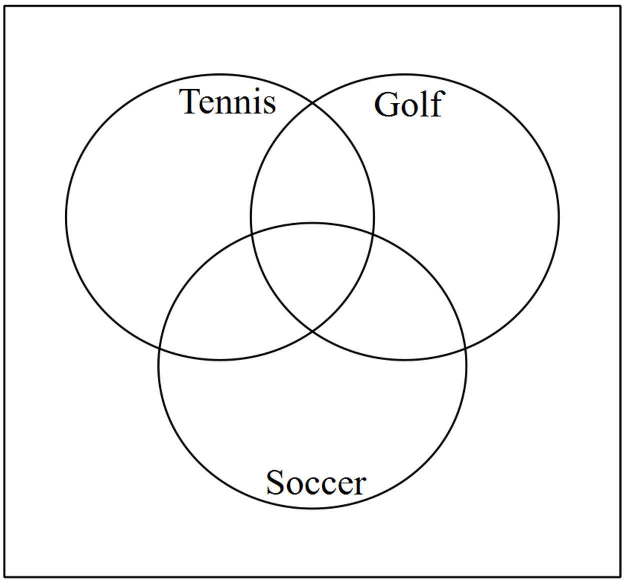
Extra Writing Space

If you use this space, clearly indicate which question you are answering.

Section II: Part 3 of 4 (22 Marks)

30. In a group of 30 students, fifteen play tennis, ten play golf and eighteen play soccer. Three student plays all three sports. Five students play both golf and tennis. Eight students play both soccer and golf. Six students play both soccer and tennis.

a) Complete the Venn diagram to illustrate the full details above. **2**



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b) Find the probability that a student selected at random plays none of these sports. **1**

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31. An object moves in a straight line so that its displacement, x metres, after t seconds, is given by $x = 2e^{-\frac{1}{2}t} - 1$, where $t \geq 0$.

a) What is the initial displacement of the object? **1**

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b) Show that the exact time taken for the displacement to equal zero is $\log_e 4$ seconds. **2**

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c) Find the exact velocity when the displacement is zero. **2**

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32. Jane plays a game using a standard 6-sided die. In each game the die is rolled three times. Each time, before the die is rolled, Jane guesses the number that will be rolled.

a) What is the probability that Jane will guess all three numbers correctly? **1**

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Jane pays \$10 to play one game. If Jane guesses the number correctly three times, she receives her money back and \$2,160.

b) What is her expected profit or loss after playing this game 1000 times? **2**

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33. Fully simplify **3**

$$\frac{\log_5\left(a^6b^4\right)-\log_5\left(a^2b^4\right)}{\log_5\left(\sqrt{a}\right)}.$$

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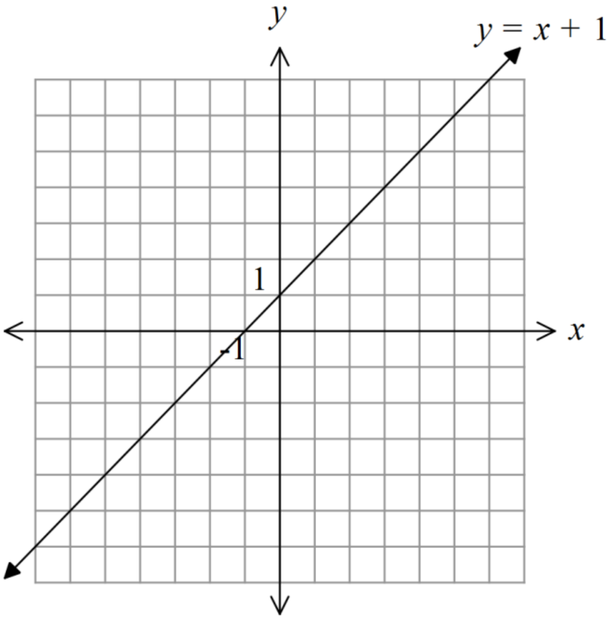
34. a) On the axes below, graph $y = |x - 3|$. **1**



b) Hence, solve graphically $x + 1 = |x - 3|$. **1**

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35. Solve $\cos x + \sin(90^\circ - x) = \sqrt{3}$ for $0^\circ \leq x \leq 360^\circ$. **3**

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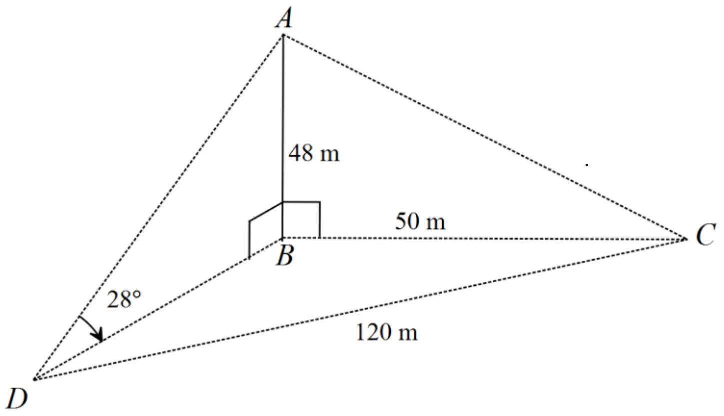
36. A 48-metre high, vertical light pole AB stands in the middle of a flat sporting ground.

Diana starts from the base of the pole, at B, and walks 50m due east to a point C where she places a peg.

She attaches a 120m long string to the peg and walks in a **roughly** south-westerly direction until the string is taut at point D.

She then measures the angle of elevation of the pole to be 28°.



Use the information provided to find the **actual** true bearing on which Diana was walking from C to D. Give your answer to the nearest degree. **3**

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- End of Section II Part 3 -

Extra Writing Space

If you use this space, clearly indicate which question you are answering.

Section II: Part 4 of 4 (23 Marks)

37. Solve $e^{2\ln x} = 8 - 2x$ **3**

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38. Let $f(x) = 3x + 1$ and $g(x) = 2x + b$, where b is a constant. **2**

Find the value of b if $g\big(f(x)\big) = f\big(g(x)\big)$. Show all working.

