Mrs Cooper	**Ms Davis**	**Name:** …….
Mr Feng	**Mrs Greenberg**	
Mrs Millar	**Ms Smith**	**Teacher:** ……..
Mrs Squires	**Mrs Ward**	

Pymble Ladies' College

YEAR 11
YEARLY EXAMINATION
2022

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using non-erasable black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning or calculations

Total marks 72

Section 1 – 10 marks (pages 1-3)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 62 marks (pages 6-19)

- Attempt Questions 11-28
- Allow about 1 hour and 45 minutes for this section

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SECTION I
10 Marks

Use the Multiple-Choice Answer sheet for Questions 1-10

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1 What is the equation of the line perpendicular to the line with equation $3y = -2x + 1$?

(A) -2

(B) $-\frac{2}{3}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

2 Which of the following is a one-to-one function?

(A) $y = x^3 - x$

(B) $y = x^2 + 4$

(C) $y = 1 - x^3$

(D) $y = \sqrt{9 - x^2}$

3 What are the radius and centre of a circle with equation $x^2 - 6x + y^2 + 10y + 18 = 0$?

(A) Radius $= 3\sqrt{2}$, Centre $(-3, 5)$

(B) Radius $= 3\sqrt{2}$, Centre $(3, -5)$

(C) Radius $= 4$, Centre $(-3, 5)$

(D) Radius $= 4$, Centre $(3, -5)$

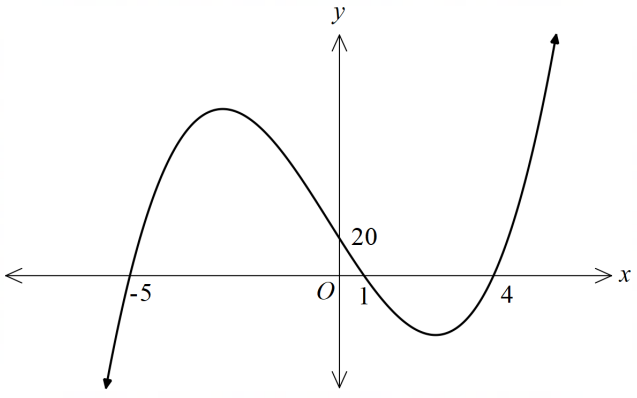
4 The sketch of $y = f(x)$ is shown. Which of the following is **NOT** a solution to $f(x) = 0$?

(A) $x = -5$

(B) $x = 1$

(C) $x = 4$

(D) $y = 20$



5 If $\tan\theta = \frac{2}{3}$ and θ is acute, which of the following is the exact value of $\cos\theta$?

(A) $\frac{2}{\sqrt{5}}$

(B) $\frac{3}{\sqrt{5}}$

(C) $\frac{3}{\sqrt{13}}$

(D) $\frac{2}{\sqrt{13}}$

6 What is the derivative of $\left(x^3+4\right)^3$?

(A) $3\left(x^3+4\right)^2$

(B) $3\left(3x^2+4\right)^2$

(C) $9x^2\left(x^3+4\right)^2$

(D) $9x^2\left(3x^2+4\right)^2$

7 Which of the following is an expression for $\cos^2 x + \cot^2 x + \sin^2 x$?

(A) 2

(B) $\sec^2 x$

(C) $\text{cosec}^2 x$

(D) $1+\text{cosec}^2 x$

8 Given $\log_3 5 = 1.46$ and $\log_3 2 = 0.63$, evaluate $\log_3 100$.

(A) 2.09

(B) 2.72

(C) 3.55

(D) 4.18

9 Let $f(x)=\frac{1}{x}$ and $g(x)=x+2$. What is the domain of $f(g(x))$?

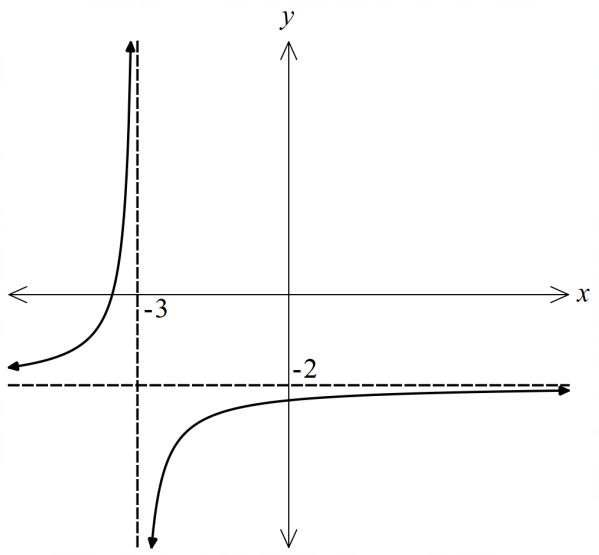
(A) All real x

(B) All real x, $x \neq -2$

(C) All real x, $x \neq 0$

(D) All real x, $x \neq 2$

10 The graph of $y=\frac{-1}{x+b}+c$ is shown.



The values for b and c are:

(A) $b=3$ $c=2$

(B) $b=3$ $c=-2$

(C) $b=-3$ $c=2$

(D) $b=-3$ $c=-2$

End of Section I

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Name: ……..