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39. Show that $\sin x \cot x - \cos x \sin^2 x = \cos^3 x$ **2**

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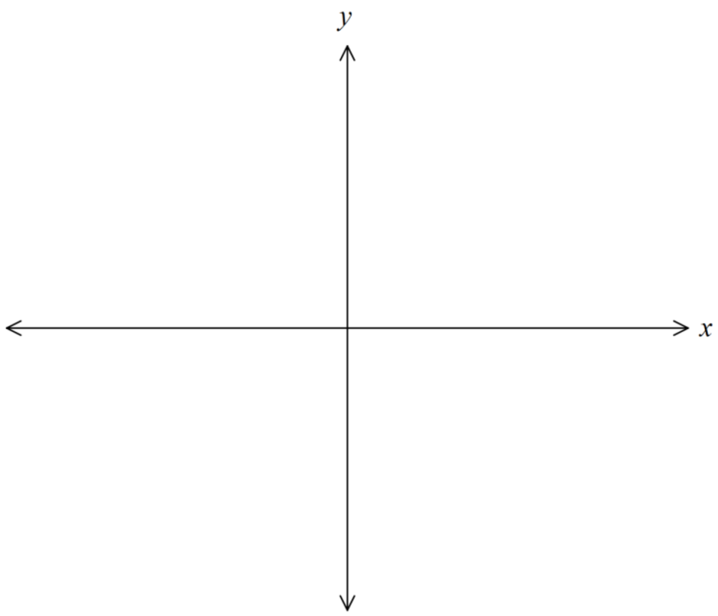
40. Consider the discrete probability distribution of the random variable X, shown below. **4**

x	1	2	3	4
$P(X = x)$	$4p$	$3p$	$2p$	p

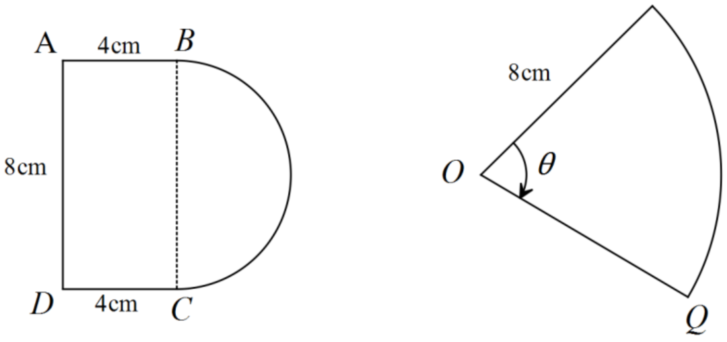
By finding the value of p, **show** that Var (X) =1.

41. Sketch the graph of $y = -\log_e(1 - x)$, showing the asymptote and all intercepts with the axes. **2**



42. The diagram below shows two farm patches. Shape X is a rectangle $ABCD$ joined to a semicircle of diameter BC. Shape Y is a sector OPQ of a circle with centre O and θ measured in radians.

Shape X **Shape Y**



(a) Given the areas of the two shapes are equal, show that $\theta = \frac{\pi}{4} + 1$. **3**

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(b) Hence, calculate the perimeter of sector OPQ. **2**
Leave your answer in exact form.

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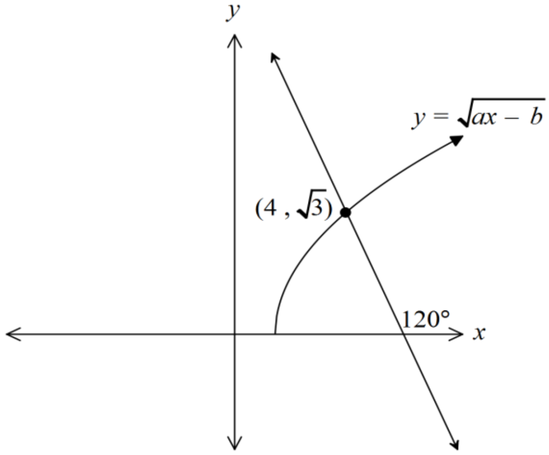
43. The normal to the function $y = \sqrt{ax - b}$ at the point $(4, \sqrt{3})$ makes a 120° angle with the positive x-axis.



a) Show that the gradient of the normal is $-\sqrt{3}$. **1**

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b) Hence, or otherwise, find the values of a and b. **3**

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44. A game show host randomly puts two balls into three separate boxes, so that each box can contain 0, 1 or 2 balls. The player chooses any two boxes to open. The player only wins if a total of two balls are in the boxes chosen. **1**

Find the probability that the player wins the game.

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- End of Section II Part 4 –

END OF EXAM

Extra Writing Space

If you use this space, clearly indicate which question you are answering.

Solutions

Name .... Teacher .... Assessment No. ....

PLC PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

Mathematics Advanced
Year 11 Yearly Examination, 2023

- Reading time – 10 minutes
- Working time – 2.5 hours
- Write using black pen
- Calculators approved by NESA may be used
- The HSC Reference Sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

	Marks
Multiple Choice	/10
Section II: Part 1	/23
Section II: Part 2	/22
Section II: Part 3	/22
Section II: Part 4	/23
Total	/100

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Section I: 10 Marks

Allow about 15 minutes for this section.

Answer Q1-10 on the **multiple-choice answer sheet** provided.

1. If $f(x) = x^2 + x + 1$, what is the value of $f(-2)$?

A. 2

B. 3

C. 5

D. 7

2. If $3^x = 7$, what is the value of x?

A. $\frac{\log 7}{\log 3}$

B. $\log\left(\frac{7}{3}\right)$

C. $\log 4$

D. $\log\left(\frac{3}{7}\right)$

3. Which of the following *correctly* describes the sketch of $y = -f(-x)$?

A. It is a reflection of $y = f(-x)$ in the x-axis

B. It is a reflection of $y = f(x)$ in the x-axis

C. It is a reflection of $y = f(-x)$ in the y-axis

D. It is a reflection of $y = f(x)$ in the y-axis

4. If $R\sin\theta = 2$ and $R\cos\theta = 3$, what is the value of R^2 ?

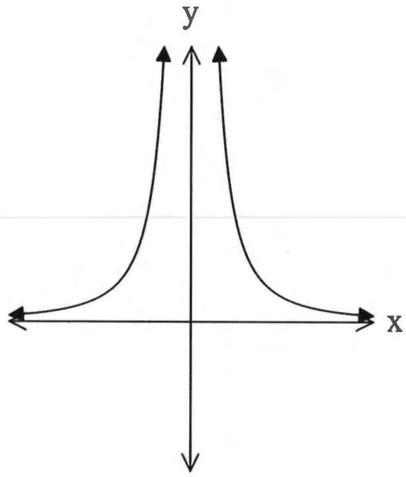
A. 1

B. 5

C. 13

D. 25

5. The diagram below shows $y = \frac{1}{x^2}$



Which of the following statements is *correct*?

A. $y = \frac{1}{x^2}$ is a one-to-one function

B. $y = \frac{1}{x^2}$ is a many-to-one function

C. $y = \frac{1}{x^2}$ is a one-to-many relation

D. $y = \frac{1}{x^2}$ is a many-to-many relation

6. What is the derivative of $\frac{e^{2x}}{2x}$?

A. $\frac{4x-2}{e^{4x}}$

B. $\frac{2-4x}{e^{4x}}$

C. $\frac{e^{2x}(1-2x)}{2x^2}$

D. $\frac{e^{2x}(2x-1)}{2x^2}$

7. If $log_a 2 = 0.36$ and $log_a 5 = 0.83$, what is the value of $log_a 20$?

A. 0.72

B. 1.19

C. 1.55

D. 2.27

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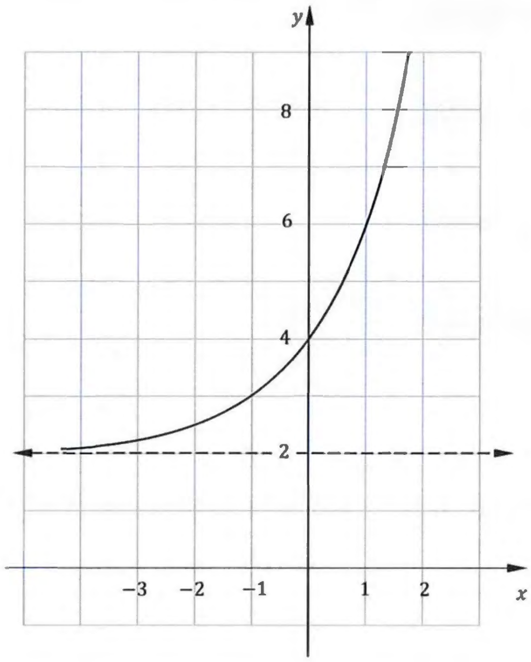


The graph of $y = ka^x + c$ is shown above.

What are the values of k and c ?

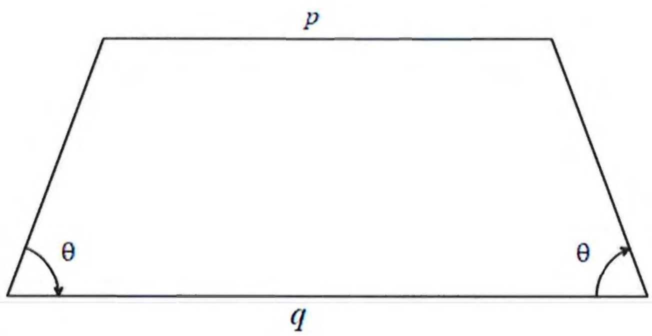
A. $k = 1,\ c = 0.$

B. $k = 1,\ c = 2.$

C. $k = 2,\ c = 2.$

D. $k = 2\ ,\ c = 0.$

9. Find an expression for the area of the trapezium below.



A. $\frac{1}{4}\left(q^2-p^2\right)\tan\theta$

B. $\frac{1}{4}\left(q^2-p^2\right)\cot\theta$

C. $\frac{1}{2}\left(q^2-p^2\right)\tan\theta$

D. $\frac{1}{2}\left(q^2+p^2\right)\tan\theta$

10. If $P(B) = 0.5$ and $P(A \cup B) = 0.8$, what is the value of $P(\bar{A}|\bar{B})$?

A. 0.2

B. 0.3

C. 0.4

D. 0.5

- End of Section I –

Section II: 90 Marks

Allow about 2 hours and 15 minutes for this section.

- Answer Q11-44 in the space provided
- Extra writing paper is provided at the end of each part
- Show relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of each booklet

Section II: Part 1 of 4 (23 Marks)

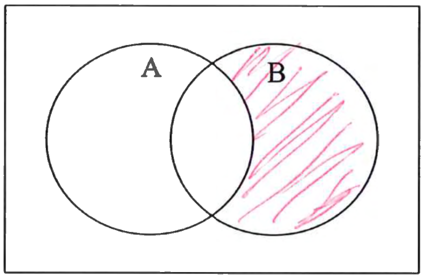
11. Show that $(2\sqrt{3}+1)(2-\sqrt{3}) = 3\sqrt{3}-4$. **1**

$= 4\sqrt{3} - 6 + 2 - \sqrt{3}$ ← must show

$= 3\sqrt{3} - 4$

12. Shade $\bar{A} \cap B$ on the Venn diagram below. **1**

Use two colours to allow you to see the intersection



13. Express $\frac{3}{x^2-2x} + \frac{2}{x^2-7x+10}$ as a single fraction in simplest form. **2**

$= \frac{3}{x(x-2)} + \frac{2}{(x-5)(x-2)}$

Best to factorise then find the lowest common denominator

$= \frac{3(x-5)+2x}{x(x-2)(x-5)}$

$= \frac{5x-15}{x(x-2)(x-5)}$

14. Find the centre and radius of the circle represented by the equation. **2**

$$x^2 - 4x + y^2 + 6y + 4 = 0$$

$x^2 - 4x + 4 + y^2 + 6y + 9 = -4 + 4 + 9$

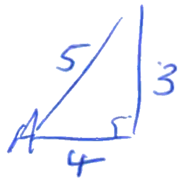
$(x-2)^2 + (y+3)^2 = 9$

$\therefore$ centre: $(2, -3)$

radius $= 3$

make sure that you add values to both sides of the equation when completing the square

15. Find the exact value of $\sec A$ if $\tan A > 0$ and $\sin A = -\frac{3}{5}$. **2**



3rd Quad. $\therefore \sec A = -\frac{5}{4}$

this determines the sign

16. Show that the line $y = 5x - 2$ is a tangent to the parabola $y = x^2 + 3x - 1$. **2**

$5x - 2 = x^2 + 3x - 1$

$x^2 - 2x + 1 = 0$

$\Delta = 4 - 4(1)(1)$

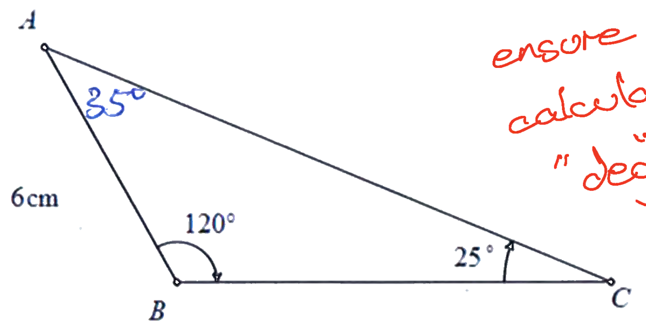
$= 0$

$\therefore$ There is one point of intersection and $y = 5x - 2$ is not a vertical line $\therefore$ it is a tangent.

There were various other valid methods that received full marks.
Remember, you must "show".