Teacher: ……

YEAR 11
YEARLY EXAMINATION
2022

Mathematics Advanced
Section II

62 marks
Attempt Questions 11-28
Allow about 1 hour and 45 minutes for this section

Instructions

- Answer in the spaces allocated for each question as these spaces provide guidance for the expected length of your response.
- Additional writing space is provided at the end of the paper.
- Write using black or blue pen. Black is preferred.
- Erasable pens are **not** to be used.
- Your responses should include relevant mathematical reasoning and/or calculations.

SECTION II

62 marks

Attempt all questions

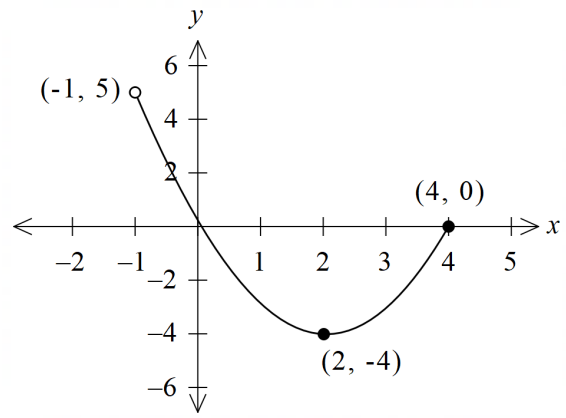
Name:……

Teacher's Name:……

Answer in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the end of this paper.

Question 11 **Marks**

The graph of the function $f(x)$ is shown below. **2**



Using interval notation, state the domain and range of $f(x)$.

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Question 12 **Marks**

$(3+\sqrt{a})(4+\sqrt{a}) = 17 + k\sqrt{a}$ where a and k are positive integers. **2**

Find the values of a and k.

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Question 13 **Marks**

Evaluate $\log_3 27$. **1**

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Question 14 **Marks**

Find the equation of the tangent to the curve $y = x^2 + 3x$ at the point $(1, 4)$. **3**

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Question 15 **Marks**

Find the derivative of $f(x) = 5x^2 - 2x$ by first principles. **3**

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Question 16 **Marks**

Show working to determine whether the function $f(x) = \dfrac{x^3}{x^2 - 1}$ is odd, even or neither. **2**

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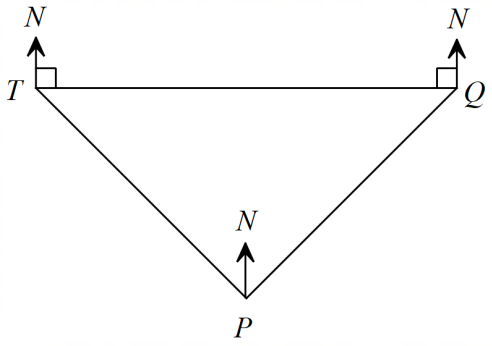
Question 17 **Marks**



N = north

NOT TO SCALE

The diagram represents the flight path of a plane. the arrows marked with N in the diagram indicate the direction of north.

The plane flies from a town, T, to a property, P, which is 83 kilometres away on a bearing of 129°. The plane then flies to another property, Q, which is 91 kilometres from P, and which is due east of T.

(a) Mark the diagram (above) with all the given information. **1**

(b) What is the bearing of Q from P correct to the nearest minute? **2**

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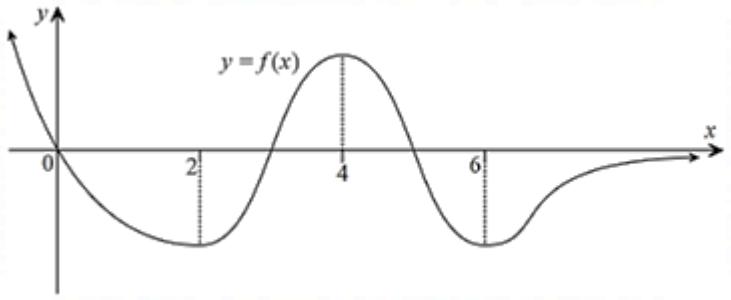
Question 18 **Marks**

Consider the function $y = f(x)$ shown below. **2**



For which values of x is the function increasing?

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Question 19 **Marks**

Solve $2\cos^2 x - \cos x = 0$ in the domain $[0, 2\pi]$. **3**

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Question 20 **Marks**

Consider the *We are Jeans* store. It costs the business $29 to make a pair of jeans. This includes materials and labour. The fixed costs come to $1000. The store sells the jeans for $49 per pair. **3**
How many pairs of jeans do they have to sell to start making a profit on them?