17. In triangle ABC below, $AB = 6\text{cm}$, $\angle ABC = 120^\circ$, $\angle ACB = 25^\circ$.

ensure that your calculator is in "deg" mode



(a) Find the length BC to the nearest cm. **2**

$$\frac{x}{\sin 35^\circ} = \frac{6}{\sin 25^\circ}$$

$$\therefore x = \frac{6\sin 35^\circ}{\sin 25^\circ} = 8.14\text{cm}$$

$$\approx 8\text{cm (to nearest cm)}$$

(b) Hence, or otherwise, calculate the area of triangle ABC.
Give your answer correct to nearest cm^2. **1**

$$A = \frac{1}{2} \times 6 \times 8 \times \sin 120$$

$$\approx 21\text{cm}^2 \text{ (to nearest cm}^2\text{)}$$

done well

18. Solve $\log_2(x+2) - \log_2(x-5) = 3$. **2**

$$\log_2\left(\frac{x+2}{x-5}\right) = 3$$

$$\therefore 2^3 = \frac{x+2}{x-5}$$

$$8x - 40 = x + 2$$

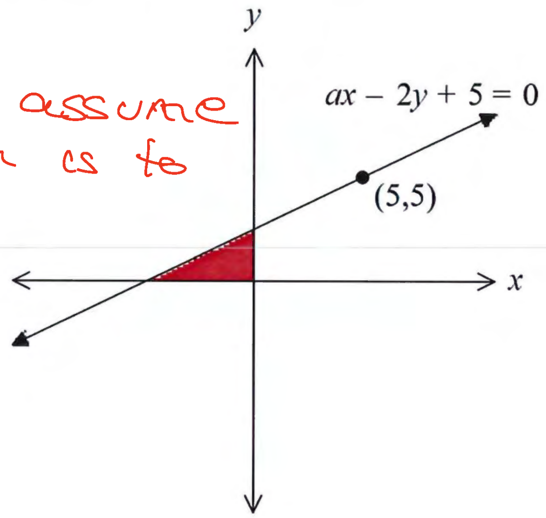
$$7x = 42$$

$$x = 6$$

Note: the difference of logs equals log of the quotient.
Students complicated the question by supposing various other identities.

19. The point (5,5) lies on the line $ax - 2y + 5 = 0$ as shown in the diagram below. **3**
Find the area of the shaded region in the diagram.

You cannot assume a diagram is to scale.



sub in (5,5): $5a - 10 + 5 = 0$

$5a = 5$

$a = 1$

x-int: $x - 2(0) + 5 = 0$
(y=0) $x = -5$

y-int $0 - 2y + 5 = 0$
(x=0) $2y = 5$

$y = \frac{5}{2}$

$\therefore A = \frac{1}{2} \times 5 \times \frac{5}{2}$

$= 6.25 \text{ units}^2$

20. State the domain and range of $f(x) = \frac{1}{2x+1}$. **2**

Domain: all real x, $x \neq -\frac{1}{2}$ can also use

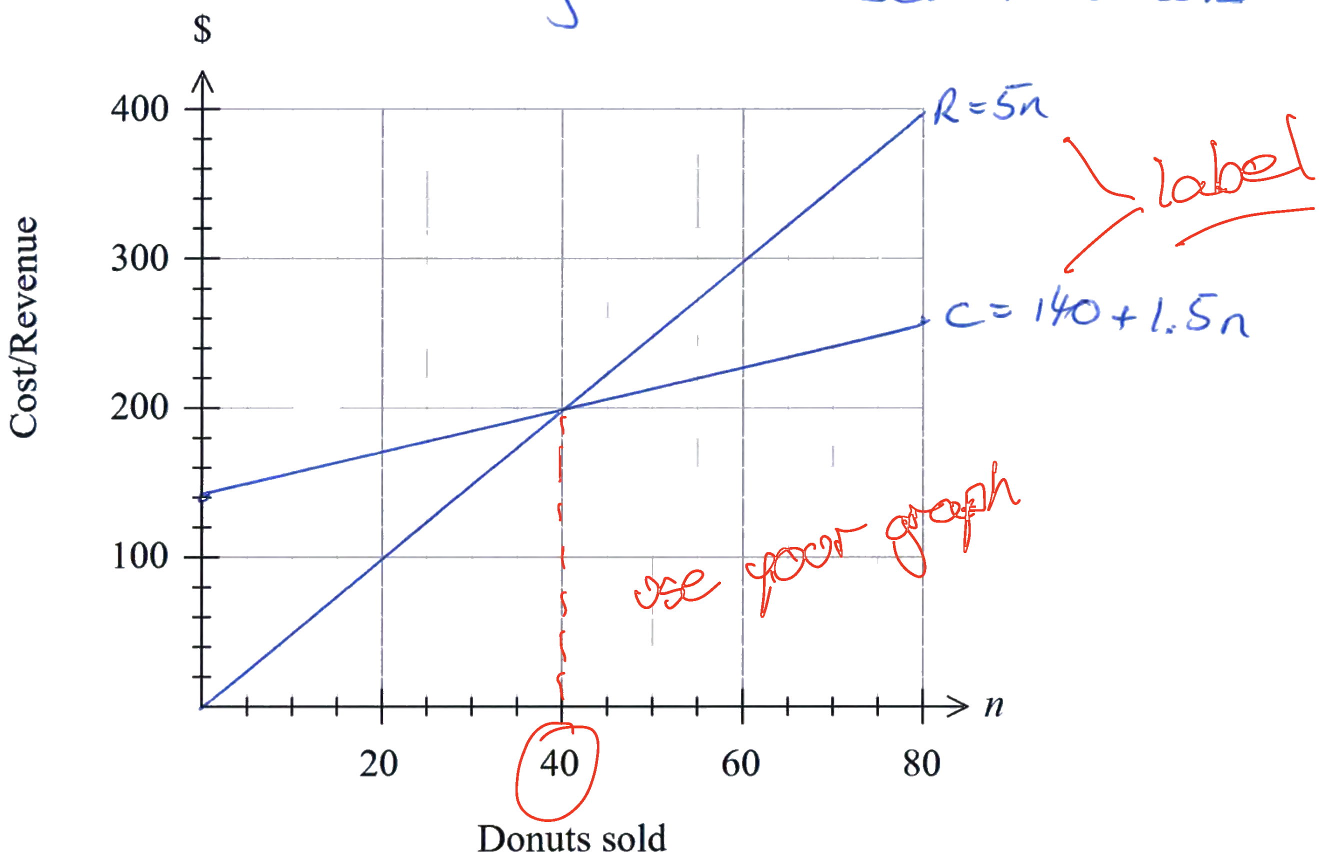
Range, all real y, $y \neq 0$ interval notation

21. Jenny decides to go into the donut business. She buys some start up equipment to the value of $140. She has worked out that it will cost her $1.50 to make each donut, and she intends to sell them for $5.00 each.

a) On the grid below graph her costs and revenue. **2**

b) Use your graphs to find the number of donuts Jenny needs to sell to break even. **1**

Jenny needs to sell 40 donuts



- End of Section II Part 1 –

Section II: Part 2 of 4 (22 Marks)

22. Determine algebraically whether the function $f(x) = \frac{1+x^2}{x^3}$ is even, odd or neither. Show all working. **2**

$f(-x) = \frac{1+(-x)^2}{(-x)^3} = \frac{1+x^2}{-x^3}$

$-f(x) = -\frac{1+x^2}{x^3}$

$= \frac{1+x^2}{-x^3}$

$f(-x) = -f(x)$

$\therefore f(x)$ is odd

must state this.

23. Differentiate the following with respect to x,

a) $y = \sqrt[3]{x^2}$ **2**

convert to index notation first

$y = x^{\frac{2}{3}}$ $\therefore y' = \frac{2}{3}x^{-\frac{1}{3}}$

b) $y = \frac{x^3-x^2-1}{x}$ **2**

simplify first

$y = x^2 - x - x^{-1}$

$y' = 2x - 1 + x^{-2}$

c) $f(x) = 4x(1-2x)^5$ **2**

Have a clear scaffold and use it with care

$u = 4x$ $v = (1-2x)^5$

$u' = 4$ $v' = -10(1-2x)^4$

$f'(x) = 4x \cdot -10(1-2x)^4 + 4(1-2x)^5$

2-marks would suggest that this is sufficient

Those students who went on to factorise will be well equipped for further applications

24. Show that $\frac{d}{dx}\left(\frac{x}{\sqrt{x-1}}\right) = \frac{x-2}{2\sqrt{(x-1)^3}}$. **3**

$u = x$ $v = (x-1)^{\frac{1}{2}}$

$u' = 1$ $v' = \frac{1}{2}(x-1)^{-\frac{1}{2}}$

$$\frac{(x-1)^{\frac{1}{2}} \cdot 1 - x \cdot \frac{1}{2}(x-1)^{-\frac{1}{2}}}{x-1}$$

$$= \frac{(x-1)^{-\frac{1}{2}}\left[x-1-\frac{x}{2}\right]}{x-1}$$

$$= \frac{\frac{1}{2}(x-2)}{(x-1)^{\frac{3}{2}}}$$

"Show" is the examiner's way of ensuring that you factorise.

$$= \frac{x-2}{2\sqrt{(x-1)^3}}$$

25. Use the result $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative of $3x^2 + 1$. **2**

$f(x) = 3x^2 + 1$

$f(x+h) = 3(x+h)^2 + 1$

$= 3x^2 + 6xh + 3h^2 + 1$

First Principles was overlooked in many students' preparation. It is worth reviewing!

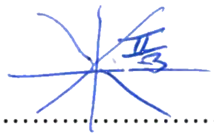
$\therefore f'(x) = \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{1} - (\cancel{3x^2} + \cancel{1})}{h}$

$= \lim_{h \to 0} \frac{6xh + 3h^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h)}{\cancel{h}}$

$= 6x$

26. Find the value(s) of θ if $\tan^2 \theta = 3$ for $0 \le \theta \le 2\pi$. **2**



$\tan\theta = \pm\sqrt{3}$

This is critical to reveal all solutions

$\theta = \frac{\pi}{3}, \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

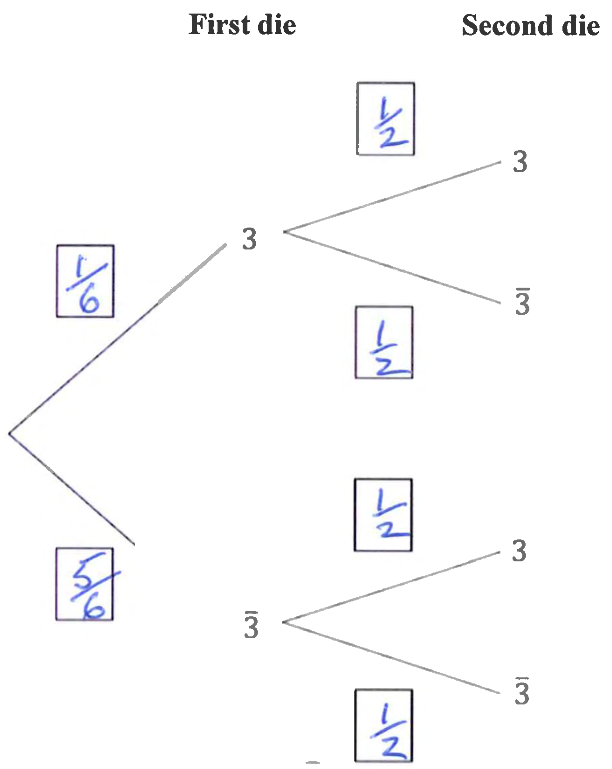
$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

27. Two dice are rolled. The first die is a standard die with numbers 1, 2, 3, 4, 5, 6 while the second die is non-standard and has the numbers 3, 3, 3, 4, 4, 4.

The number of threes is then recorded.

a) Insert the correct probabilities into the six boxes on the probability tree below, to show the probability of whether or not a 3 is rolled. **2**

Well done



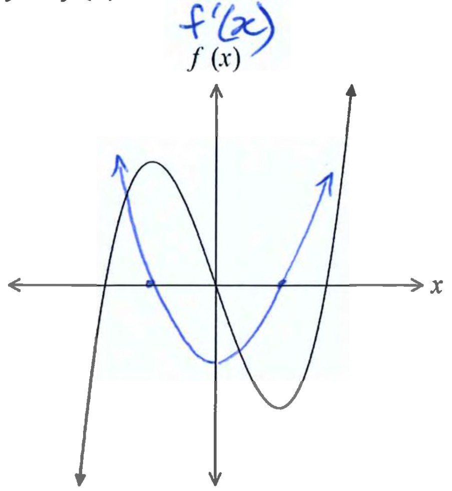
b) Calculate the probability of rolling at least one 3. **1**

$= 1 - P(\text{no } 3\text{s})$

$= 1 - \left(\frac{5}{6} \times \frac{1}{2}\right)$

$= \frac{7}{12}$

28. Consider the graph of $y = f(x)$ shown on the axes below.



1

Sketch the derivative function, $f'(x)$, on the same axes.

29. Find the equation of the tangent of $y = 2e^{3x}$ when $x = 0$, leaving your answer in general form. **3**

You need....

if $x=0$ $\quad y = 2e^{3(0)}$

$= 2 \quad (0,2) \Leftarrow$ a point

&

$y' = 6e^{3x}$ @ $x=0$ $\quad m_T = 6e^{3(0)}$

$= 6 \Leftarrow$ a gradient (at that point)

$\therefore$ eq$^{\underline{n}}$ of Tangent: $y - 2 = 6(x - 0)$

$6x - y + 2 = 0$

Note: General Form.