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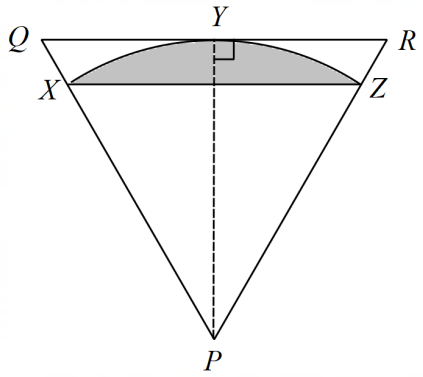
Question 21 **Marks**



NOT TO SCALE

The diagram shows an equilateral triangle PQR with sides 6 cm in length, and XYZ is an arc of a circle with centre P. The point Y lies on QR.

(a) Show that the length of the radius $PY = 3\sqrt{3}$ cm. **1**

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(b) Find the length of the arc XYZ. **1**

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(c) Find the **EXACT** area of the shaded segment XYZ. **2**

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Question 22 **Marks**

Solve each of the following:

(a) $|1-5x|=16$. **2**

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(b) $x^2+24=11x$. **2**

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(c) $2^{x-3}=\frac{1}{4}$. **2**

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(d) $\log_x 4=\log_3 \sqrt{3}$. **2**

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Question 23 **Marks**

Differentiate each of the following with respect to x.

(a) $y = 7 - \frac{3}{x^2}$ **2**

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(b) $y = \frac{2x-5}{3x+7}$ **2**

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(c) $y = \frac{1}{\sqrt{5-x}}$ **2**

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Question 24 **Marks**

Prove $\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}=2\sec^2\theta.$ **2**

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Question 25 **Marks**

Air pressure P, measured in kPa, at an altitude of h metres above sea level can be approximated using the formula $P=101e^{-kh}$, where k is a constant. The air pressure is 90 kPa at an altitude of 1000 m.

(a) Find the value of k correct to 4 significant figures. **2**

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(b) Find the depth of a mine below sea level, where the air pressure is 103 kPa. **3**

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Question 26 **Marks**

A particle is moving in a straight line. At time t seconds its displacement, in metres, from a fixed point O on the line is given by $x = t^2 - 2t$.

(a) When is the particle at rest? **2**

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(b) When does the particle return to its starting position? **1**

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(c) What is the velocity of the particle at the time it has returned to its starting position? **1**

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Question 27 **Marks**

A swimming pool is to be emptied for maintenance. The quantity of water, Q litres, remaining in the pool at a time, t minutes after it starts to drain, is given by

$$Q(t) = 2000(25 - t)^2, \quad t \geq 0$$

(a) What is the initial amount of water in the pool? **1**

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(b) At what rate is the water flowing out of the pool at 2 minutes? **2**

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(c) At what time does the rate of flow of water from the pool reach 20 000 L/minute? **2**

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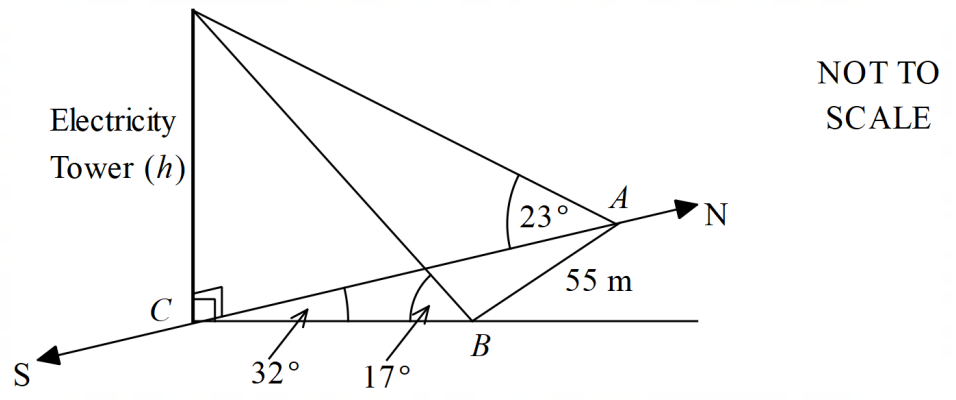
Question 28 **Marks**

A person walks on level ground, in a northerly direction, away from an electricity tower. When they arrive at a point A, the angle of elevation to the top of the tower is $23°$. Another person walks on level ground on a bearing of $032°\text{T}$ from the same tower, until he reaches point B, and notices that the angle of elevation is $17°$. It is known that points A and B are 55 m apart. Let h be the height of the tower and assume that the tower, base C, is perpendicular to the ground.



(a) Find BC in terms of h. **1**

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(b) Given $AC = h\cot 23°$, show that $h^2 = \dfrac{3025}{\cot^2 23° + \cot^2 17° - 2\cot 23° \cot 17° \cos 32°}$. **2**

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Question 28 continues on page 19

Question 28 continued.

(c) Find h correct to the nearest metre. **1**

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END OF PAPER

If you use this space, clearly indicate which question you are answering.