- End of Section II Part 2 –

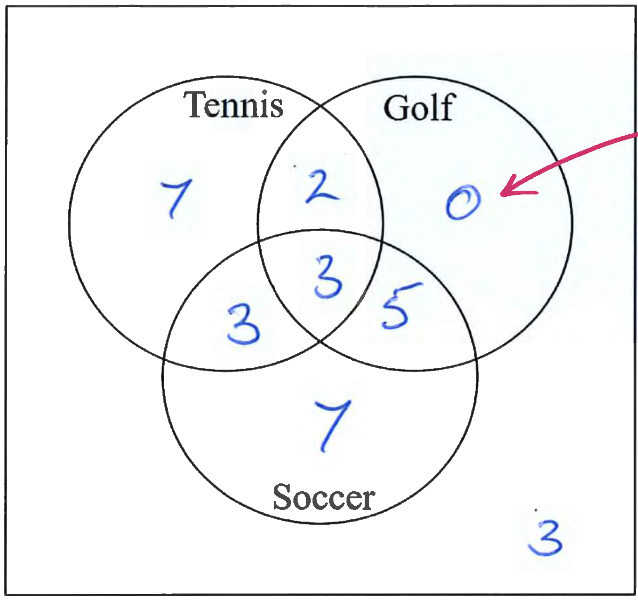
Section II: Part 3 of 4 (22 Marks)

check your answer: do your numbers all add up to 30?

30. In a group of $\boxed{\text{30 students}}$ fifteen play tennis, ten play golf and eighteen play soccer. Three student plays all three sports. Five students play both golf and tennis. Eight students play both soccer and golf. Six students play both soccer and tennis.

a) Complete the Venn diagram to illustrate the full details above. **2**

Fairly well attempted.



include 0 (don't leave it blank)

1

\

Good.

b) Find the probability that a student selected at random plays none of these sports.

$P(\text{none of these sports}) = \frac{3}{30}$

$= \frac{1}{10}$

31. An object moves in a straight line so that its displacement, x metres, after t seconds, is given by $x = 2e^{-\frac{1}{2}t} - 1$, where $t \geq 0$.

a) What is the initial displacement of the object? **1**

$t=0 \quad x = 2e^{-\frac{1}{2}(0)} - 1$

$= 2-1$

$= +1\,m$

well done

Ensure you include a lot of details and don't just have $\log_e 4$ appear out of nowhere.

b) $\boxed{\text{Show that}}$ the exact time taken for the displacement to equal zero is $\log_e 4$ seconds. **2**

$x=0 \quad 0 = 2e^{-\frac{1}{2}t} - 1$

$e^{-\frac{1}{2}t} = \frac{1}{2}$

$\log_e \frac{1}{2} = -\frac{1}{2}t$

$t = -2\log_e \frac{1}{2}$

$= \log_e \left(\frac{1}{2}\right)^{-2}$

$= \log_e 2^2 = \log_e 4$ #

Some mistakes made here. Practise showing a clear and correct substitution in every worded question, and ask your teacher if you're unsure.

c) Find the exact velocity when the displacement is zero. **2**

$v = x' = -e^{-\frac{1}{2}t}$

@ $t = \log_e 4 \quad v = -e^{-\frac{1}{2}\times \log_e 4}$

$= -e^{\log_e 4^{-\frac{1}{2}}}$

$= -e^{\log_e \frac{1}{2}}$

$= -\frac{1}{2}\,m/s$

Quite well done. Calculator helped for answering or for checking.

32. Jane plays a game using a standard 6-sided die. In each game the die is rolled three times. Each time, before the die is rolled, Jane guesses the number that will be rolled.

a) What is the probability that Jane will guess all three numbers correctly? **1**

$\left(\frac{1}{6}\right)^3 = \frac{1}{216}$

quite good

Jane pays \$10 to play one game. If Jane guesses the number correctly three times, she receives her money back and \$2,160.

wording is a bit tricky; read and interpret carefully.

b) What is her expected profit or loss after playing this game 1000 times? **2**

$\text{Profit} = \left[\frac{1}{216} \times 2160 - \frac{215}{216} \times 10\right] \times 1000$

$= +46.30$

Fully correct answers were rare.

Use of a prob. dist. table was a good idea by many students.

33. Fully simplify **3**

$$\frac{\log_5\left(a^6b^4\right) - \log_5\left(a^2b^4\right)}{\log_5\left(\sqrt{a}\right)}$$

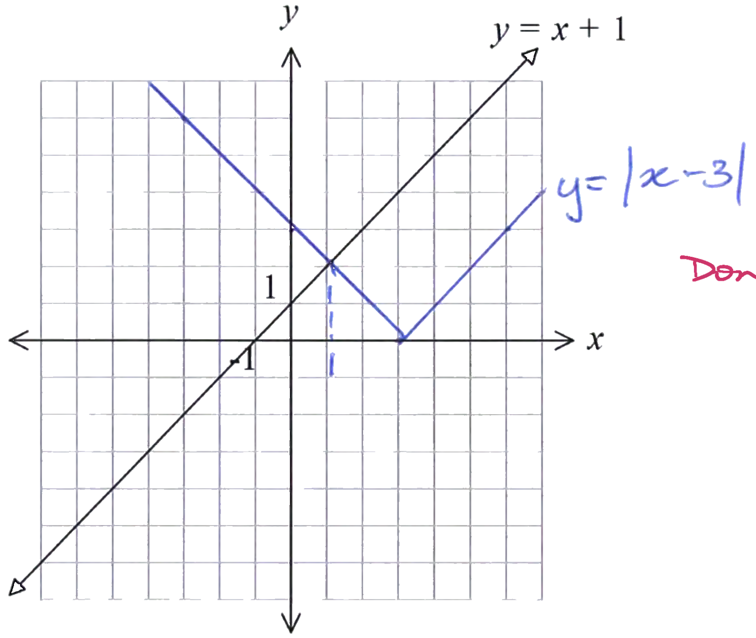
$= \dfrac{\log_5 \frac{a^6b^4}{a^2b^4}}{\log_5 a^{\frac{1}{2}}}$

$= \dfrac{\log_5 a^4}{\log_5 a^{\frac{1}{2}}}$

$= \dfrac{4 \cancel{\log_5 a}}{\frac{1}{2} \cancel{\log_5 a}} = 8$

Good progress made by many, but a huge variety of mistakes made. Practise your log laws!

34. a) On the axes below, graph $y = |x - 3|$. **1**



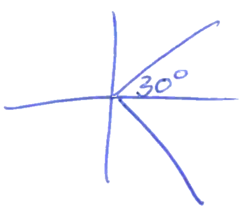
Done quite well.

b) Hence, solve graphically $x + 1 = |x - 3|$. **1**

Algebraic solutions were not accepted.

$x = 1$

35. Solve $\cos x + \sin(90^o - x) = \sqrt{3}$ for $0^o \leq x \leq 360^o$. **3**



$\cos x + \cos x = \sqrt{3}$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = 30^\circ$ or 330°

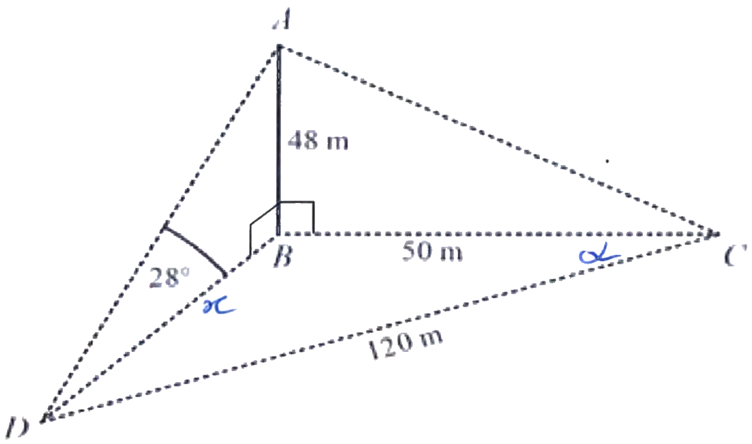
Students need to be, or become, very comfortable with every step of this solution.
Quite a few started with $\sin(90^\circ - x) + \sin(90^\circ - x) = \sqrt{3}$.
This is legitimate, but it's harder to proceed correctly from this starting point.

36. A 48-metre high, vertical light pole AB stands in the middle of a flat sporting ground.

Diana starts from the base of the pole, at B, and walks 50m due east to a point C where she places a peg.

She attaches a 120m long string to the peg and walks in a **roughly** south-westerly direction until the string is taut at point D.

She then measures the angle of elevation of the pole to be 28°.



Use the information provided to find the **actual** true bearing on which Diana was walking from C to D. **3**

In ΔABD, $\tan 62^\circ = \frac{DB}{48}$ $DB = 48 \tan 62^\circ$

$\doteqdot 90.27\ldots$

In ΔBCD, $\cos\alpha = \frac{120^2 + 50^2 - DB^2}{2(120)(50)}$

Quite well done. Congratulations!

Note the succinct and neat working here and try to emulate it.

$\cos\alpha = 0.7292\ldots$

$\alpha = 43^\circ$ $\therefore$ Bearing $= 270^\circ - 43^\circ$

$= 227^\circ$ (to n.d.) ✓

- **End of Section II Part 3** -

Section II: Part 4 of 4 (23 Marks)

37. Solve $e^{2\ln x} = 8 - 2x$ **3**

$\log_e(8-2x) = 2\log_e x$

$8 - 2x = x^2$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4$ or 2

But $x > 0$ $\therefore$ $x = 2$

Unfamiliarity with log laws meant that many students were unable to reduce to a quadratic.

The quadratic was solved well, but students should be cautious to check that all solutions are valid for the original equation $(x>0)$.

38. Let $f(x) = 3x + 1$ and $g(x) = 2x + b$, where b is a constant. **2**

Find the value of b if $g(f(x)) = f(g(x))$. Show all working.

$2(3x+1) + b = 3(2x+b) + 1$

$\cancel{6x} + 2 + b = \cancel{6x} + 3b + 1$

$1 = 2b$

$b = \frac{1}{2}$

Generally well done.

39. Show that $\sin x \cot x - \cos x \sin^2 x = \cos^3 x$ **2**

$LHS = \cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}} - \cos x \sin^2 x$

$= \cos x(1 - \sin^2 x)$

$= \cos x \cdot \cos^2 x$

$= \cos^3 x = RHS$ #

Mostly well done. Many students successfully used trig. identities but were undone by arithmetic errors relating to fractions or signs.

40. Consider the discrete probability distribution of the random variable X, shown below. **4**

x	1	2	3	4
$P(X = x)$	$4p$	$3p$	$2p$	p
$x \cdot P(x)$	0.4	0.6	0.6	0.4
$x^2 \cdot P(x)$	0.4	1.2	1.8	1.6

When the answer is given, take extra care to show ALL steps and details.

By finding the value of p, **show** that Var $(X) = 1$.

$10p = 1 \quad p = \frac{1}{10}$

$E(x) = \mu = 0.4 + 0.6 + 0.6 + 0.4 = 2$

Most students successfully found p and μ.

$E(x^2) = 0.4 + 1.2 + 1.8 + 1.6 = 5$

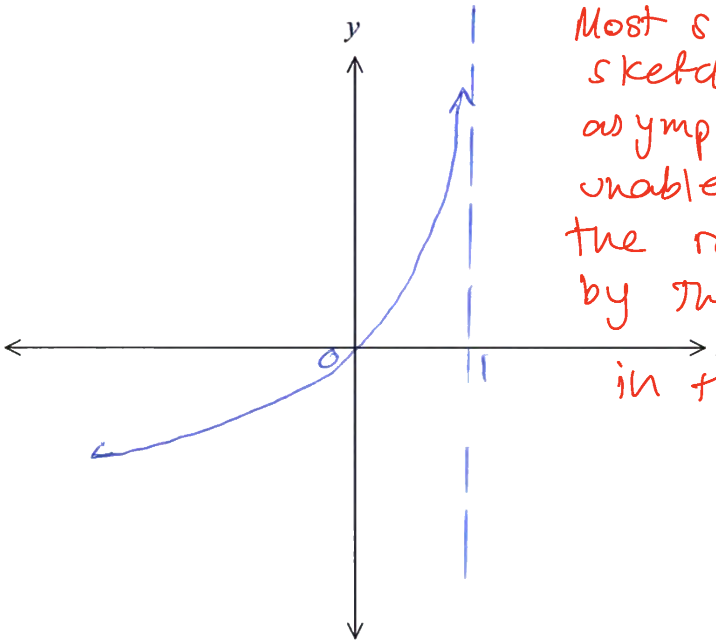
$\text{Var} = E(x^2) - \mu^2$

$= 5 - 2^2$

$= 1$

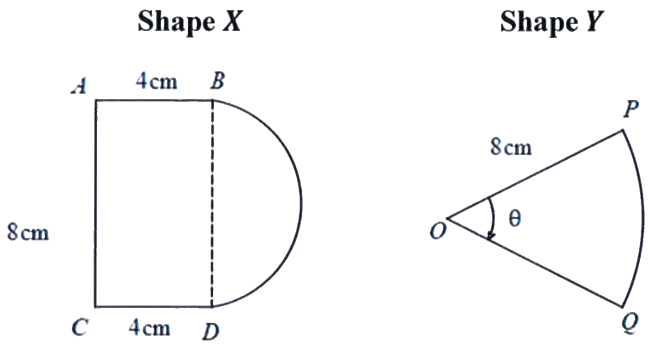
This was poorly done. Students need to be familiar with the formula $Var(X) = \sum x^2 p(x) - \mu^2$ which does not appear explicitly on the Ref. Sheet.