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NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

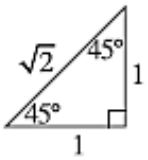
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

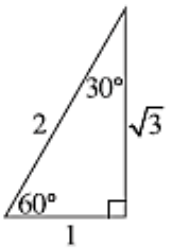
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

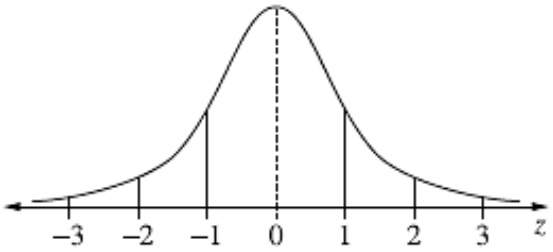
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

Mrs Cooper	**Ms Davis**	**Name:** Answers
Mr Feng	**Mrs Greenberg**	
Mrs Millar	**Ms Smith**	**Teacher:** .....
Mrs Squires	**Mrs Ward**	

Pymble Ladies' College

YEAR 11
YEARLY EXAMINATION
2022

Mathematics Advanced

General Instructions

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Total marks 72

Section 1 – 10 marks (pages 1-3)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 62 marks (pages 6-19)

- Attempt Questions 11-28
- Allow about 1 hour and 45 minutes for this section

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SECTION I

10 Marks

Use the Multiple-Choice Answer sheet for Questions 1-10

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1 What is the equation of the line perpendicular to the line with equation $3y = -2x + 1$?

(A) -2

(B) $-\frac{2}{3}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

2 Which of the following is a one-to-one function?

(A) $y = x^3 - x$

(B) $y = x^2 + 4$

(C) $y = 1 - x^3$

(D) $y = \sqrt{9 - x^2}$

3 What are the radius and centre of a circle with equation $x^2 - 6x + y^2 + 10y + 18 = 0$?

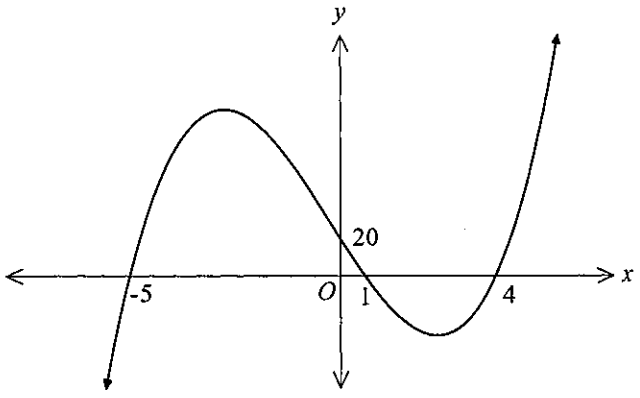
(A) Radius $= 3\sqrt{2}$, Centre $(-3, 5)$

(B) Radius $= 3\sqrt{2}$, Centre $(3, -5)$

(C) Radius $= 4$, Centre $(-3, 5)$

(D) Radius $= 4$, Centre $(3, -5)$

4 The sketch of $y = f(x)$ is shown.
Which of the following is **NOT** a solution to $f(x) = 0$?



(A) $x = -5$

(B) $x = 1$

(C) $x = 4$

(D) $y = 20$

5 If $\tan\theta = \frac{2}{3}$ and θ is acute, which of the following is the exact value of $\cos\theta$?

(A) $\frac{2}{\sqrt{5}}$

(B) $\frac{3}{\sqrt{5}}$

(C) $\frac{3}{\sqrt{13}}$

(D) $\frac{2}{\sqrt{13}}$

6 What is the derivative of $\left(x^3+4\right)^3$?

(A) $3\left(x^3+4\right)^2$

(B) $3\left(3x^2+4\right)^2$

(C) $9x^2\left(x^3+4\right)^2$

(D) $9x^2\left(3x^2+4\right)^2$

7 Which of the following is an expression for $\cos^2 x + \cot^2 x + \sin^2 x$?

(A) 2

(B) $\sec^2 x$

(C) $\operatorname{cosec}^2 x$

(D) $1+\operatorname{cosec}^2 x$

8 Given $\log_3 5 = 1.46$ and $\log_3 2 = 0.63$, evaluate $\log_3 100$.

(A) 2.09

(B) 2.72

(C) 3.55

(D) 4.18

9 Let $f(x)=\frac{1}{x}$ and $g(x)=x+2$. What is the domain of $f(g(x))$?

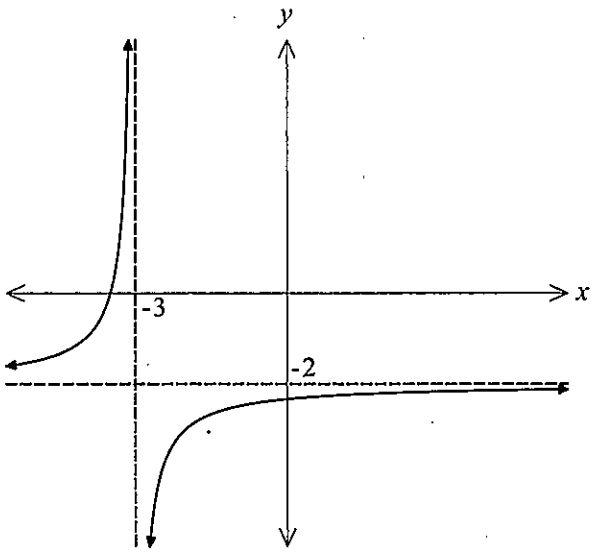
(A) All real x

(B) All real x, $x \neq -2$

(C) All real x, $x \neq 0$

(D) All real x, $x \neq 2$

10 The graph of $y=\frac{-1}{x+b}+c$ is shown.



The values for b and c are:

(A) $b=3$ $c=2$

(B) $b=3$ $c=-2$

(C) $b=-3$ $c=2$

(D) $b=-3$ $c=-2$

End of Section I

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Name: .....

Teacher: .....

Pymble Ladies' College

YEAR 11
YEARLY EXAMINATION
2022

Mathematics Advanced
Section II

62 marks
Attempt Questions 11-28
Allow about 1 hour and 45 minutes for this section

Instructions

- Answer in the spaces allocated for each question as these spaces provide guidance for the expected length of your response.
- Additional writing space is provided at the end of the paper.
- Write using black or blue pen. Black is preferred.
- Erasable pens are **not** to be used.
- Your responses should include relevant mathematical reasoning and/or calculations.

SECTION II **Name:**.....

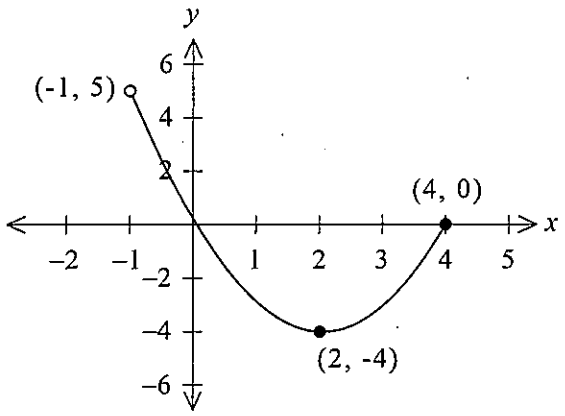
62 marks

Attempt all questions **Teacher's Name:**.....

Answer in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the end of this paper.

Question 11 **Marks**

The graph of the function $f(x)$ is shown below. **2**



Using interval notation, state the domain and range of $f(x)$.

Domain $(-1, 4]$ Range $[-4, 5)$ ..... 1 for each

If not interval notation but correct, -1

Question 12 **Marks**

$(3+\sqrt{a})(4+\sqrt{a}) = 17 + k\sqrt{a}$ where a and k are positive integers. **2**

Find the values of a and k.

$12 + 7\sqrt{a} + a = 17 + k\sqrt{a}$ * ..... 2 | Correct.

$\therefore 12 + a = 17 \rightarrow a = 5$ ..... | Getting to *

$k = 7$ 1

Question 13 **Marks**

Evaluate $\log_3 27$. **1**

$\log_3 3^3 = 3\log_3 3$ I.R./W

$= 3$

Question 14 **Marks**

Find the equation of the tangent to the curve $y = x^2 + 3x$ at the point $(1, 4)$. **3**

$\frac{dy}{dx} = 2x + 3$ When $x = 1$ $\frac{dy}{dx} = 2 + 3$

$= 5$

Eqn of tangent: $y - 4 = 5(x - 1)$

$y = 5x - 1$

3	Correct
2	Derivative, substitution but error in finding tangent
1	Derivative and substitution

Question 15 **Marks**

Find the derivative of $f(x) = 5x^2 - 2x$ by first principles. **3**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5(x+h)^2 - 2(x+h) - (5x^2 - 2x)}{h}$$

$$= \lim_{h \to 0} \frac{5x^2 + 10xh + 5h^2 - 2x - 2h - 5x^2 + 2x}{h}$$

$$= \lim_{h \to 0} \frac{h(10x + 5h - 2)}{h}$$

$$= \lim_{h \to 0} 10x + 5h - 2$$

$$= 10x - 2$$

3	correct working and answer
2	correct algebraic expansion and attempts to apply lim as $h \to 0$
1	correct formula stated.