41. Sketch the graph of $y = -\log_e(1 - x)$, showing the asymptote and all intercepts with the axes. **2**



Most students correctly sketched the $x = 1$ asymptote, but were unable to recognise the reflections indicated by the negative signs in the equation.

42. The diagram below shows two farm patches. Shape X is a rectangle $ABDC$ joined to a semicircle of diameter BD. Shape Y is a sector OPQ of a circle with centre O and θ measured in radians.



Many responses indicated that students are still uncomfortable with working in radians. Familiarity with the formula, $A = \frac{1}{2}r^2\theta$ greatly simplified question.

(a) Given the areas of the two shapes are equal, show that $\theta = \frac{\pi}{4} + 1$. **3**

$A_x = 32 + \frac{1}{2} \times \pi \times 4^2$ $\qquad A_y = \frac{1}{2} \times 8^2 \theta$

$= 32 + 8\pi$

$A_x = A_y$

$\therefore 32 + 8\pi = 32\theta$

$1 + \frac{\pi}{4} = \theta$

Students should work in exact form rather than finding a decimal (approximate) value for $A_x = 32 + 8\pi$.

(b) Hence, calculate the perimeter of sector OPQ. Leave your answer in exact form. **2**

$l = \theta r = \left(1 + \frac{\pi}{4}\right) \times 8$

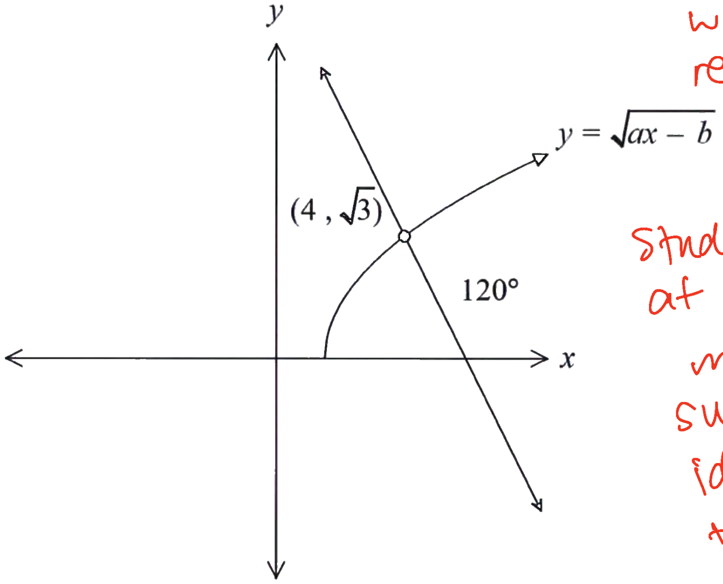
$= 8 + 2\pi$

$\therefore P = 8 + 2\pi + 8 + 8$

$= (24 + 2\pi)$ cm

Generally well done although common errors were incorrect expansion of $\left(1 + \frac{\pi}{4}\right) \times 8$ and forgetting to add radii to find perimeter.

43. The normal to the function $y = \sqrt{ax - b}$ at the point $(4, \sqrt{3})$ makes a 120^o angle with the positive x-axis.



Many students were not able to recall the relationship $m = \tan\theta$.

Students who arrived at the expression $m = \tan 120^\circ$ were successfully able to identify that the tangent ratio is negative in the 2nd quadrant.

a) Show that the gradient of the normal is $-\sqrt{3}$. **1**

$m_N = \tan 120 = -\tan 60$

$= -\sqrt{3}$

b) Hence, or otherwise, find the values of a and b. **3**

$m_T = \frac{1}{\sqrt{3}}$

sub in $(4, \sqrt{3})$

$\sqrt{3} = \sqrt{4a - b}$ (1)

$y = (ax - b)^{\frac{1}{2}}$

$y' = \frac{a}{2}(ax - b)^{-\frac{1}{2}}$ @ $x = 4$ $y' = \frac{1}{\sqrt{3}}$

$\therefore \frac{1}{\sqrt{3}} = \frac{a}{2} \cdot (4a - b)^{-\frac{1}{2}}$ (2)

$2\sqrt{4a - b} = \sqrt{3}a$

from (1): $2\sqrt{3} = \sqrt{3}a$

$a = 2$

sub into (1). $\sqrt{3} = \sqrt{4(2) - b}$

$3 = 8 - b$ $b = 5$

Many candidates struggled here.

A common hurdle was recognising that $m_T = -\frac{1}{m_N} = \frac{1}{\sqrt{3}}$ in order to make use of the derivative.

Many students also struggled to solve the two equations simultaneosly.

Students need to be confident in their understanding of the derivative and its meaning in order to apply it to an unfamiliar context.

44. A game show host randomly puts two balls into three separate boxes, so that each box can contain 0, 1 or 2 balls. The player chooses any two boxes to open. The player only wins if a total of two balls are in the boxes chosen. **1**

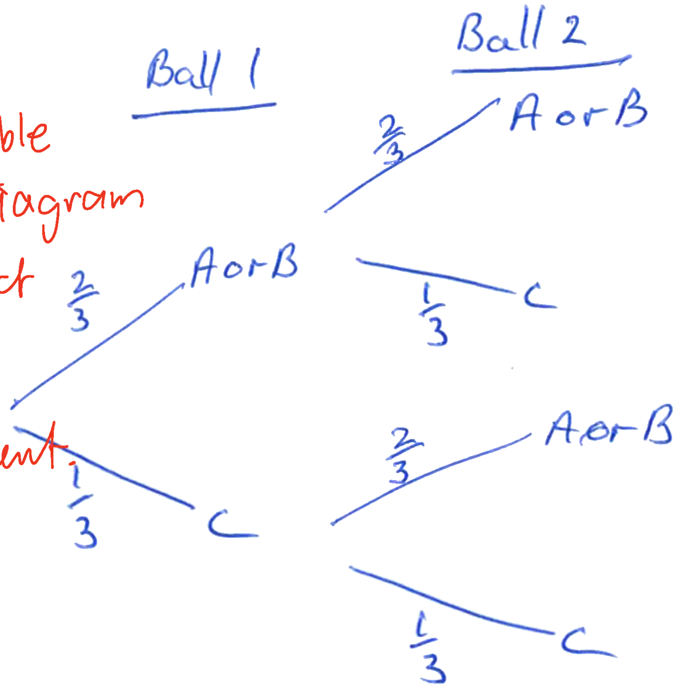
Find the probability that a player wins the game.

Boxes A B C

* say player chooses A&B

Arbitrary assumptions are useful tool to simplify complex problems.

Very poorly done. Students were unable to use a tree diagram to correctly depict the problem as a multi-stage experiment.



√ win!

$$\therefore P(\text{Win}) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

- End of Section II Part 4 –

END OF EXAM

* The probability of winning is unchanged if the player chose A&C or B&C