Question 16 **Marks**

Show working to determine whether the function $f(x) = \frac{x^3}{x^2 - 1}$ is odd, even or neither. **2**

$$f(a) = \frac{a^3}{a^2 - 1}$$

$$f(-a) = \frac{(-a)^3}{(-a)^2 - 1}$$

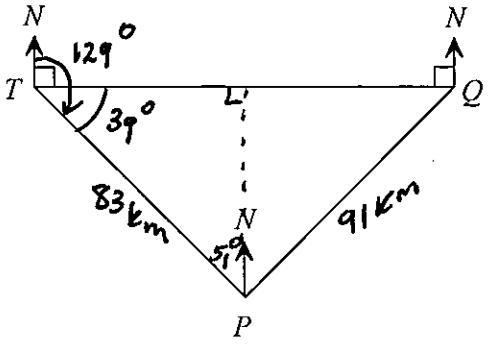
$$= \frac{-a^3}{a^2 - 1}$$

$$= -f(a)$$

since $f(-a) = -f(a)$ the function is odd

2	reaches $f(-x) = -f(x)$ correctly • no conclusion, no mark
1	substitutes correctly with brackets for $f(-x)$

Question 17 **Marks**



N = north

NOT TO SCALE

The diagram represents the flight path of a plane. the arrows marked with N in the diagram indicate the direction of north.

The plane flies from a town, T, to a property, P, which is 83 kilometres away on a bearing of 129°. The plane then flies to another property, Q, which is 91 kilometres from P, and which is due east of T.

(a) Mark the diagram (above) with all the given information. 1 r/w 1

(b) What is the bearing of Q from P correct to the nearest minute? 2

In $\triangle TPQ$ $\frac{\sin Q}{83} = \frac{\sin 39^\circ}{91}$

$\sin Q = \frac{83 \sin 39}{91}$

$Q = 35^\circ 1' 46''$ or $144^\circ 58' 14''$

Not possible

$\therefore Q = 35^\circ 1' 46''$

$\therefore \angle TPQ = 180 - (39 + 35^\circ 1' 46'')$

$= 105^\circ 58' 14'$

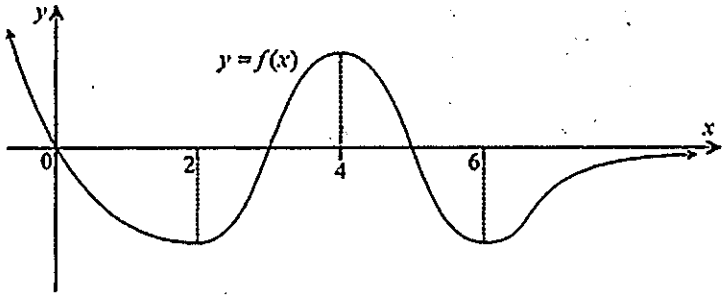
Bearing of Q from P : $105^\circ 58' 14' - 51^\circ$

$= 054^\circ 58'$ (nearest minute)

2	Correct answer $054^\circ 58'$
1	Works towards $\angle Q = 35^\circ 1' 46''$ using Sine Rule (or other correct method)

Question 18 **Marks**

Consider the function $y = f(x)$ shown below. **2**



For which values of x is the function increasing?

1 for each.

$2 < x < 4$ or $x > 6$

OR $x \in (2;4) \cup (6;\infty)$

2	correct numbers
1	correct parentheses

Question 19 **Marks**

Solve $2\cos^2 x - \cos x = 0$ in the domain $[0, 2\pi]$. **3**

$\cos x (2\cos x - 1) = 0$

$\therefore \cos x = 0$ or $\cos x = \frac{1}{2}$

$x = 0^\circ \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = \frac{\pi}{3}, \frac{5\pi}{3}$

3	4 correct angles
2	2 correct angles
1	Factorises correctly

Question 20 **Marks**

Consider the *We are Jeans* store. It costs the business $29 to make a pair of jeans. This includes materials and labour. The fixed costs come to $1000. The store sells the jeans for $49 per pair. **3**
How many pairs of jeans do they have to sell to start making a profit on them?

Let the number of jeans be n

$49n = 29n + 1000$

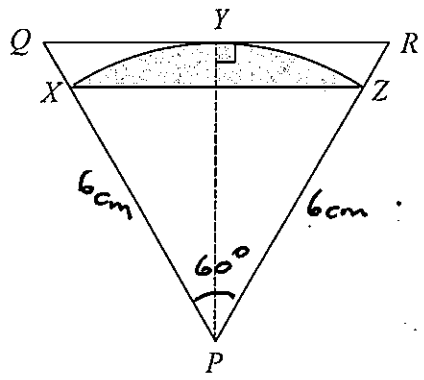
$20n = 1000$

$n = 50$

They need to sell more than 50 pairs to begin to make a profit.

3	$n > 50$
2	$n = 50$
1	Works towards $20n = 1000$

Question 21 **Marks**



NOT TO SCALE

The diagram shows an equilateral triangle PQR with sides 6 cm in length, and XYZ is an arc of a circle with centre P. The point Y lies on QR.

(a) Show that the length of the radius $PY = 3\sqrt{3}$ cm. **1**

In $\triangle PYQ$ $\cos 30° = \frac{PY}{6}$ 1 r/w

$PY = 6 \times \frac{\sqrt{3}}{2}$

$= 3\sqrt{3}$ cm

(b) Find the length of the arc XYZ. **1**

$\angle QPR = \pi/3$ $\ell_{XYZ} = 3\sqrt{3} \times \frac{\pi}{3}$ 1 r/w

$= \sqrt{3}\pi$ cm

(c) Find the **EXACT** area of the shaded segment XYZ. **2**

Shaded area = Area of sector − Area $\triangle XPZ$

$= \frac{1}{2} \times (3\sqrt{3})^2 \times \frac{\pi}{3} - \frac{1}{2} \times (3\sqrt{3})^2 \sin \pi/3$

$= \frac{9\pi}{2} - \frac{27}{2} \times \frac{\sqrt{3}}{2}$

$= \left(\frac{9\pi}{2} - \frac{27\sqrt{3}}{4}\right)$ cm^2 2 | Correct answer

1 | A(sector) − A(ΔPXZ)
OR
uses $A = \frac{1}{2}r^2(\theta - \sin\theta)$
(with errors)

Question 22 **Marks**

Solve each of the following:

(a) $|1-5x|=16$. 2

$1-5x = \pm 16$

$1-5x = 16$ or $1-5x = -16$

$5x = -15$ $\quad -5x = -17$

$x = -3$ $\quad x = 17/5$

2 | both correct

1 | one correct

(b) $x^2+24=11x$. 2

$x^2-11x+24=0$

$(x-8)(x-3)=0$

$\therefore x = 3, 8$

2 | correct solutions

1 | correct factorisation

(c) $2^{x-3}=\frac{1}{4}$. 2

$2^{x-3} = 2^{-2}$

$\therefore x-3 = -2$

$x = 1$

2 | correct answer.

1 | progress to equate indices OR correctly uses identity

(d) $\log_x 4 = \log_3 \sqrt{3}$. 2

$\log_x 4 = \frac{1}{2}\log_3 3$ $\quad x = 16$

$\log_x 4 = \frac{1}{2}$

$x^{1/2} = 4$

2 | correct answer

1 | progress using identity

Question 23 **Marks**

Differentiate each of the following with respect to x.

(a) $y = 7 - \frac{3}{x^2}$ $y = 7 - 3x^{-2}$ 2

$\frac{dy}{dx} = \frac{6}{x^3}$ 2 | Correct differentiation

OR $\frac{dy}{dx} = 6x^{-3}$ 1 | Rewrite as $y = 7 - 3x^{-2}$

(b) $y = \frac{2x-5}{3x+7}$ 2

$\frac{dy}{dx} = \frac{(3x+7)(2) - (2x-5)(3)}{(3x+7)^2}$

$= \frac{6x + 14 - 6x + 15}{(3x+7)^2}$ 2 | Correct simplified answer

1 | Correct numerator OR correct denominator

$= \frac{29}{(3x+7)^2}$

(c) $y = \frac{1}{\sqrt{5-x}}$ $y = (5-x)^{-1/2}$ 2

$\frac{dy}{dx} = -\frac{1}{2}(5-x)^{-3/2} \times -1$ 2 | Correct simplified answer

$= \frac{1}{2\sqrt{(5-x)^3}}$ 1 | Correct coefficient $(-\frac{1}{2})$ and index $(-\frac{3}{2})$ OR correctly multiplies by chain rule factor

Question 24 **Marks**

Prove $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$. 2

LHS : $\frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$=\frac{2}{1-\sin^2\theta}$ $1-\sin^2\theta=\cos^2\theta$

$=\frac{2}{\cos^2\theta}$

$=2\sec^2\theta$

$=$ RHS

2	fully complete proof
1	combines or breaks apart fractions (showing numerator) using $(1+\sin\theta)(1-\sin\theta)=1-\sin^2\theta$ OR correctly uses $\frac{1}{\cos^2\theta}=\sec^2\theta$

Question 25 **Marks**

Air pressure P, measured in kPa, at an altitude of h metres above sea level can be approximated using the formula $P=101e^{-kh}$, where k is a constant. The air pressure is 90 kPa at an altitude of 1000 m.

(a) Find the value of k correct to 4 significant figures. 2

$90=101e^{-1000k}$

$e^{-1000k}=\frac{90}{101}$

$e^{-1000k}=0.\overline{891}$

$k=-\frac{1}{100}\ln(0.\overline{891})$

$k=0.001153$

2	Correctly solves using logs.
1	correct substitution and rearranges to index form

(b) Find the depth of a mine below sea level, where the air pressure is 103 kPa. 3

$103=101e^{-kh}$

$e^{-kh}=\frac{103}{101}$

$-kh=\ln\left(\frac{103}{101}\right)$

$h=-\frac{1}{k}\ln\left(\frac{103}{101}\right)$

$h=17.00488027$

$\doteq 17$ m.

3	correctly solves for $h=-170$ or answers in context (170m BELOW)
2	Writes a correct equation using logs.
1	substitutes $P=103$ and correctly rounded value of k.

Question 26 **Marks**

A particle is moving in a straight line. At time t seconds its displacement, in metres, from a fixed point O on the line is given by $x = t^2 - 2t$.

(a) When is the particle at rest? **2**

$\frac{dx}{dt} = 2t - 2$ $\quad 2t - 2 = 0$

$t = 1$

When $\frac{dx}{dt} = 0$ $\quad \therefore$ particle is at rest when $t = 1$s.

2 | correct answer.

1 | correct $\frac{dx}{dt}$

(b) When does the particle return to its starting position? **1**

When $t = 0$ $\quad x = 0$ $\quad \therefore$ starting position is $x = 0$ $\quad t^2 - 2t = 0$

$t(t-2) = 0$ $\quad \therefore$ particle returns to starting position after 2s

$\therefore t = 0$ or $t = 2$

1/r/w

(c) What is the velocity of the particle at the time it has returned to its starting position? **1**

When $t = 2$ $\quad \frac{dx}{dt} = 2 \times 2 - 2$

$= 2$ m/s

1 r/w

Question 27 **Marks**

A swimming pool is to be emptied for maintenance. The quantity of water, Q litres, remaining in the pool at a time, t minutes after it starts to drain, is given by

$$Q(t) = 2000(25-t)^2, \quad t \geq 0$$

(a) What is the initial amount of water in the pool? **1**

When $t = 0$ $Q(t) = 2000 \times 25^2$ 1 c/w

$= 1250000$ litres

(b) At what rate is the water flowing out of the pool at 2 minutes? **2**

$\frac{dQ}{dt} = 2000 \times 2(25-t) \times -1$

$= -4000(25-t)$ 2 correct answer.

1 correct $\frac{dQ}{dt}$

When $t = 2$ $\frac{dQ}{dt} = -4000(25-2)$

$= -92000$ L/min

(c) At what time does the rate of flow of water from the pool reach 20 000 L/minute? **2**

When $\frac{dQ}{dt} = -20000$

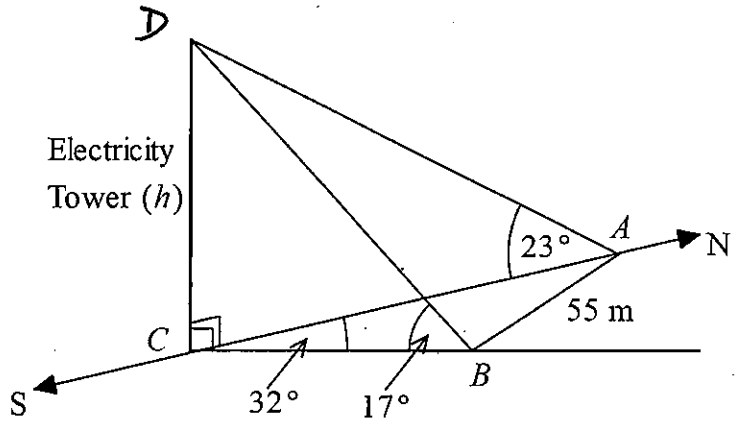
$-20000 = -4000(25-t)$ 2 Substituting -20000 getting 20 min

$25 - t = 5$

$t = 20$ min 1 Substituting 20000, giving 30 mins

Question 28 **Marks**

A person walks on level ground, in a northerly direction, away from an electricity tower. When they arrive at a point A, the angle of elevation to the top of the tower is 23°. Another person walks on level ground on a bearing of 032°T from the same tower, until he reaches point B, and notices that the angle of elevation is 17°. It is known that points A and B are 55 m apart. Let h be the height of the tower and assume that the tower, base C, is perpendicular to the ground.



NOT TO SCALE

(a) Find BC in terms of h. **1**

In $\triangle BCD$ $\tan 17^\circ = \frac{h}{BC}$ 1 c/w

$BC = h\cot 17^\circ$

(b) Given $AC = h\cot 23°$, show that $h^2 = \dfrac{3025}{\cot^2 23° + \cot^2 17° - 2\cot 23°\cot 17°\cos 32°}$. **2**

In $\triangle ABC$

$55^2 = (h\cot 17^\circ)^2 + (h\cot 23^\circ)^2 - 2\times(h\cot 17^\circ)(h\cot 23^\circ)\cos 32^\circ$

$3025 = h^2(\cot^2 17^\circ + \cot^2 23^\circ - 2\cot 17^\circ\cot 23^\circ\cos 32^\circ)$

$h^2 = \dfrac{3025}{(\cot^2 17^\circ + \cot^2 23^\circ - 2\cot 17^\circ\cot 23^\circ\cos 32^\circ)}$

As req'd

2	Answer
1	OK this line. including h throughout on the RHS.

Question 28 continues on page 19

Question 28 continued.

(c) Find h correct to the nearest metre. 1

$h = 30.847\,31622$ 1 n/w

$\doteq 31$ m

END OF PAPER